# Theory of economic growth

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## 10 Endogenous growth: An overview

Required reading: Romer (1990), sec. 1-2., BSiM: 4.1-4.2

### 10.1 What do we mean by endogenous growth?

- In the neo-classical growth models we ended up with the following conclusion:
  1. Factor-accumulation can not give rise to growth in the long run, i.e. it can not explain perpetual growth.
  2. The only source of long-run growth in the model is if we postulate a process of changes in technology, i.e. if we introduce exogenous technological progress.
- We will now turn to models falling under the heading "endogenous growth". This terminology is somewhat misleading, since we will be focusing on models that departure from the neo-classical models with respect to either of the two conclusions above.
- Our models will follow either of four paths:
  1. Relaxing the upper Inada-condition
  2. Externalities and/or public goods (Abandoning CRS)
  3. Two-sector models, giving 'effective' increasing returns to the accumulated factors.
  4. Models of research and technological progress (Endogenizing $T$)
- The first three gives rise to perpetual (long-run) growth from factor-accumulation. Only the last one explicitly endogenizes something that was exogenous in the neo-classical model (i.e. technological progress).
- It is thus not a clear distinction in terms of exogenity/endogenity between these models and the neo-classical models.
- The term endogenous growth theory is, however, fairly well established as a caption for the full array of models.

## 10.2 Inessential non-reproducible factors

- We consider the case with only two inputs to production, characterized by the production function

$$Y = F(K, L)$$

- Note that in the models we have looked at so far there is the following important distinction between the two inputs:

  1. Capital, $K$, is reproducible, i.e. it can be made from $Y$.
  2. Labor, $L$, is non-reproducible, i.e. it can not be produced/taken from $Y$.

- In our previous models also human capital has been reproducible, and more specifically been produced by the same technology as $Y$ and $K$.

- Remember that

$$\dot{k}/k = sf(k)/k - (n + \delta)$$

  and we have the graph in $(\dot{k}, k)$ space

  In order to have perpetual growth in $k$ (and hence in $y$) we must have

$$\lim_{k\to\infty} [f(k)/k] > (n + \delta)/s \tag{1}$$

  For this to hold we must have that the average productivity of capital, $f(k)/k$, approaches a positive number when $k$ approaches infinity. (Note that since $f(k)/k = Y/K$ this also corresponds to $\lim_{K\to\infty}[Y/K] > (n + \delta)/s$ for any given $L$).

- Since $\lim_{k\to\infty} f(k) = \infty$ (Show this!) we must use l'Hopitals rule on the limit in expression (1), that is

$$\lim_{k\to\infty} [f(k)/k] = \lim_{k\to\infty} f'(k)$$

  We hence see that when the upper Inada-condition holds, i.e.

$$\lim_{k\to\infty} f'(k) = 0$$

  condition (1) cannot hold and we cannot have perpetual growth.

- Hence the upper Inada condition for the reproducible input, is the property of the neo-classical production function that is fundamental in securing convergence to a steady state, and thereby preventing long-run growth.

- It is also illustrating to see this in the familiar graph in $(\dot{k}, k)$ space

- That is: When we relax the upper Inada condition we can get perpetual growth by. This is a form of endogenous growth as explained above.

- Remember from Lecture 2 that the upper Inada-conditions ensures that capital and labor are essential for production, i.e. $F(0, L) = F(K, 0) = 0$ for all $K, L$.

- Thus we can conclude that

  - *When the non-reproducible factors of production are inessential to production we can get perpetual growth.*

### 10.2.1 Two examples

- Let us therefore investigate some cases where $\lim_{K\to\infty} F_K = \mu > 0$.

- These are cases where only the reproduced factor of production ($K$) is essential to production, meaning $F(K, 0) > 0$ for all $K > 0$, while $F(0, L) = 0$ for all $L$.

- Consider a Constant Elasticity of Substitution (CES) production function:

$$Y = F(K, L) = A\left\{a(bK)^{\psi} + (1-a)[(1-b)L]^{\psi}\right\}^{1/\psi} \tag{2}$$

  where $0 < a < 1$, $0 < b < 1$ and $\psi < 1$. It is easy to show that the production function exhibits constant returns to scale (do this!). When

$\psi \to -\infty$ the production function approaches $Y = \min[bK, (1-b)L]$ i.e. a limitation law as in the Harrod-Domar case, while it approaches a cobb-Douglas, $Y = CK^a L^{1-a}$ when $\psi \to 0$. For $\psi = 1$ the function is linear in $L$ and $K$.

- For simplicity we let $A$ be a constant. Dividing by $L$ we then get output per capita:

$$y = f(k) = A\left\{a(bk)^{\psi} + (1-a)(1-b)^{\psi}\right\}^{1/\psi} \tag{3}$$

and hence

$$f'(k) = Aab^{\psi}\left\{ab^{\psi} + (1-a)(1-b)^{\psi}k^{-\psi}\right\}^{(1-\psi)/\psi} \tag{4}$$

and

$$f(k)/k = A\left\{ab^{\psi} + (1-a)(1-b)^{\psi}k^{-\psi}\right\}^{1/\psi} \tag{5}$$

- Consider the case with a high degree of substitutability between labor and capital, that is, $0 < \psi < 1$. Then the limits of the expressions above are

$$\lim_{k\to\infty}[f'(k)] = \lim_{k\to\infty}[f(k)/k] = Aba^{1/\psi} > 0$$

and

$$\lim_{k\to 0}[f'(k)] = \lim_{k\to 0}[f(k)/k] = \infty$$

- What happens if $\psi < 0$? We will get convergence, but can have quite problematic solutions. Consider what happens with $sAba^{1/\psi}$ as an exercise. This resembles what we have seen for the Harrod-Domar model.

- Hence the CES-production function gives an example of perpetual growth even if we have constant returns to scale.

- As another example consider the production function

$$Y = F(K, L) = AK + BK^{\alpha}L^{1-\alpha}$$

or in per capita terms

$$y = f(k) = Ak + Bk^{\alpha}$$

Note that this is a mixture of a $AK$-production function, and a Cobb-Douglas. The function exhibits constant returns to scale. In the limit the latter part becomes relatively unimportant.

- We now have

$$f(k)/k = A + Bk^{-(1-\alpha)}$$

which approaches $A$ as $k$ tends to infinity. We are thus approaching $AK$-like behavior and have perpetual growth.

## 10.3 Externalities and public provision

- Consider an economy characterized by a continuum of representative agents/workers. (For simplicity we keep their number fixed, i.e. there is no population growth). Each agent's output is given by a constant returns to scale production function

$$y = F(k, El)$$

  where for each agent, $k$ is the stock of available capital and $l$ is the input of labor. $E$ is the efficiency of each worker and is common to all workers.

- Assume that the total amount of capital in the economy has a positive impact on the productivity of each worker, i.e.

$$E = A(K) \tag{6}$$

  where $A(K)$ is increasing in $K$.

- We will later look more closely at examples where we justify this assumption based on externalities in the use of capital (learning by doing/social knowledge) or by public provision of services (the level depending on $K$) that improve productivity and are provided as public goods.

- The relationship (6) adds to

$$Y = F(K, A(K)L)$$

  and the growth rate is then

$$\dot{Y}/Y = [F_K + F_L A'(K)L]\dot{K}/Y$$

- With a constant savings rate $s = \dot{K}/Y$, we get

$$\dot{Y}/Y = sF_K + sF_L A'(K)L$$

- Due to the usual upper Inada-condition the term involving $F_K$ vanishes when $K$ approaches infinity, however $\dot{Y}/Y$ might still be positive if $sF_L A'(K)L$ is bounded away from zero.

- Since we have CRS, the partial derivative $F_L(K, A(K)L)$ is homogenous of degree zero, so we have

$$F_L(K, A(K)L)A'(K)L = F_L(1, A(K)L/K)A'(K)L$$

- Assume that
$$\lim_{K\to\infty} A'(K)L = b \tag{7}$$
then (since $\lim_{K\to\infty} A'(K) = c \Rightarrow \lim_{K\to\infty} A(K)/K = c$) we also have that
$$\lim_{K\to\infty} \frac{A(K)}{K} \cdot L = b$$
and hence
$$\lim_{K\to\infty} F_L(1, \frac{A(K)}{K}L)A'(K)L = F_L(1, b)b$$
giving
$$\lim_{K\to\infty} \frac{\dot{Y}}{Y} = sF_L(1, b)b > 0$$
and perpetual growth.

- Hence given the framework described by (6) condition (7) is a sufficient condition for growth in the long run.

- We will be considering conditions (6) and (7) in detail in the more specific models to follow under topic 11 and 12.

## 10.4 Two-sector approaches

- We now consider another approach which takes a minimum departure from the neo-classical model. We show that

  - *long-run growth is possible even with a neo-classical production function if at least one of the accumulated factors are produced only by use of reproducible factors.*

- To see an example of this consider an economy with the following two-sector structure.

- Output $C$ of the consumption good is produced by a Cobb-Douglas production function
$$C = K_C^\alpha L^{1-\alpha}$$
where $K_C$ is the amount of capital used in this production sector.

- The sector producing capital, the investment sector, uses only capital $(K_I)$ but exhibits constant returns to scale
$$\dot{K} = aK_I$$
and where $K = K_C + K_I$.

- Assume that a constant fraction of capital goes to the investment sector, $K_I = \phi K$ (and hence $K_C = (1-\phi)K$). Note that this replaces the assumption of a fixed savings rate.

- For simplicity we keep population constant. Then we see that

$$\frac{\dot{C}}{C} = \alpha \frac{\dot{K}_C}{K_C} = \alpha \frac{\dot{K}}{K} = \alpha a \phi$$

  So the growth rate of consumption is constant and positive. We will see long-run growth even if we have not introduced any departures from constant returns to scale.

- The most interesting example of this model is one with a separate sector for production of human capital. We will return to this in more detail as topic 13.

## 10.5 Why is explaining technological progress so hard?

- The final approach we will follow is to try to explain the growth process of $T$ where $T$ is interpreted as a stock of knowledge.

- A typical feature of these models is that previous inventions (increases in the level of $T$) can be used for future new inventions.

- A central feature of these models is a production function for knowledge (i.e. characterizing the R&D-sector)

$$\dot{T} = G(T, K, L)$$

- A serious problem for the models is that it is very hard to have a good idea about the nature of this production function (in particular the role of $T$ as an input). This is serious since results tend to be strongly dependent upon parameters of the production function.

- Why is it so difficult to explain such research and development activities? At this point we should note two things:

  1. There are several reasons to expect the ideas/knowledge embodied in $T$ to have the character of a public good. It is non-rival in its use (as reflected by the production function $Y = F(K, TL)$) and to a certain extent non-excludable. (Cf. the excellent discussion in Romer (1990), sec. 1-2.

2. Due to constant returns to scale in the rival inputs $K$ and $L$ we know that if we have a perfectly competitive market structure we have
$$RK + wL = Y$$
Hence, there is no income left to remunerate production of $T$.

- Hence, under perfect competition private agents/firms will have no incentives to do research, and we will not get growth in $T$.
- Therefore we must seek models that introduces markets with imperfect competition.
- In topic 14 and 15, we will pursue two such paths
  1. Monopolistic competition in a market with a variety of products.
  2. Quality ladders with temporary monopoly power for the use of the newest technology.

## 10.6 Endogenous growth: Does it matter?

- An important lesson of our study of the neo-classical growth model is that there is basically no room for policy in promoting growth. This is so for two reasons
  1. The decentralized solution is pareto-optimal
  2. Policy can not affect the growth in the long run
- We now briefly sketch why this in general will change in models with endogenous growth.
- Consider the AK-model where
$$Y = AK$$
- The AK-model is in many ways the reduced form of several of the models we will encounter. The parameter $A$ then summarizes various parameters (e.g. those related to policy), and should be thought of more broadly as characterizing the level of technology.
- An important rationale for the AK-model is that it can be seen as an alternative representation of a model with
$$Y = F(K, H)$$

that exhibits neoclassical properties including CRS in $K$ and $H$, i.e. CRS in broad capital. See BSiM 4.2 for the details.

- Using our usual setup for household preferences, household behavior in this technological environment is characterized by the Euler-equation

$$\frac{\dot{c}}{c} = (1/\theta)(r - \rho) \tag{8}$$

- But we now have that the rental price must bee a constant $R = A$. Hence, the interest rate is also constant

$$r = A - \delta \tag{9}$$

- So we have

$$\frac{\dot{c}}{c} = (1/\theta)(A - \delta - \rho) \tag{10}$$

- The growth rate of consumption is therefore constant (and we assume $A - \delta > \rho$, so that it is positive).

- Capital is evolving according to

$$\dot{k} = (sA - \delta - n)k - c \tag{11}$$

- It is easy to show (cf. BSiM p. 207) that in the steady state $c/k$ must be fixed, so $k$ must grow at the same (constant) rate as $c$. Further, if we investigate the transversality condition more carefully (see BSiM 4.1.4) we see that (not surprisingly) there is no transitional dynamics in this model, and

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = (1/\theta)(A - \delta - \rho) = \text{Constant} \tag{12}$$

at all times.

- We can also see this in the phase diagram.

- Notice that in the AK-model changes in parameters such as $A, \delta$ and $\rho$ will affect both levels **and** growth in the long run.

- Remembering that $A$ might in turn depend on parameters of policy, we see that in this framework there is a much more important role for policy.

# References

[1] **Romer, Paul, M.** Endogenous Technological Change, *Journal of Political Economy,* October 1990, 98 (5), 71-102, Part 2.