# Theory of economic growth

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# 14 Growth through increasing variety

Required reading: Romer (1990)
Secondary reading: BSiM 6

## 14.1 Ideas: Rivalry, excludability and nonconvexities

- Are knowledge/ideas public goods?
- We should distinguish between two aspects: Rivalry and excludability.
- Rivalry is a technological attribute of the good itself.
- An idea is in general non-rival.
- That is of course not to say that e.g. one firms sales of a product produced using the idea is not competing with (is rival to) another firm's sales.
- Note further that there is a fundamental difference between knowledge in the form of ideas/designs (our $A$) and knowledge in the form of the human capital of individuals.
- The former is in general non-rival and only partially excludable, the latter has a high degree of both rivalry and excludability.
- Note further that nonrival knowledge can be accumulated without bound on a per capita basis, while human capital can not because it is constrained by the life time of each separate individual.
- If a nonrival input has productive value this inevitably violates the replication argument underlying constant returns to scale and leads to nonconvexities.
- If $K$ and $L$ are all rival inputs and $A$ is a nonrival input such as ideas and the production function is $F(A, K, L)$, the standard replication argument leads to CRS in $K, L$:

$$F(A, sK, sL) = sF(A, K, L) \tag{1}$$

which also implies (as a special case of Euler's formula)

$$F(A, K, L) = KF_K + LF_L \tag{2}$$

But if $A$ is productive we must have

$$F(sA, sK, sL) > sF(A, K, L) \tag{3}$$

and

$$F(A, K, L) < AF_A + KF_K + LF_L \tag{4}$$

- Note that (4) implies that a firm having this production function could not survive as price taker remunerating all inputs at their marginal product.
- Thus, under perfect competition we can not explain why firms remunerate $A$ and hence not explain why private agents do research.
- We have looked at two approaches: Either treating $A$ as a proper public good (provided by the government) or letting production of $A$ follow as a by-product such as through learning by doing.
- We now look at a new approach: Letting knowledge be partially excludable and introducing market power.

Ideas $\Rightarrow$ Nonrivalry $\Rightarrow$ Increasing returns $\Rightarrow$ Imperfect competition

## 14.2 Monopolistic competition in horizontally differentiated products

### 14.2.1 The final-goods sector

- The following is a somewhat simplified version of the model in Romer (1990).
- The production function for the representative firm producing the final good $Y$ is

$$Y = L_1^{1-\alpha} \sum_{i=1}^{N} x_i^{\alpha} \tag{5}$$

where $L_1$ is labor input and $\{x_i\}_{i=1}^{A}$ are different capital goods (intermediate goods).

- Note that the production function exhibits CRS with resepct to all inputs $L_1$ and $\{x_i\}_{i=1}^{A}$. Thus the assumption of a single representative price-taking firm is legitimate.

- Invention of ideas in the model corresponds to the creation of new capital goods that can be used in the final-goods sector.

- Profit maximization of representative firm

$$\max_{L_1,\{x_i\}_{i=1}^{N}} \sum_{i=1}^{N} (L_1^{1-\alpha} x_i^{\alpha} - wL_1 - p_i x_i)$$

leads to F.O.C

$$w = (1-\alpha)\frac{Y}{L_1}$$

$$p_i = \alpha L_1^{1-\alpha} x_i^{\alpha-1}$$

where $w$ is wage paid for labor and $p_i$ is rental price for capital goods.

Interpretation is as usual: the firm hires labor until the marginal product of labor equals the wage and rent capital goods until the marginal product equals the rental price.

- Note that marginal product of $x_i$ is $MP = \alpha L_1^{1-\alpha} x_i^{\alpha-1}$ which is infinite at $x_i = 0$. Hence the firm will want to use all $A$ inputs available.

- Note that if we regard $x_i$ as different capital goods (5) assumes that these have additively separable effects on production. The marginal product of capital good $i$ (tractor) does not depend on the amount used of other capital goods (e.g. computers)

- This contrasts with the traditional case where we measure the total use of capital goods simply as the sum $((\sum_{i=1}^{A} x_i)^{\alpha} = K^{\alpha})$. This assumes perfect substitutability between tractors and computers.

### 14.2.2 Production of intermediate goods

- (Inverse) demand function for intermediate good $x_i$ follows from the FOC of previous profit-maximization problem

$$\alpha L_1^{1-\alpha} x_i^{\alpha-1} = p_i(x_i) \tag{6}$$

- Each input $x_i$ is supplied by a single firm that has monopoly power in the production of the intermediate good $x_i$.
- This monopoly power is assumed to be based on the ownership/renting of a patented design for the production of $x_i$.
- To simplify we assume that the firm producing intermediate output $x_i$ can do so simply by converting a unit of capital/consumption good $Y$ into $x_i$. (In this sense, all $x_i$ have the same production function as $Y$.)
- This implies that the $x_i$ is produced with a fixed unit price equal to the interest rate $r$.
- Thus, inserting for (6), each monopolist faces the profit maximizing problem

$$\pi = \max_{x_i} \; p(x_i)x_i - rx_i \;\; = \;\; \max_{x_i} \alpha L_1^{1-\alpha} x_i^{\alpha-1} x_i - rx_i$$

giving

$$\alpha L_1^{1-\alpha} x_i^{\alpha-1} = r/\alpha \tag{7}$$

or

$$p_i = r/\alpha \tag{8}$$

- Thus intermediate-goods firm charges price as a markup over marginal cost $r$
- Due to the symmetry of inputs $x_i$ in (5), we must have $x_i = x$ and $p_i = p$ for all $i$ in equilibrium. Each capital good is employed in the same amount.
- The profit for the monopolists producing intermediate inputs is hence

$$\pi = \frac{r}{\alpha}x - rx \;\; = \;\; \frac{1-\alpha}{\alpha} rx > 0 \tag{9}$$

- The total demand for capital must equal total capital stock in the economy

$$\sum_{i=1}^{A} x_i = K$$

Since the capital goods are used in the same amount we can determine $x$

$$x = \frac{K}{A}$$

Hence the final-goods production function can be rewritten as

$$Y = L_1^{1-\alpha} A x^{\alpha} = L_1^{1-\alpha} (Ax)^{\alpha} A^{1-\alpha} \quad (10)$$

Which is finally usual production function

$$Y == L_1^{1-\alpha} (K)^{\alpha} A^{1-\alpha} = K^{\alpha} (AL_1)^{1-\alpha}$$

We see that the production function exhibits CRS in $L_1$ and $K$.

- Technological progress is captured by an increase in the number/variety of available inputs to production, that is, through increases in $A$.
- An interpretation: As a broader diversity of inputs become available production can increase by using more specialized inputs.

### 14.2.3 Research and development

- Designs for new intermediate goods are developed through research and development. The production of new ideas for designs is characterized by the deterministic production function

$$\dot{A} = BL_2A \quad (11)$$

where $L_2$ is the number of workers in R&D, satisfying the constraint $L_1 + L_2 = L$, where the total supply of labor is fixed.

- Note that ideas, $A$, are partially excludable. Inventors own the patent for the designs on how to produce $x_i$, but the ideas underlying each design is non-excludable in the production of new ideas and designs. Hence, $A$ is a public good in (11).
- We do not need to be explicit as to who conduct the R&D activities (be it the monopolistic producers of intermediate goods or someone else) as long as the patent can be rented.
- The value $V_a$ of inventing one new design is thus equal to the net present value of receiving the profit in (9) in all future periods.

- Method of arbitrage: put the money in the bank (in this model is equivalent to purchasing unit of capital) or to buy patent for one period, earn the profit and sell the patent. In equilibrium it must be the case that these two investment equal.

- The arbitrage condition leads to

$$rV_a = \pi + \dot{V}_a$$

LHS = interest earned from investin $V_a$ in the bank, RHS = profit plus the capital gain or loss from change in the price of the patent.

$$r = \frac{\pi}{V_a} + \frac{\dot{V}_a}{V_a}$$

Along BGP, $r$ is constant, but $V_a$ is also constant, thus $\dot{V}_a = 0$. It follows

$$V_a = \frac{\pi}{r} = \frac{(1-\alpha)}{\alpha} x \tag{12}$$

- Next, it must be the case that individuals are indifferent between working in the final-goods sector and working in the research sector.

- The wage in R & D sector is equal to marginal product ($BA$) multiplied by the value of the new ideas created $V_a$.

$$w_{R\&D} = BAV_a$$

- Equating the wages in the two sectors we then have

$$BAV_a = (1-\alpha) L_1^{-\alpha} x^{\alpha} A$$

$$V_a = \frac{1-\alpha}{B} L_1^{-\alpha} x^{\alpha}$$

which together with (7) and (12) gives

$$r = B\alpha L_1 \tag{13}$$

i.e. the interest rate as a function of $L_1$.

### 14.2.4 Consumers

- To close the model we need to add the consumer side. We adopt the usual case of intertemporal utility maximization for given market prices ($w$ and $r$), using a CRRA functional form.
- Population is constant ($n = 0$).
- We skip the details of familiar utility maximization problem, turning directly to the Euler equation

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\theta} \tag{14}$$

where $\rho$ and $\theta$ are the now common parameters of the life-time utility expression.

- For simplicity we neglect depreciation, so

$$\dot{K}(t) = Y(t) - C(t)$$

### 14.2.5 The balanced growth path

- We will now show that we get a balanced growth path where $r$ and $L_1$ are constant.
- We do not solve for the transitional dynamics because this is difficult, and not all that interesting.
- Start by assuming that $r$ is constant. Then $L_1$ is also constant according to (13), and hence $x$ is also constant according to (7). It then follows from (10) that the growth rate of $Y$ is the same as the growth rate of $A$.
- Further, since $K = Ax$ and $x$ is constant $K$ also grows at the same constant rate as $A$. Hence also $K/Y$ is constant.
- It then follows that

$$\frac{C}{Y} = \frac{Y - \dot{K}}{Y} = 1 - \frac{\dot{K}}{K}\frac{K}{Y}$$

is constant since $\dot{K}/K$ and $K/Y$ are both constant.

- We then finally have

$$\gamma = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = BL_2$$

$$\gamma = BL - BL_1 = BL - \frac{r}{\alpha} \tag{15}$$

- Thus, the existence of this balanced growth path depends on $r$ being constant. But the constancy of $r$ was what we assumed at the outset and showed that it implied this balanced growth path. That there indeed is consistency.

  By setting (14) and (15) equal we get

$$r = \frac{\alpha}{\alpha + \theta} (\theta BL + \rho)$$

  i.e. a constant as required.

- Thus, there exists a balanced growth path

$$\gamma = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \frac{\alpha BL - \rho}{\alpha + \theta} \tag{16}$$

- Note the following about the steady state growth rate:

  1. There is positive and constant growth in equilibrium (The model is misbehaved if $\alpha BL < \rho$).
  2. This growth is driven by private agents doing research ($L_2 > 0$), so we have endogenized technological progress.
  3. The growth depends on the willingness to save reflected by $\rho$ and $\theta$. Hence there is room for policy to affect the long run growth rate.
  4. The growth rate increases in population $L$. Hence there is a scale effect.
  5. It can be shown that the two last properties relies heavily on the properties of the R&D production function (11), or more specifically from $\dot{A}$ being linear in $A$.

## 14.3 Implications for policy

- We now look at the allocation a social planner would have chosen in the setting with an increasing variety of intermediate inputs.
- The social planner will always choose $x_i = x$ for all $i$.
- Production is given by

$$Y = L_1^{1-\alpha} A x^\alpha = L_1^{1-\alpha} A x^\alpha = A^{1-\alpha} L_1^{1-\alpha} K^\alpha$$

  In the third equality we have defined $K = Ax$ as the total amount of resources used on capital goods.

- This definition of the accounting stock $K$ ('capital') has the following law of motion:

$$\dot{K} = Y - C = A^{1-\alpha} L_1^{1-\alpha} K^\alpha - C$$

- We now can formulate the problem of finding a socially optimal balanced growth path (i.e. we again ignore transitional dynamics) by solving the social planners problem:

$$\max \int_0^\infty \frac{C^{1-\theta}}{1-\theta} e^{-\rho t} dt \tag{17}$$

$$\dot{K} = A^{1-\alpha} L_1^{1-\alpha} K^\alpha - C \tag{18}$$

$$\dot{A} = B L_2 A \tag{19}$$

$$L_1 + L_2 \leq L \tag{20}$$

- Notice that (19) is the R&D production function.
- The control variables are $C$ and $L_2$, and there are two state variables $K$ and $A$. The extension of optimal control theory to this two dimensional case is straightforward.
- The current-value Hamiltonian for the problem is

$$\mathcal{H} = \frac{C^{1-\theta}}{1-\theta} + p[A^{1-\alpha}(L - L_2)^{1-\alpha} K^\alpha - C] + qBL_2A \tag{21}$$

- The co-state variables $p$ and $q$ must satisfy

$$\dot{p} = \rho p - \frac{\partial \mathcal{H}}{\partial K} \tag{22}$$

$$\dot{q} = \rho q - \frac{\partial \mathcal{H}}{\partial A} \tag{23}$$

- While we have the usual first order conditions for the control variables

$$\frac{\partial \mathcal{H}}{\partial C} = C^{-\theta} - p = 0 \tag{24}$$

$$\frac{\partial \mathcal{H}}{\partial L_2} = -(1-\alpha)pA^{1-\alpha}(L-L_2)^{-\alpha}K^{\alpha} + qBA = 0 \tag{25}$$

- Since we are only looking at balanced growth path solutions we must have $\gamma^* = \frac{\dot{C}}{C} = \frac{\dot{A}}{A}$. Since (19) gives $\frac{\dot{A}}{A} = BL_2$ we have $\gamma^* = BL_2$ and for determining the common growth rate $\gamma^*$ we need only to determine the share of labor used for research $L_2$.

- Note that (dividing by $A$ and multiplying by $(L - L_2)$) (25) implies

$$(1-\alpha)pA^{-\alpha}(L-L_2)^{1-\alpha}K^{\alpha} = qB(L-L_2)$$

and since

$$\frac{\partial \mathcal{H}}{\partial A} = (1-\alpha)pA^{-\alpha}(L-L_2)^{1-\alpha}K^{\alpha} + qBL_2$$

we have

$$\frac{\partial \mathcal{H}}{\partial A} = qB(L-L_2) + qBL_2 = qBL$$

- Joining this with (23) we have

$$\frac{\dot{q}}{q} = \rho - BL \tag{26}$$

- As usual (24) gives

$$-\theta \frac{\dot{C}}{C} = \frac{\dot{p}}{p}$$

- Since on a balanced growth path we have $\frac{\dot{C}}{C} = \frac{\dot{A}}{A}$ and must also have $\frac{\dot{q}}{q} = \frac{\dot{p}}{p}$, this translates to

$$-\theta \frac{\dot{A}}{A} = \frac{\dot{q}}{q}$$

Inserting for $\frac{\dot{A}}{A} = BL_2$ and (26) we get

$$-\theta BL_2 = \rho - BL$$

which we can finally solve for $L_2$

$$L_2 = \frac{L}{\theta} - \frac{\rho}{B\theta} \tag{27}$$

which in turn implies that the growth rate on the balanced growth path is

$$\gamma^* = \frac{\dot{C}}{C} = \frac{\dot{A}}{A} = \frac{BL - \rho}{\theta} \tag{28}$$

- Since $\alpha \in (0,1)$ it follows that the growth rate in the decentralized solution we found last time ($\gamma$) falls short of the growth rate in the socially optimal solution ($\gamma^*$). This, of course, follows from too little research being done in the decentralized solution.

- There are two reasons why there is too little research in the decentralized solution

  1. There are positive externalities in research. The private agents do not take into consideration that new inventions also makes it more easy to come up with yet newer inventions (that is, by doing research they contribute to $A$ which is a public good in the R&D production function (19)).
  2. There is monopolistic price setting for the intermediate goods, the firm set price as a mark-up over marginal cost. Its incentive to innovate is the monopoly profit, which is less than the gain to society (consumper surplus). The producer does not have as strong incentives to come up with a new design as does a social planner maximizing the consumer (social) surplus.

- Note that market power is necessary for there to be private incentives to invest, but that monopoly pricing gives rise to sub-optimal growth.

# 15 Growth through increasing quality

Required reading: Aghion and Howitt (1998), Chapter 2.1–2.3.

## 15.1 The Schumpeterian perspective

- Rather than assuming monopolistic competition in a variety of intermediate inputs, we now look at a case where innovations lead to new and better intermediate inputs which replace the old ones.

- As long as the good is the best one available, its innovator has a monopoly for sales of inputs (enforced by a patent). But since the new good only survives until it itself is replaced, a new innovation only gives a temporary monopoly.

- The profit enjoyed while in the monopoly position secures incentives for doing R&D.

- This framework reflects Schumpeter's ideas about creative destruction. New innovations are creative in increasing productivity. However, they are destructive in removing the ground for older products. But then again the fact that one can benefit from a monopoly position after having destroyed ones competitors is what induces the creative behavior in the first place.

## 15.2 A simple model without capital accumulation

- We now formulate a simple model of Aghion and Howitt (1998).

- Output of the consumption good per unit of calendar time is given by

$$Y_t = A_t x_t^{\alpha} \tag{29}$$

  where $x_t$ is the amount used of the newest intermediate input and is the only input to production.

- Note that the index $t$ does not denote time but the number of innovations that have occured so far.

- There is a given stock of labor $L$ which is either used to do research ($n$) or to produce $x$. One unit of labor produces one unit of the intermediate input. Thus we have labor market clearing given by

$$L = n_t + x_t \tag{30}$$

- When $n$ units of labor are used in research this gives an expected $\lambda n$ new innovations. $\lambda$ follows Poisson proces. It is probability that new innovation appears (in some time) for one researcher.

  A new innovation gives rise to a new intermediate input $x_{t+1}$ with a higher productivity $A_{t+1} = \gamma A_t$, where $\gamma > 1$ is a parameter characterizing the magnitude of innovations.

- The most essential relation of the model is the arbitrage equation

$$w_t = \lambda V_{t+1} \tag{31}$$

  This says that labor must have the same expected value in its two uses. It can be used in producing the intermediate good giving the wage $w_t$ per hour, or to do research which gives an expected $\lambda$ new innovations per extra hour. Each new innovation leads to the creation of a monopoly market for the new intermediate input $x_{t+1}$ which gives rise to an income stream with net present value $V_{t+1}$.

- Since we do neglect capital accumulation in the model the interest rate, $r$, is exogenous.

- The discounted value $V_{t+1}$ must then obey

$$rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} V_{t+1} \tag{32}$$

  A patent on good number $t+1$ is an asset with expected return $rV_{t+1}$ per unit calendar time. On the right hand side we have the profit from production and monopoly sales of $x_{t+1}$ given by $\pi_{t+1}$ per unit calendar time. At some future point the asset will become worthless, since the good is obsolete after a yet newer innovation. This occurs with probability $\lambda n_{t+1}$ per unit of calendar time, so $\lambda n_{t+1} V_{t+1}$ is the expected capital loss per unit of calendar time. Equality between the two sides is thus a standard asset equation.

- Equation (32) can be reorganized to

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}} \tag{33}$$

  that is, the income stream is discounted by a factor which is larger than the interest rate $r$ because at some future time (characterized by the probability $\lambda n_{t+1}$) the income stream will stop.

- The final (consumption) good sector is competitive. It follows that the price of $x_t$ will equal its marginal product in production of $Y$, which gives an inverse demand function for the input $x_t$

$$p_t(x_t) = A_t \alpha x_t^{\alpha-1}$$

Note that $\alpha$ is the elasticity of this demand curve, thus also characterizing the market power of the intermediate monopolist.

- Now we have to determine the profit flow $\pi_t$ and the allocation of labor $x_t$. This is found from the profit maximization of the monopolist producer of the intermediate good

$$\pi_t = \max_{x_t}[p_t(x_t)x_t - w_t x_t]$$

- The monopolist then chooses

$$x_t = \left(\frac{\alpha^2}{w_t/A_t}\right)^{1/(1-\alpha)} \tag{34}$$

or equivalently

$$A_t \alpha^{\alpha-1} = \frac{w}{\alpha}$$

and hence the profit is

$$\pi_t = \left(\frac{1}{\alpha} - 1\right) w_t x_t = A_t \tilde{\pi}(\omega_t) \tag{35}$$

where $\omega_t = w_t/A_t$ is the productivity adjusted wage rate, and

$$\tilde{\pi}(\omega_t) \equiv \left[\left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} \omega_t^{\frac{\alpha}{\alpha-1}}\right] / A_t$$

so $\tilde{\pi}'(\omega_t) < 0$. Note from (34) that also $x_t'(\omega_t) < 0$.

- Inserting in (33) and rearranging (remember that $A_{t+1} = \gamma A_t$) we get

$$\omega_t = \lambda \frac{\gamma \tilde{\pi}(\omega_{t+1})}{r + \lambda n_{t+1}} \tag{36}$$

which together with equilibrium in the labor market

$$L = n_t + \tilde{x}(\omega_t) \tag{37}$$

where $x_t = \tilde{x}(\omega_t)$ determine the model by determining the paths of $\omega_t$ and $n_t$.

- We focus on the steady state where $\omega_t = \omega$ and $n_t = n$ for all $t$. Since (36) is downward sloping and (37) upward sloping in the $(n, \omega)$-plane we can determine a unique $\hat{n}$.

- From straightforward algebra it follows that the steady state value $\hat{n}$ satisfies

$$1 = \lambda \frac{\gamma \frac{1-\alpha}{\alpha}(L - \hat{n})}{r + \lambda \hat{n}} \tag{38}$$

- Notice that the amount of research ($\hat{n}$) determines the growth. Aghion and Howitt (1998) (Section 2.2.2)

$$g = \lambda \hat{n} \ln \gamma$$

## 15.3 Incentives for R&D and implications for policy

- We have the following results

  1. An decrease in the interest rate ($r$) raises the benefit to research (future income more valuable) and raises $\hat{n}$.
  2. An increase in available labor ($L$) reduces the wage and hence the marginal cost of research as well as raising the expected monopoly profit (increased demand).
  3. The more likely are new innovations the less costly is research (higher $\lambda$). But the less worth is a new innovation because it increases creative destruction. As specified here, the former effect is dominant so an increased $\lambda$ leads to a rise in $\hat{n}$.
  4. Increasing the size of each innovation ($\gamma$) raises the next intervals monopoly profit relative to today's productivity and hence raises the incentives to do research.
  5. The amount of research decreases in the elasticity of the demand curve facing the monopolist ($\alpha$). So product market competition is bad for growth.

- When comparing the decentralized solution with that chosen by a social planner which is

$$1 = \lambda \frac{(\gamma - 1)\frac{1}{\alpha}(L - \hat{n}^*)}{r - \lambda \hat{n}^*(\gamma - 1)} \tag{39}$$

  there are three effects.

  1. *The intertemporal spillover effect.* The social planner takes into account that the benefit to the next innovation will continue forever, whereas the private reserch firm attaches no weight to the benefits that arrive beyond the succeeding innovation. This effects lead to too little private research.
  2. *The appropriability effect.* As in the Romer model, the private monopolist can not appropriate the entire increase in social surplus from the new intermediate good and hence has too weak incentives for research.
  3. *The business-stealing effect.* The private firm does not internalize that the monopolist it replaces looses its entire profit. This leads to too strong incentives for research.

- Which effect dominates is an empirical question. However, this model has an important difference form the Romer model in that it opens up the possibility that laissez-faire growth can be too high.

# References


[1] **Aghion, P., Howitt, P.** *Endogenous growth theory*, Cambridge and London: MIT Press, 1998, Chapter 2.

[2] **Romer, Paul M.** Endogenous Technological Change, *Journal of Political Economy,* October 1990, 98 (5) . Pp. 71-102. Part 2.