If we interpret A as the asymptotic value of $f'(k)$, then the conditions derived in this appendix are satisfied by all the models that we discussed in this chapter. In particular, the steady-state per capita growth rate is given in equation (4.76), and the steady-state level of c/k is given in equation (4.78).

4.8 Problems

4.1 The *AK* model as the limit of the neoclassical model. Consider the neoclassical growth model discussed in chapter 2. Imagine that the production function is Cobb–Douglas, $\hat{y} = A\hat{k}^{\alpha}$.

a. How does an increase in α affect the transition equations for $\hat{k}$ and $\hat{c}$ in equations (2.23) and (2.24)? How, therefore, does the increase in α affect the loci for $\dot{\hat{c}} = 0$ and $\dot{\hat{k}} = 0$ in figure 2.1? How does it affect the steady-state values, $\hat{k}^*$ and $\hat{c}^*$?

b. What happens, for example, to $\hat{k}^*$ as α approaches 1? How does this result relate to the *AK* model that was discussed in this chapter?

4.2 Oversaving in the *AK* model (based on Saint-Paul, 1992). We know from chapter 1 that an economy oversaves if it approaches a steady state in which the rate of return, r, is smaller than the growth rate. Suppose that the technology is $Y = AK$, and the ratio c/k approaches the constant $(c/k)^*$ in the steady state.

a. Use equation (4.8) to determine the steady-state growth rate of K (and, hence, of Y and C). Can this steady-state growth rate exceed the interest rate, r, given in equation (4.7)? Is it possible to get oversaving if the economy approaches a steady state and the technology is $Y = AK$?

b. Suppose that we combine the *AK* technology with the model of finite-horizon consumers of Blanchard (1985), as described in section 3.7. Is it possible to get oversaving in this model? What if we combine the *AK* technology with an overlapping-generations model, as described in the appendix to chapter 3?

4.3 Transitional dynamics. Show that in the model of learning by doing with knowledge spillovers presented in section 4.3 there is no transitional dynamics. That is, output and capital always grow at the constant consumption growth rate given in equation (4.28).

4.4 Spillovers from average capital per worker. In the model presented in section 4.3, assume that the firm's productivity parameter, A_i, depends on the economy's average capital per worker, K/L, rather than on the aggregate capital stock, K. The production function is assumed to be Cobb–Douglas:

$$Y_i = A \cdot (K_i)^{\alpha} \cdot [(K/L) \cdot L_i]^{1-\alpha}$$

Derive the growth rates for the decentralized economy and for the social planner. Comment on how the scale effect discussed in section 4.3 does not appear with this new specification.

4.5 Distorting taxes in the public-goods model. Suppose, in the model of section 4.4.1, that public expenditures, G, are financed by a tax on household asset income at the rate τ_a. How does this change affect the relation between the growth rate and G/Y, that is, how does equation (4.42) change?

4.6 Congestion of public services (based on Barro and Sala-i-Martin, 1992c). In the congestion model discussed in section 4.4.2, suppose that output for firm i is given by

$$Y_i = AK_i \cdot f(G/K)$$

that is, the congestion of public services involves G in relation to K, rather than Y. How do the results change under this revised specification of congestion? Consider, in particular, the growth rates that arise in the decentralized economy and in the social planner's solution.

4.7 Adjustment costs with an AK technology (based on Barro and Sala-i-Martin, 1992c). Imagine that firms face an AK technology, but that investment requires adjustment costs as described in section 3.3. The unit adjustment-cost function is $\phi(i/k) = (b/2)\cdot(i/k)$, so that the total cost of purchasing and investment for 1 unit of capital is $1 + (b/2) \cdot (i/k)$. Producers maximize the present value of cash flows,

$$\int_0^\infty \{AK - I \cdot [1 + (b/2) \cdot (I/K)]\} \cdot e^{-rt} \cdot dt$$

where $r = A - \delta$. The maximization is subject to the constraint $\dot{K} = I - \delta K$.

a. Set up the Hamiltonian and work out the first-order conditions for the representative firm. Find the relation between the interest rate and the growth rate of capital. Is this relation monotonic? Explain.

b. Assume that consumers solve the usual infinite-horizon Ramsey problem, so that the growth rate of consumption relates positively to the interest rate. Suppose that the growth rate of consumption equals the growth rate of the capital stock. Does this condition pin down the growth rate? If not, can one of the solutions be ruled out from the transversality condition?

c. Show that the growth rate of consumption equals the growth rate of the capital stock. What does this finding imply about the model's transitional dynamics? Explain.

4.8 Growth in a model with spillovers (based on Romer, 1986). Assume that the production function for firm i is

$$Y_i = AK_i^{\alpha} \cdot L_i^{1-\alpha} \cdot K^{\lambda}$$

where $0 < \alpha < 1$, $0 < \lambda < 1$, and K is the aggregate stock of capital.

a. Show that if $\lambda < 1 - \alpha$ and L is constant, the model has transitional dynamics similar to those of the Ramsey model. What is the steady-state growth rate of Y, K, and C in this case?

b. If $\lambda < 1 - \alpha$ and L grows at the rate $n > 0$, what is the steady-state growth rate of Y, K, and C?

c. Show that if $\lambda = 1 - \alpha$ and L is constant, the steady state and transitional dynamics are like those of the AK model.

d. What happens if $\lambda = 1 - \alpha$ and L grows at the rate $n > 0$?