Theory of economic growth

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11 Learning by doing

Required reading: BSiM: 4.3
Secondary reading: Irwing Klenow (1994), Thompson (2001)

11.1 Technological progress as an externality

- We remember from the last lecture that if the disaggregated production function is
$$Y_i = F(K_i, EL_i)$$
with CRS in K_i and L_i while
$$E = A(K) \tag{1}$$
is taken as given by all agents (e.g. a public good) and satisfies
$$\lim_{K\to\infty} A'(K)L = b > 0 \tag{2}$$
then we have
$$\lim_{K\to\infty} \frac{\dot{Y}}{Y} = F_L(1,b)b > 0$$
and perpetual growth.

- One rationale for (1) is learning by doing effects.

- That is, knowledge and efficiency follow from production and investments. The higher is production, the more experience and know-how have been generated. Since the level of production can be associated with the level of capital, we can postulate a relationship like (1).

- Implicit in this formulation is an assumption of complete spill-over of knowledge in the sense that each firms knowledge is a public good to all other firms.

- There is thus a positive externality from firm i's use of capital on the production of all other firms.

- We thus associate the parameter E with the total stock of capital in the economy. For convenience we use the functional form

$$E = A(K) = K^{\sigma} \tag{3}$$

where $\sigma > 0$.

- The aggregate production function is then found by summing over all firms

$$Y = F(K, EL) = F(K, K^{\sigma}L)$$

- If $\sigma < 1$ we have $A(K)/K = K^{\sigma-1} \to 0$ as $K \to \infty$, so condition (2) is not met.

- However this condition was only a sufficient condition, and derived for a case with no population growth. Assume that Y and K grow at a common rate g. Then we have

$$Y_0e^{gt} = F(K_0e^{gt}, K_0L_0e^{(g\sigma+n)t})$$

Due to CRS this is possible if and only if

$$g = g\sigma + n \Rightarrow g = n/(1-\sigma)$$

Hence for $\sigma < 1$ we can have growth in Y forever only if $n > 0$, that is if we have population growth. If this is the case also have that Y/L grows at rate $g - n = n(1/(1-\sigma) - 1) > 0$.

- However, if the learning curve does not exhibit decreasing returns to K, that is $\sigma \geq 1$ the conclusion is altered.

- Assume there is no population growth and $\sigma = 1$. Then $A(K)/K = 1$ so so condition (2) is met, and we will get perpetual growth. This is the case discussed by Romer (1986).

- Notice that BSiM only consider the case $\sigma = 1$.

11.2 Implications for policy

- The individual firm takes E as given. Assuming that the production function $F(\cdot, \cdot)$ is CD, the private marginal product of capital for firm i is

$$MPK_{p,i} = \frac{\partial F(K_i, EL_i)}{\partial K} = F_1(K_i, EL_i) = \alpha K_i^{\alpha-1}(EL_i)^{1-\alpha} \tag{4}$$

or in the aggregate

$$MPK_p = \alpha K^{\alpha-1+(1-\alpha)\sigma}L^{1-\alpha} \tag{5}$$

- Notice that if $\sigma = 1$
$$MPK_p = \alpha L_i^{1-\alpha}$$
so the marginal product will be independent of K and we have an AK-model.

- In this case, increases in the size of the population will shift the constant marginal product up leading to stronger growth (i.e. like a shift in A in the AK-model). This is what we call a scale effect.

- A social planner, on the other hand, will evaluate the marginal product of capital to be

$$\begin{aligned} MPK_s &= \frac{\partial F(K, A(K)L)}{\partial K} = F_1(K, A(K)L) + LA'(K)F_2(K, A(K)L) \\ &= (\alpha + (1-\alpha)\sigma)K_i^{\alpha-1+(1-\alpha)\sigma} L_i^{1-\alpha} = (\alpha + (1-\alpha)\sigma)/\alpha \cdot MPK_p > MPK_p \end{aligned}$$

and hence choose a path with higher growth (of $\dot{c}/c$ and $\dot{k}/k$).

- Hence the decentralized solution will end up with too low investments and sub-optimal growth.

- A subsidy on investments (or production) can go at least some way in making the firms internalize the externality. However, this might have to be financed by taxes introducing other distortions.

11.3 Some evidence on learning by doing

- Learning appears to be particularly strong in early phases of a firms life. This might suggest that $\sigma < 1$.

- There are important case studies suggesting substantial learning effects.

- A famous example is that of the manufacturing of the Liberty-class ships in the US during WW2.

- The amount of worker-days required to build each ship declined dramatically as did the time used on each ship. Suggesting productivity gains of up to of 40 per cent per year (averages).

- A large share of this productivity gain has been attributed to learning by doing.

- However, more recent research show that previous estimations exaggerated the effect by not taking sufficiently care of increases in capital and capital deepening. In addition there was a strong tendency for increasing the output of ships at the expense if quality.
- Other important case studies has focused on US airframe manufacturing.

11.4 The scope of the externality: Local or global?

- An important thing to note is that the Liberty-ship experience only provides evidence on the learning curve at the level of an individual firm, and does not tell us much about the degree of spill-over.
- The extent of spill-over effects is of course crucial for the implications both for how learning by doing affects growth and for the role of policy.
- It can be worth considering how spill-overs might matter to varying degrees within industries vs across industries, within countries/regions or between countries/region.
- This in turn has important consequences for the extent of scale effects, that is how the population/scale of the relevant units affect growth.

12 Government investments

Required reading: BSiM: 4.4

12.1 Public provision of non-rival infrastructure

- An alternative justification of (1) is to regarding the efficiency parameter E as a reflection of public services which enhance labor productivity.
- Assume that these services are produced with the same production function as private services, and that the flow of services promoting labor productivity is equal to government spending, i.e. $E = G$.
- Let G be financed by a proportional output tax, τ and assume for simplicity that the government always runs a balanced budget. Then

$$E = G = \tau Y = \tau F(K, EL)$$

- Due to CRS we have
$$1 = \tau F(K/E, L)$$
which implicitly determines
$$E = A(K) = cK \tag{6}$$
where c is a positive constant (for a given τ) determined by $1 = \tau F(1/c, L)$

- Equation (6) of course satisfies (2) so we will get perpetual growth in this model.

12.2 Public goods model

- G is nonrival and nonexcludable (pure public good)

- production function
$$Y_i = AL_i^{1-\alpha} K_i^{\alpha} G^{1-\alpha} = L_i[Ak_i^{\alpha} G^{1-\alpha}]$$

- proportional tax on output
$$G = \tau Y$$

- Firms have to maximize after tax income (per capita income)
$$\max_{k_i} (1-\tau)Ak_i^{\alpha} G^{1-\alpha} - (r+\delta)k_i - w$$
F.O.C.
$$(1-\tau)\alpha A k_i^{\alpha-1} G^{1-\alpha} = r + \delta = R \tag{7}$$
$$(1-\tau)(1-\alpha) A k_i^{\alpha-1} G^{1-\alpha} = w \tag{8}$$

- Household's optization is unchanged (taxation only affects firms). Equilibrium conditions. Every firm choosed the same capital-labor ratio. We can rewrite (aggregated) production function as
$$G = \tau L A k^{\alpha} G^{1-\alpha} = (\tau L A)^{1/\alpha} k$$

- After inserting into (7) for G we get
$$R = (1-\tau)\alpha(\tau L)^{\frac{1-\alpha}{\alpha}} A^{1/\alpha}$$
which is independent of K and therefore constant over time. There is again scale effect. After inserting into Euler equation we get
$$\gamma_c = \gamma_k = \gamma_y = \frac{1}{\theta}\left[(1-\tau)\alpha(\tau L)^{\frac{1-\alpha}{\alpha}} A^{1/\alpha} - \delta - \rho\right]$$

- If the government wants to maximiye the growth rate, what is the optimal tax rate? Maximum growth rate is attained when the first derivative wrt τ is equal to 0.

- The result is
$$\tau = 1 - \alpha$$
Therefore, equilibrium in decentralized economy with benevolent government is
$$\gamma_c = \gamma_k = \gamma_y = \frac{1}{\theta}\left[\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}} L^{\frac{1-\alpha}{\alpha}} A^{1/\alpha} - \delta - \rho\right]$$

- Larger population causes larger growth. The economy benefits from greater scale because government services are assument to be nonrival. That is at odd with the data.

12.3 Crowding in and crowding out

- We will now look more closely at slightly different model where govenment services are partly rival. The model is due to Barro (1990).

- For simplicity we abstract from population growth, and any exogenous growth in technology. The economy is closed.

- We incorporate public services in the production function in the following way
$$y = \Phi(k, g)$$
where $g = G/L$ is the amount of public services per worker. Note that this implicitly assumes that G is rival for the users, though in a rather narrow sense.

- The general idea of including g as a separate argument is that it is not a close substitute for private inputs (k), i.e. public services/activity represents something that would not be replaced by corresponding private activities if it vanished. This is again based on non-excludability which would make private incentives weak/non-existent.

- The production function $\Phi()$ is assumed to exhibit CRS to the two inputs k and g, so
$$y = \Phi(k, g) = k\phi(g/k)$$
with $\phi' > 0$ and $\phi'' < 0$.

- Whenever it is simpler, we work with the Cobb-Douglas case:

$$y/k = \phi(g/k) = \left(\frac{g}{k}\right)^{1-\alpha}$$

- The balanced government budget is:

$$g = \tau y = \tau k \phi(g/k)$$

- It follows directly that both g/y and g/k only depend upon the tax-rate τ. For a given τ they will therefore be constant.

- The marginal product of capital as seen by the individual agents who regard g/k as independent of their decision is

$$\frac{\partial y}{\partial k} = \phi\left(\frac{g}{k}\right)(1-\eta) = \phi\left(\frac{g}{k}\right)\alpha$$

where $\eta \equiv \phi' \frac{g}{y} = \frac{\partial y}{\partial g}\frac{g}{y} = 1-\alpha$, i.e. is the elasticity of y with respect to g which is equal to the constant $1-\alpha$ in the CD-case.

- As usual, we consider a representative household-producer who maximizes:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\theta}}{1-\theta} dt \tag{9}$$

- Since the after-tax private returns to capital is $(1-\tau)(\partial y/\partial k)$, it should by now be familiar that the Euler-equation now reads

$$\gamma = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[(1-\tau)\phi\left(\frac{g}{k}\right)\alpha - \delta - \rho\right] \tag{10}$$

- We can now see how the size of government, as characterized by $\tau = g/y$ impinges on the growth rate.

- There are two offsetting effects. The direct effect of an increase in τ is to reduce the incentive to invest. However, increasing g/y will also increase g/k and hence increase the marginal product of capital. This promotes incentives to invest.

- In the Cobb-Douglas case we get

$$\frac{d\gamma}{d(\tau)} = \frac{1}{\theta}\phi\left(\frac{g}{k}\right)(\phi' - 1)$$

thus we get the following hump where the growth rate increases in the size of government when g/k is so low that $\phi' > 1$ and declines when g/k is so high that $\phi' < 1$

- Note that the hump is a type of Laffer-curve. We can refer to the raising part of the curve as a case where public expenses are 'crowding-in' by increasing private productivity, while in the declining part they are 'crowding-out' by taxation affecting the marginal product of capital too strongly.
- Note that $\phi' = 1$ is a natural condition for productive efficiency.
- When η is constant as in the CD case it can be shown that increasing the growth rates monotonically rises life-time utility. So the benevolent government should indeed choose $\tau = g/y$ such that $\phi' = 1$.
- However, this is only a second best solution.
- Consider a social planner who fixes $\tau = g/y$ and then is able to dictate each household-producers consumption over time. For a given value of g/y, the social marginal returns to capital is $(1 - g/y)\phi(g/k)$, where the adjustment $-g/y$ is required for holding g/y constant.
- Hence the growth rate of consumption chosen by the social planner is

$$\gamma_s = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}\left[\left(1 - \frac{g}{y}\right)\phi\left(\frac{g}{k}\right) - \delta - \rho\right] \tag{11}$$

- Since $\tau = g/y$, the only difference between (10) and (11) is the absence of the term α in (11). Hence, for all levels of τ the growth rate is lower in the decentralized case.

- The private agents invest sub-optimally in capital because they do not internalize the externality of also being able to increase g and hence increase production. The public provision of these services are financed by a distortional tax on income (τ reduces the private returns to capital).

13 Human capital accumulation reconsidered

Required reading: BSiM 5.1, 5.3
Secondary reading:BSiM 5.2.1-5.2.2, Lucas(1988)

13.1 The one-sector model

- In our previous encounters with human capital we have considered production functions of the following type

$$Y = F(K, H, L) \tag{12}$$

  with CRS in K, H and L.

- We know that as long as Y, K and H are all produced by the same technology (12), the inclusion of human capital does not alter the qualitative conclusions of the neo-classical model (but significantly affects the quantitative predictions).

- We have seen this both in models where equality of returns to the two types of capital requires that H/K is constant, and in the model in MRW.

- Consider now instead a production function of the form

$$Y = F(K, H) \tag{13}$$

  with CRS in broad capital K and H.

- Note that we now should perhaps think of human capital as $H = hL$, i.e. as the number of workers, L, multiplied by the human capital of the typical worker, h. Implicit in the formulation of (13) as opposed to (12) is then the assumption that it is only the total stock of human capital that matters for production, not how this stock is made up of L and h. Stated loosely, labor quantity (L) and quality (h) are perfect substitutes.

- An important consequence of this is that we now have CRS in accumulated factors. This gives us a source of perpetual growth of the AK-type.

- To see this rewrite (13) as

$$Y = K \cdot f(H/K) \tag{14}$$

- We then have

$$\begin{aligned} r + \delta_K = R_K &= \frac{\partial Y}{\partial K} = f(H/K) - (H/K) \cdot f'(H/K) \\ r + \delta_H = R_H &= \frac{\partial Y}{\partial H} = f'(H/K) \end{aligned}$$

or

$$f(H/K) - f'(H/K) \cdot (1 + H/K) = \delta_K - \delta_H$$

which determines a fixed value for the ratio $H/K = 1/\omega^*$.

- Defining $A \equiv f(1/\omega^*)$ the production function (14) can be written

$$Y = AK$$

so we have a model of the AK-type. See BSiM 5.1.1 for additional details.

- In conclusion, we see that the consequences of adding human capital to the (one sector) model can depend crucially on which of the technologies (12) or (13) is the relevant one.

13.2 Imbalance effects

- In several of the models we have encountered it transpires that the ratio $\omega = K/H$ should be constant to secure equality of returns.

- So far we have assumed that it is always possible to achieve this instantly. However, this requires that we at any time can convert existing K to H and vice versa. This is obviously not very realistic.

- Assume this was not possible, and that changes to the ratio ω could only be made by distributing positive gross investments I_H and I_K on $\dot{H} = I_H - \delta_H H$ and $\dot{K} = I_K - \delta_K K$. In such situations we would get so called imbalance effects.

- More precisely we are now constrained by $I_H \geq 0$ and $I_K \geq 0$.

- If we initially had too much human capital relative to physical capital, so that $\omega < \omega^*$ the fastest way we can increas ω is by setting $I_H = 0$ (i.e. the constraint on I_H is binding), so that $\dot{H}/H = -\delta_H$.

- This process will lead to a transitional phase where ω is increasing towards it steady state value. It reaches this value in finite time, after which Y grows at a constant rate γ_Y^* according to the AK-model behavior discussed above.

- During this transition H essentially takes the role of L in the neoclassical growth model, and since the marginal product of K is decreasing in K/H we get the usual property that the growth rate of output falls during the transition. Hence, the growth rate falls monotonically towards γ_Y^*.

- If $\omega > \omega^*$ we get the same story only that the roles of H and K are reversed.

- In sum we get that the growth rate γ_Y falls monotonically towards γ_Y^* as ω is approaching ω^* from either side.

- This implies that if we get an imbalance such as a reduction of ω from a war that destroys physical capital rather than human capital (WW2), we get a transitional phase with strong growth.

- According to the model we should get the same effect from e.g. a plague which destroys H but leaves K intact. This is, however, not in line with what we see (cf. also WW1).

- An important source of possible asymmetries in the imbalance effect is that adjustment costs are typically higher for human capital. The effects also tend to be asymmetric in more elaborate models such as the two-sector model below.

13.3 A two-sector model. Uzawa-Lucas.

- As stated several times earlier in the course we should also be sceptical to the assumption that human capital is produced by the same technology as consumption and physical capital.

- In particular we are lead to expect that production of new human capital is considerably more intense in the use of existing human capital than is the production of physical capital.

- To highlight this effect consider the following two-sector model

$$Y = C + \dot{K} = F(K, uH) \tag{15}$$
$$\dot{H} = B(1-u)H \tag{16}$$

  where u is the share of human capital used in production of consumption goods/physical capital. We neglect depreciation for simplicity.

- Here the schooling sector produces new human capital using only human capital (teachers, student time/vigilance) as an input. This is obviously an extreme version of what we pointed at above.

- Note that (16) can also be interpreted as primarily capturing that individuals accumulate human capital more easily when the economy's level of human capital is larger, i.e. an externality.

- The important new feature of this model is that one of the capital inputs (H) is produced using only reproducible inputs.

- Also now we can get perpetual growth, even if there are no changes in technology over time. To see this assume that we are on a balanced growth path with

$$\gamma = \dot{C}/C = \dot{K}/K = \dot{H}/H = B(1-u)$$

which is possible if u and the savings rate s are chosen such that

$$\gamma K = \dot{K} = Y - C = sY = sF(K, uH)$$

at all times.

- The full analysis of the model requires that we endogenize allocation of resources, and let the savings rate s and the fraction u follow from optimization of a representative household-producer.
- This analysis will primarily teach us something new about the short run by giving a richer description of transitionary adjustments. Note however, that since u^* determines the long run growth rate, determining u^* will also tell us something about what determines growth.
- The results are primarily able to tell us something interesting about how the economy reacts to imbalances in the ratio of the two stocks of capital, ω.
- The full analysis is quite involved and is left for cursory self-study. (See BSiM 5.2).
- The most important new result is that there is now an asymmetry in the imbalance effect. If $\omega > \omega^*$ (i.e. we have too little human capital) the marginal product of human capital in the goods sector is high, implying a high wage rate. This makes operation costs in the sector that is intensive in the use of this input, i.e. education, high. This motivates an allocation of resources to production of goods rather than education. This retards the production of the scarce factor (H), thus retarding the economy's growth rate.
- Thus the model predicts that we should see faster recovery after an event that destroys physical capital, than if it had destroyed human capital.

13.4 Conditions for perpetual/endogenous growth

- We now generalize the setup of the Uzawa-Lucas model to

$$\begin{aligned} Y = C + \dot{K} + \delta K &= A(vK)^{\alpha_1}(uH)^{\alpha_2} && (17) \\ \dot{H} + \delta H &= B \cdot [(1-v)K]^{\eta_1}[(1-u)H]^{\eta_2} && (18) \end{aligned}$$

- We now look for a steady state where u and v are constant, and C, K, H, and Y grow at constant (but not necessarily equal) rates.
- Divide (18) by H and transform both sides to growth rates. This gives

$$\eta_1\gamma_K^* + (\eta_2 - 1)\gamma_H^* = 0 \tag{19}$$

- Divide (17) by K and transform both sides to growth rates. This gives

$$\left(\frac{C/K}{C/K + \gamma_K^* + \delta}\right) \cdot (\gamma_C^* - \gamma_K^*) = (\alpha_1 - 1)\gamma_K^* + \alpha_2\gamma_H^* \tag{20}$$

- It can be shown that we must have $\gamma_K^* = \gamma_C^*$. This in turn implies that $\gamma_K^* = \gamma_Y^*$, so Y, K and C grow at the same rate.
- Therefore (20) simplifies to

$$(\alpha_1 - 1)\gamma_K^* + \alpha_2\gamma_H^* = 0 \tag{21}$$

- Equations (19) and (21) constitutes a homogenous system of two linear equations in the two unknown γ_K^* and γ_H^*. It should be a familiar result that the only way the solution to this problem could be different from $\gamma_K^* = \gamma_H^* = 0$ is if the characteristic matrix is singular.
- This implies that the only way we can have endogenous growth ($\gamma_K^* > 0$,$\gamma_H^* > 0$ as postulated above) is if the parameters satisfy

$$\alpha_2\eta_1 = (1 - \eta_2)(1 - \alpha_1) \tag{22}$$

- It is worth considering some special cases where the condition is satisfied
  - CRS in both sectors ($\alpha_1 + \alpha_2 = \eta_1 + \eta_2 = 1$). Then we also have $\gamma_H^* = \gamma_K^*$ so K/H is constant.
  - Uzawa-Lucas: $\eta_1 = 0$, η_2=1. Then (22) holds irrespective of the size $\alpha_1+\alpha_2$, so we can have both decreasing and increasing returns to scale in the final goods sector.
  - Decreasing returns in one sector is offset by increasing returns in the other. Assume all elasticities are positive. If $\alpha_1 + \alpha < 1$ condition (22) can be satisfied provided $\eta_1 + \eta_2 > 1$, and vice versa.

- The AK-model with $\alpha_1 = 1$ and $\alpha_2 = 0$. Notice that H is just a waste here.
- Notice that as long as $\alpha_1 \neq 1$ we have from (21) that

$$\gamma_K^* = \left(\frac{\alpha_2}{1-\alpha_1}\right)\gamma_H^*$$

so $\gamma_K^* > \gamma_H^*$ if $\alpha_1 + \alpha_2 > 1$ and vice versa. In particular, only with constant returns to scale $(\alpha_1 + \alpha_2 = 1)$ do we have $\gamma_K^* = \gamma_H^*$ and a constant H/K on the balanced growth path.
- It follows that the only situation where we get growth and where H/K is constant is if we have CRS in both sectors.

- So what have we learned? There are many ways we can get perpetual/endogeneous growth in this two-sector setup. However, the predictions depend crucially on the values of the parameters describing the technologies. Since we know rather little about these values, the predictive power of the model is limited. This is a serious short-coming, and a challenge for future research.

References

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