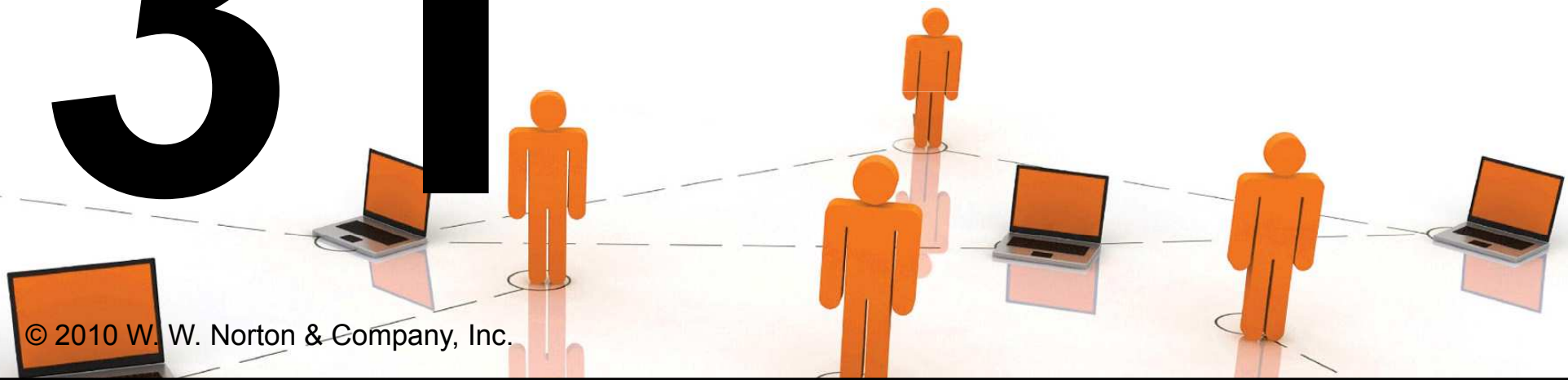# INTERMEDIATE

MICROECONOMICS

HAL R. VARIAN

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Exchange

Exchange

◆ **Two consumers, A and B.**

◆ **Their endowments of goods 1 and 2 are** $\omega^A = (\omega_1^A, \omega_2^A)$ **and** $\omega^B = (\omega_1^B, \omega_2^B)$.
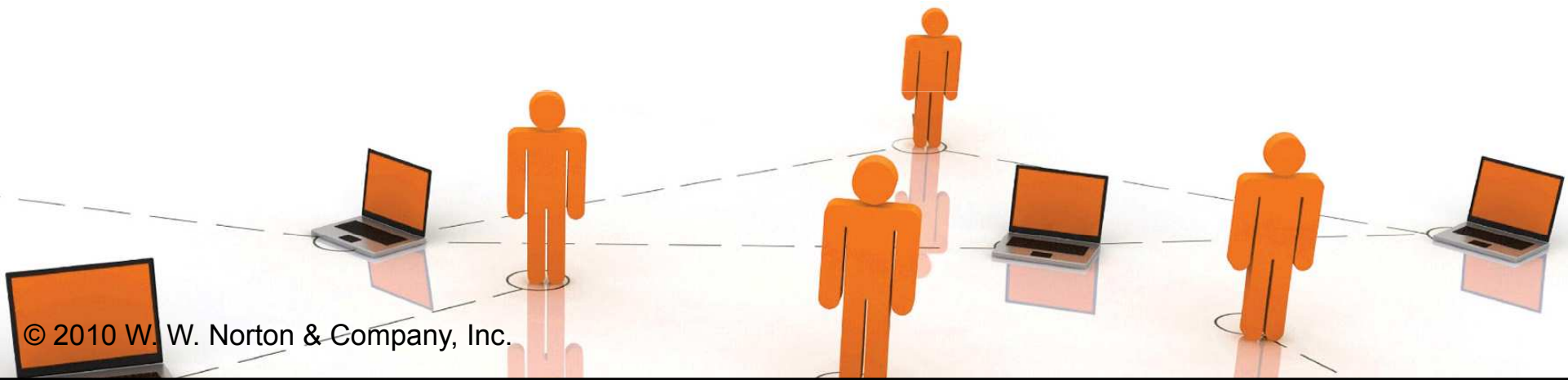
◆ **E.g.** $\omega^A = (6,4)$ **and** $\omega^B = (2,2)$.

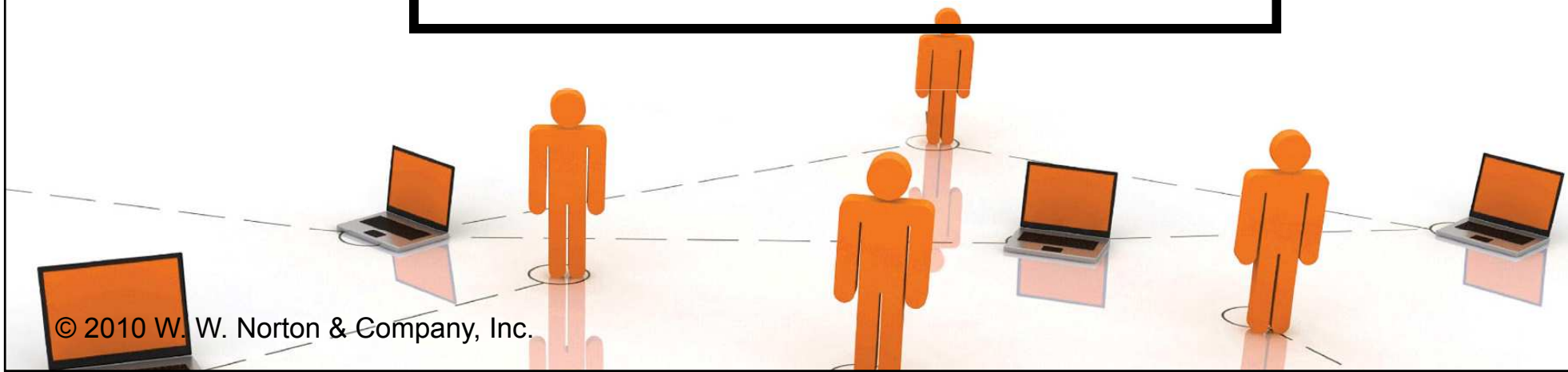◆ **The total quantities available are** $\omega_1^A + \omega_1^B = 6 + 2 = 8$ **units of good 1 and** $\omega_2^A + \omega_2^B = 4 + 2 = 6$ **units of good 2.**
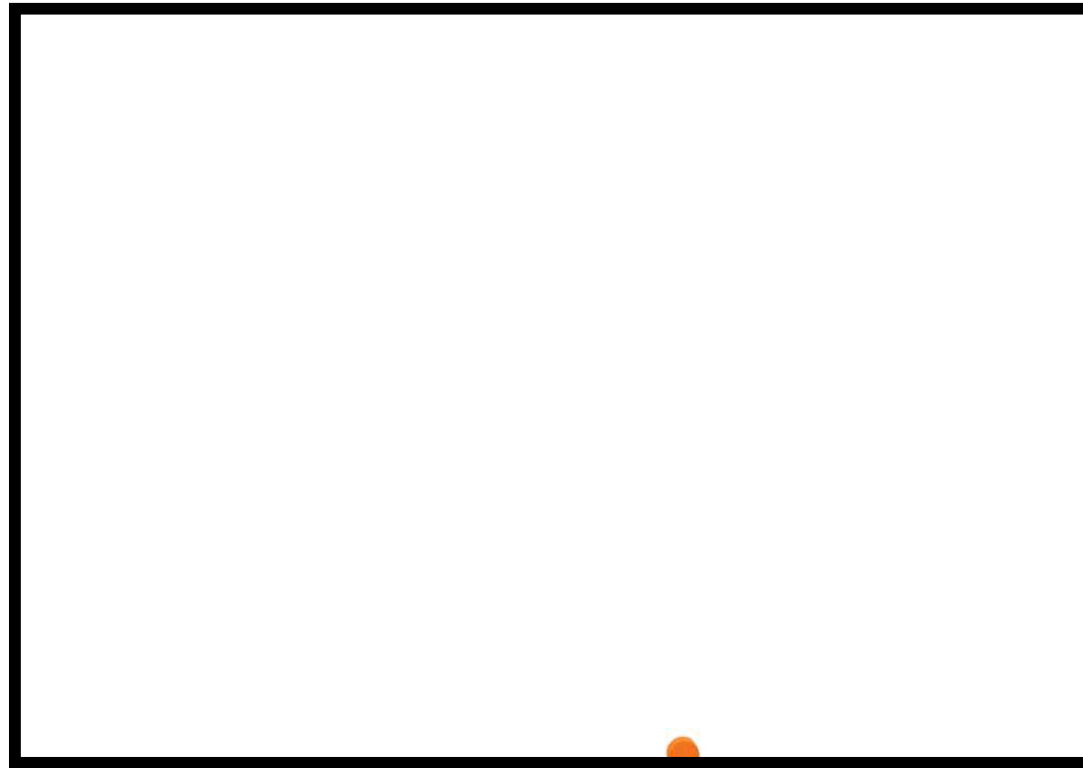
2

Exchange

◆ **Edgeworth and Bowley devised a diagram, called an Edgeworth box, to show all possible allocations of the available quantities of goods 1 and 2 between the two consumers.**

3

Starting an Edgeworth Box

Starting an Edgeworth Box

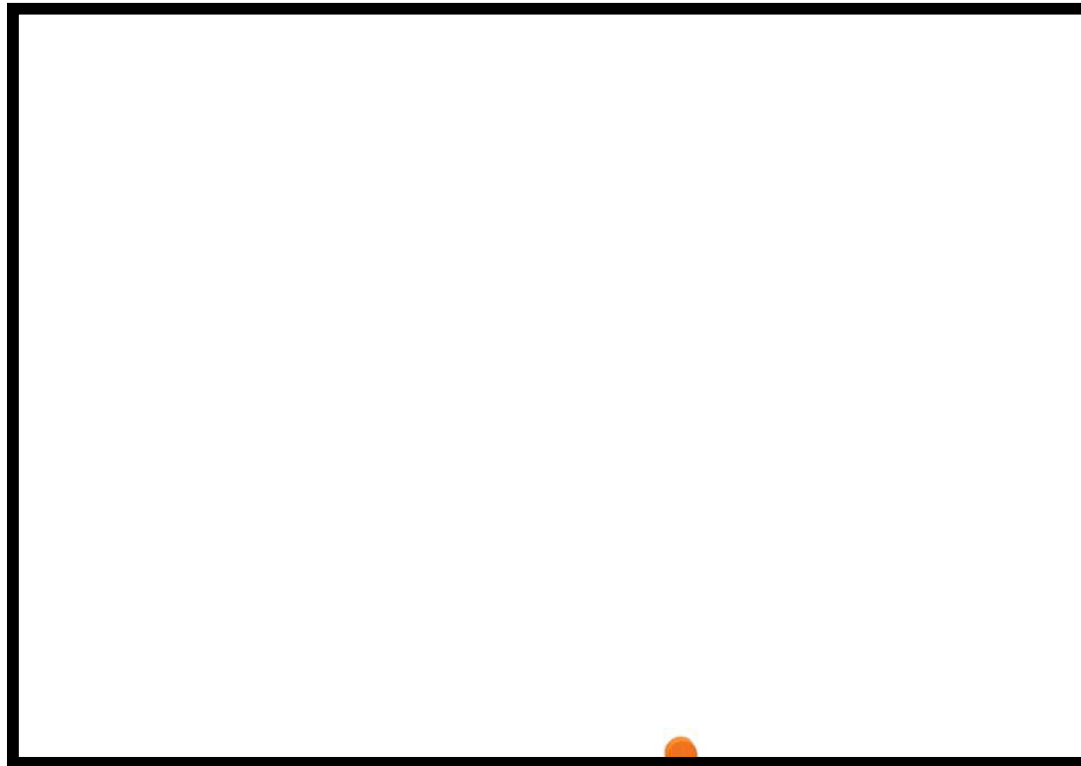$$\textbf{Width} = \omega_1^A + \omega_1^B = 6 + 2 = 8$$

Starting an Edgeworth Box

Height =

$$\omega_2^A + \omega_2^B$$

$$= 4 + 2$$

$$= 6$$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Starting an Edgeworth Box

Height =

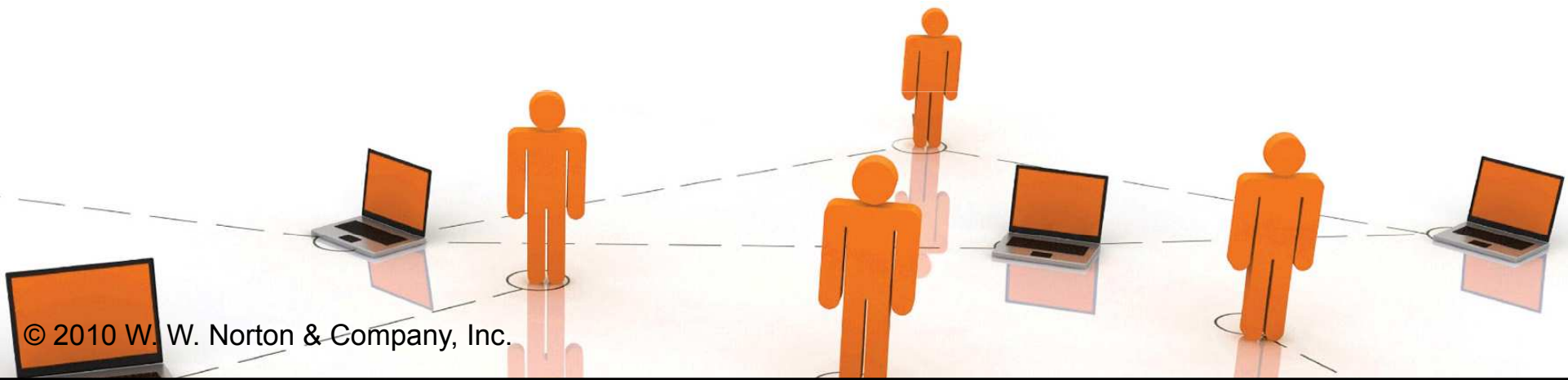$$\omega_2^A + \omega_2^B$$

$$= 4 + 2$$

$$= 6$$

The dimensions of the box are the quantities available of the goods.

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

Feasible Allocations

◆ **What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?**

◆ **How can all of the feasible allocations be depicted by the Edgeworth box diagram?**

Feasible Allocations

◆ **What allocations of the 8 units of good 1 and the 6 units of good 2 are feasible?**

◆ **How can all of the feasible allocations be depicted by the Edgeworth box diagram?**

◆ **One feasible allocation is the before-trade allocation; i.e. the endowment allocation.**

The Endowment Allocation

Height =

$$\omega_2^A + \omega_2^B$$

$$= 4 + 2$$

$$= 6$$

The endowment allocation is

$$\omega^A = (6,4)$$

and

$$\omega^B = (2,2).$$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

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The Endowment Allocation

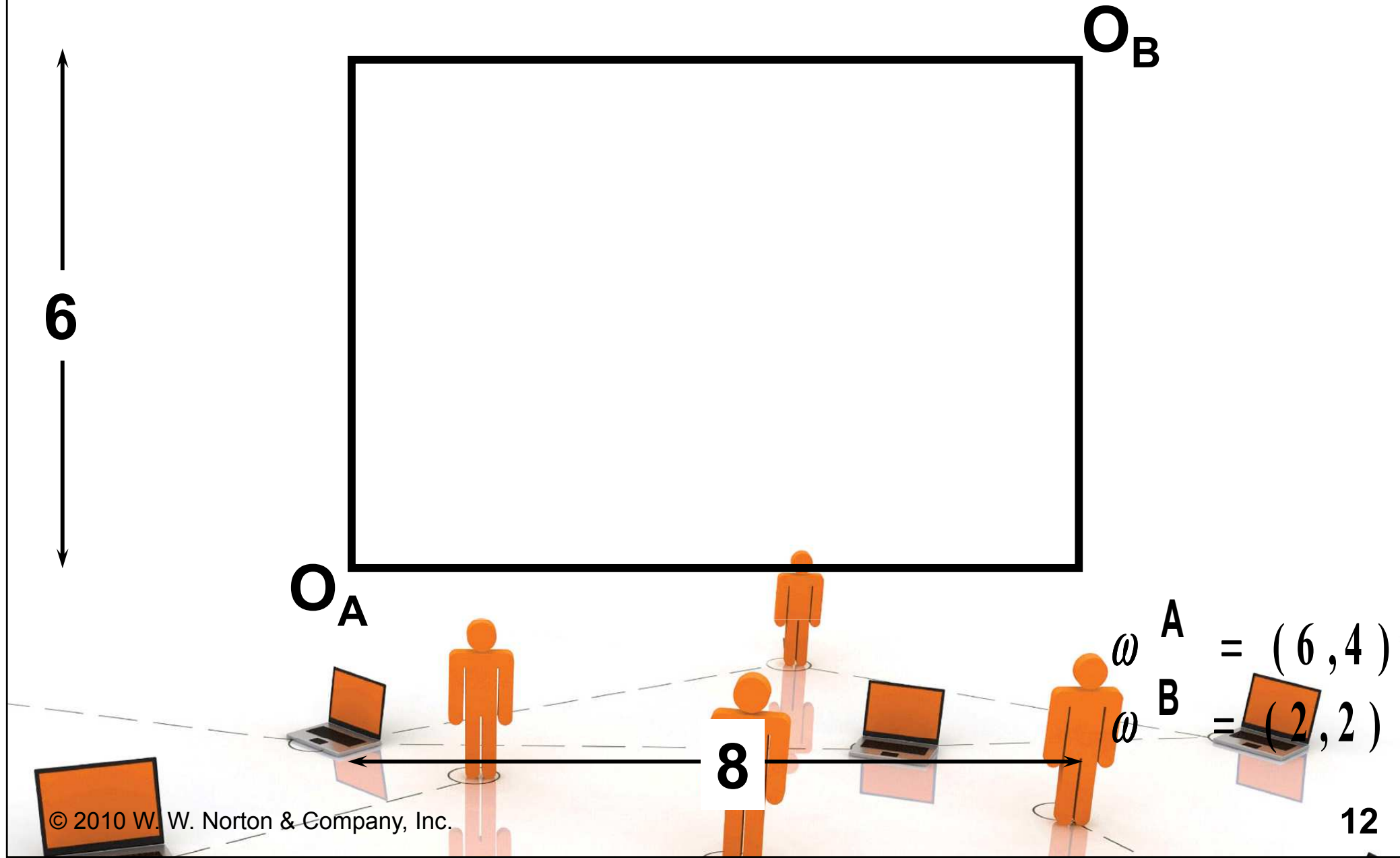**Height =**

$\omega_2^A + \omega_2^B$

$= 4 + 2$
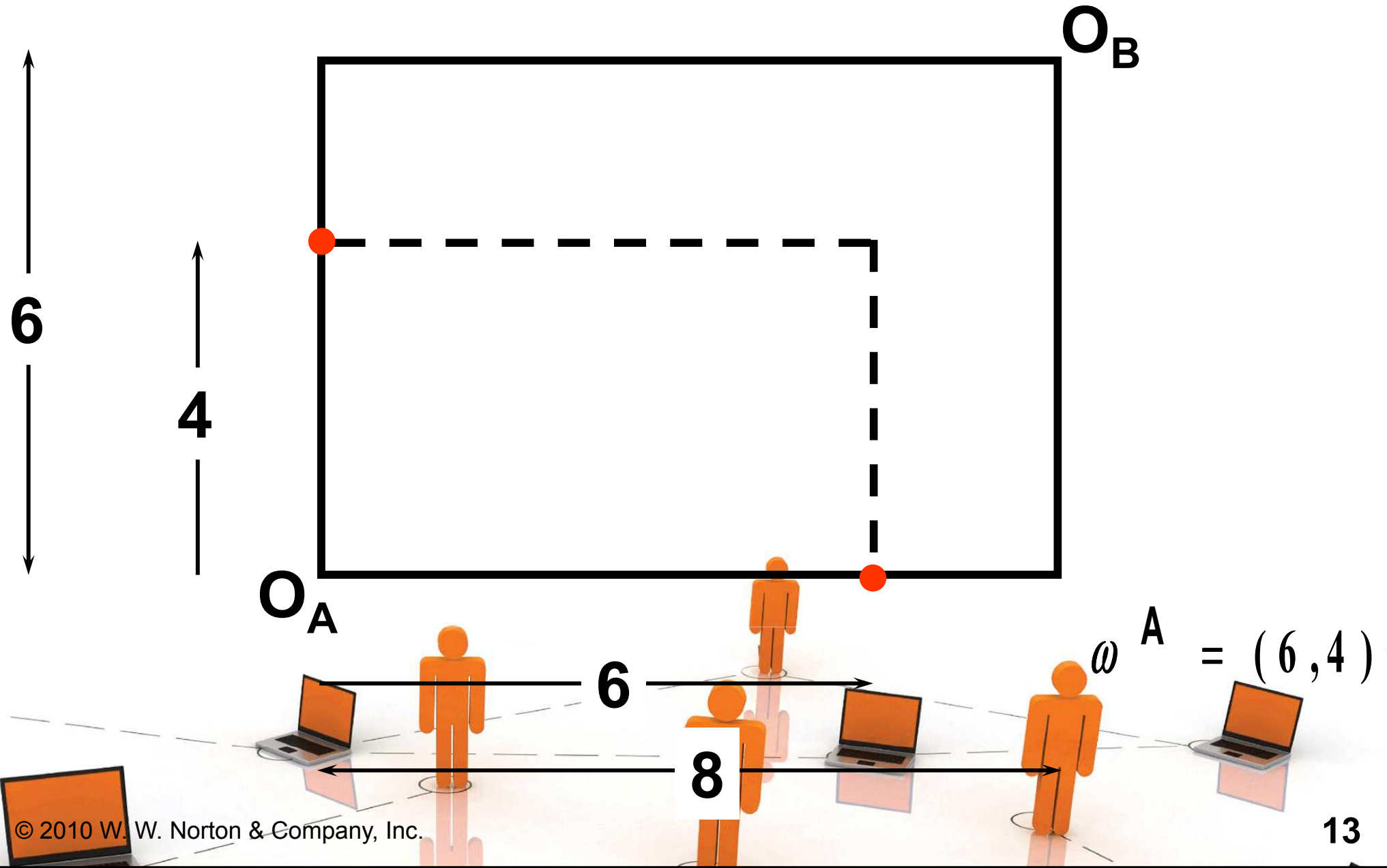
$= 6$

Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$
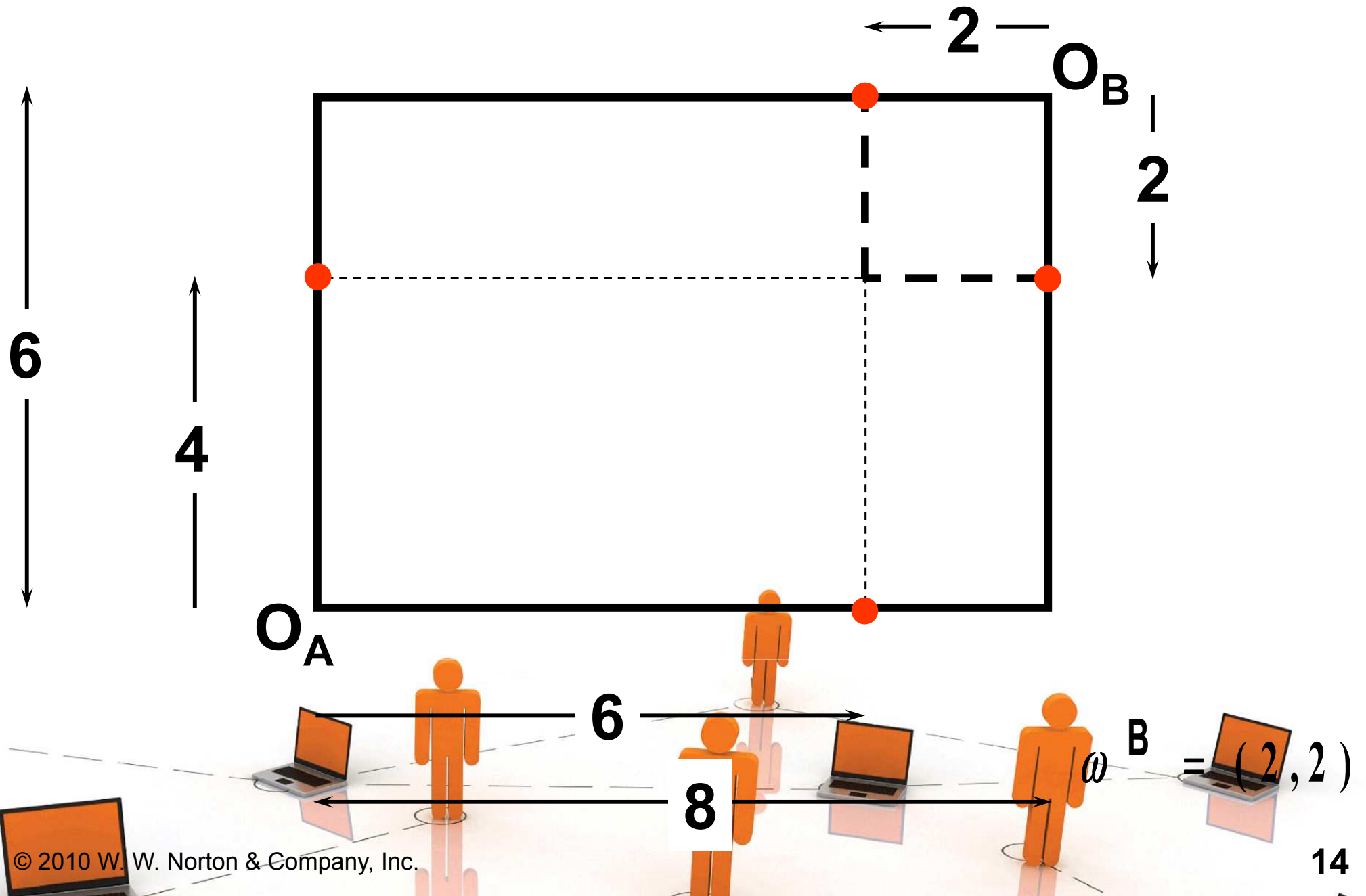
$\omega^A = (6,4)$

$\omega^B = (2,2)$

The Endowment Allocation

O_B

6

8

O_A

$\omega^A = (6,4)$

$\omega^B = (2,2)$

12

The Endowment Allocation

O_B

6

4

O_A

6

8

$\omega^A = (6,4)$

13

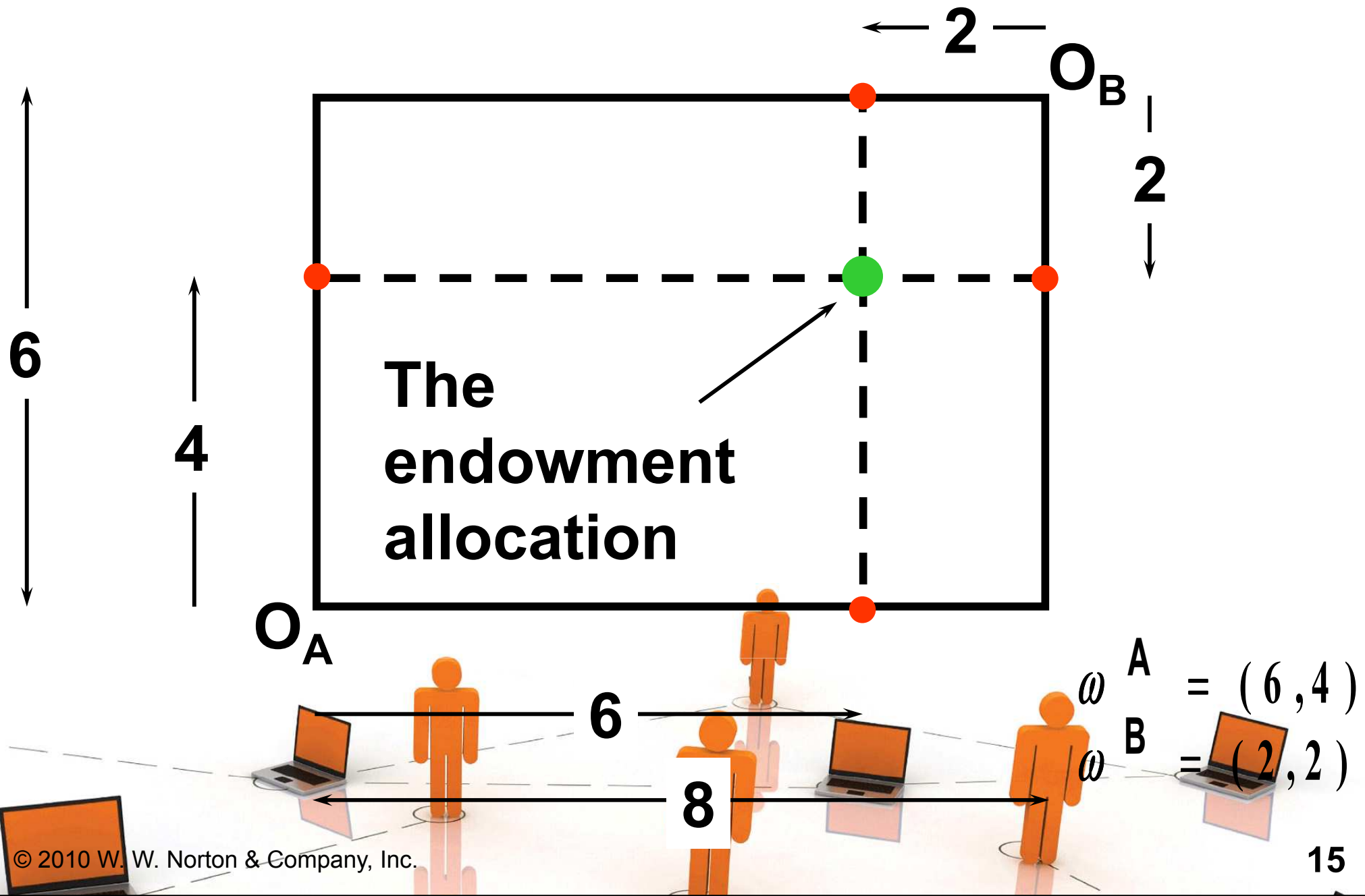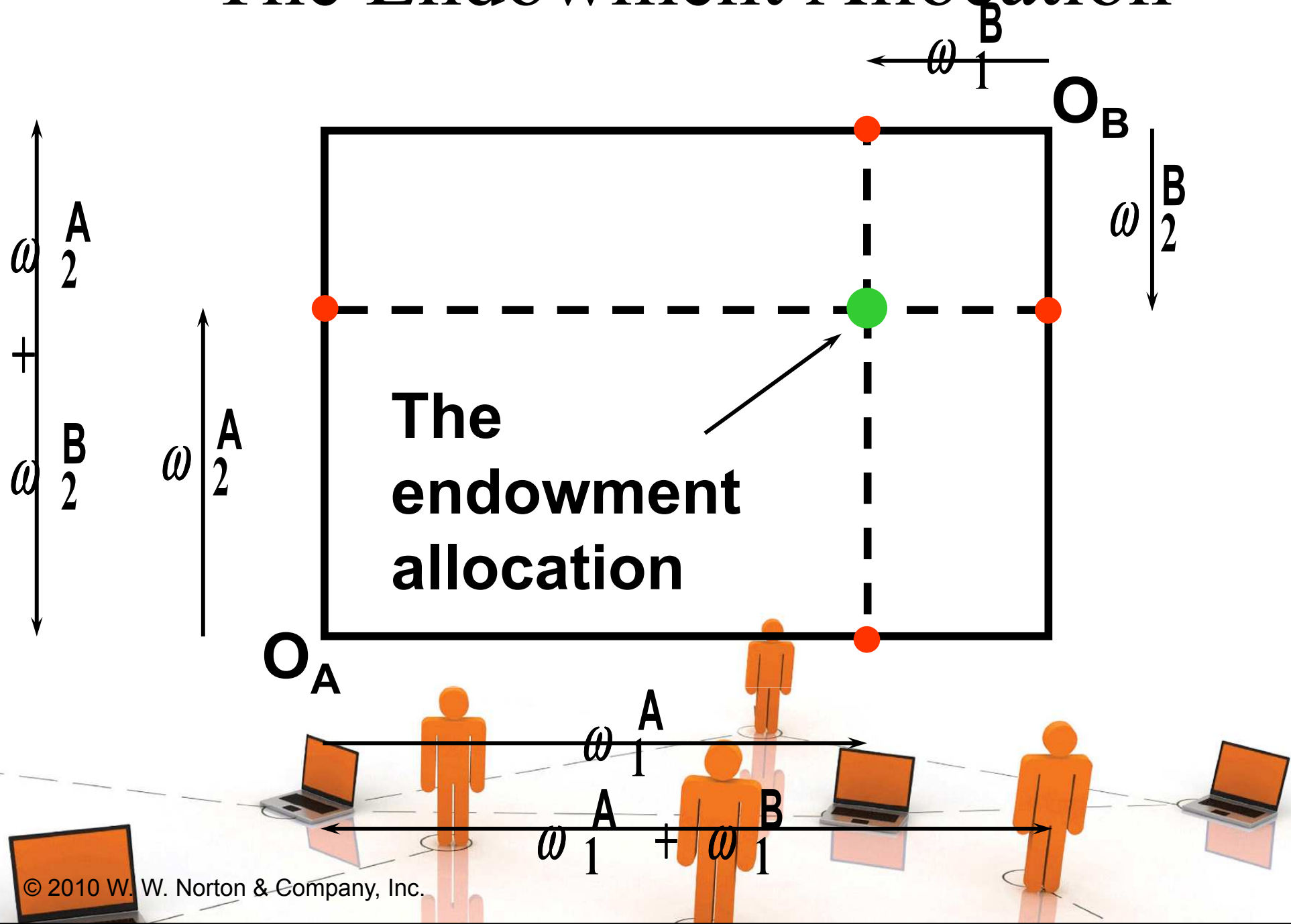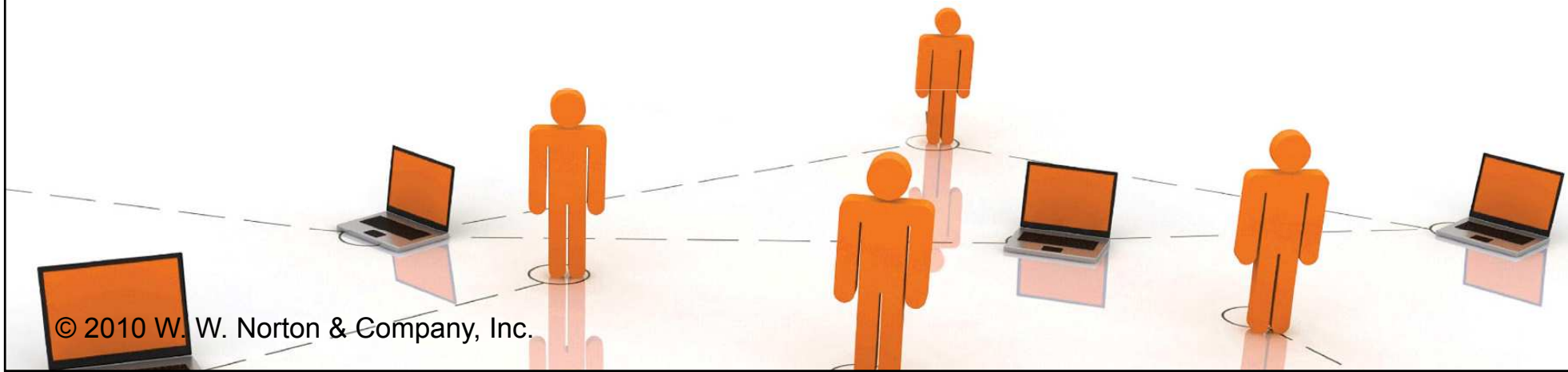# The Endowment Allocation

The Endowment Allocation

The Endowment Allocation

More generally, …

The Endowment Allocation
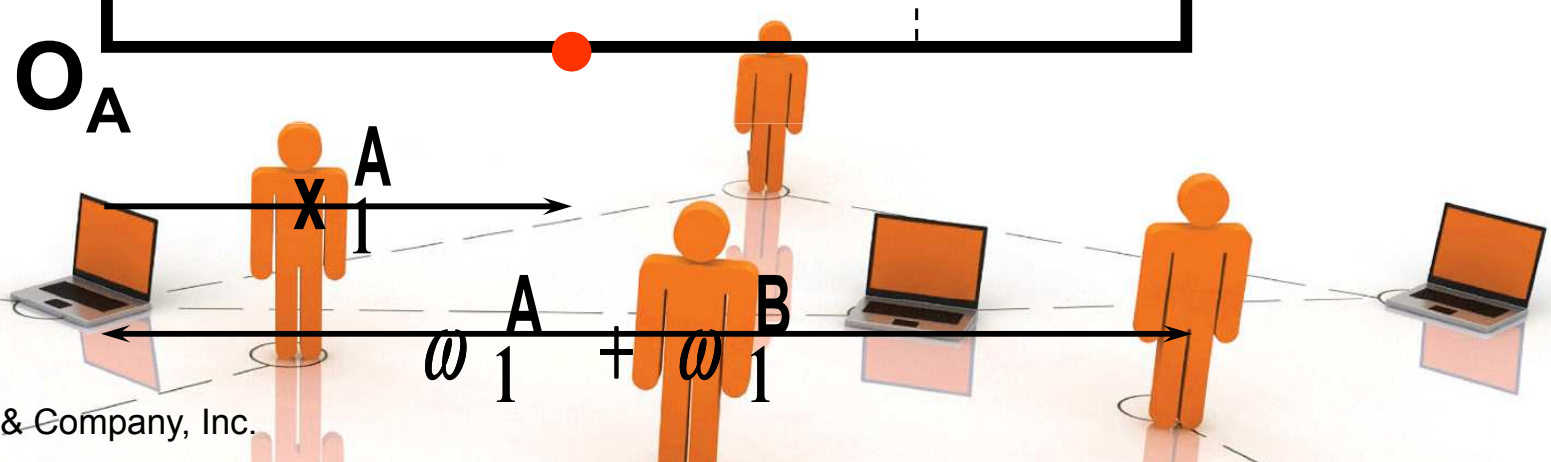
Other Feasible Allocations

◆ (x_1^A, x_2^A) **denotes an allocation to consumer A.**

◆ (x_1^B, x_2^B) **denotes an allocation to consumer B.**

◆ **An allocation is feasible if and only if**

$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$

and $x_2^A + x_2^B \leq \omega_2^A + \omega_2^B.$

Feasible Reallocations
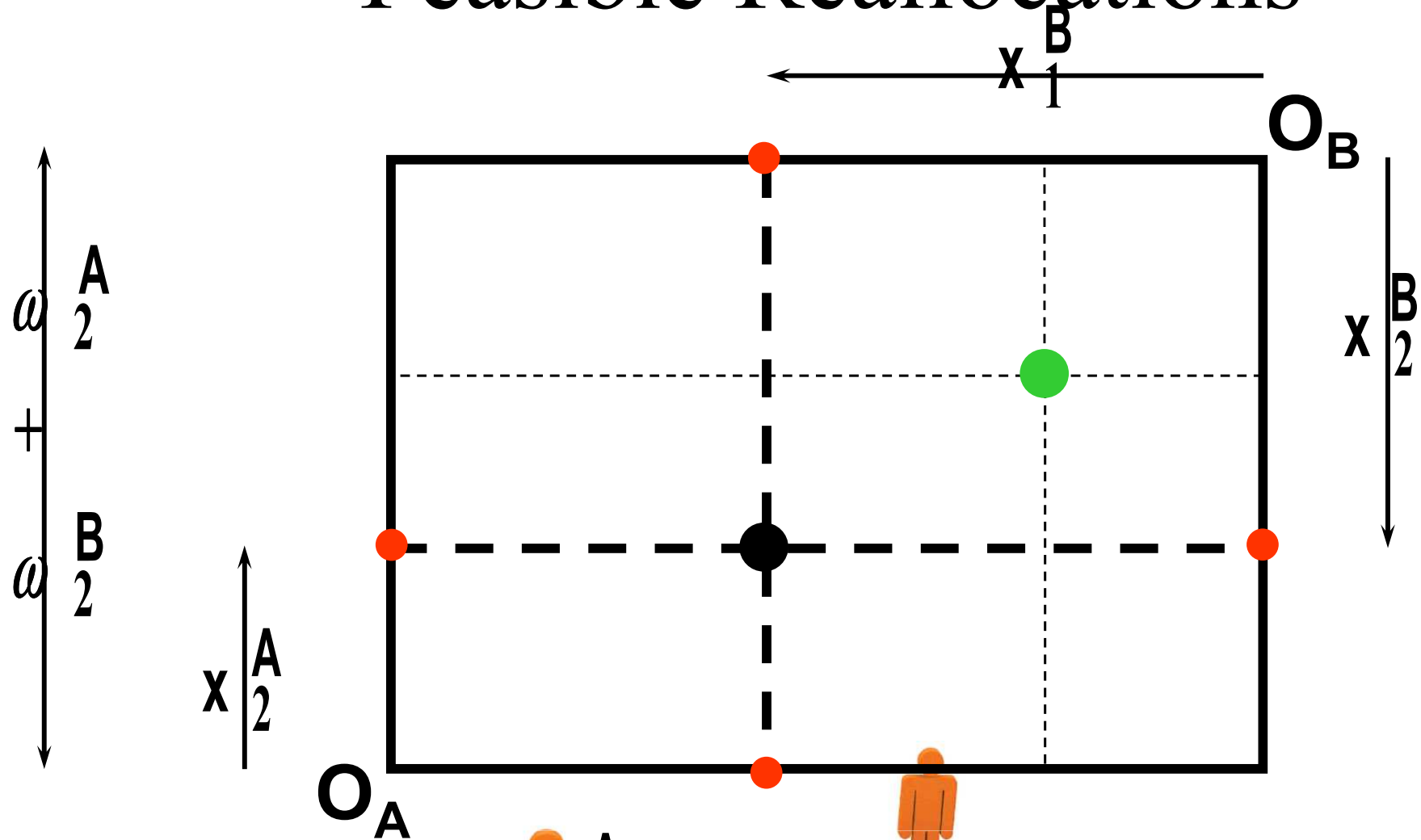
Feasible Reallocations

Feasible Reallocations

◆ **All points in the box, including the boundary, represent feasible allocations of the combined endowments.**

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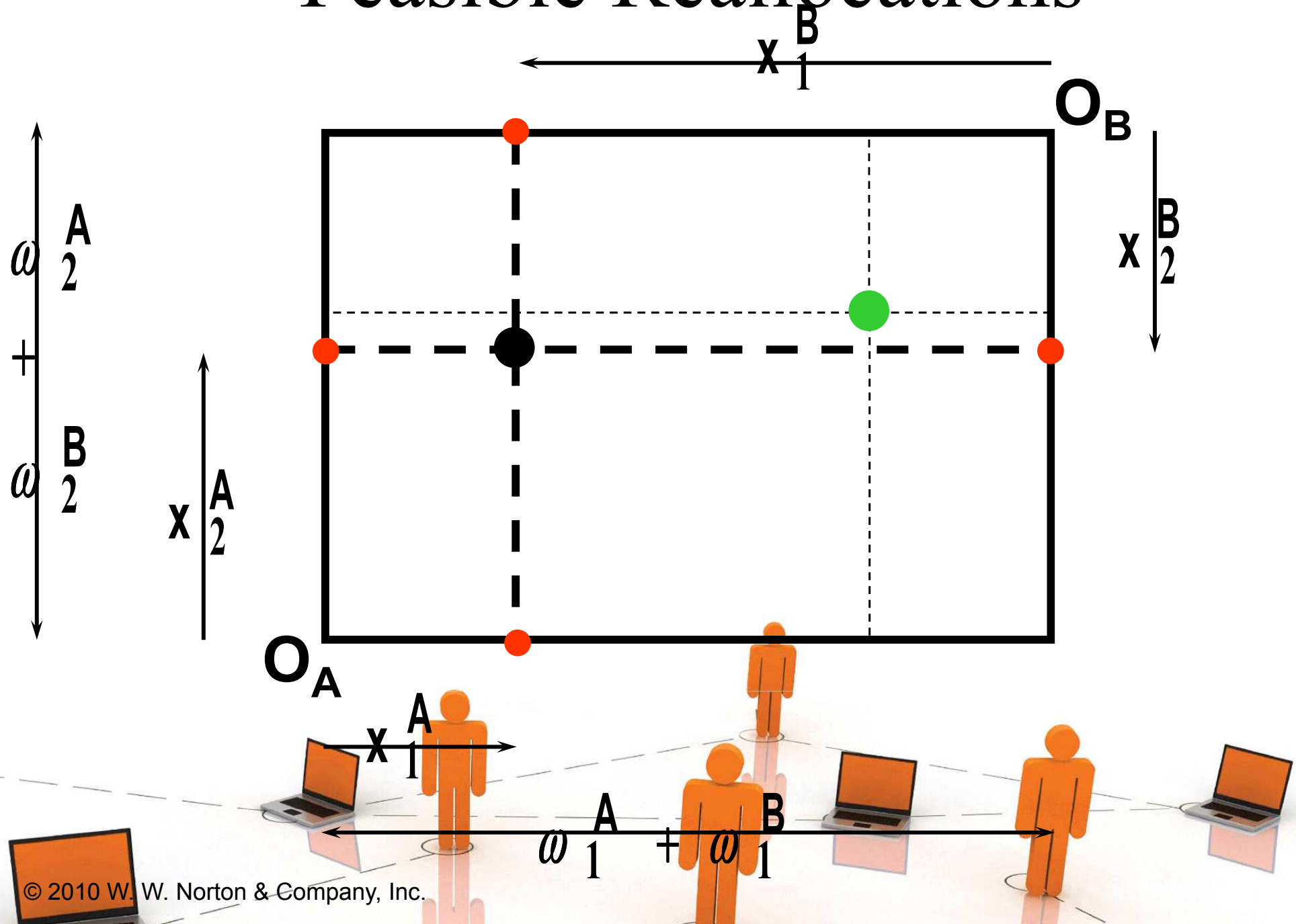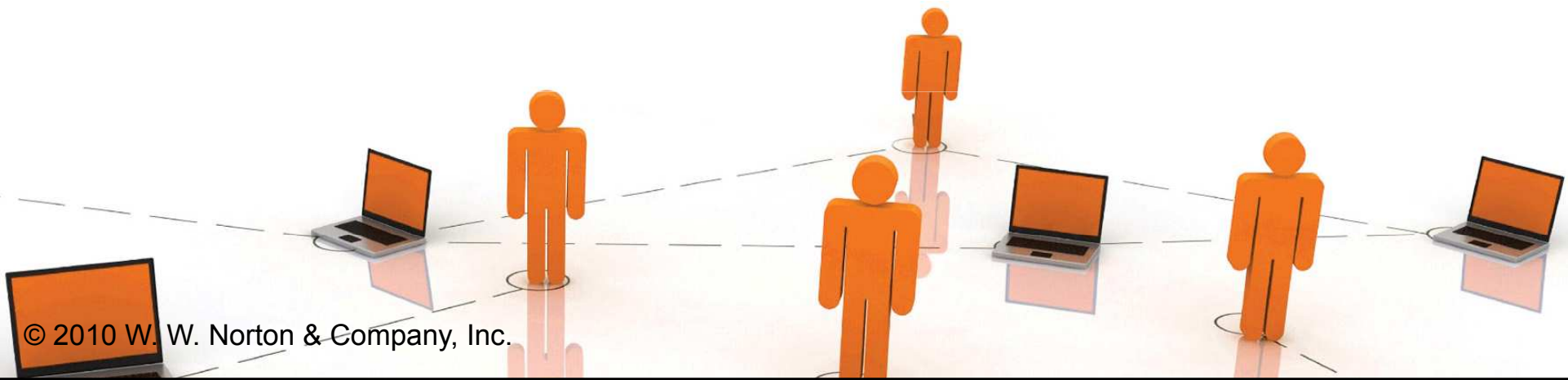# Feasible Reallocations

- **All points in the box, including the boundary, represent feasible allocations of the combined endowments.**
- **Which allocations will be blocked by one or both consumers?**
- **Which allocations make both consumers better off?**
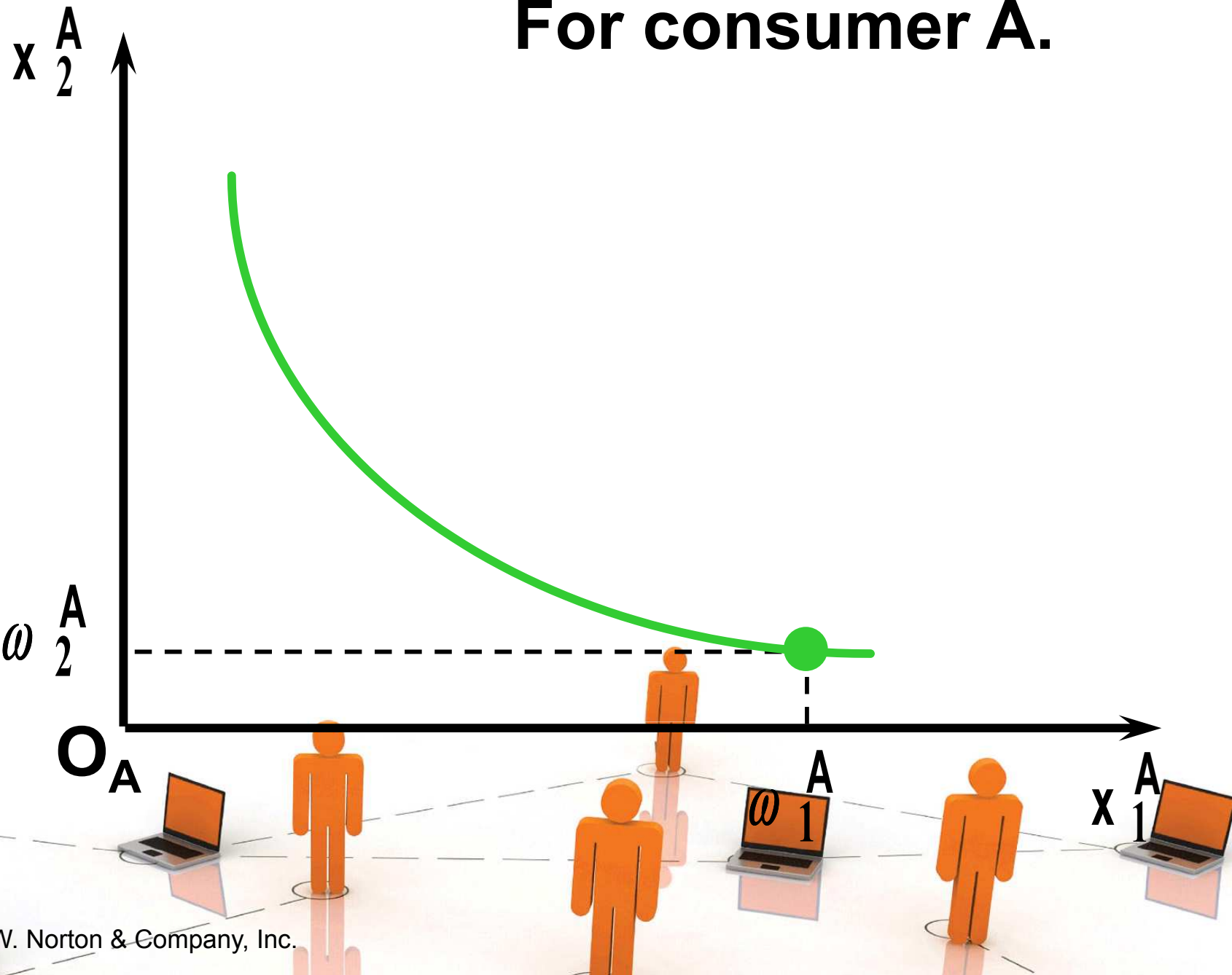
Adding Preferences to the Box

For consumer A.

Adding Preferences to the Box

For consumer A.

More preferred

X_2^A

ω_2^A

O_A

ω_1^A

X_1^A
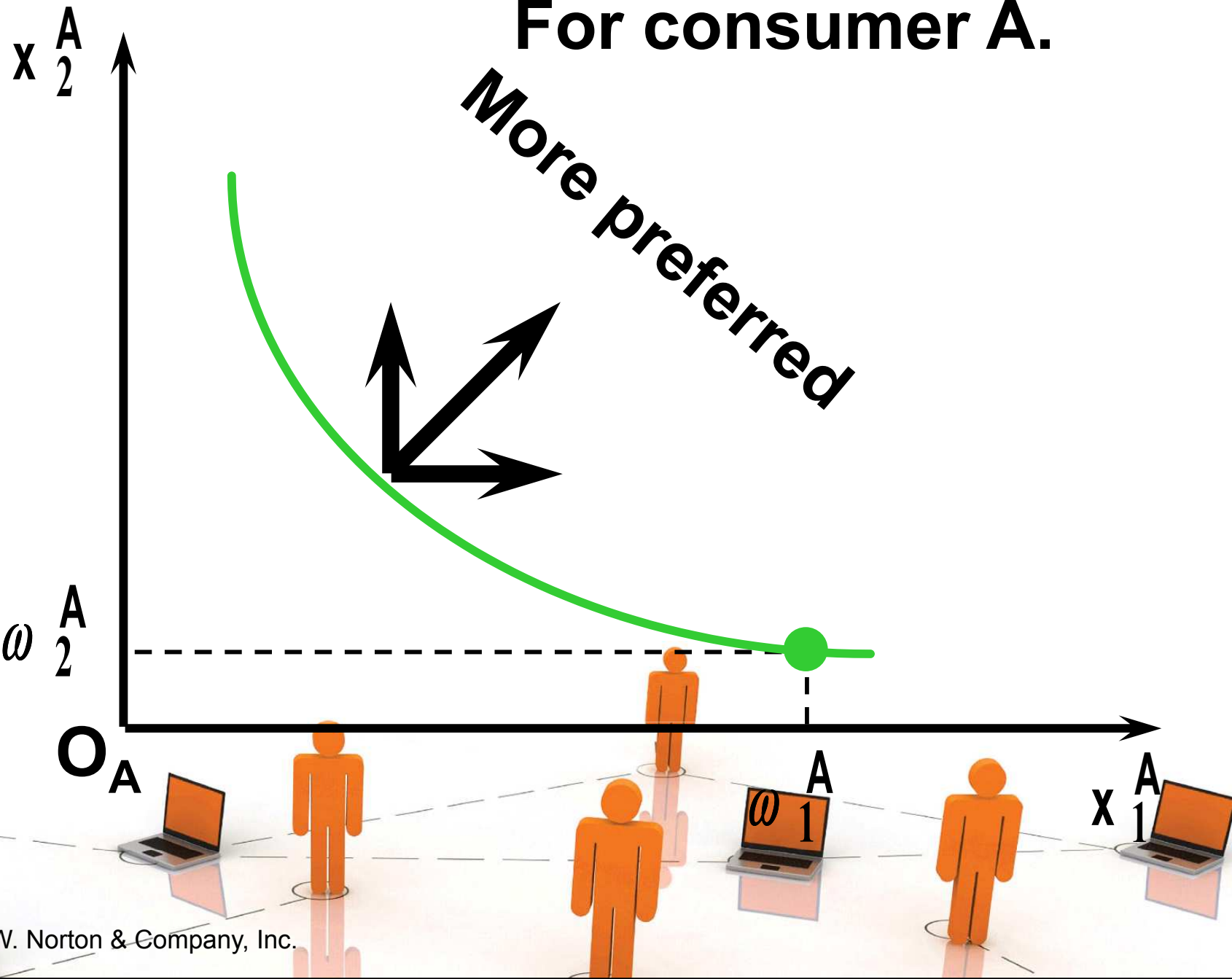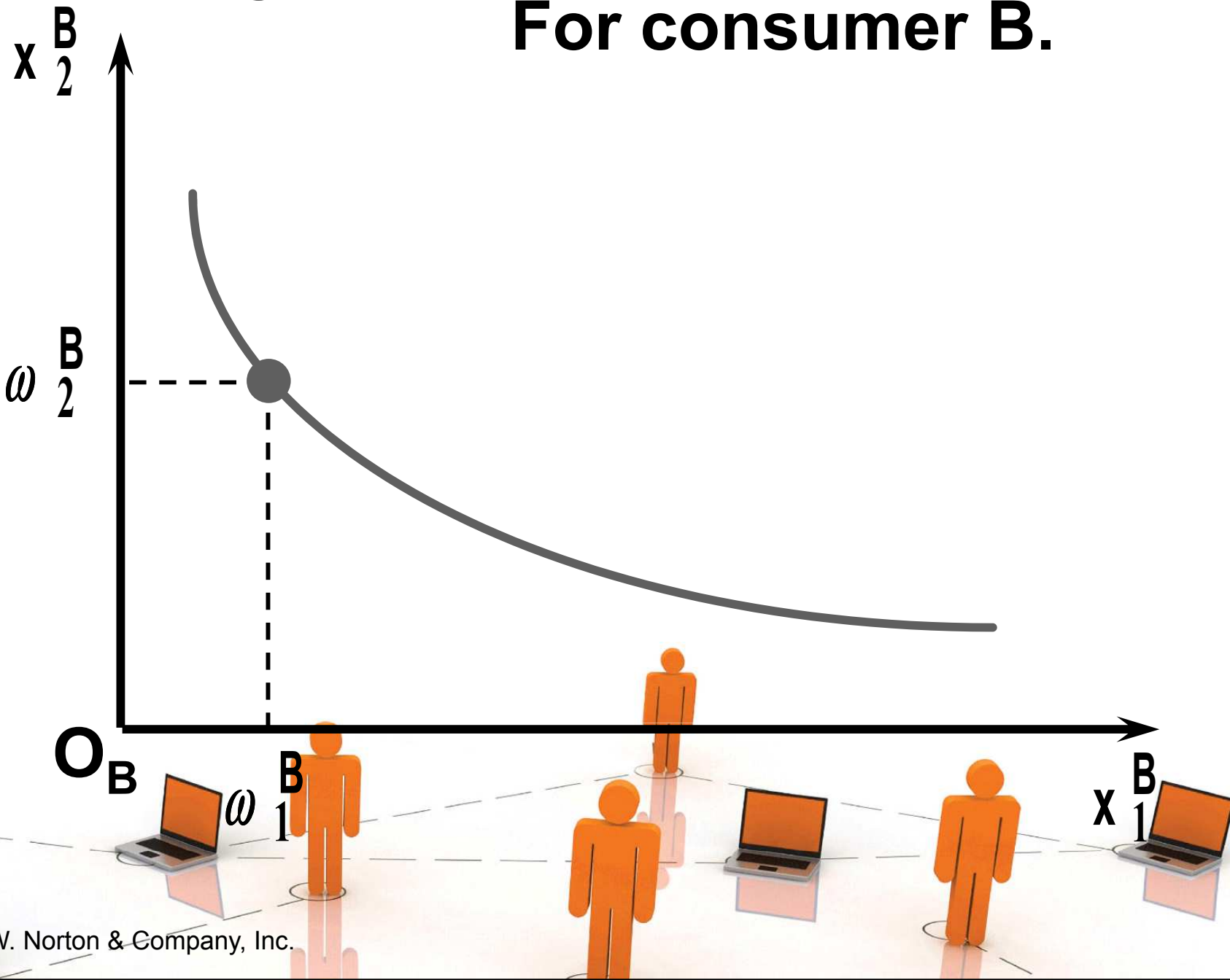
Adding Preferences to the Box

For consumer B.

Adding Preferences to the Box

For consumer B.

More preferred

x_2^B

ω_2^B

O_B

ω_1^B

x_1^B
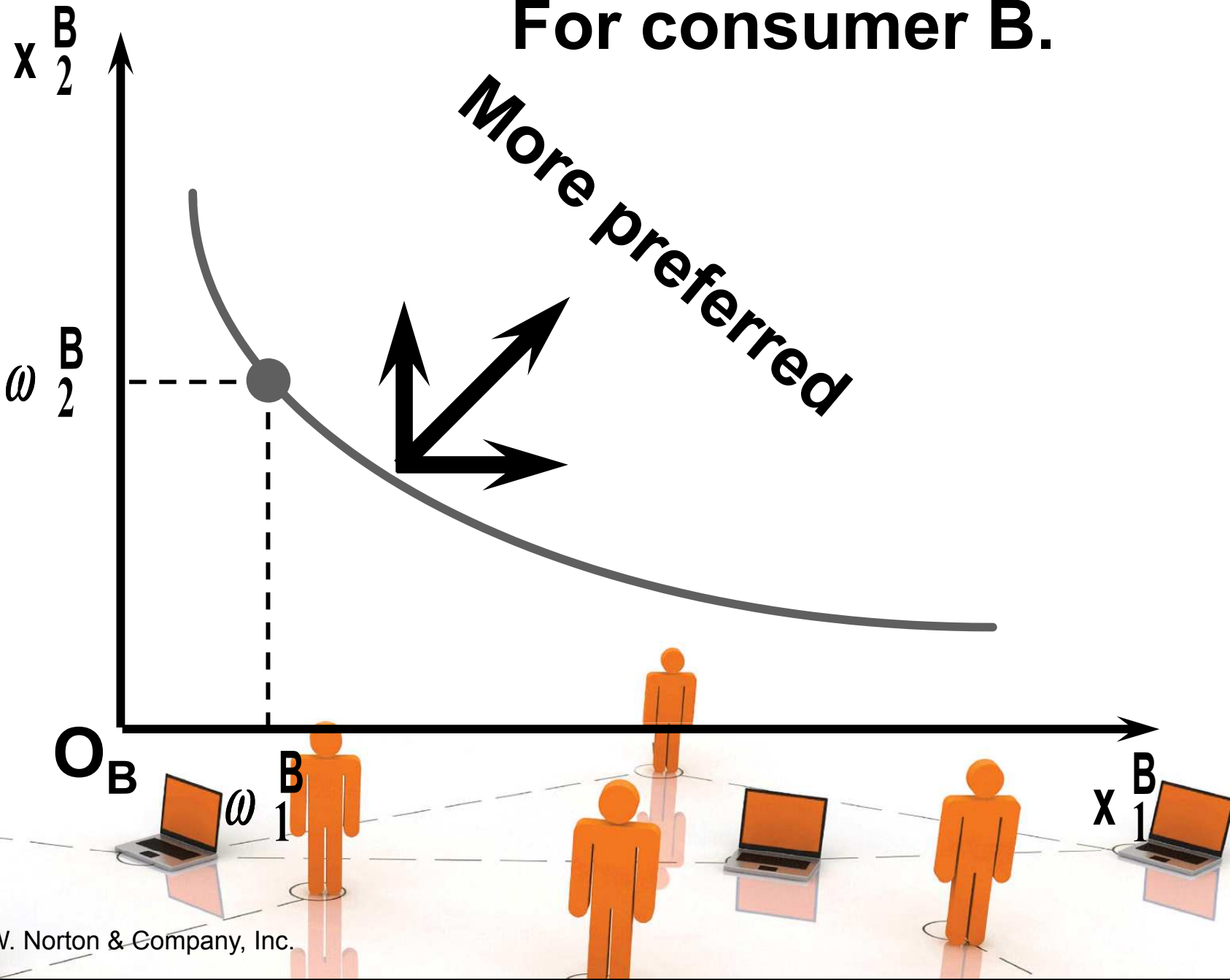
Adding Preferences to the Box

For consumer B.

$$x_1^B \quad \omega_1^B \quad O_B$$

$$\omega_2^B$$

$$x_2^B$$

More preferred

Adding Preferences to the Box

For consumer A.

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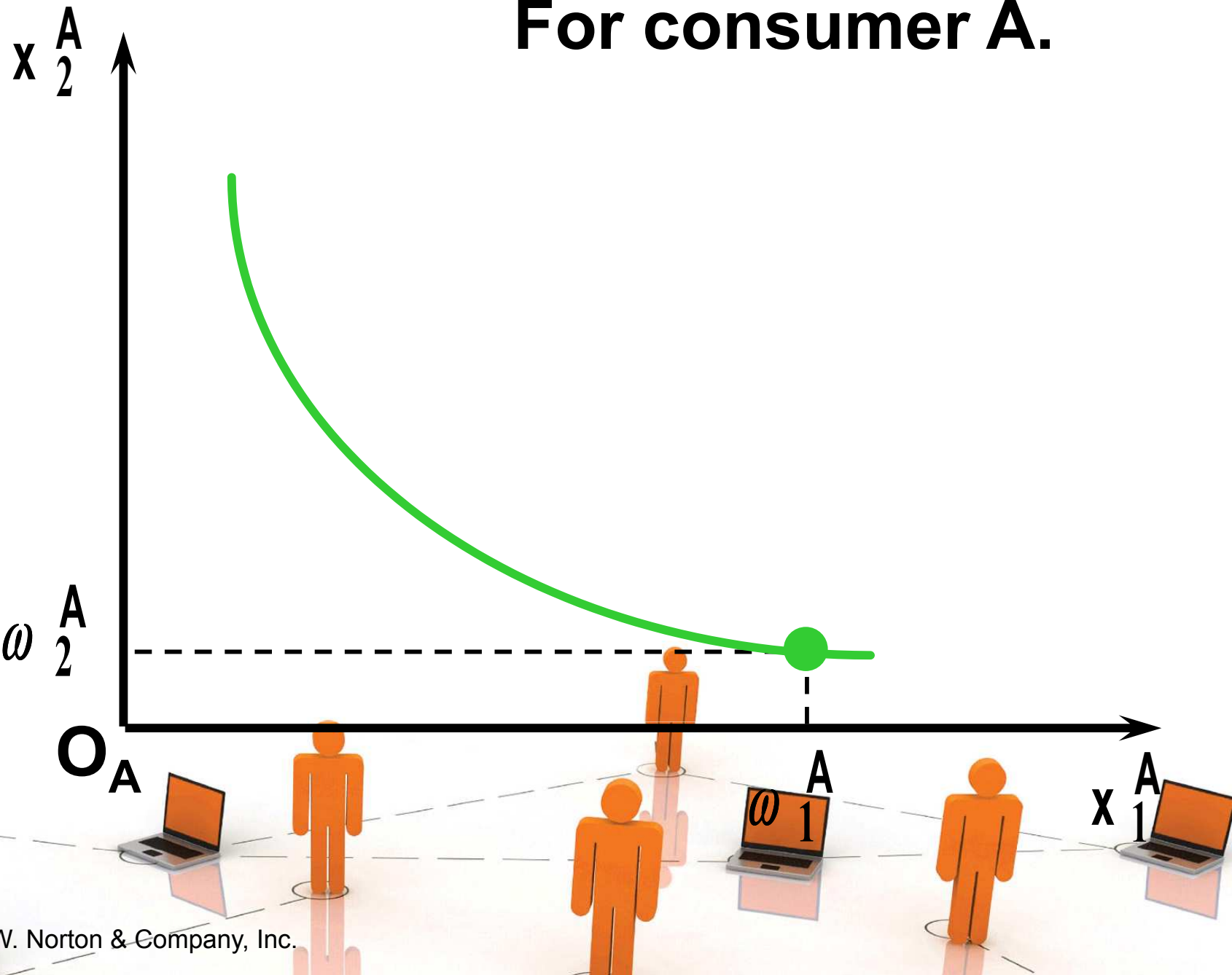
Adding Preferences to the Box
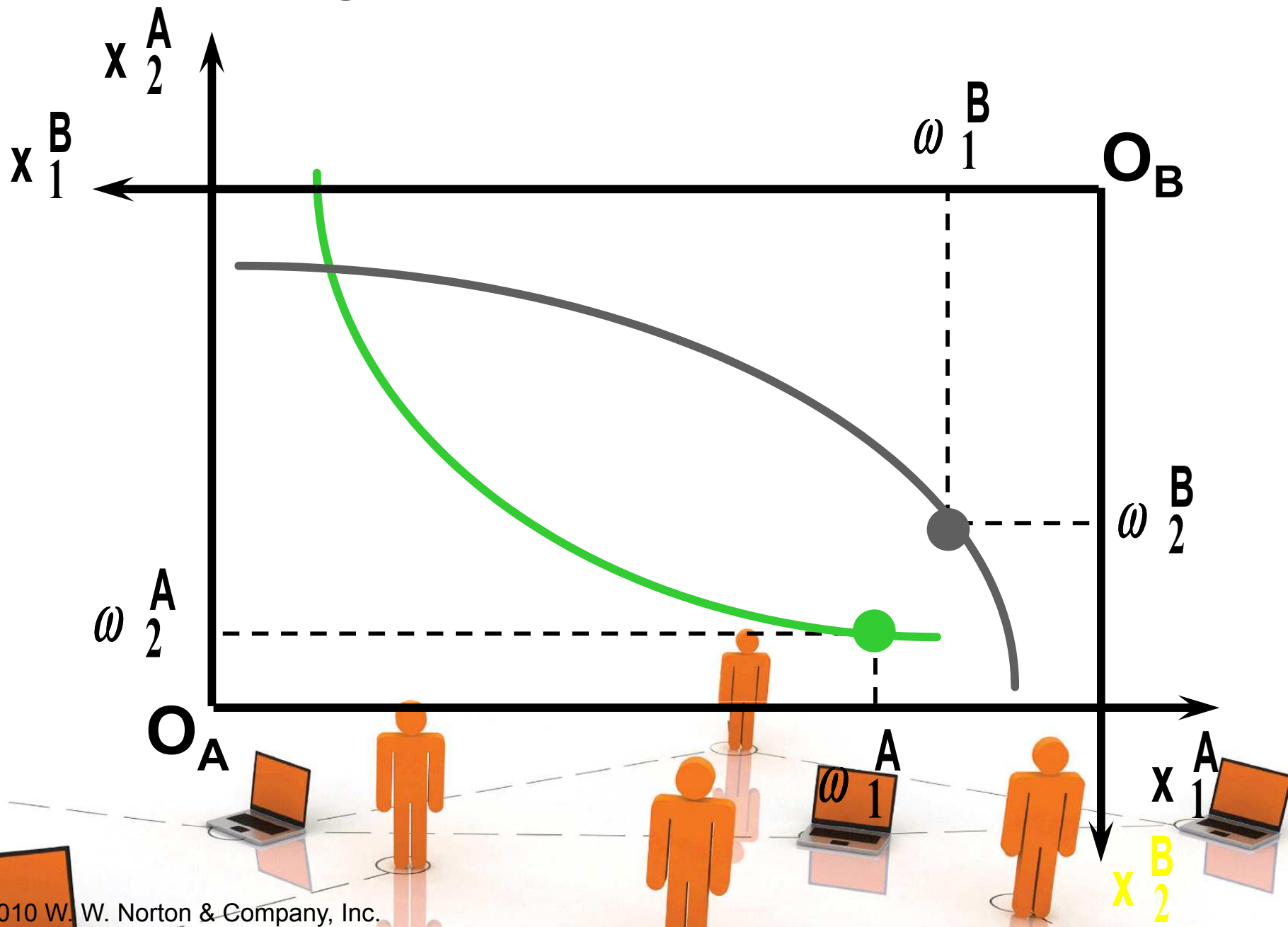
Edgeworth's Box

Pareto-Improvement

◆ **An allocation of the endowment that improves the welfare of a consumer without reducing the welfare of another is a Pareto-improving allocation.**

◆ **Where are the Pareto-improving allocations?**

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Edgeworth's Box

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Pareto-Improvements

The set of Pareto-improving allocations

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Pareto-Improvements

◆ **Since each consumer can refuse to trade, the only possible outcomes from exchange are Pareto-improving allocations.**

◆ **But which particular Pareto-improving allocation will be the outcome of trade?**

Pareto-Improvements
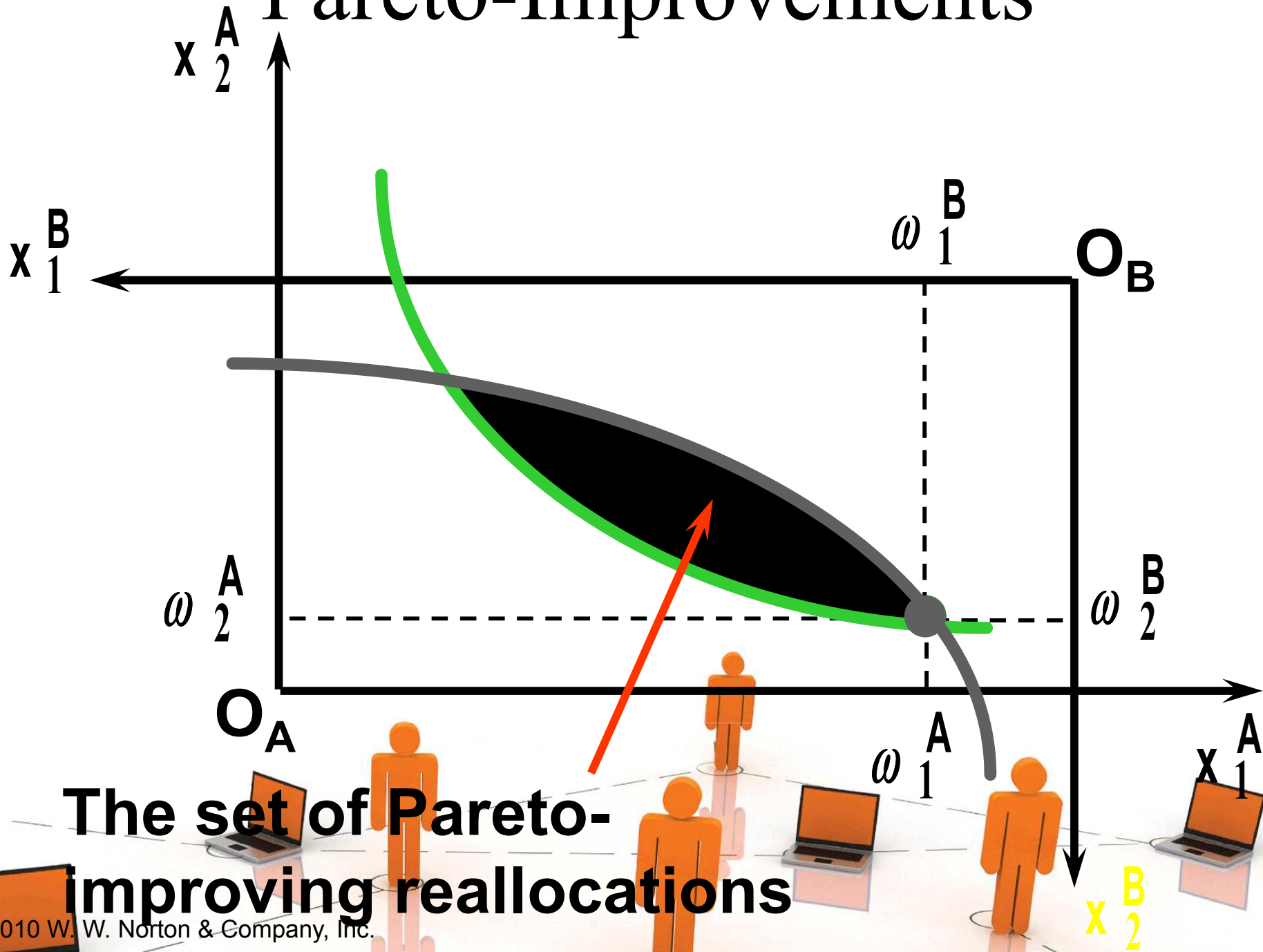
The set of Pareto-improving reallocations

Pareto-Improvements
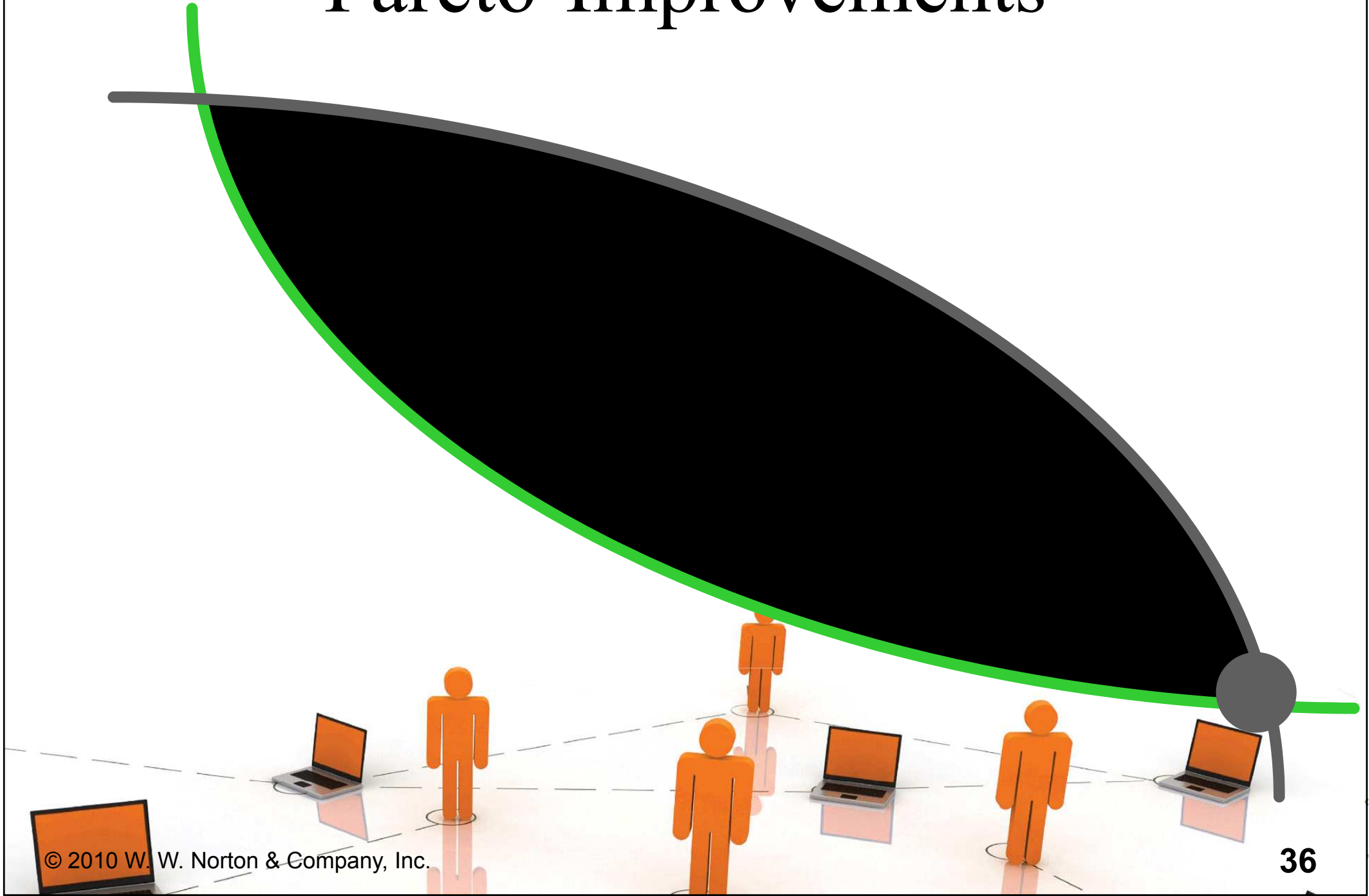
Pareto-Improvements
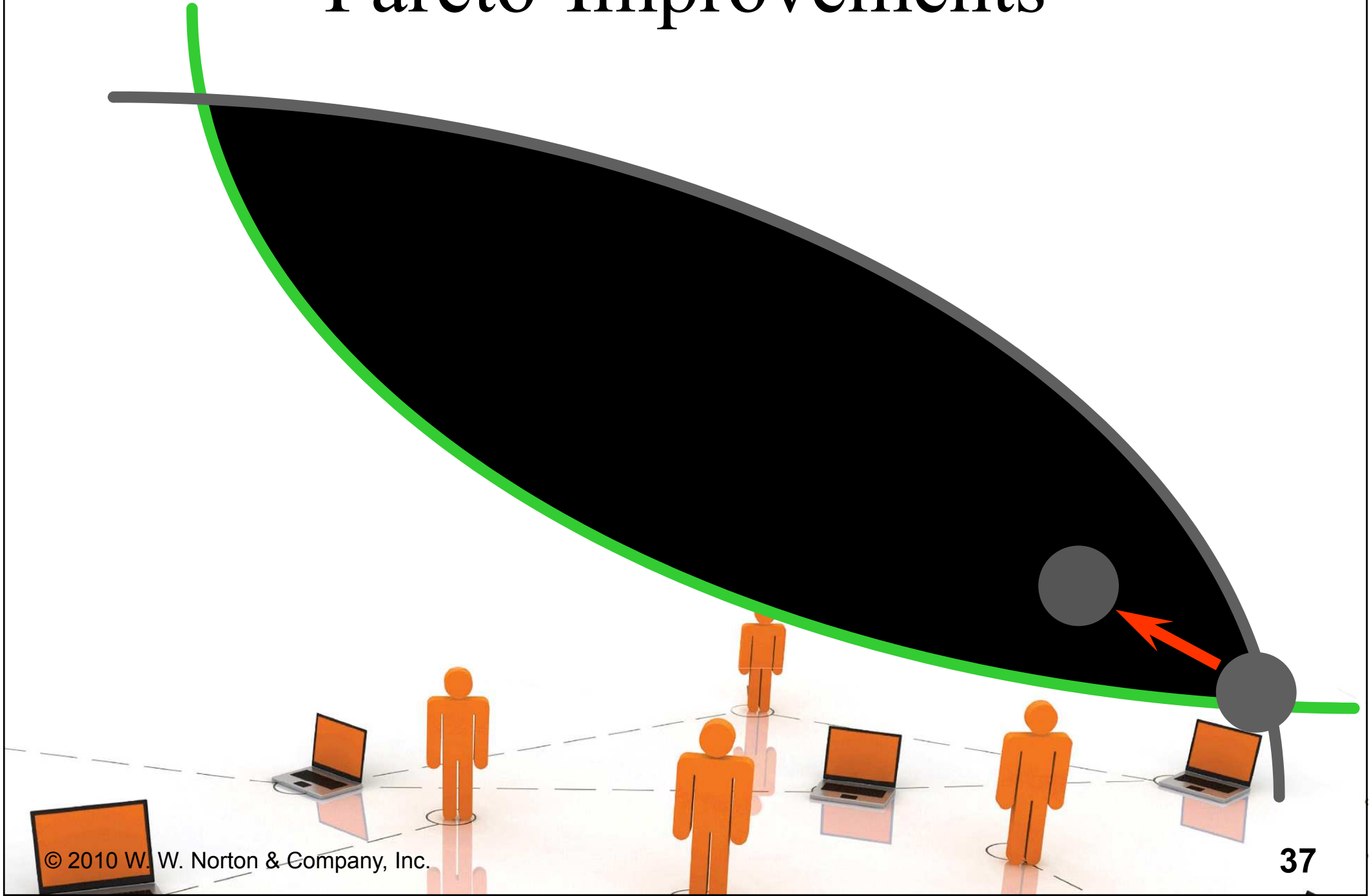
Pareto-Improvements

Trade improves both A's and B's welfares. This is a Pareto-improvement over the endowment allocation.

Pareto-Improvements

New mutual gains-to-trade region is the set of all further Pareto-improving reallocations.

Trade improves both A's and B's welfares. This is a Pareto-improvement over the endowment allocation.
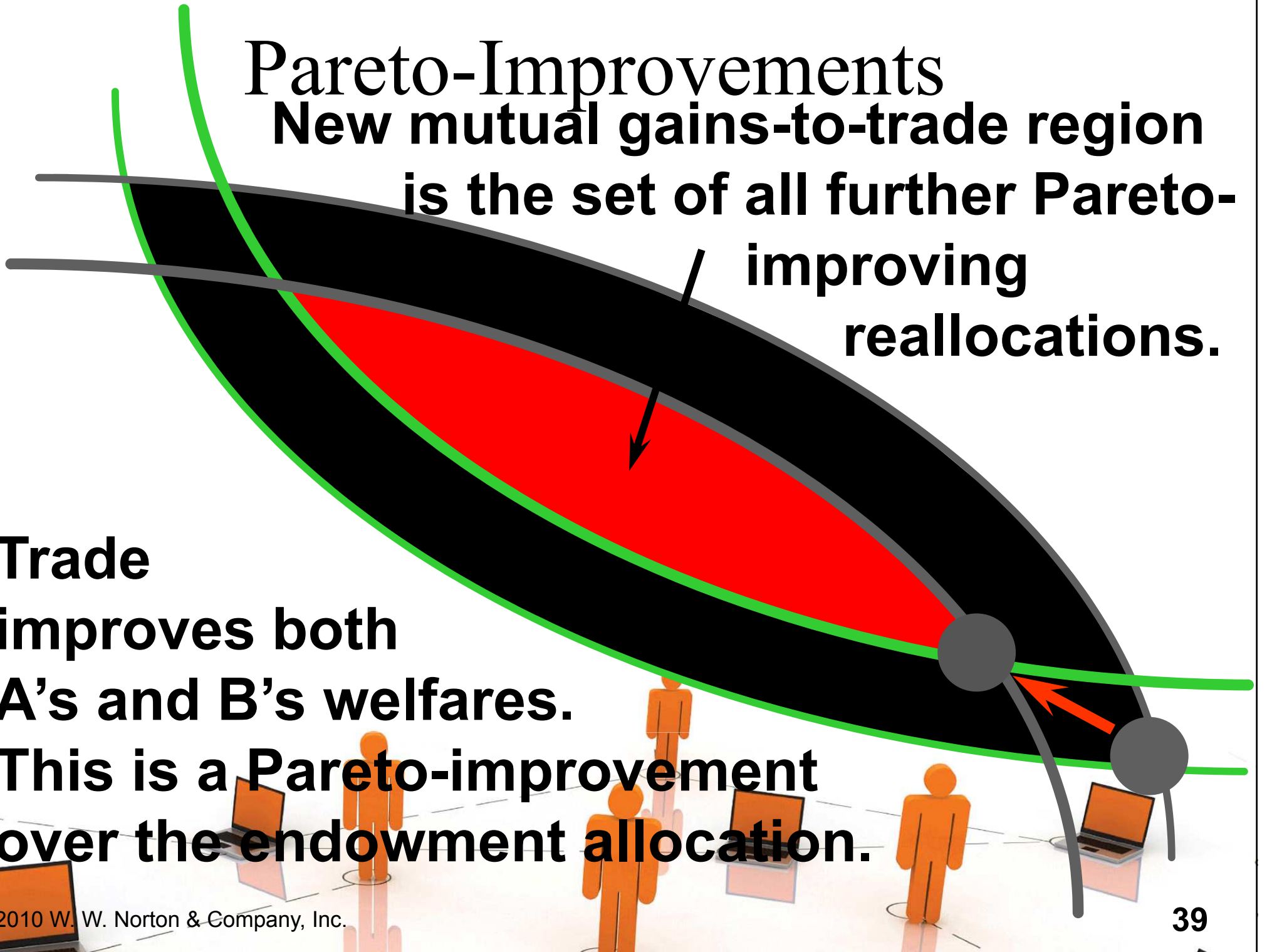
Pareto-Improvements

Further trade cannot improve both A and B's welfares.
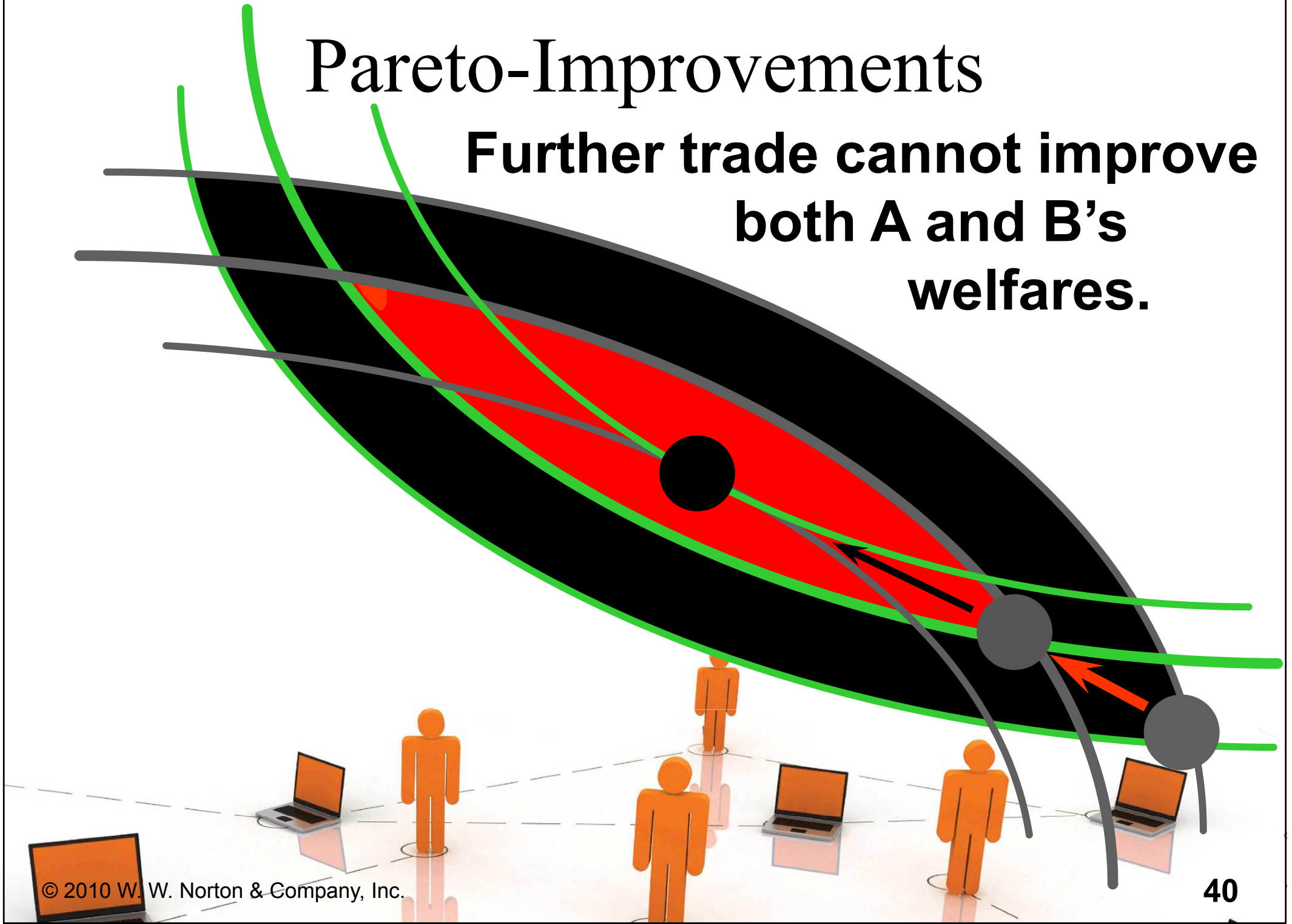
Pareto-Optimality

Better for consumer A

Better for consumer B
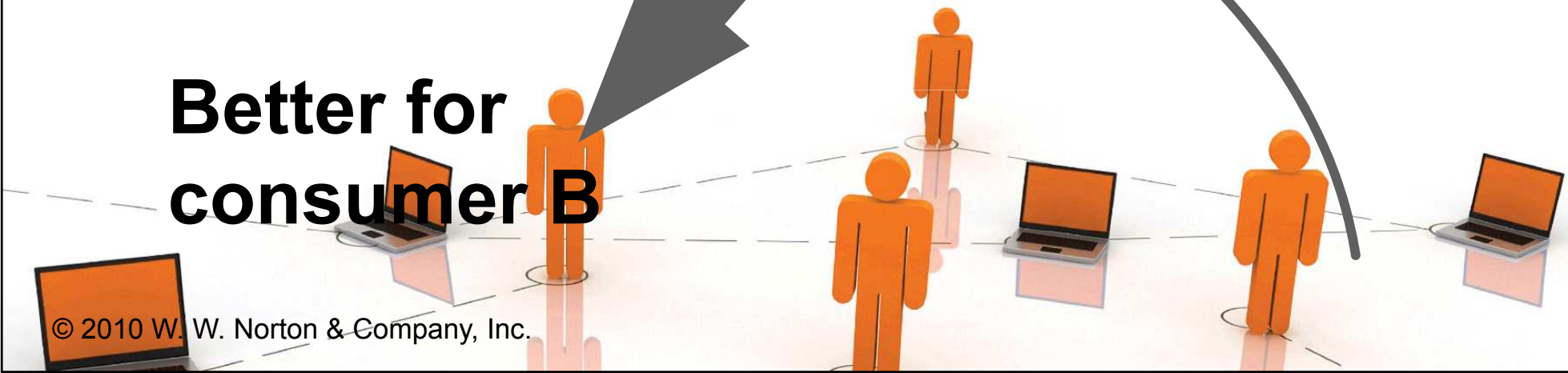
Pareto-Optimality
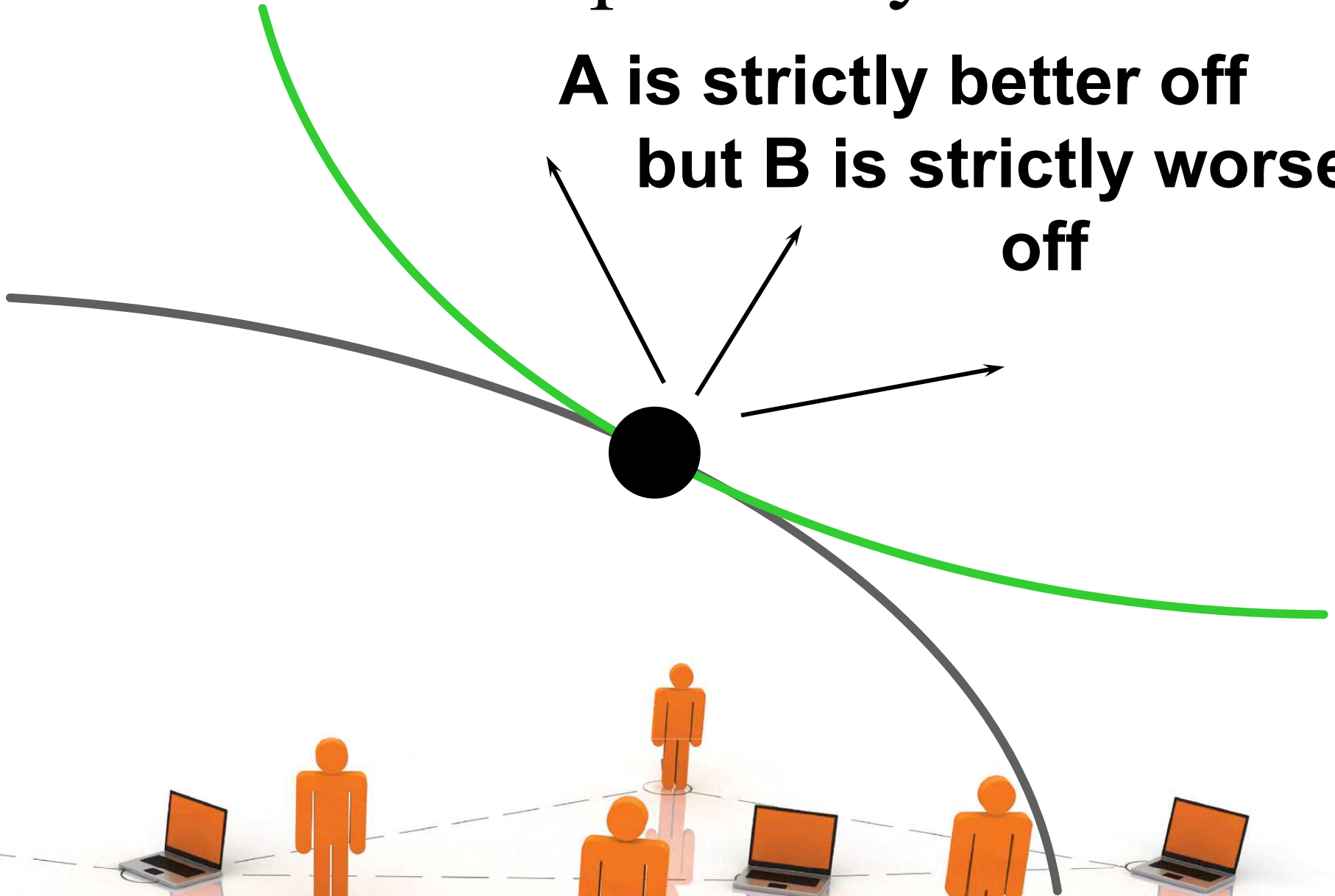
A is strictly better off but B is strictly worse off

Pareto-Optimality

A is strictly better off but B is strictly worse off

B is strictly better off but A is strictly worse off

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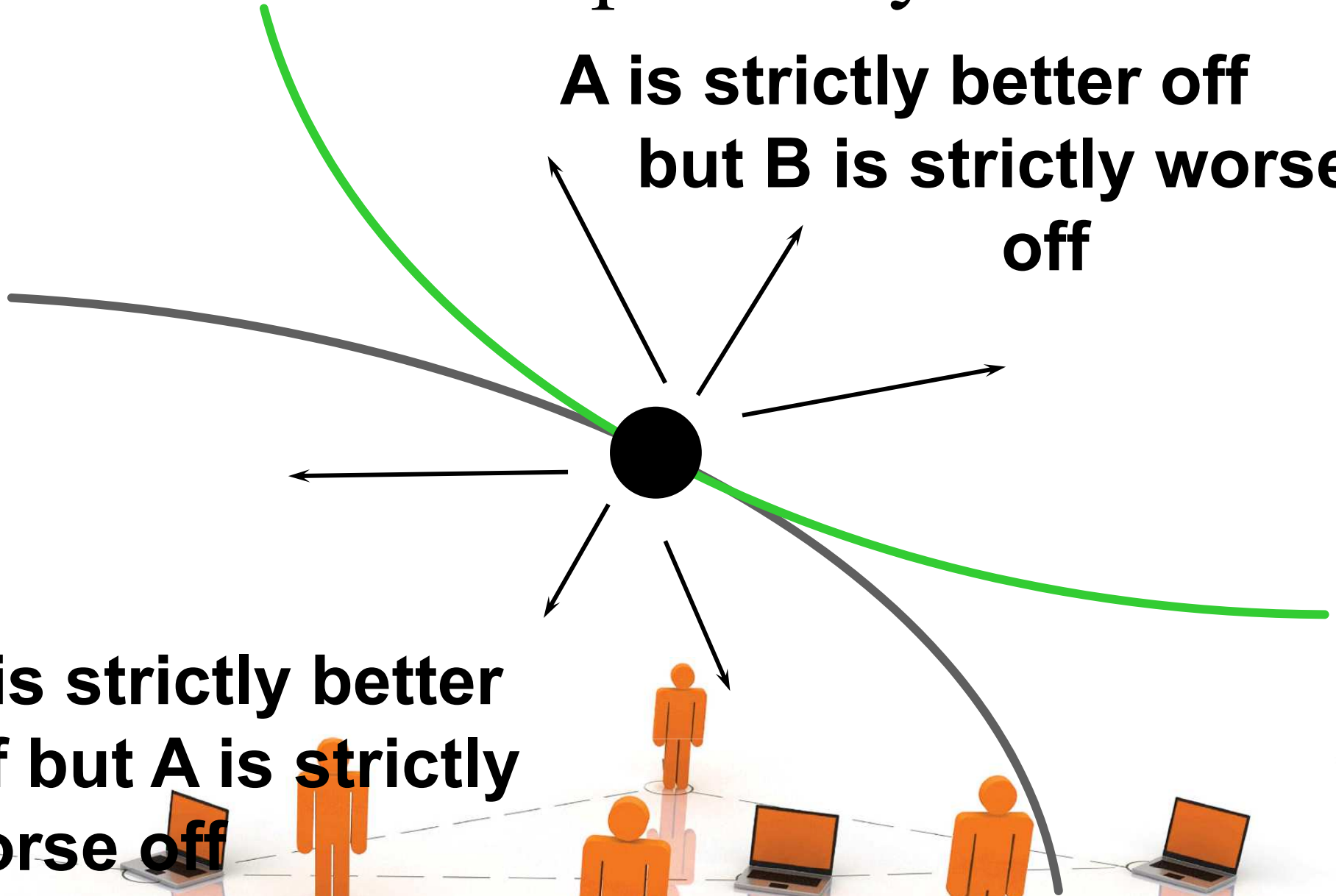
Pareto-Optimality

Both A and B are worse off

A is strictly better off but B is strictly worse off

B is strictly better off but A is strictly worse off
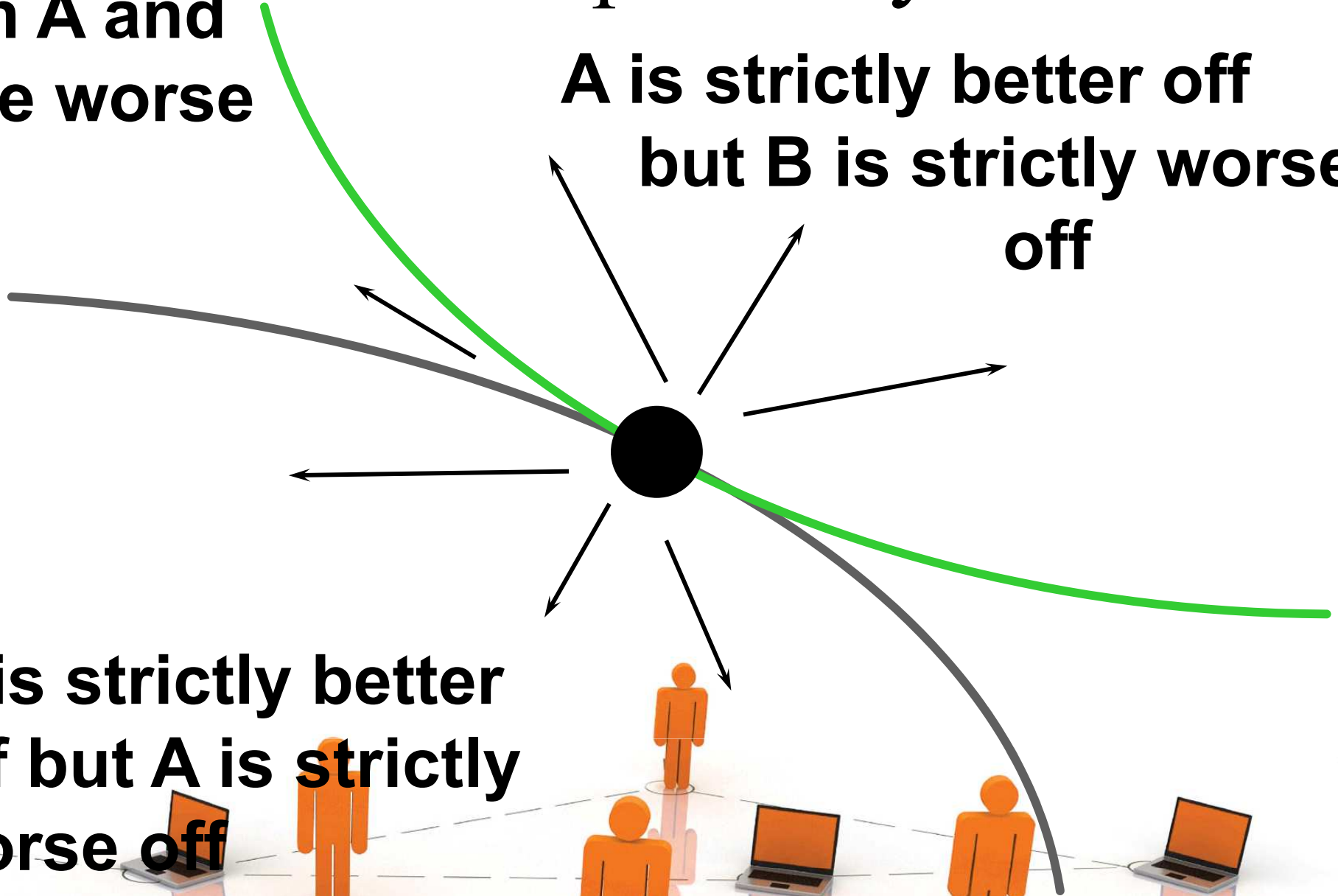
Pareto-Optimality

Both A and B are worse off

A is strictly better off but B is strictly worse off

B is strictly better off but A is strictly worse off

Both A and B are worse off

45
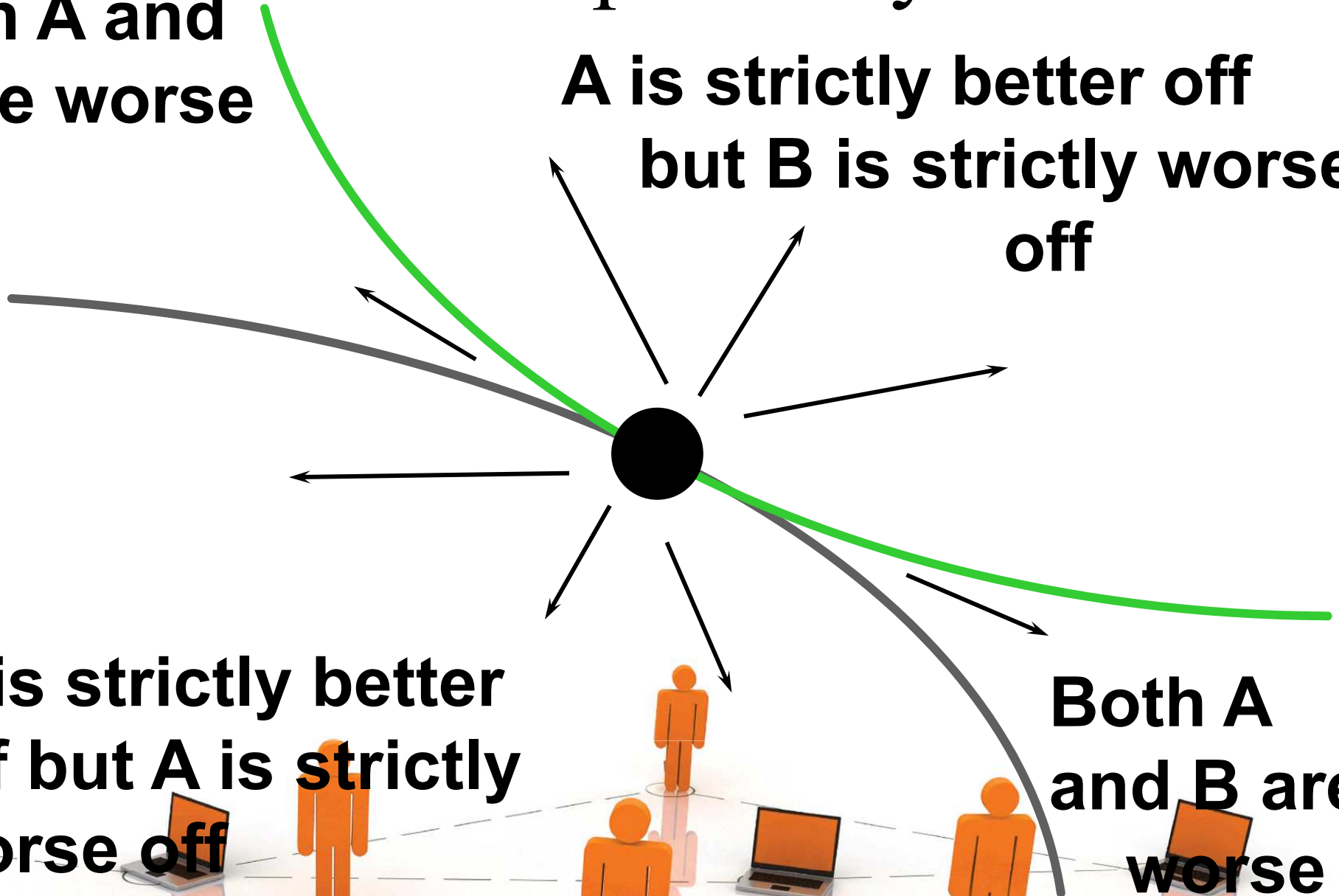
Pareto-Optimality

The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

An allocation where convex indifference curves are "only just back-to-back" is Pareto-optimal.

The allocation is Pareto-optimal since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

Pareto-Optimality

◆ **Where are all of the Pareto-optimal allocations of the endowment?**

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Pareto-Optimality

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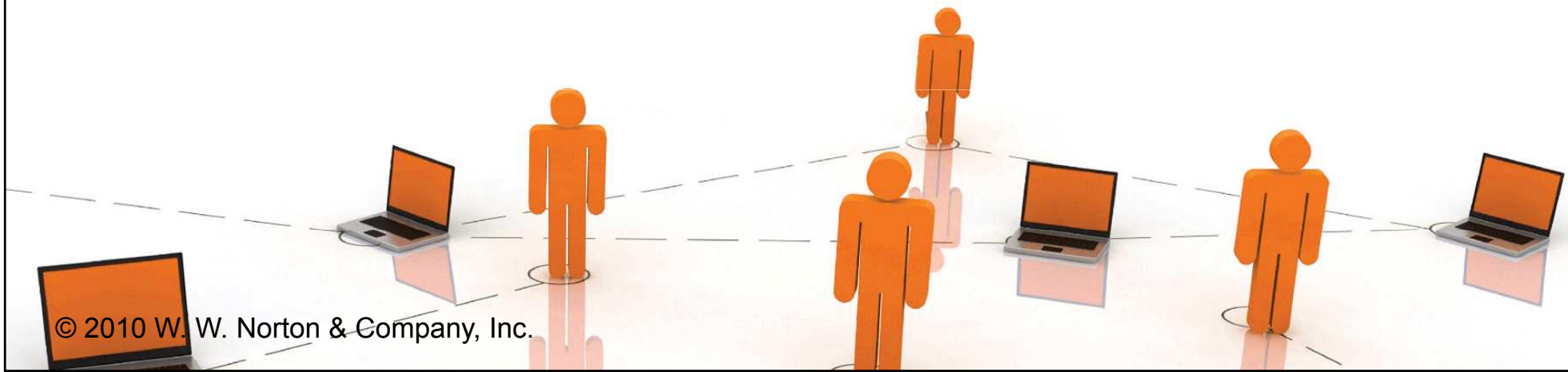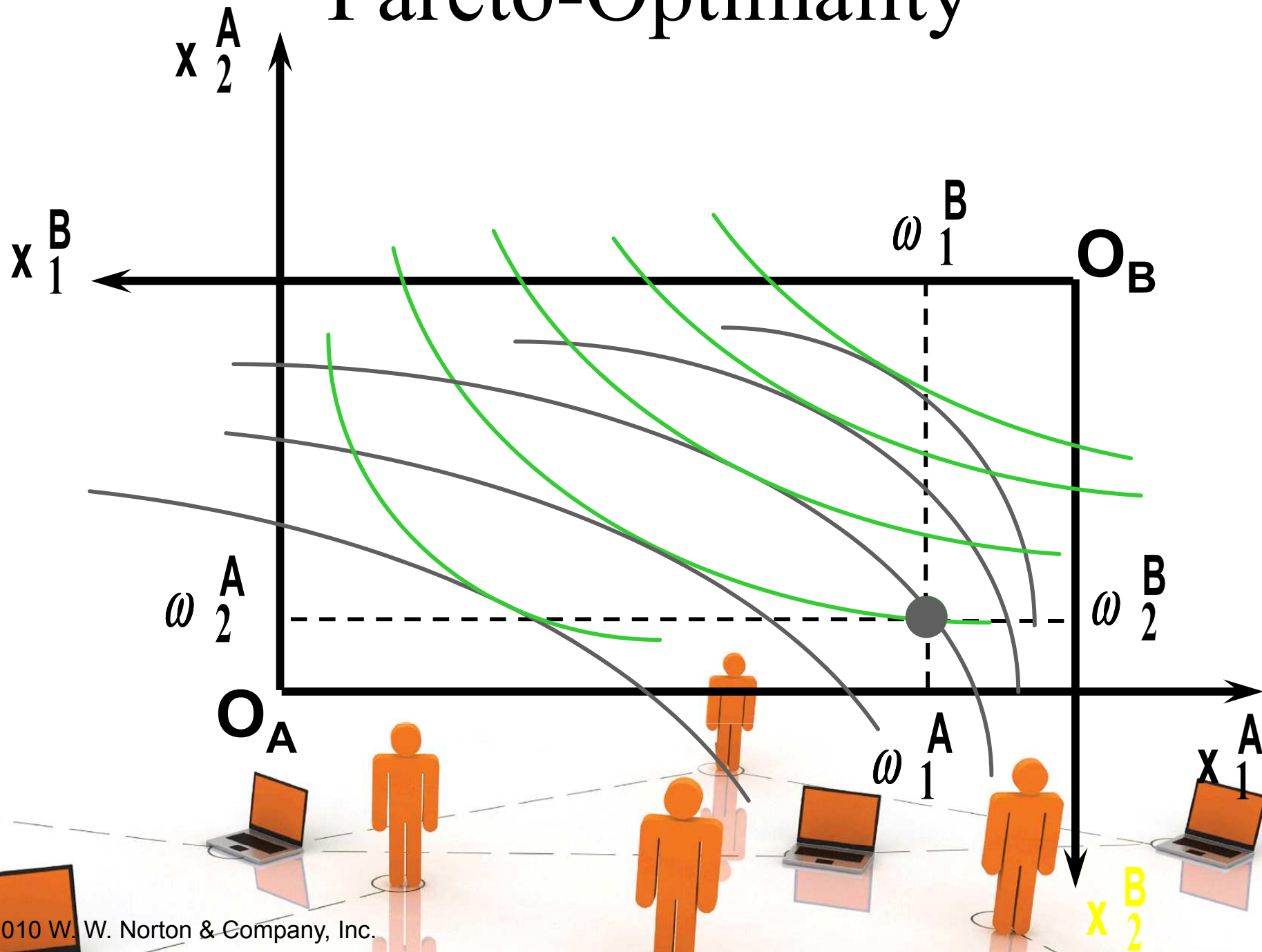
Pareto-Optimality

All the allocations marked by a ● are Pareto-optimal.

Pareto-Optimality

◆ **The contract curve is the set of all Pareto-optimal allocations.**

Pareto-Optimality

x_2^A

All the allocations marked by a ● are Pareto-optimal.

x_1^B

ω_1^B

O_B

ω_2^A

ω_2^B

O_A

ω_1^A

The contract curve

x_1^A

x_2^B
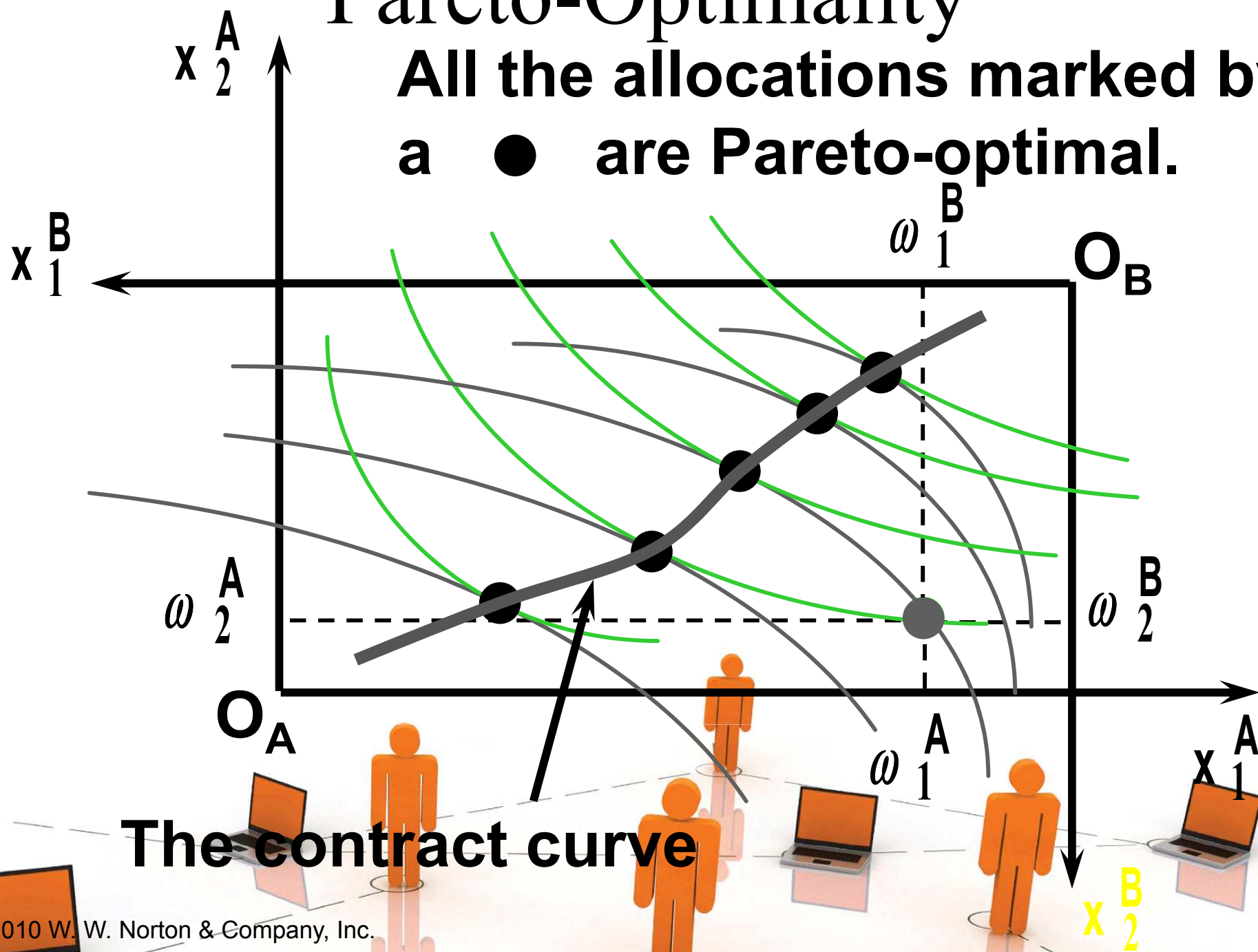
Pareto-Optimality

◆ **But to which of the many allocations on the contract curve will consumers trade?**

◆ **That depends upon how trade is conducted.**

◆ **In perfectly competitive markets? By one-on-one bargaining?**

The Core
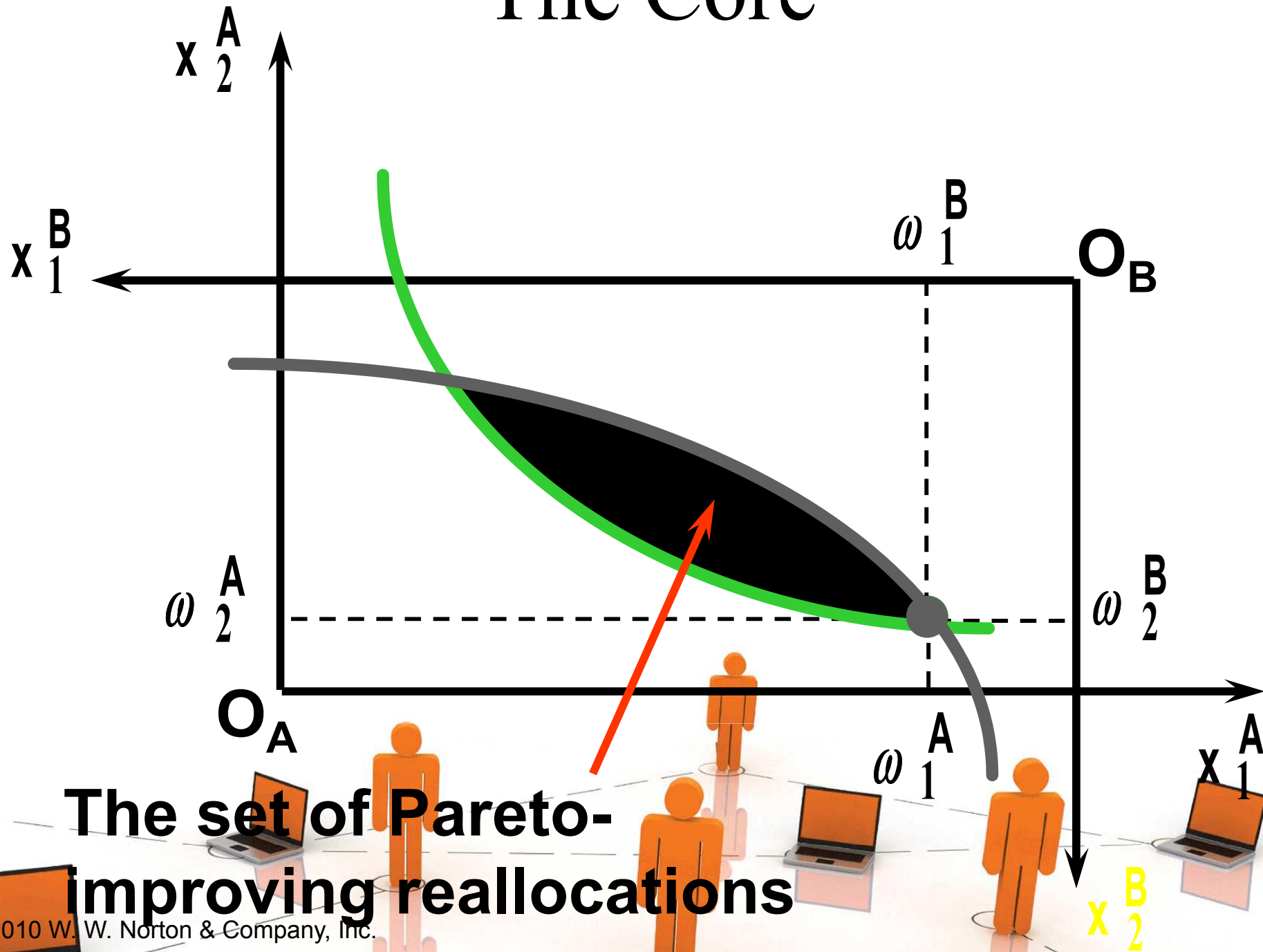
The set of Pareto-improving reallocations

The Core

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The Core

Pareto-optimal trades blocked by B

x_2^A

x_1^B

ω_1^B

O_B

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^A

x_2^B

Pareto-optimal trades blocked by A

56
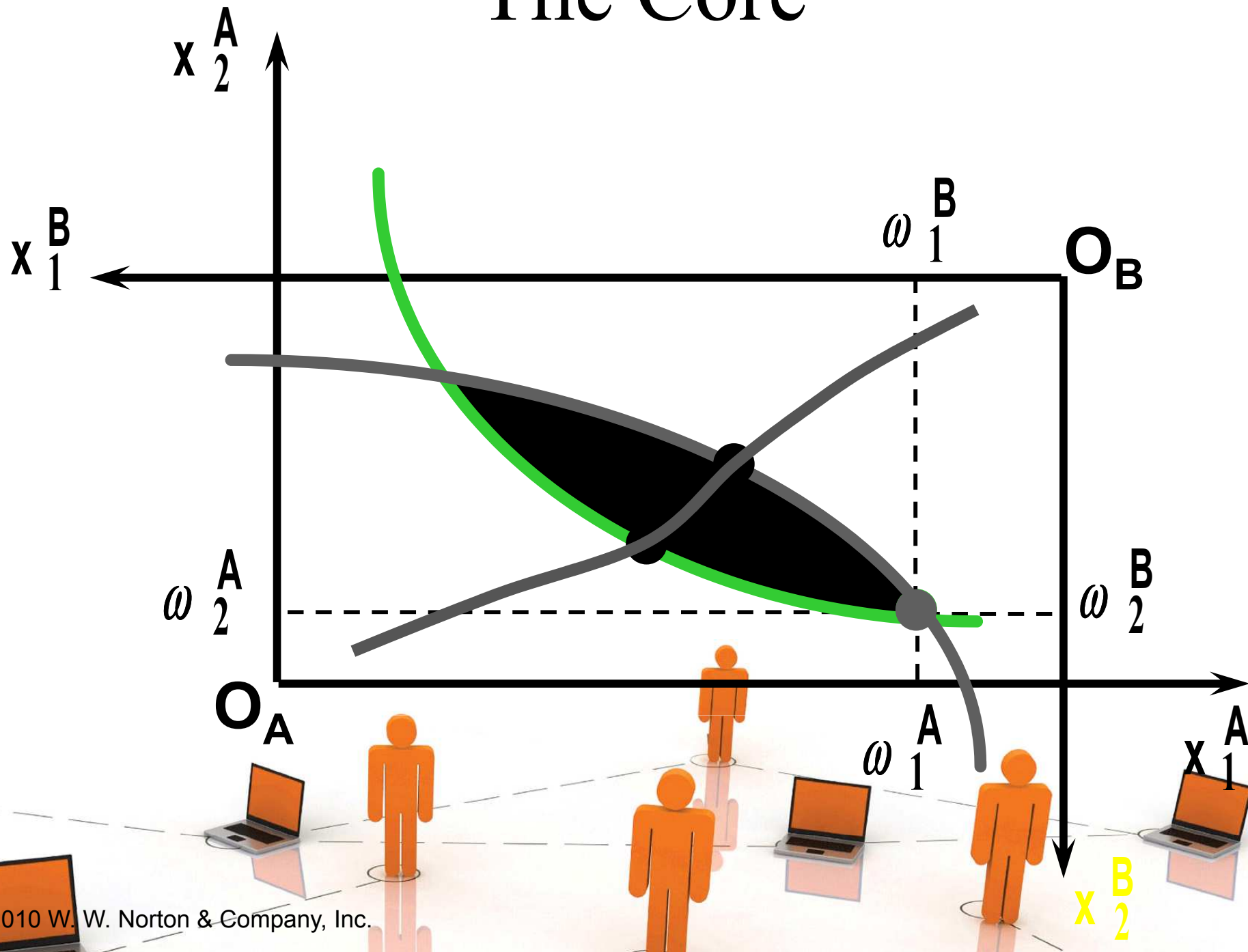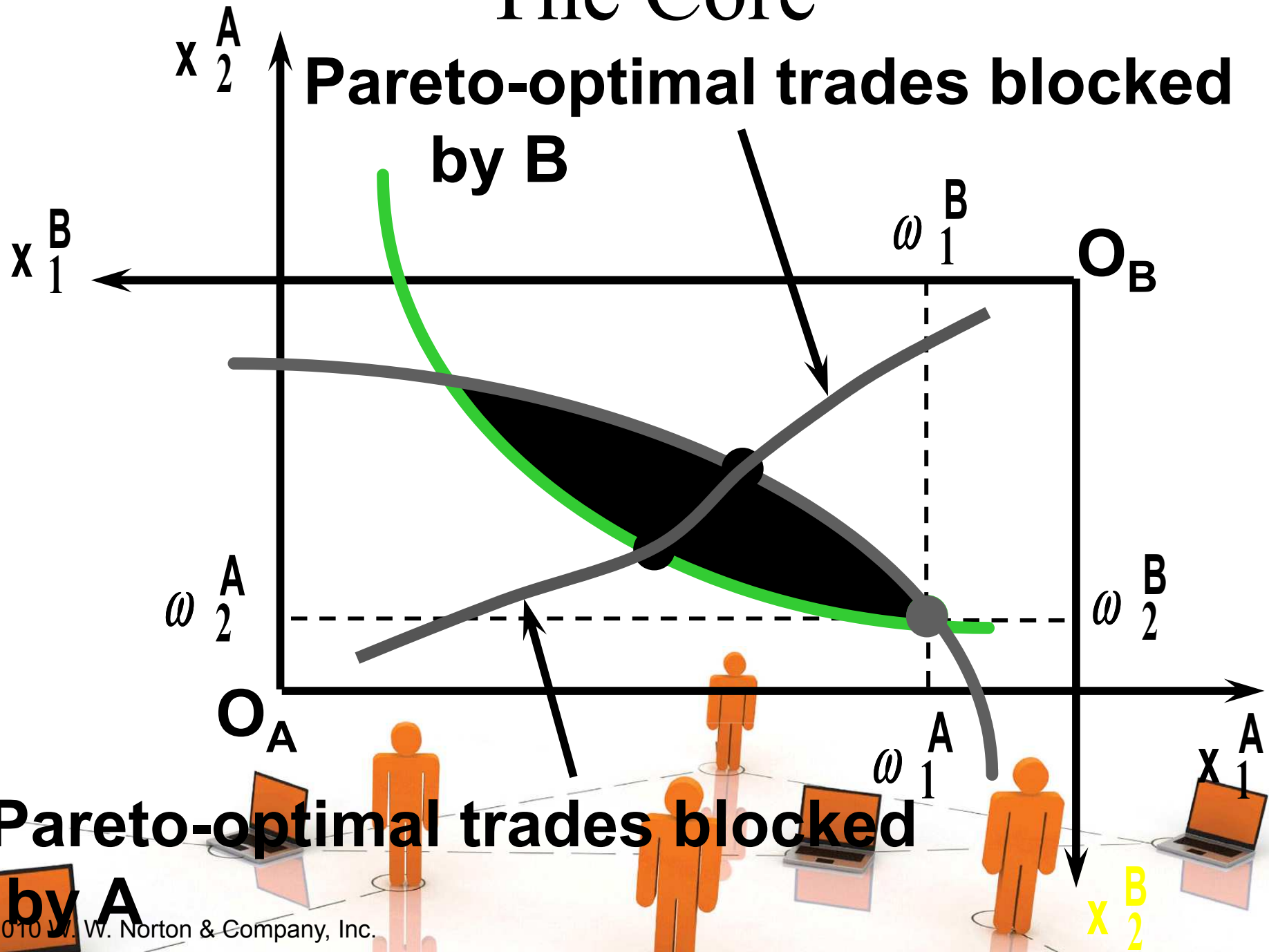
The Core
Pareto-optimal trades not blocked by A or B
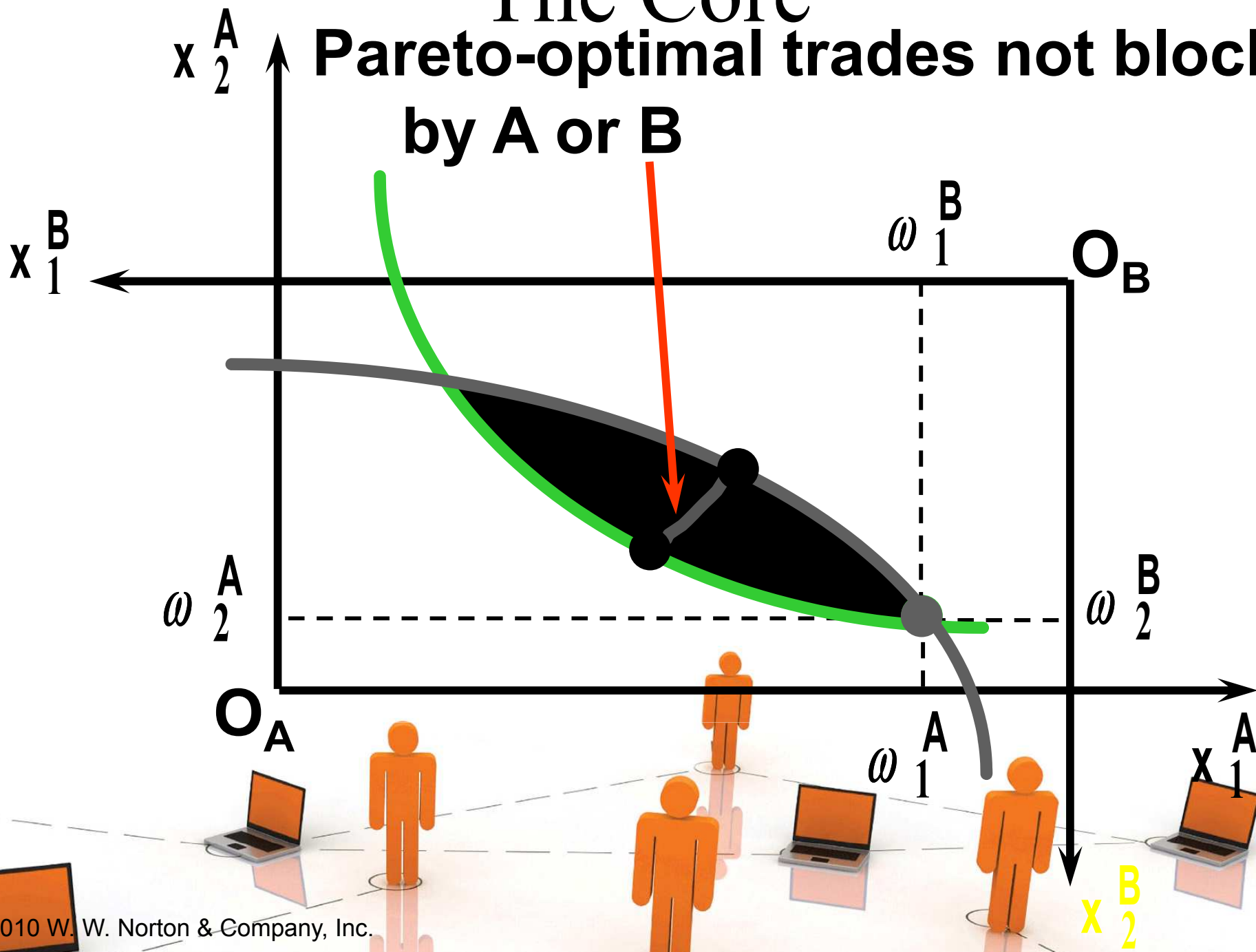
The Core

Pareto-optimal trades not blocked by A or B are the core.

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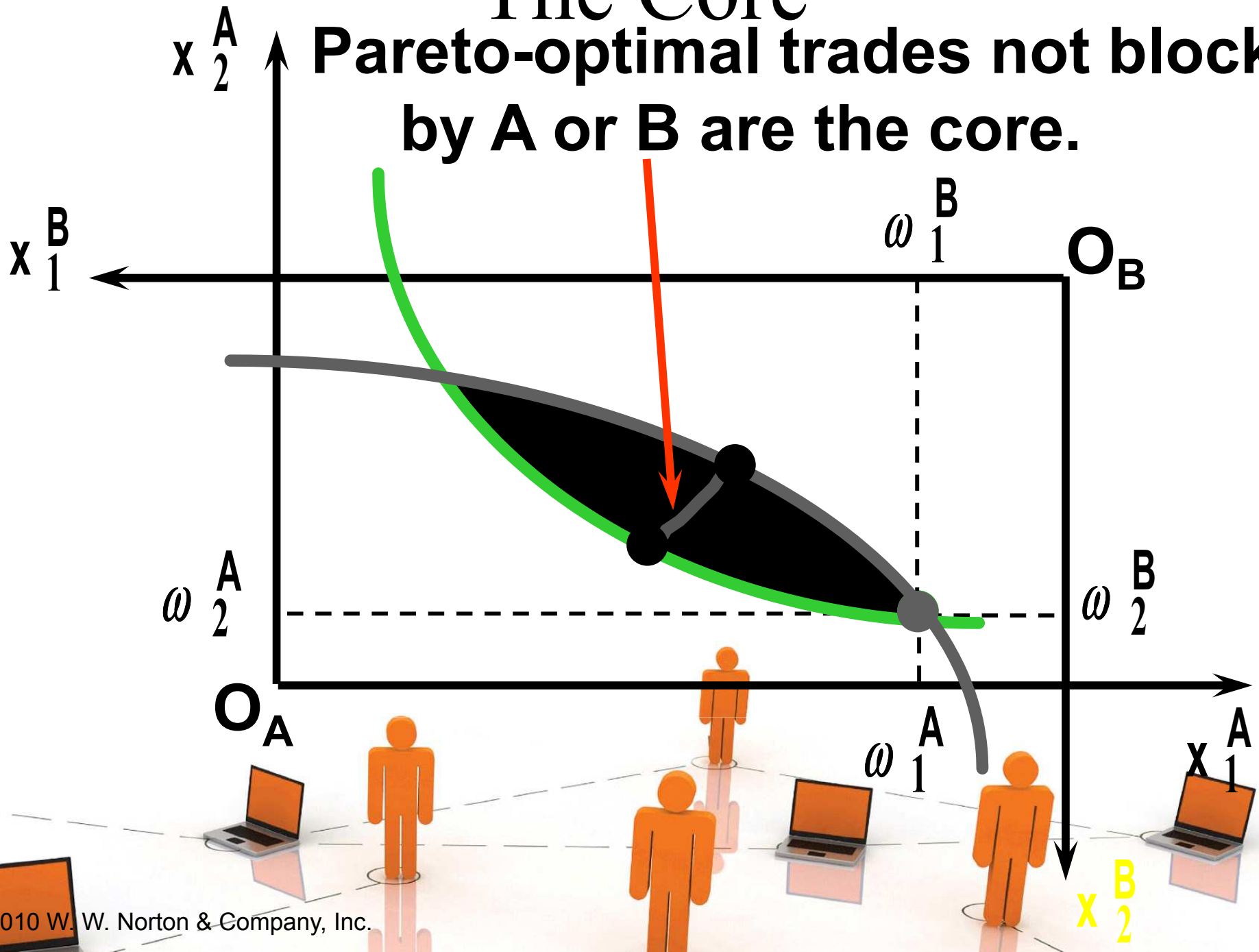
The Core

◆ **The core is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.**

◆ **Rational trade should achieve a core allocation.**

The Core

◆ **But which core allocation?**

◆ **Again, that depends upon the manner in which trade is conducted.**
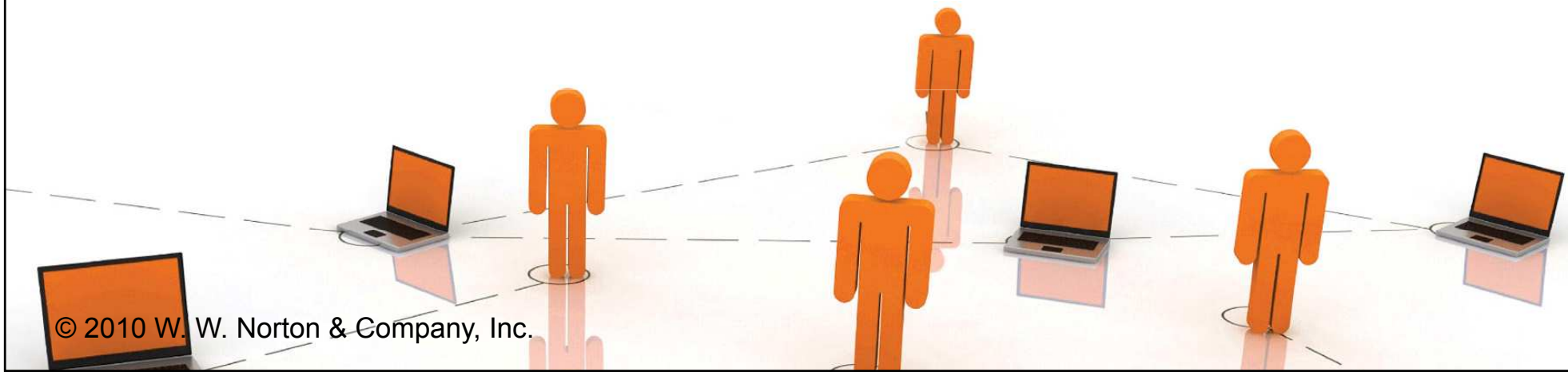
Trade in Competitive Markets

- ◆ **Consider trade in perfectly competitive markets.**
- ◆ **Each consumer is a price-taker trying to maximize her own utility given p_1, p_2 and her own endowment. That is, ...**

Trade in Competitive Markets

For consumer A.

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$

x_2^A

x_2^{*A}

ω_2^A

O_A

x_1^{*A}

ω_1^A

x_1^A
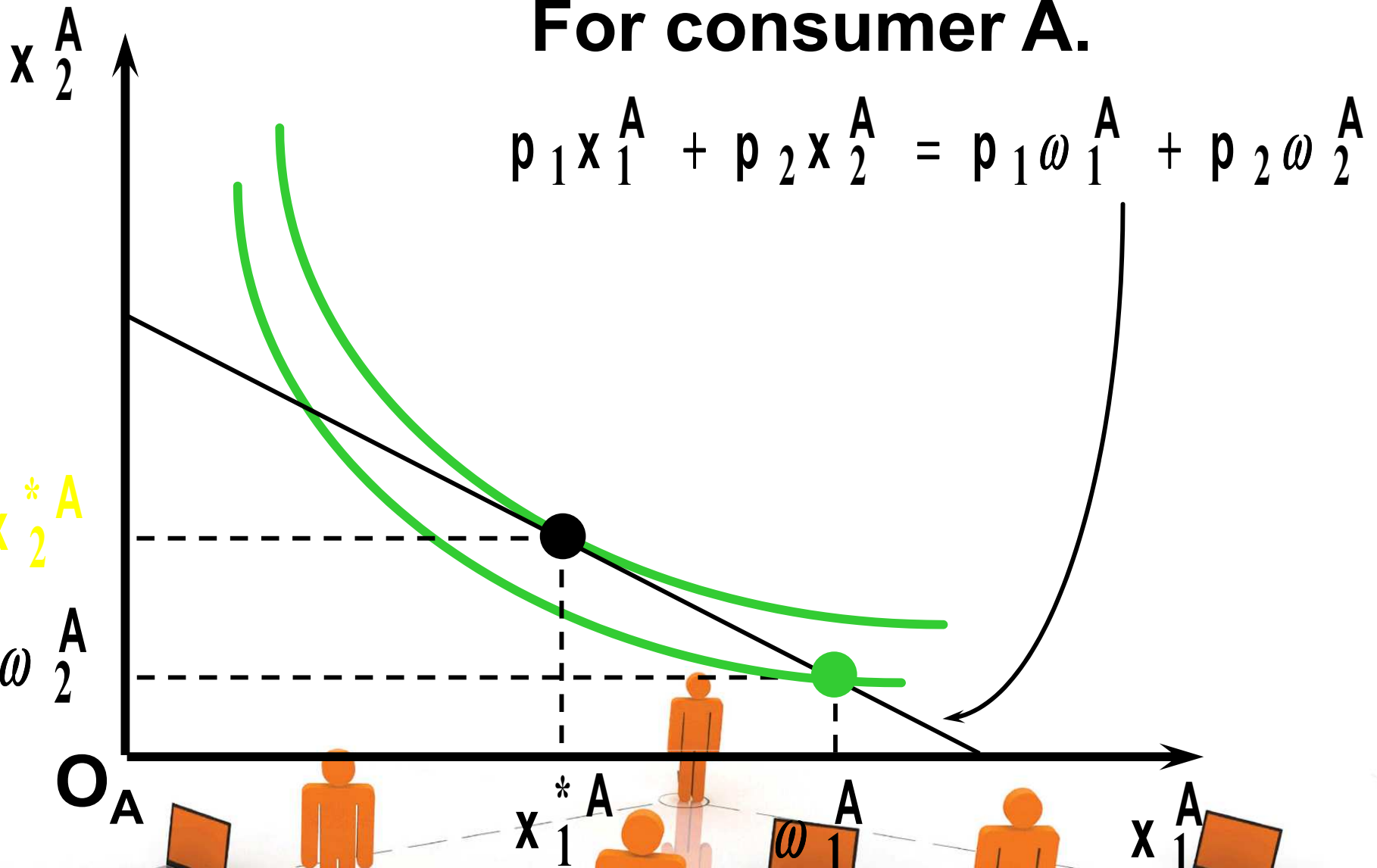
Trade in Competitive Markets

◆ **So given p_1 and p_2, consumer A's net demands for commodities 1 and 2 are**

$$x_1^{*A} - \omega_1^A \quad \textbf{and} \quad x_2^{*A} - \omega_2^A.$$
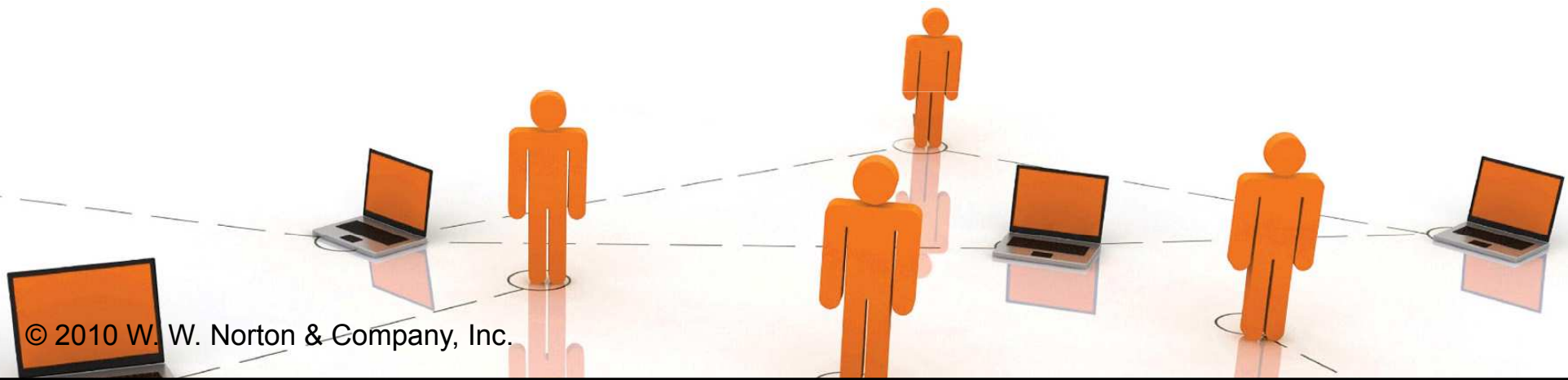
Trade in Competitive Markets

◆ **And, similarly, for consumer B …**

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Trade in Competitive Markets

For consumer B.

$$p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B$$

x_2^B

ω_2^B

x_2^{*B}

O_B

ω_1^B

x_1^{*B}

x_1^B
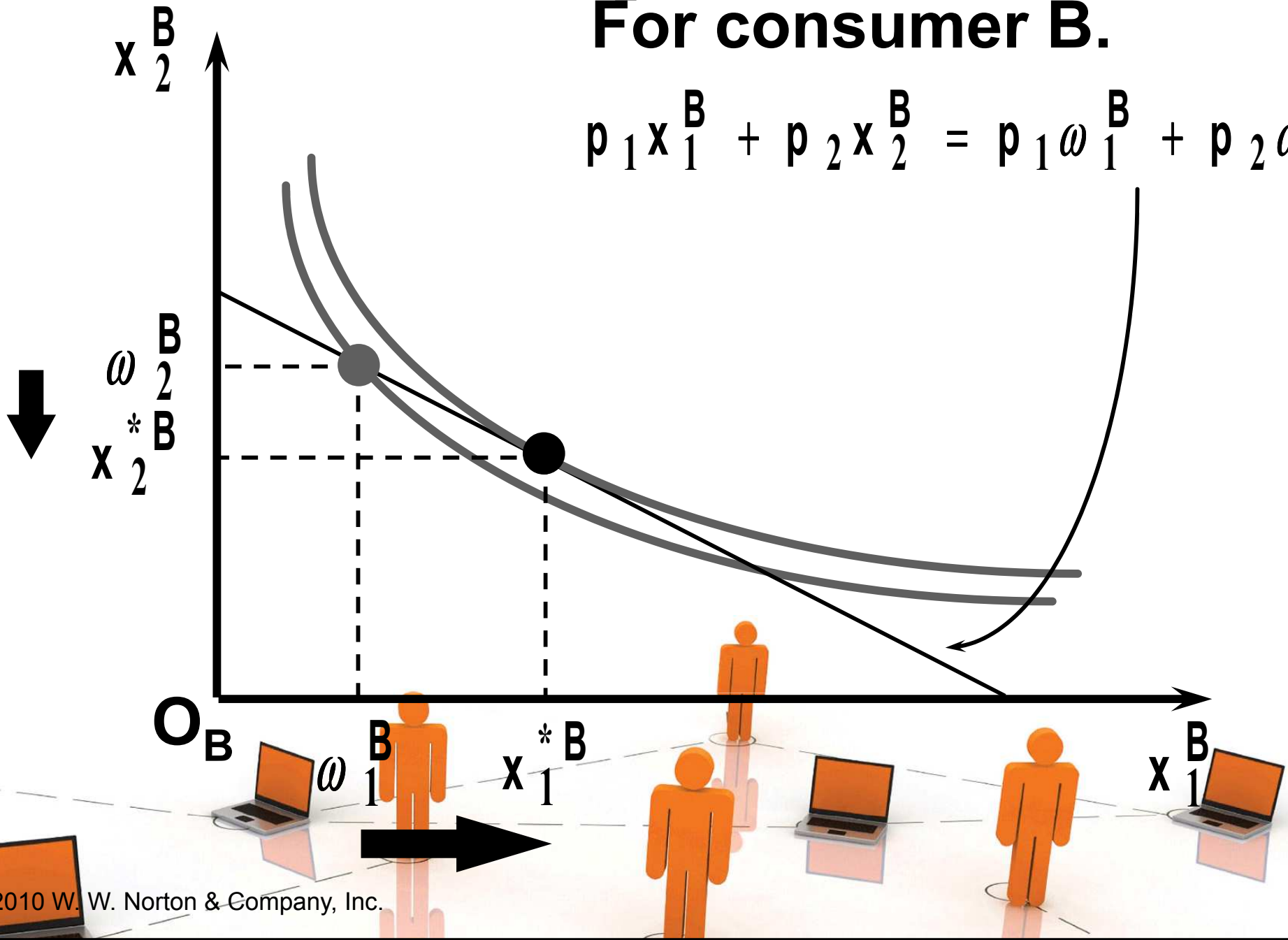
Trade in Competitive Markets

◆ **So given p$_1$ and p$_2$, consumer B's net demands for commodities 1 and 2 are**

$$x_1^{*B} - \omega_1^B \quad \textbf{and} \quad x_2^{*B} - \omega_2^B.$$
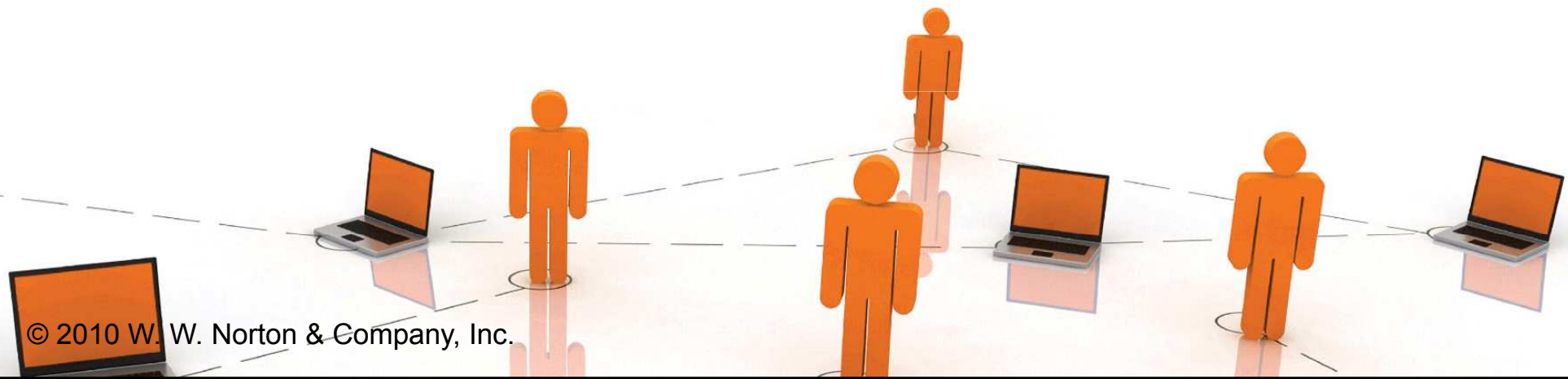
Trade in Competitive Markets

◆ **A general equilibrium occurs when prices p_1 and p_2 cause both the markets for commodities 1 and 2 to clear; i.e.**

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

and $\quad x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B .$

Trade in Competitive Markets

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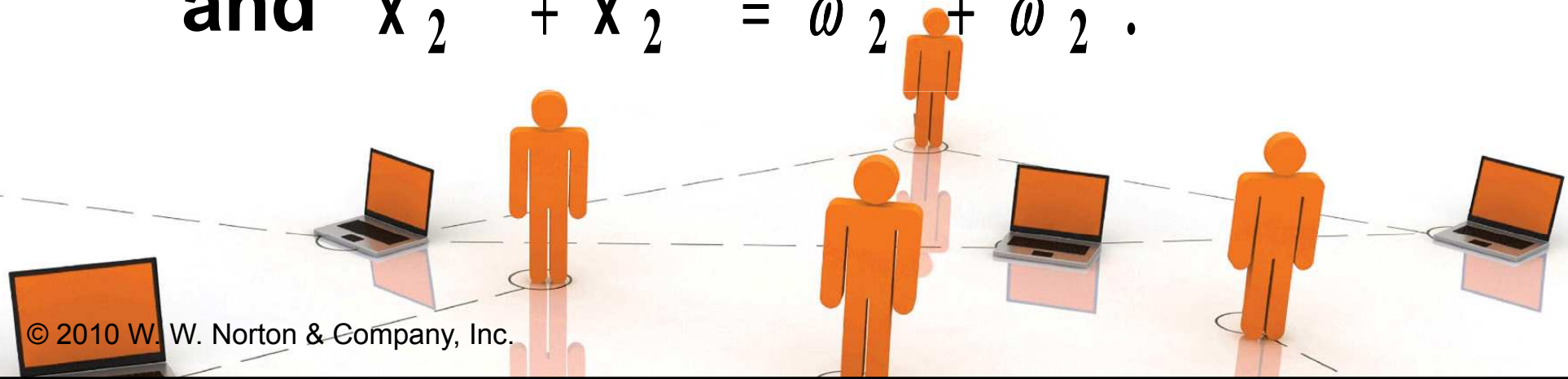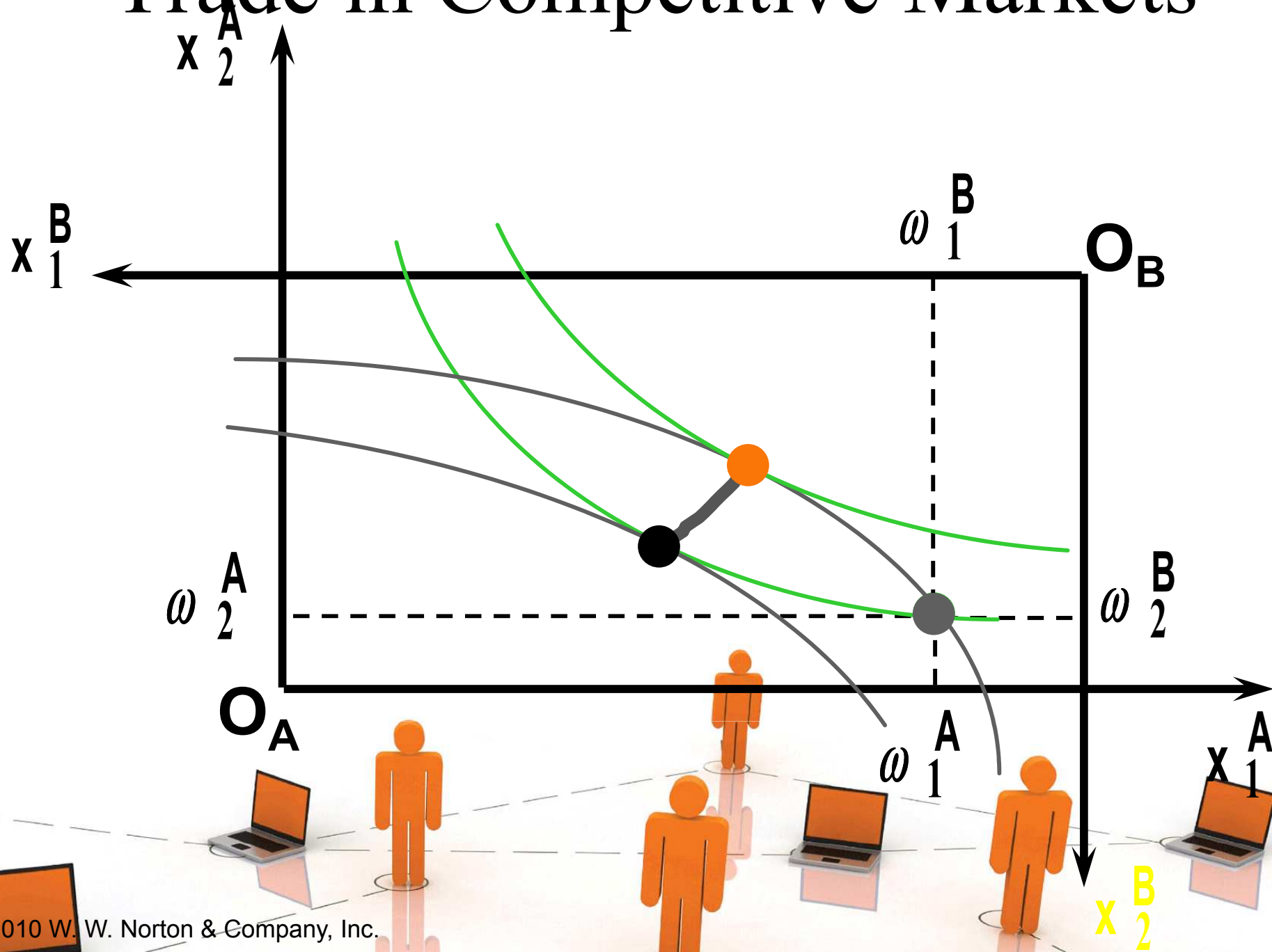
Trade in Competitive Markets

Can this PO allocation be achieved?

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Trade in Competitive Markets

Budget constraint for consumer A

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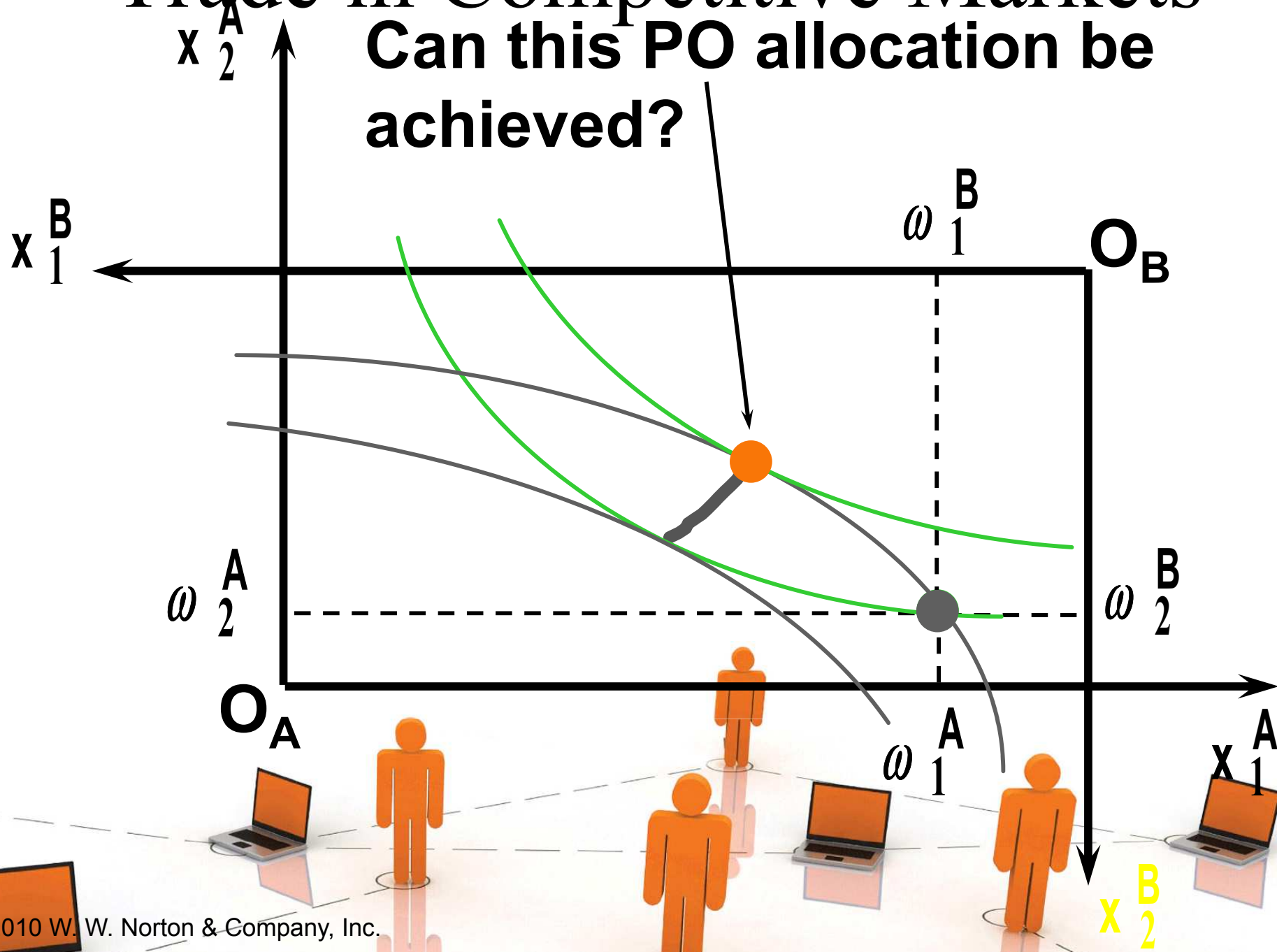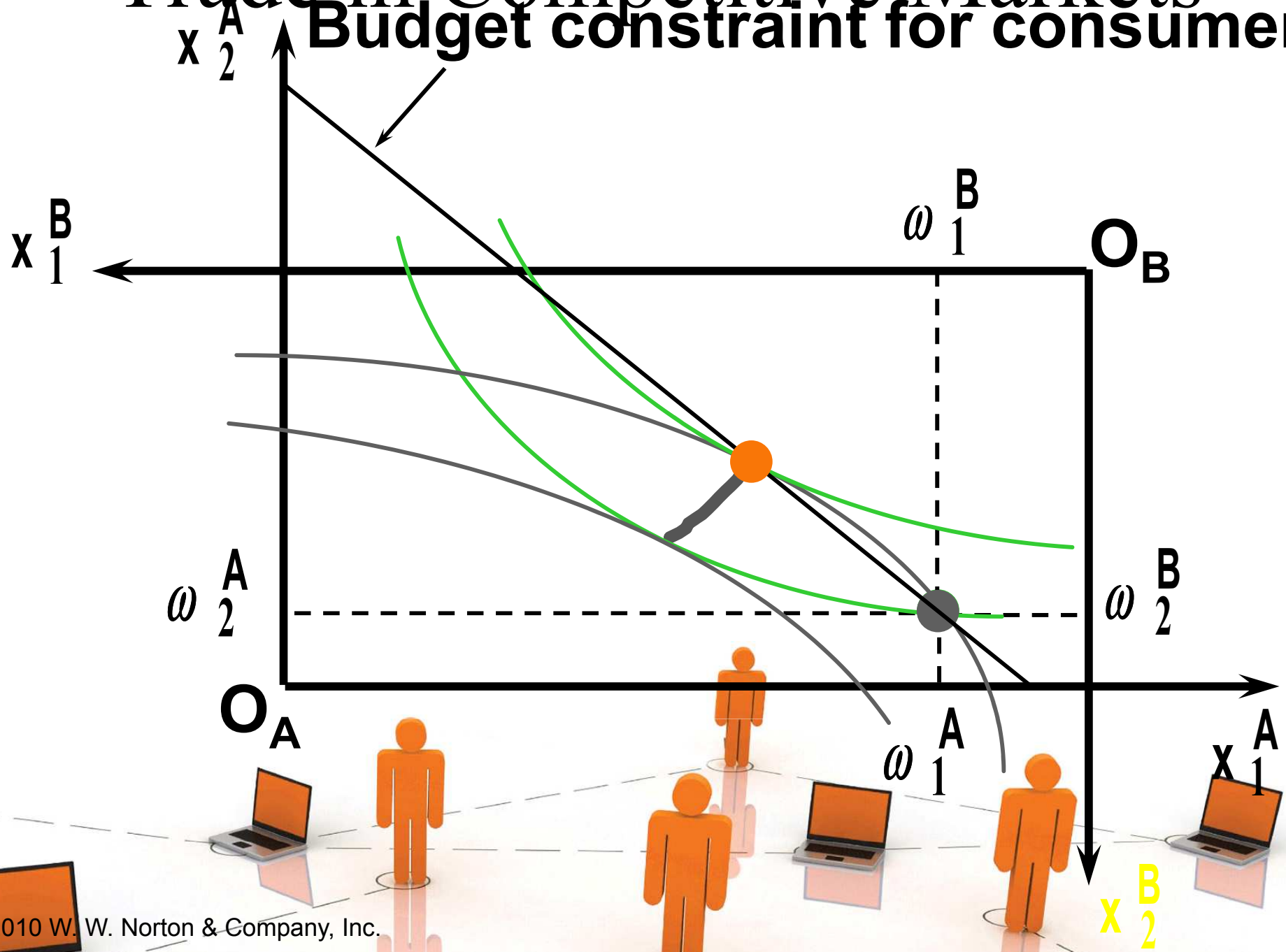
Trade in Competitive Markets

Budget constraint for consumer A

Trade in Competitive Markets
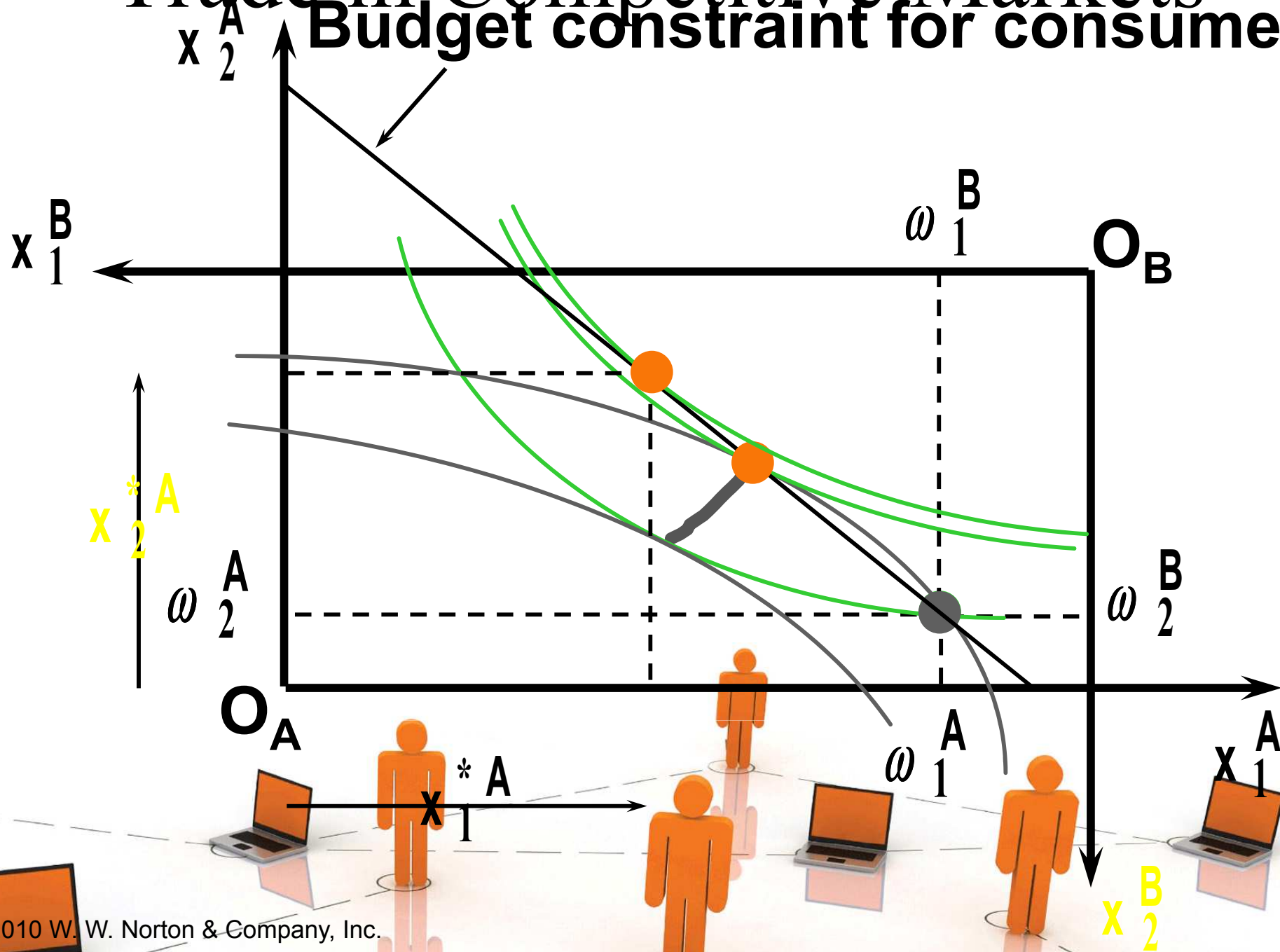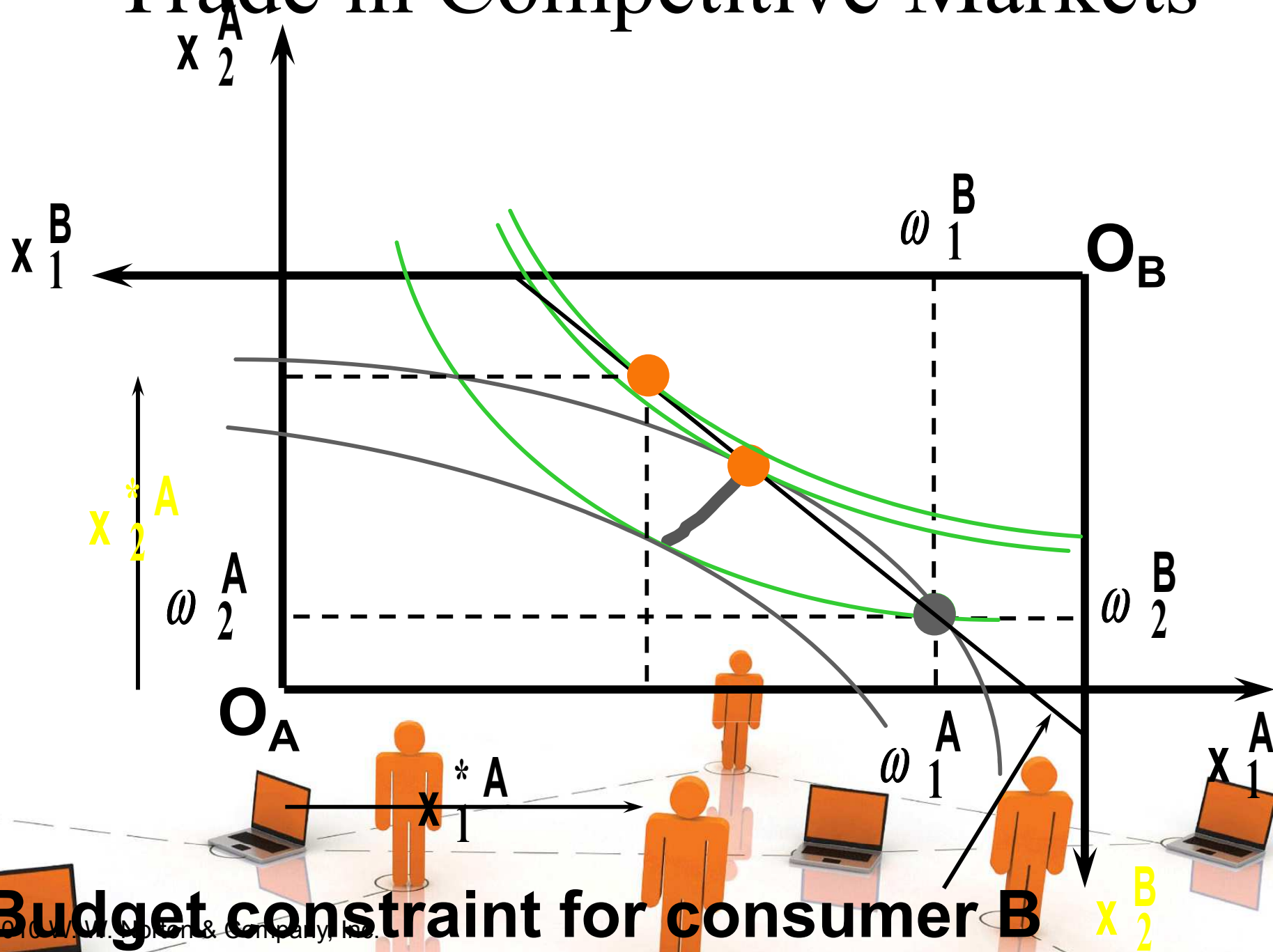
Budget constraint for consumer B

Trade in Competitive Markets

Budget constraint for consumer B

x_2^A

x_1^{*B}

ω_1^B

O_B

x_2^{*B}

x_1^B

x_2^{*A}

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^A

x_1^{*A}

x_2^B

Trade in Competitive Markets

$$x_2^A$$

$$\overset{*B}{x_1} \longleftarrow$$

$$x_1^B \longleftarrow$$

$$\omega_1^B$$

$$O_B$$

$$\overset{*B}{x_2}$$

$$\overset{*A}{x_2}$$

$$\omega_2^A$$

$$\omega_2^B$$

$$O_A$$

$$\omega_1^A$$

$$x_1^A$$

$$\overset{*A}{x_1} \longrightarrow$$

But

$$\overset{*A}{x_1} + \overset{*B}{x_1} < \omega_1^A + \omega_1^B$$

$$x_2^B$$

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Trade in Competitive Markets

x_2^A

x_1^B

x_1^{*B}

ω_1^B

O_B

x_2^{*B}

x_2^{*A}

ω_2^A

O_A

ω_2^B

ω_1^A

x_1^{*A}

x_1^A

and $x_2^{*A} + x_2^{*B} > \omega_2^A + \omega_2^B$

x_2^B

Trade in Competitive Markets

◆ **So at the given prices p_1 and p_2 there is an**

– **excess supply of commodity 1**

– **excess demand for commodity 2.**

◆ **Neither market clears so the prices p_1 and p_2 do not cause a general equilibrium.**
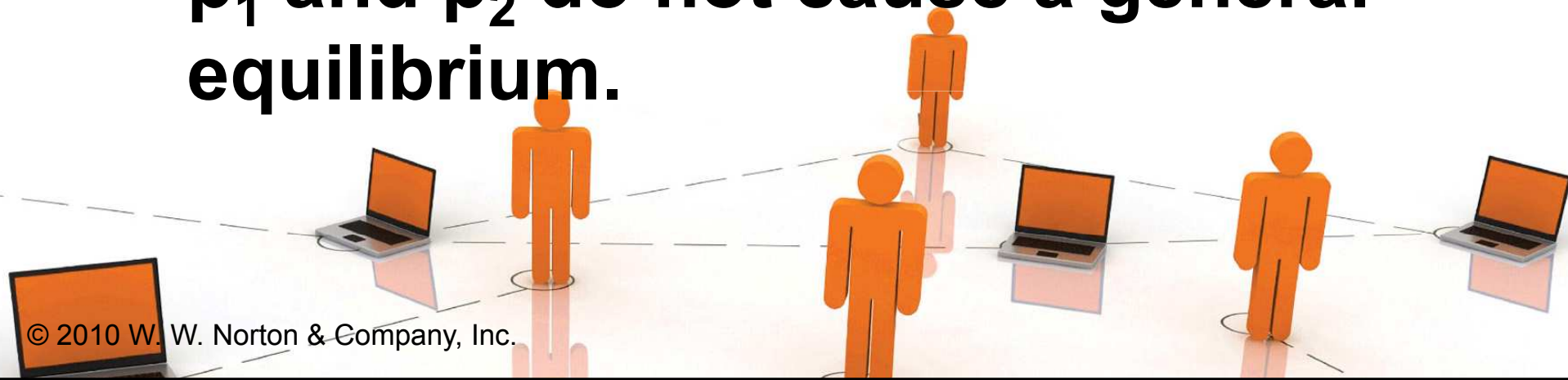
Trade in Competitive Markets

So this PO allocation cannot be achieved by competitive trading.
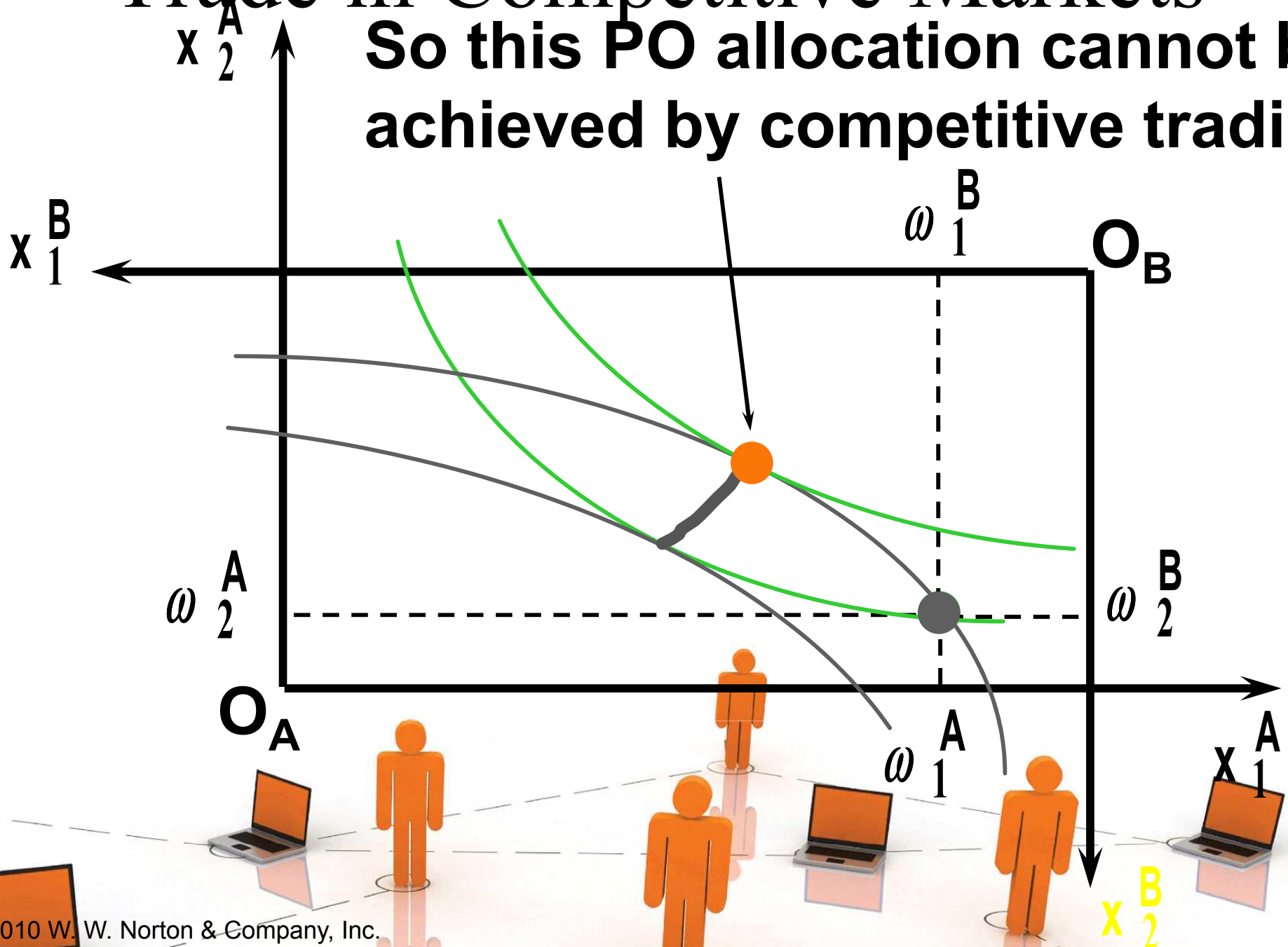
x_2^A

x_1^B

ω_1^B

O_B

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^A

x_2^B

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Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?
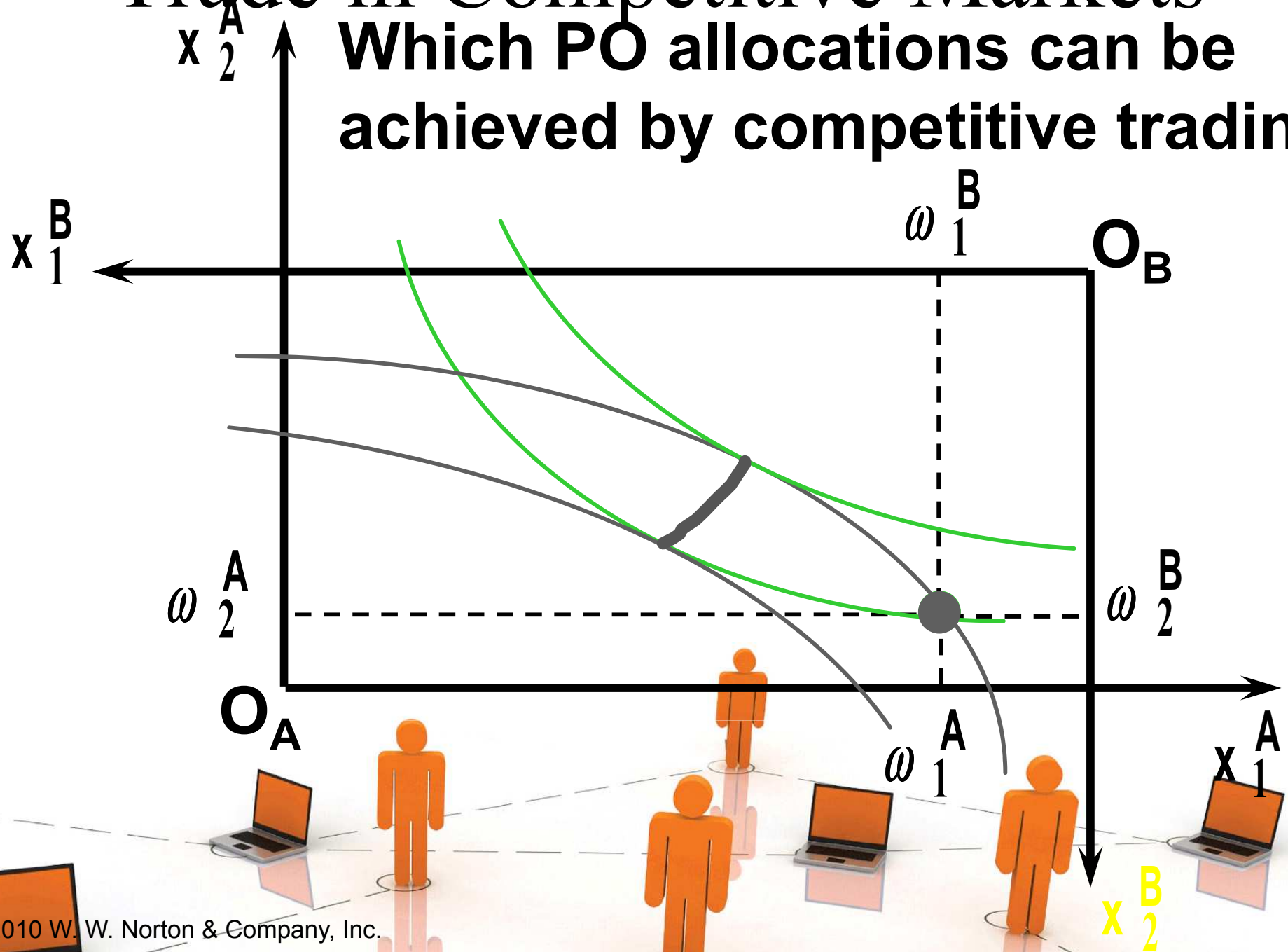
Trade in Competitive Markets

- **Since there is an excess demand for commodity 2, p_2 will rise.**

- **Since there is an excess supply of commodity 1, p_1 will fall.**

- **The slope of the budget constraints is $-p_1/p_2$ so the budget constraints will pivot about the endowment point and become less steep.**

Trade in Competitive Markets

Which PO allocations can be achieved by competitive trading?

Trade in Competitive Markets

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Trade in Competitive Markets

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Trade in Competitive Markets

Budget constraint for consumer A

x_2^A

x_1^B

ω_1^B

O_B

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^A

x_2^B

Trade in Competitive Markets

Budget constraint for consumer A

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Trade in Competitive Markets

x_2^A

x_1^B

ω_1^B

O_B

O_A

ω_2^A

x_2^{*A}

x_2^B

ω_2^B

x_1^{*A}

ω_1^A

x_1^A

x_2^B

Budget constraint for consumer B

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Trade in Competitive Markets

x_2^A

x_1^B

x_1^{*B}

ω_1^B

O_B

x_2^{*B}

x_2^A ω_2^A

x_2^{*A}

ω_2^B

O_A

ω_1^A

x_1^{*A}

x_1^A

Budget constraint for consumer B

x_2^B

Trade in Competitive Markets

x_2^A

x_1^B

x_1^{*B}

ω_1^B

O_B

x_2^{*B}

x_2^{*A}

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^{*A}

x_1^A

So $x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$

x_2^B

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Trade in Competitive Markets

x_2^A

x_1^{*B}

x_1^B

ω_1^B

O_B

x_2^{*B}

x_2^{*A}

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^{*A}

x_1^A

and $x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$

x_2^B
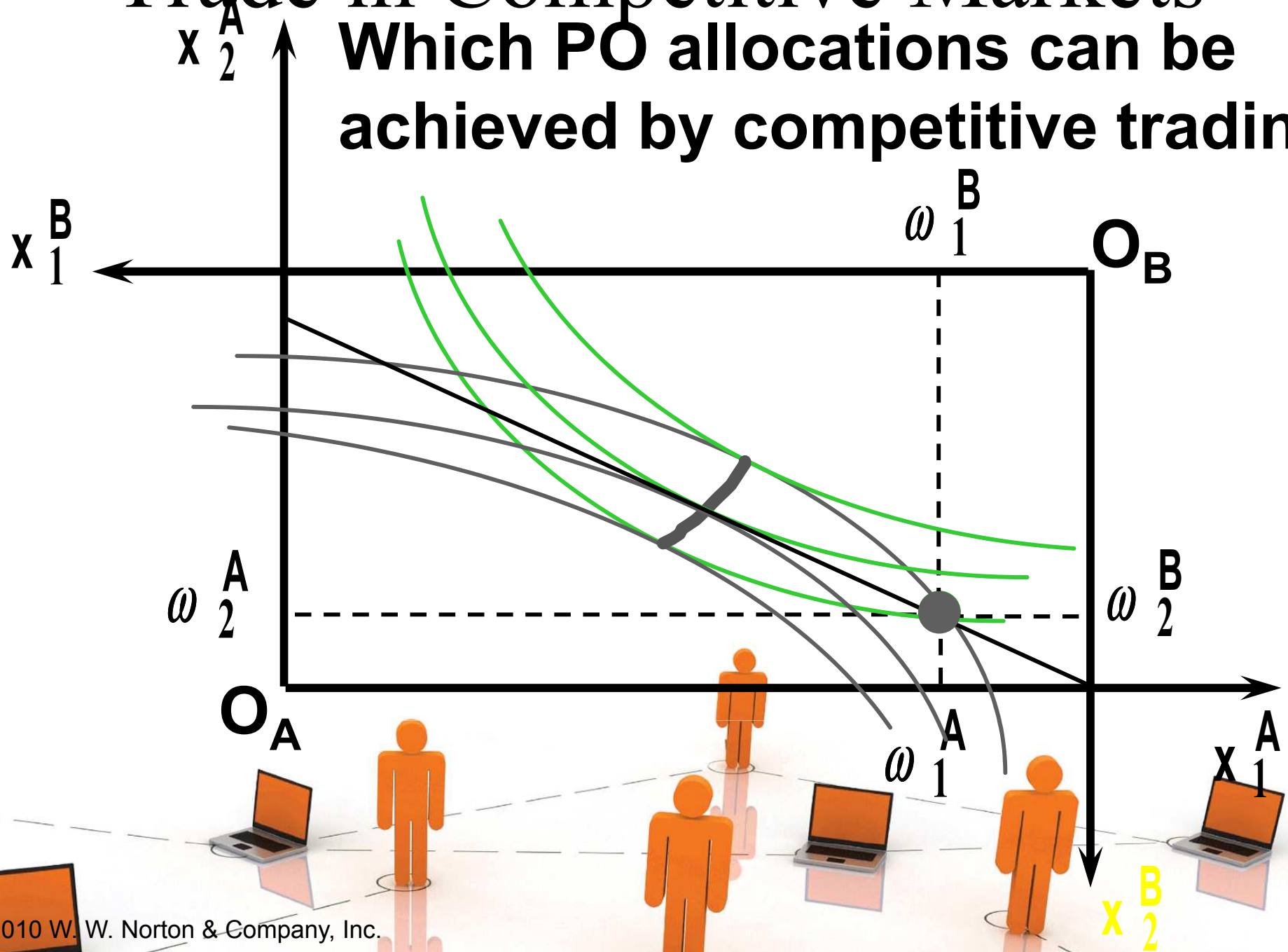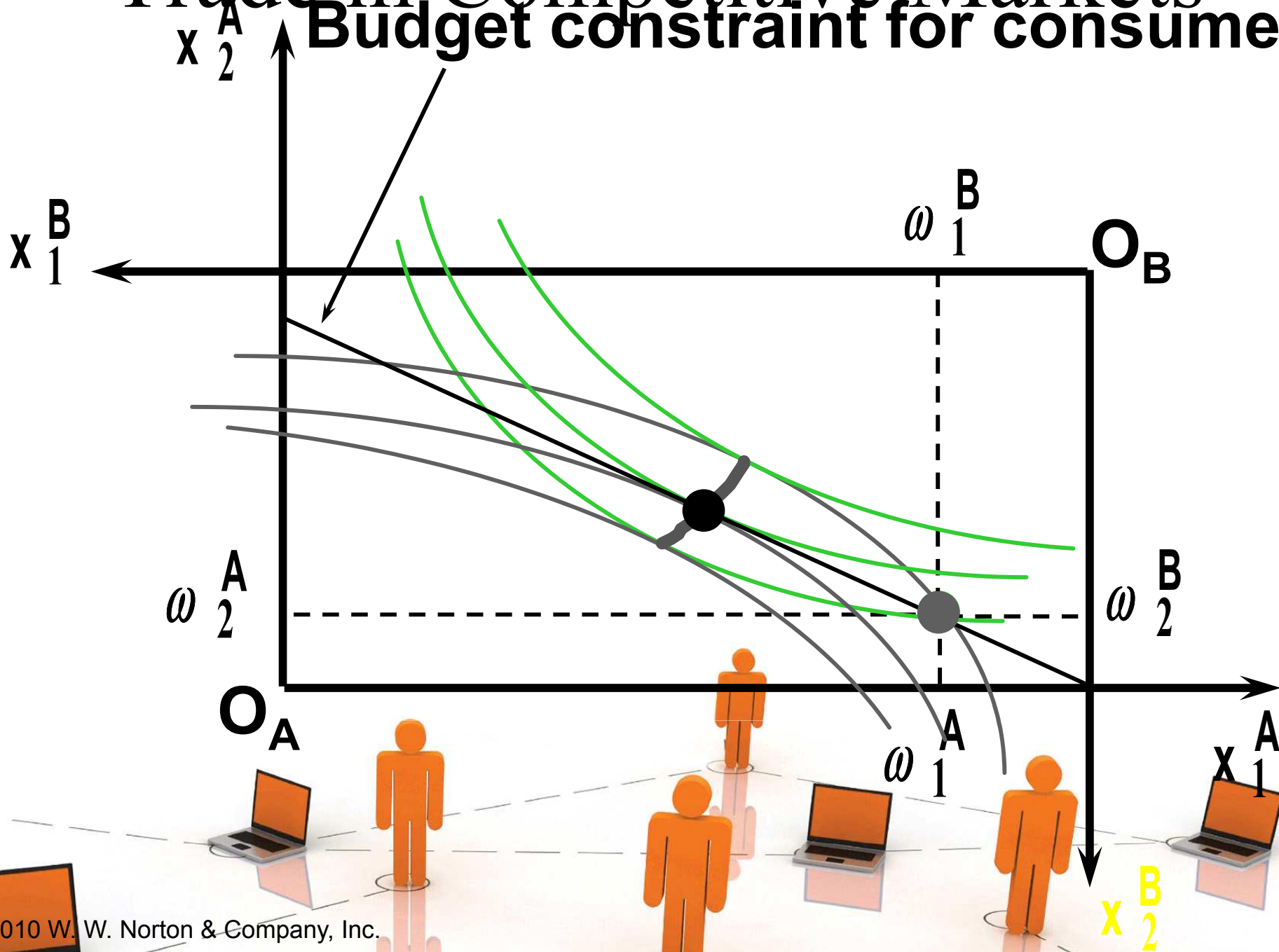
Trade in Competitive Markets

◆ **At the new prices p_1 and p_2 both markets clear; there is a general equilibrium.**

◆ **Trading in competitive markets achieves a particular Pareto-optimal allocation of the endowments.**

◆ **This is an example of the First Fundamental Theorem of Welfare Economics.**
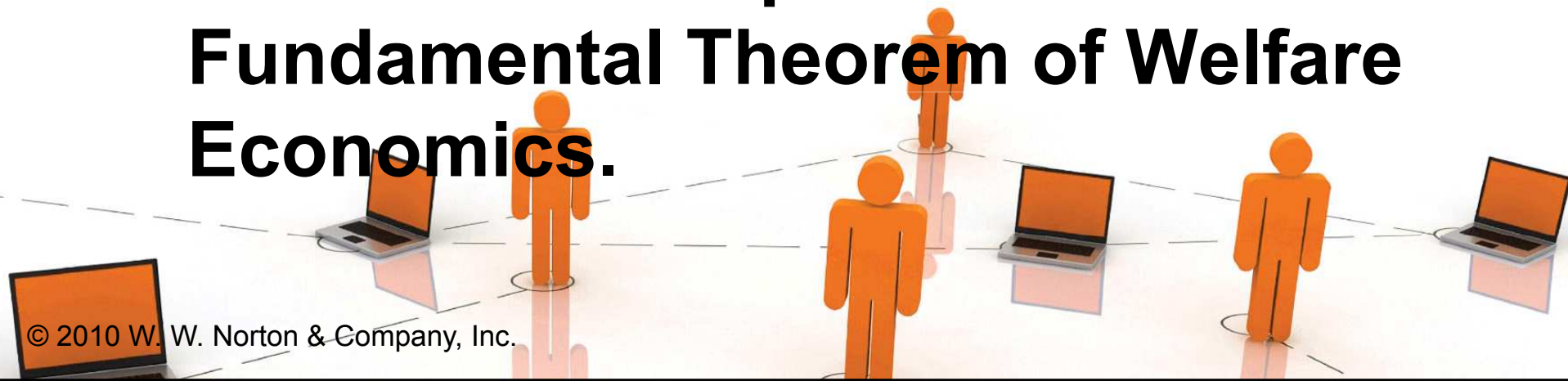
First Fundamental Theorem of Welfare Economics

◆ **Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.**
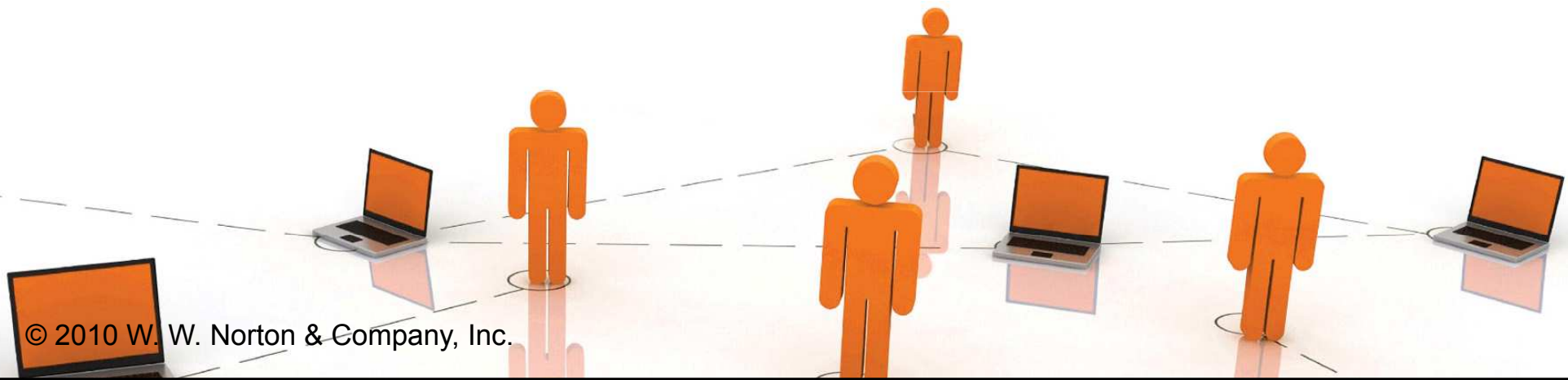
Second Fundamental Theorem of Welfare Economics

◆ **The First Theorem is followed by a second that states that any Pareto-optimal allocation (i.e. any point on the contract curve) can be achieved by trading in competitive markets provided that endowments are first appropriately rearranged amongst the consumers.**

Second Fundamental Theorem of Welfare Economics

◆ **Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.**

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Second Fundamental Theorem

The contract curve

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Second Fundamental Theorem

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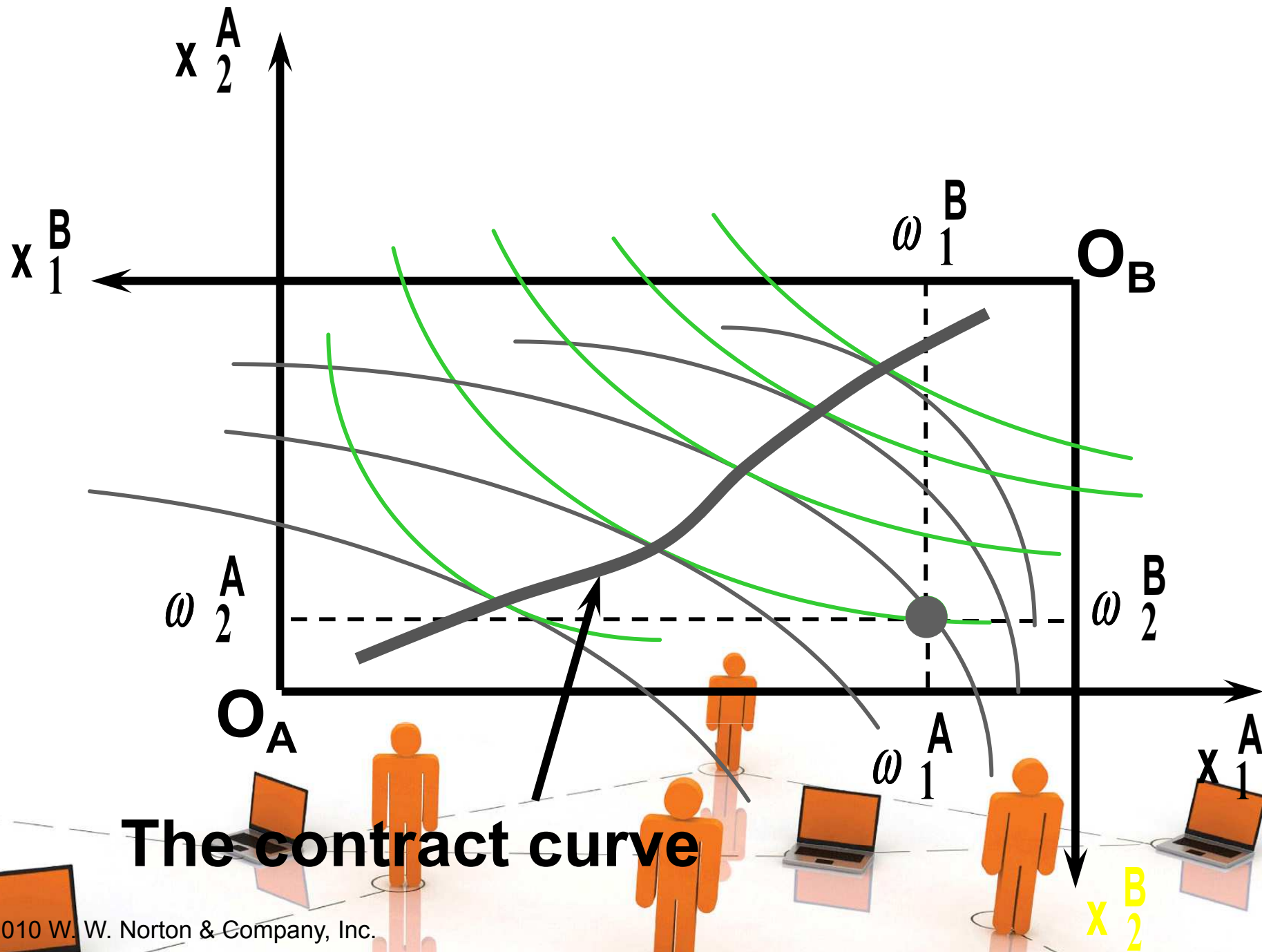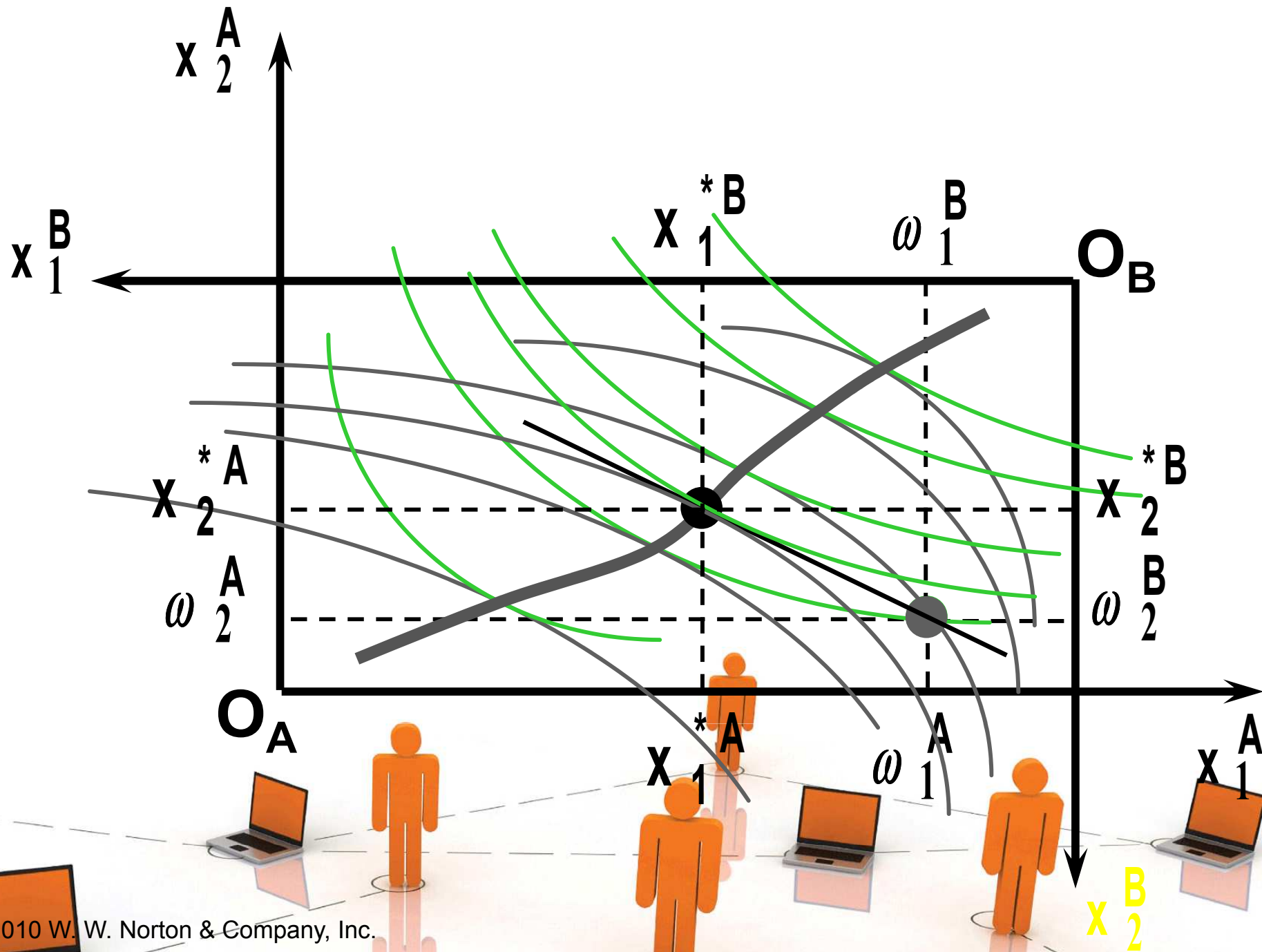
Second Fundamental Theorem

Implemented by competitive trading from the endowment ω.
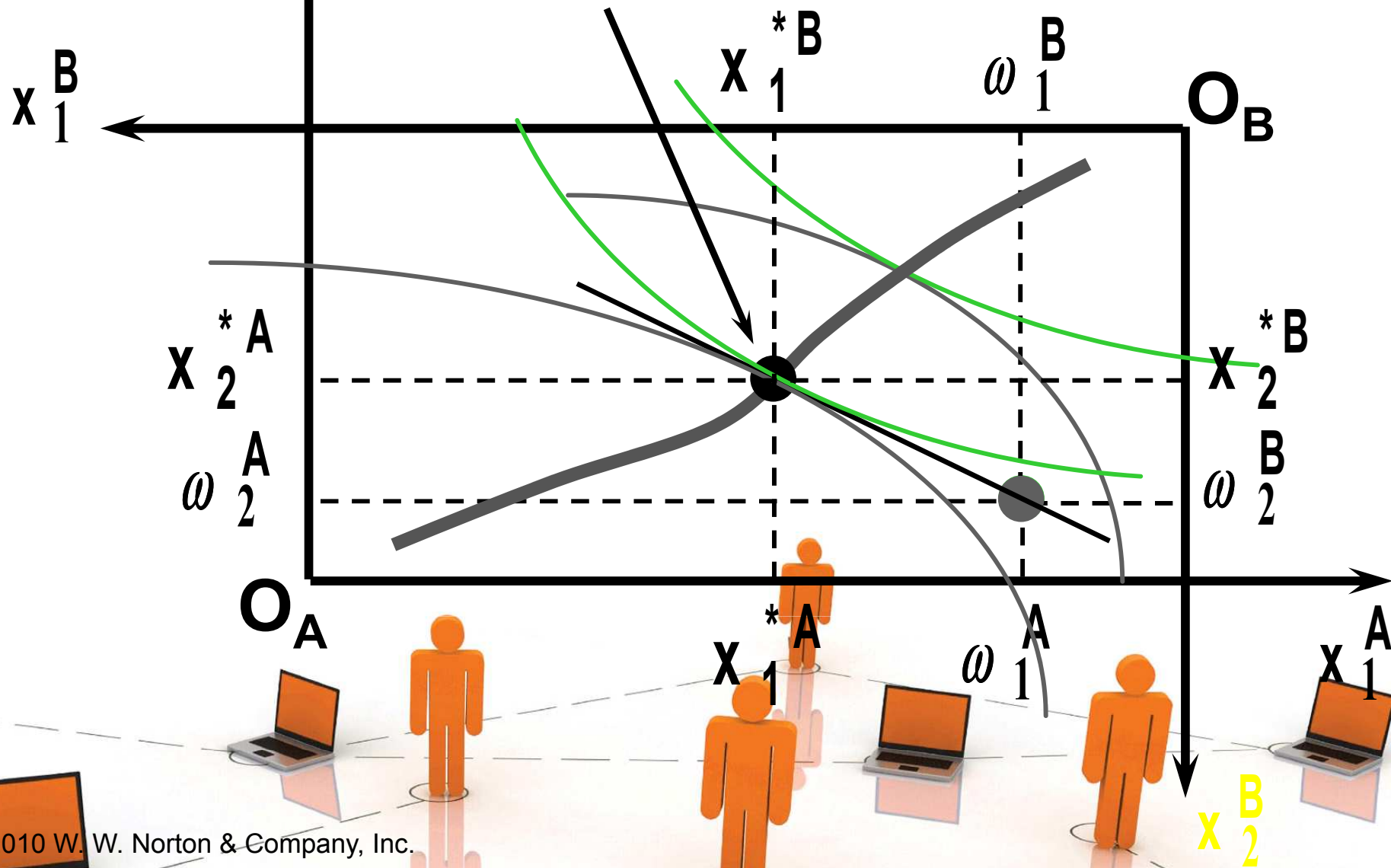
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Second Fundamental Theorem

x_2^A

Can this allocation be implemented by competitive trading from ω?

x_1^B

ω_1^B

O_B

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^A

x_2^B

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Second Fundamental Theorem

x_2^A

Can this allocation be implemented by competitive trading from ω? No.

x_1^B

ω_1^B

O_B

ω_2^A

ω_2^B

O_A

ω_1^A

x_1^A

x_2^B

Second Fundamental Theorem

But this allocation is implemented by competitive trading from θ.

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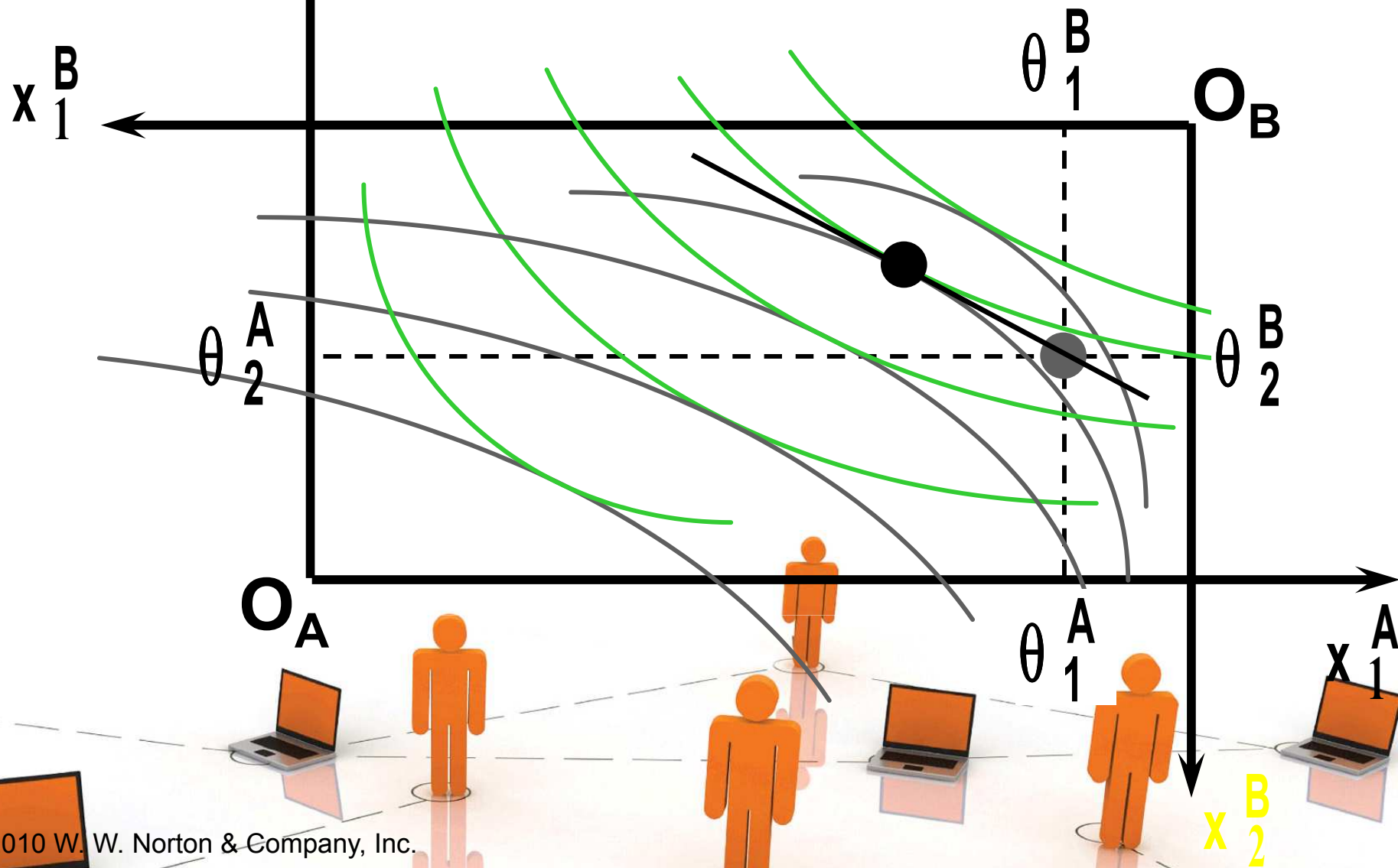
Walras' Law

◆ **Walras' Law is an identity; i.e. a statement that is true for any positive prices (p_1,p_2), whether these are equilibrium prices or not.**

Walras' Law

◆ **Every consumer's preferences are well-behaved so, for any positive prices (p_1, p_2), each consumer spends all of his budget.**

◆ **For consumer A:**

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Walras' Law

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

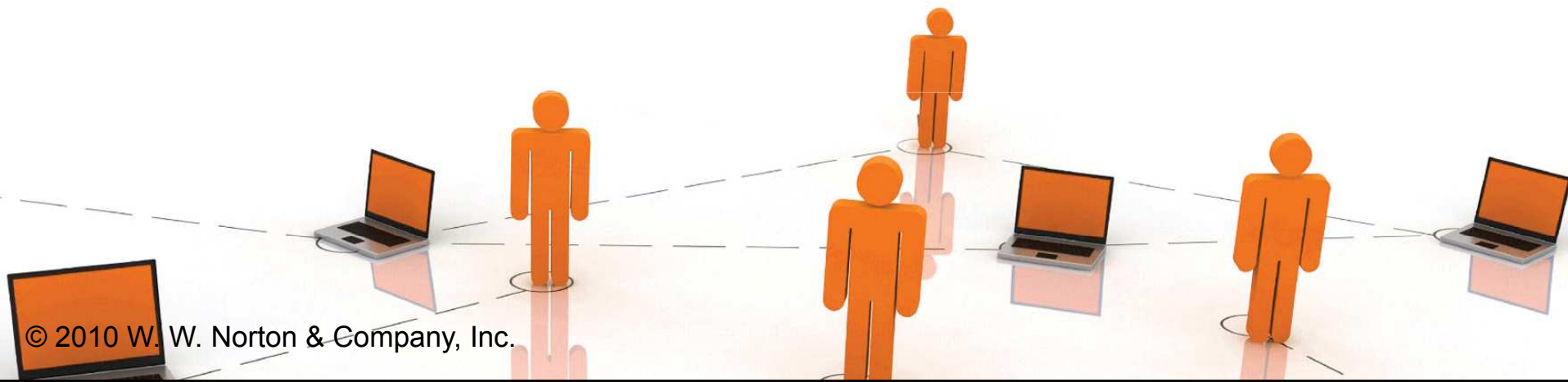$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Summing gives

$$p_1 (x_1^{*A} + x_1^{*B}) + p_2 (x_2^{*A} + x_2^{*B})$$

$$= p_1 (\omega_1^A + \omega_1^B) + p_2 (\omega_2^B + \omega_2^B).$$

Walras' Law

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Rearranged,

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0.$$

That is, ...

Walras' Law

$$p_1 \left(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B \right) +$$

$$p_2 \left(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B \right)$$

$$= 0.$$
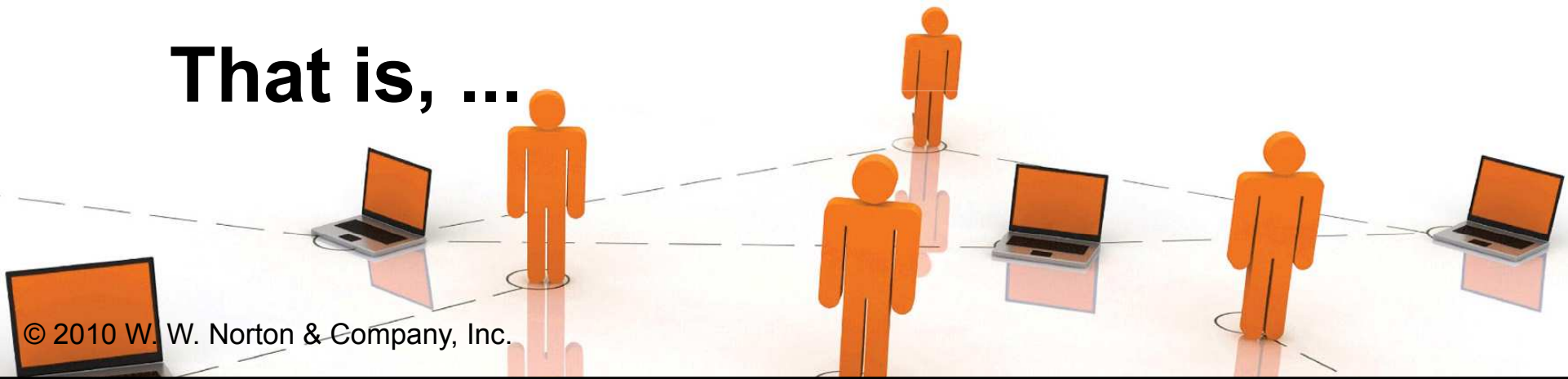
This says that the summed market value of excess demands is zero for any positive prices p_1 and p_2 -- this is Walras' Law.

Implications of Walras' Law

Suppose the market for commodity A is in equilibrium; that is,

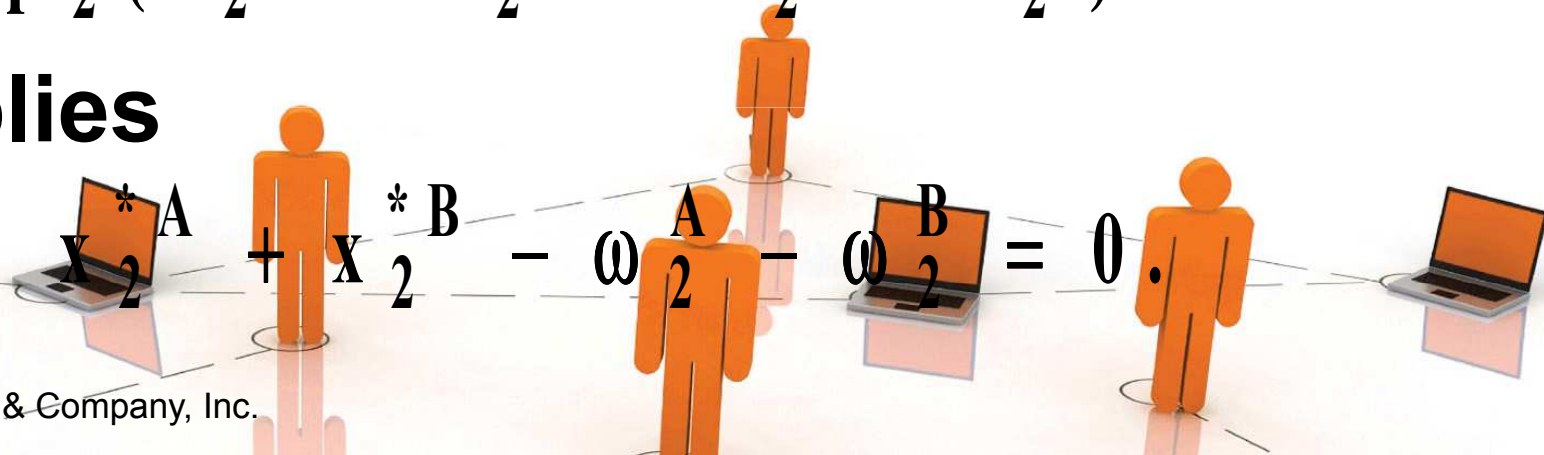$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then

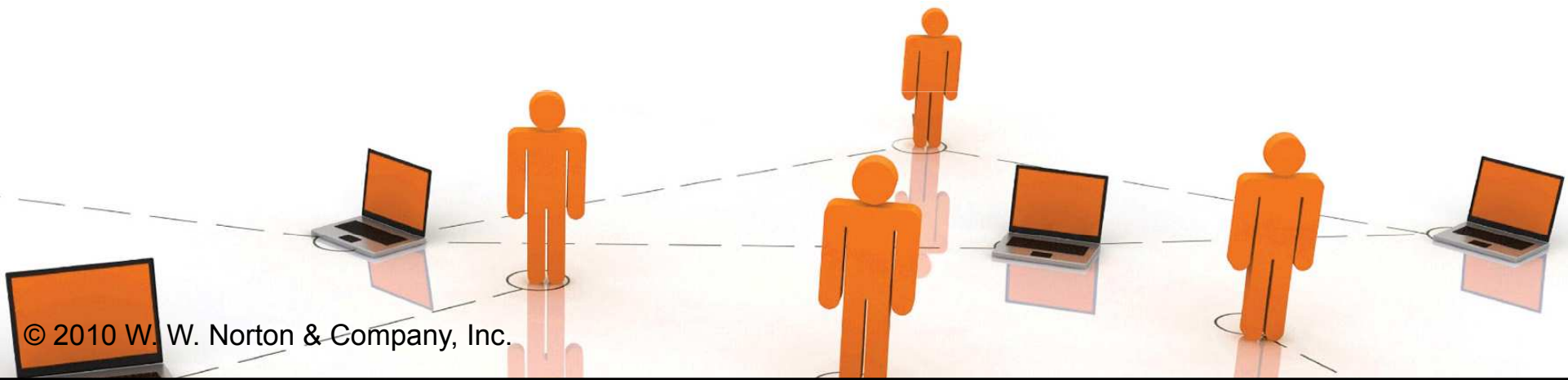$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

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Implications of Walras' Law

So one implication of Walras' Law for a two-commodity exchange economy is that if one market is in equilibrium then the other market must also be in equilibrium.

Implications of Walras' Law

What if, for some positive prices p_1 and p_2, there is an excess quantity supplied of commodity 1? That is,

$$x_1^{*A} + x_1^{*B} - \omega_1^{A} - \omega_1^{B} < 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^{A} - \omega_1^{B}) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^{A} - \omega_2^{B}) = 0$$

implies

$$x_2^{*A} + x_2^{*B} - \omega_2^{A} - \omega_2^{B} > 0.$$

Implications of Walras' Law

So a second implication of Walras' Law for a two-commodity exchange economy is that an excess supply in one market implies an excess demand in the other market.