

INTERMEDIATE

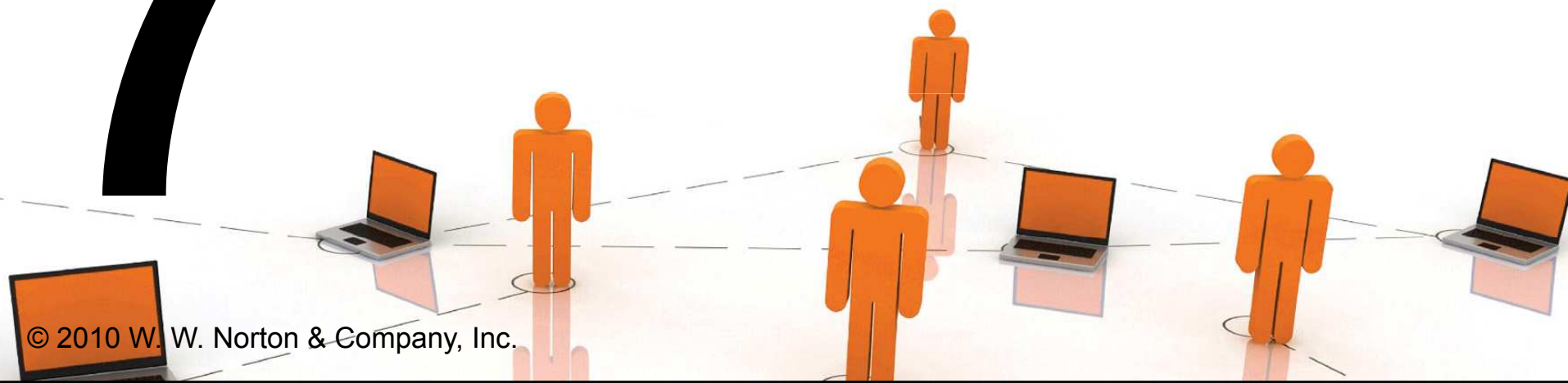
8TH EDITION

# MICROECONOMICS

HAL R. VARIAN

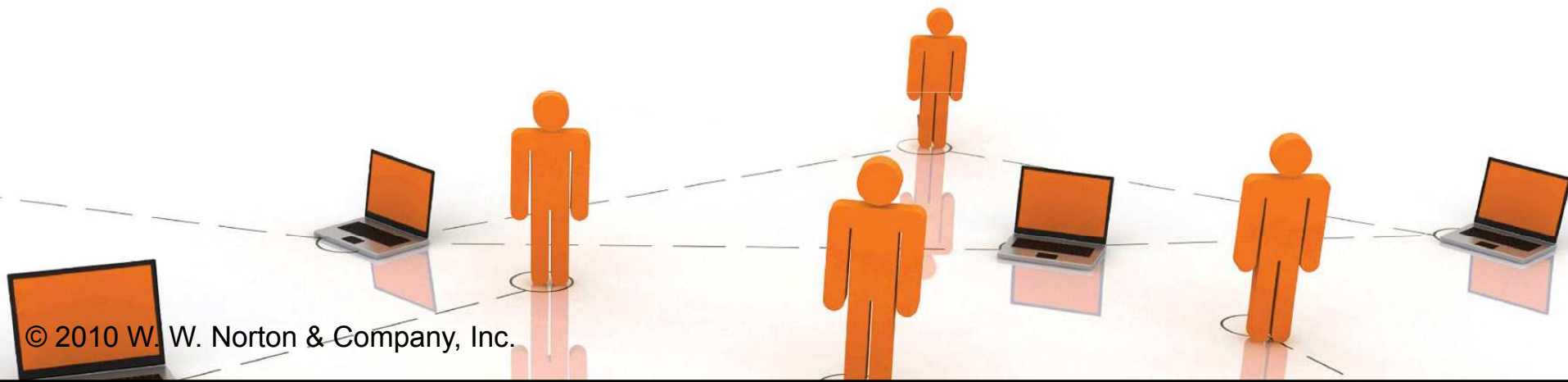
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## Revealed Preference



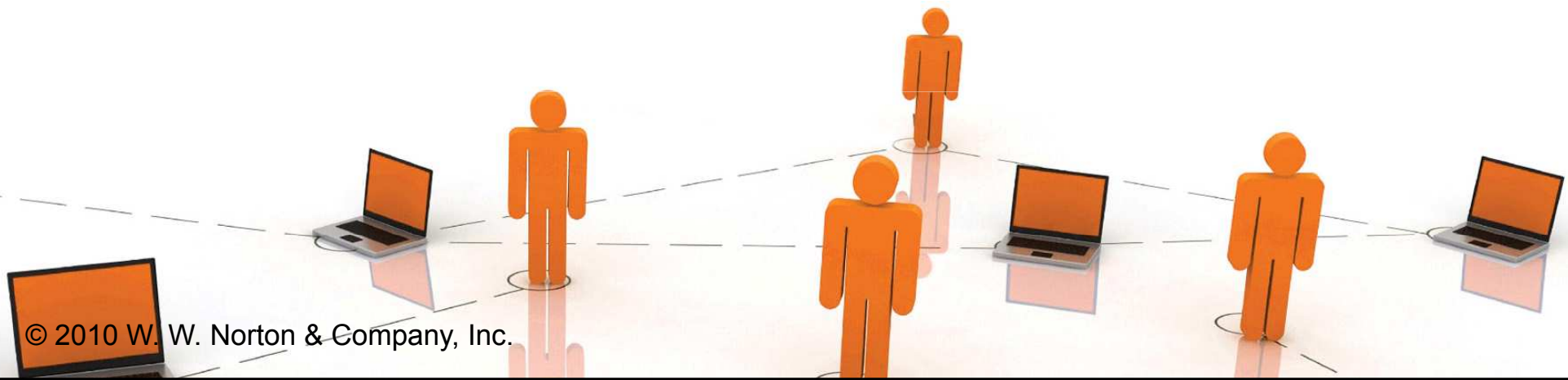
# Revealed Preference Analysis

- ◆ **Suppose we observe the demands (consumption choices) that a consumer makes for different budgets. This reveals information about the consumer's preferences. We can use this information to ...**



# Revealed Preference Analysis

- **Test the behavioral hypothesis that a consumer chooses the most preferred bundle from those available.**
- **Discover the consumer's preference relation.**



# Assumptions on Preferences

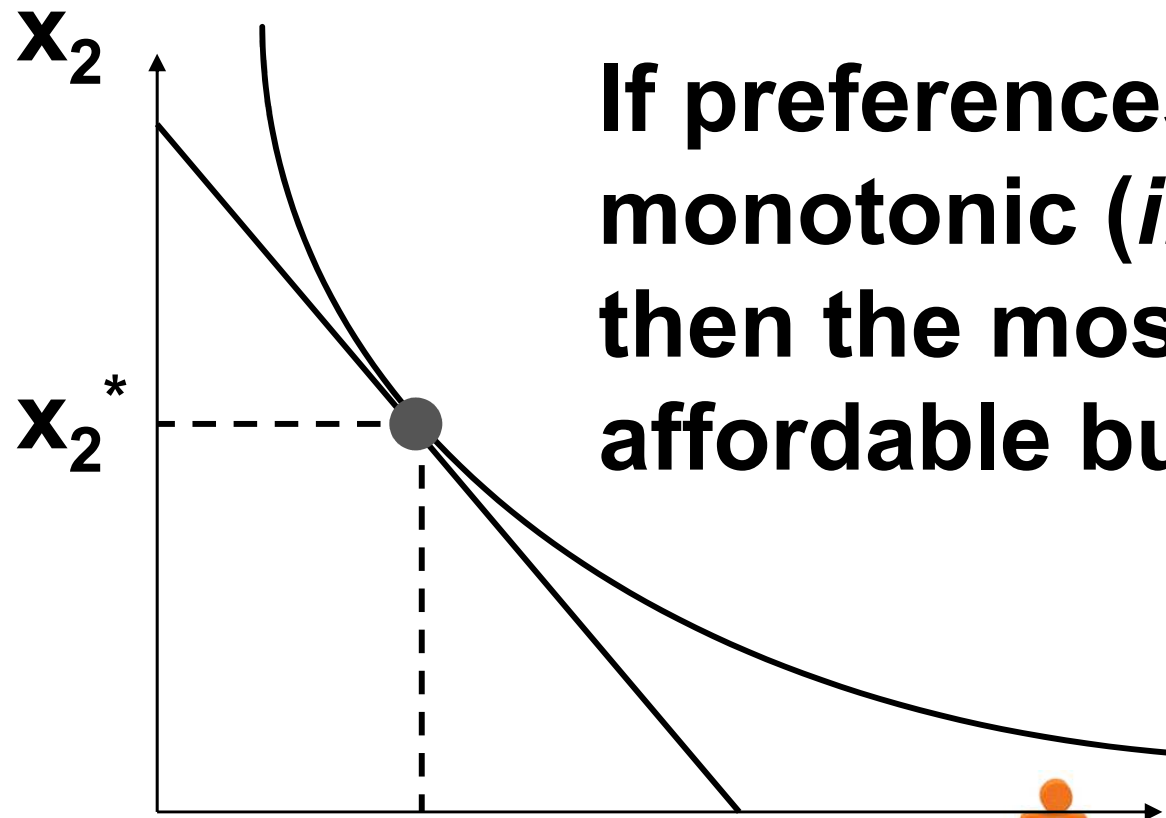
## ◆ Preferences

- do not change while the choice data are gathered.
- are strictly convex.
- are monotonic.

## ◆ Together, convexity and monotonicity imply that the most preferred affordable bundle is unique.



# Assumptions on Preferences



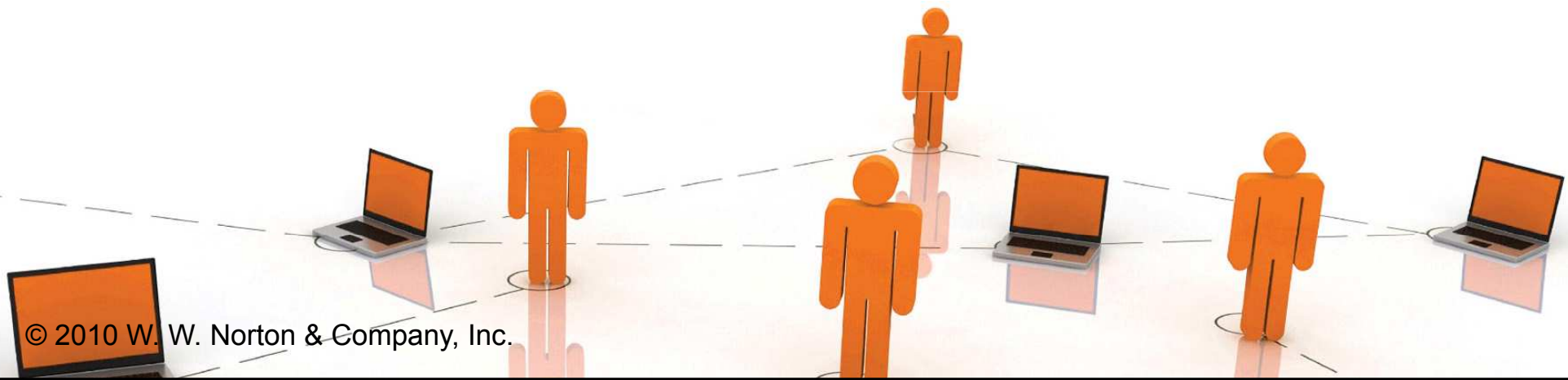
**If preferences are convex and monotonic (*i.e.* well-behaved) then the most preferred affordable bundle is unique.**

$x_1^*$

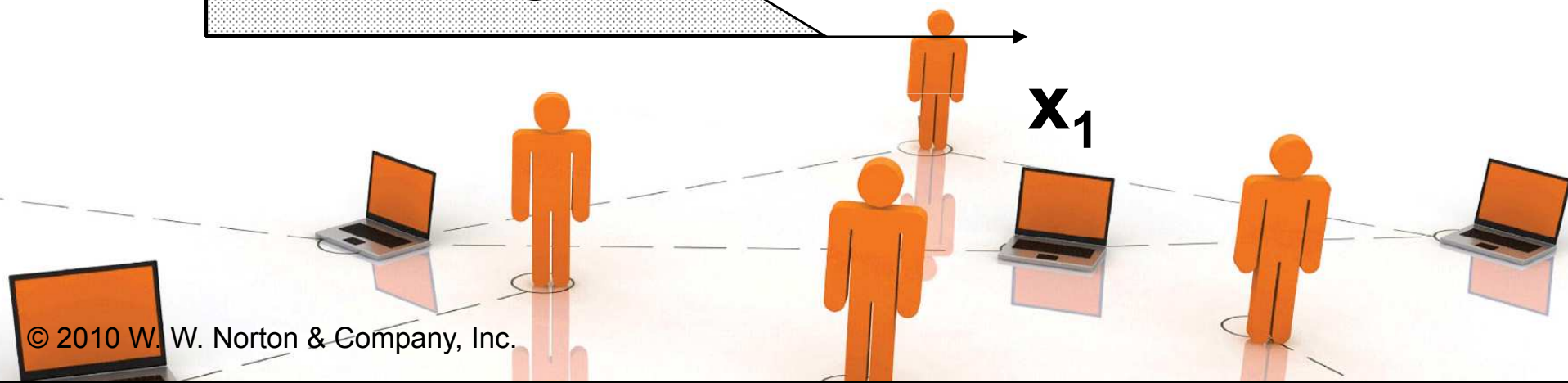
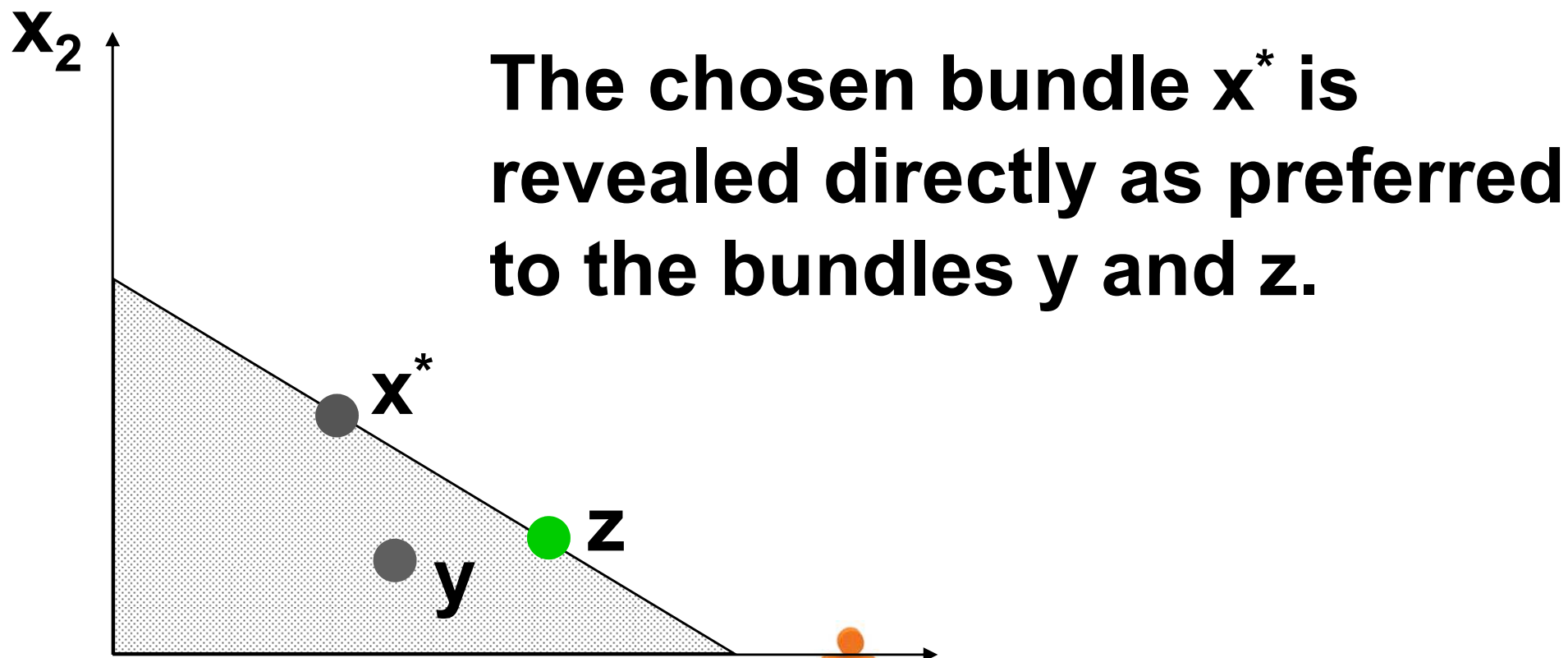
$x_1$

# Direct Preference Revelation

- ◆ **Suppose that the bundle  $x^*$  is chosen when the bundle  $y$  is affordable. Then  $x^*$  is revealed directly as preferred to  $y$  (otherwise  $y$  would have been chosen).**



# Direct Preference Revelation



# Direct Preference Revelation

- ◆ That  $x$  is revealed directly as preferred to  $y$  will be written as

$$x \succ_D y.$$



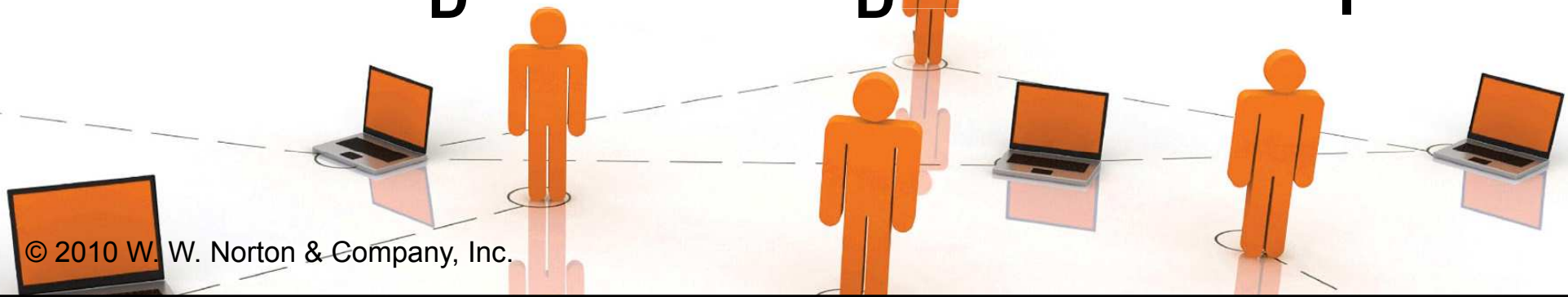


# Indirect Preference Revelation

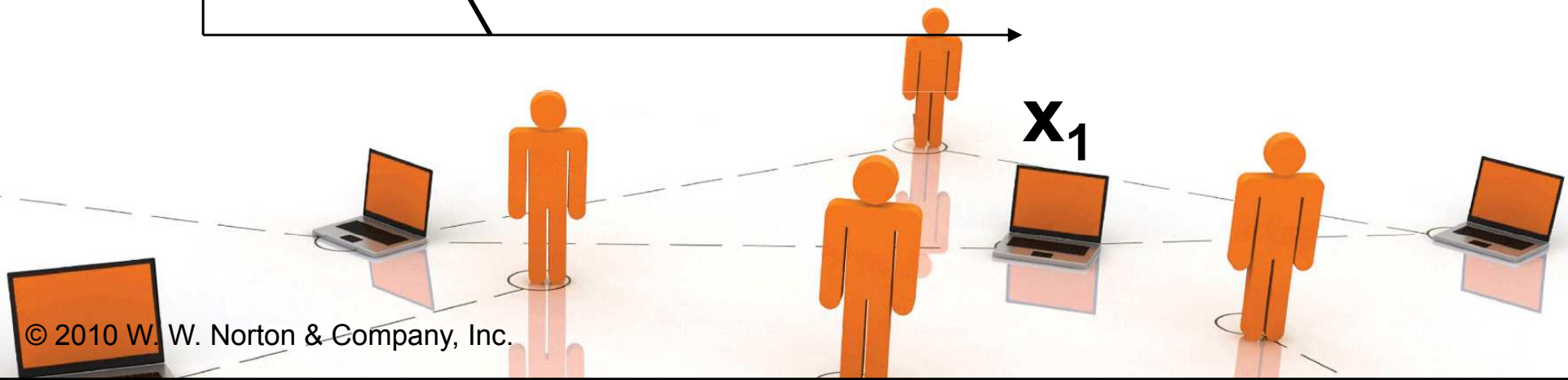
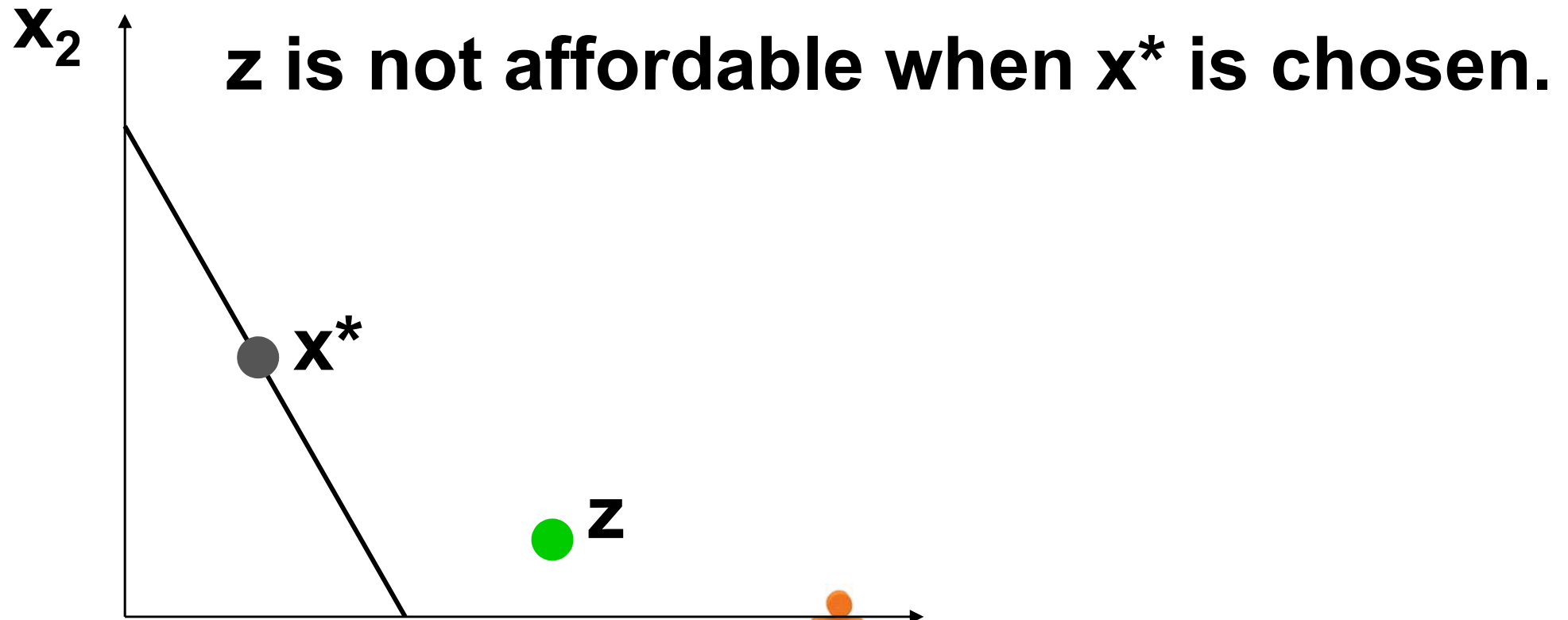
- ◆ Suppose  $x$  is revealed directly preferred to  $y$ , and  $y$  is revealed directly preferred to  $z$ . Then, by transitivity,  $x$  is revealed indirectly as preferred to  $z$ . Write this as

$$x \succsim_I z$$

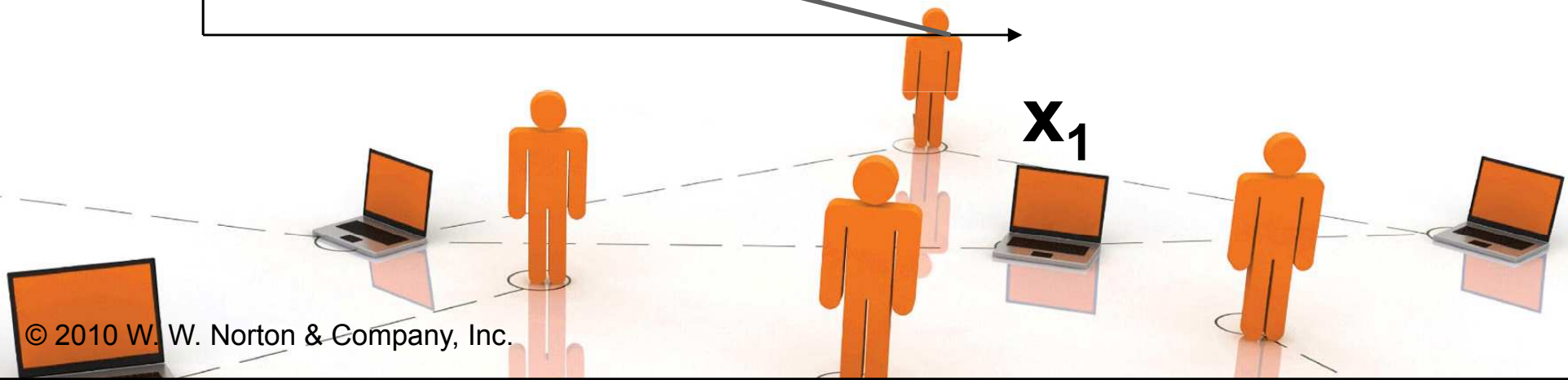
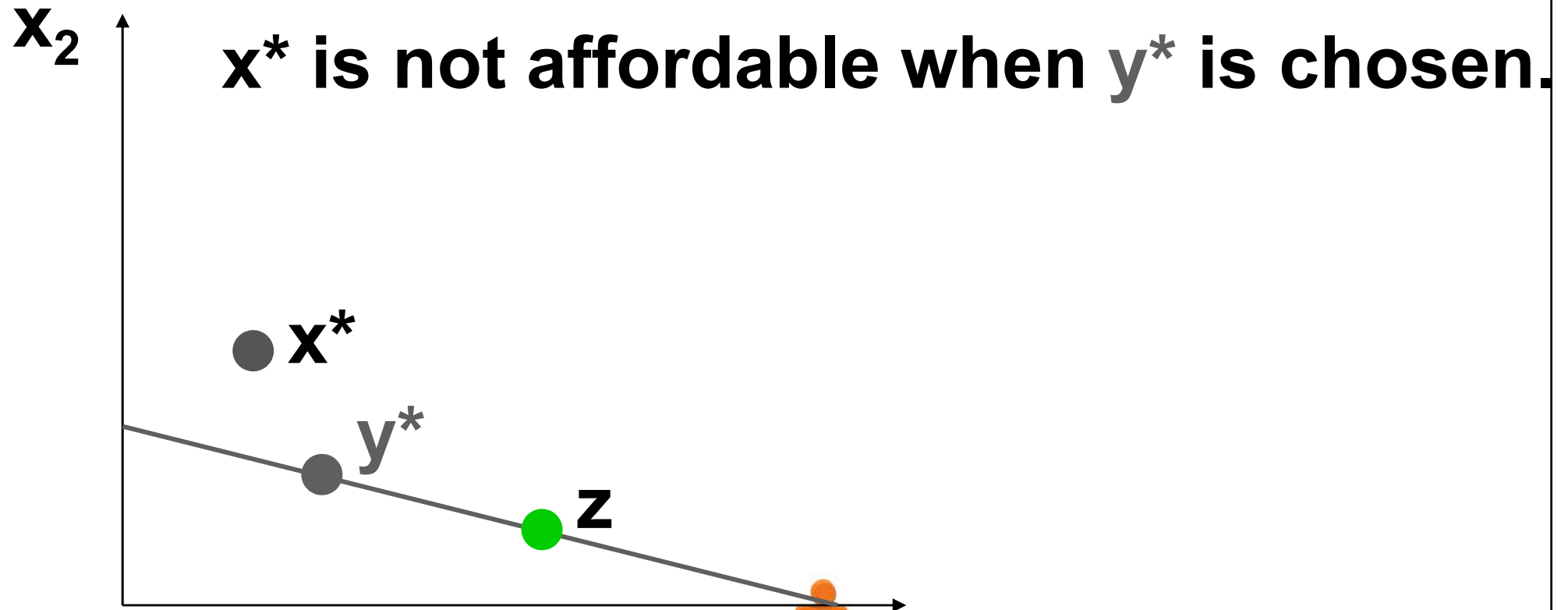
so  $x \succ_D y$  and  $y \succ_D z \Rightarrow x \succ_I z$ .



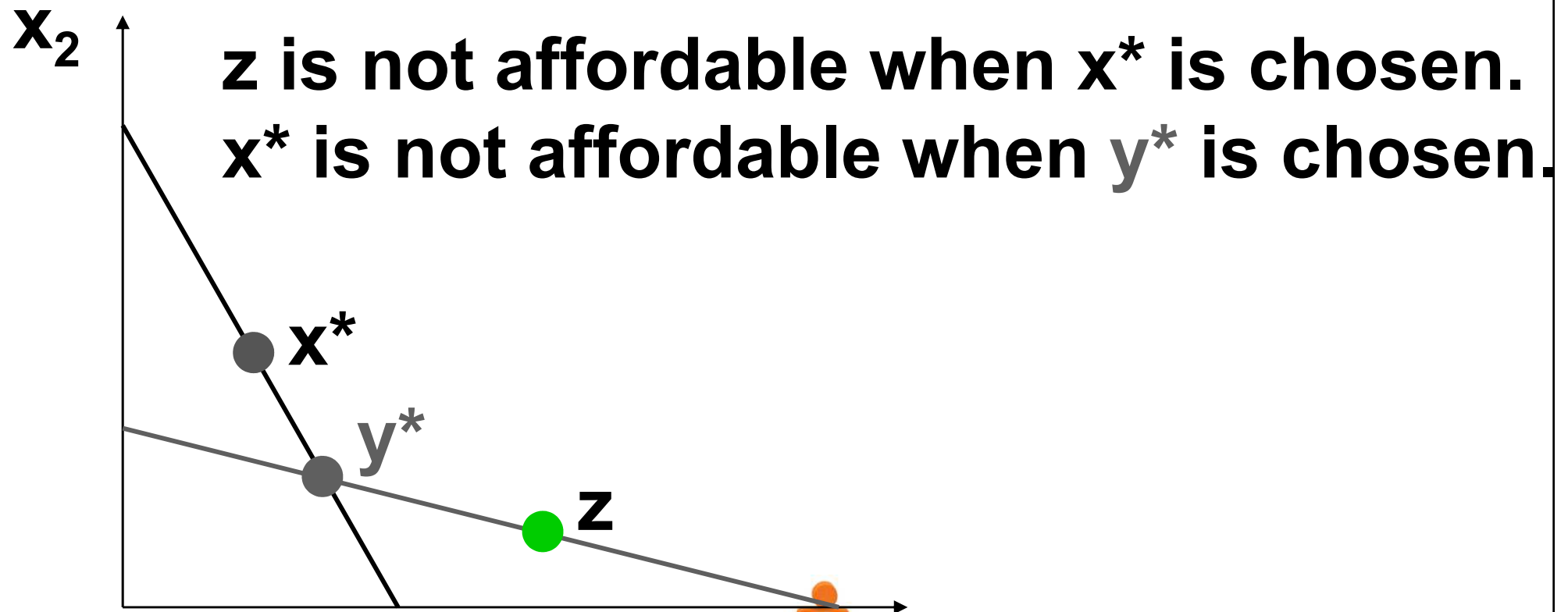
# Indirect Preference Revelation



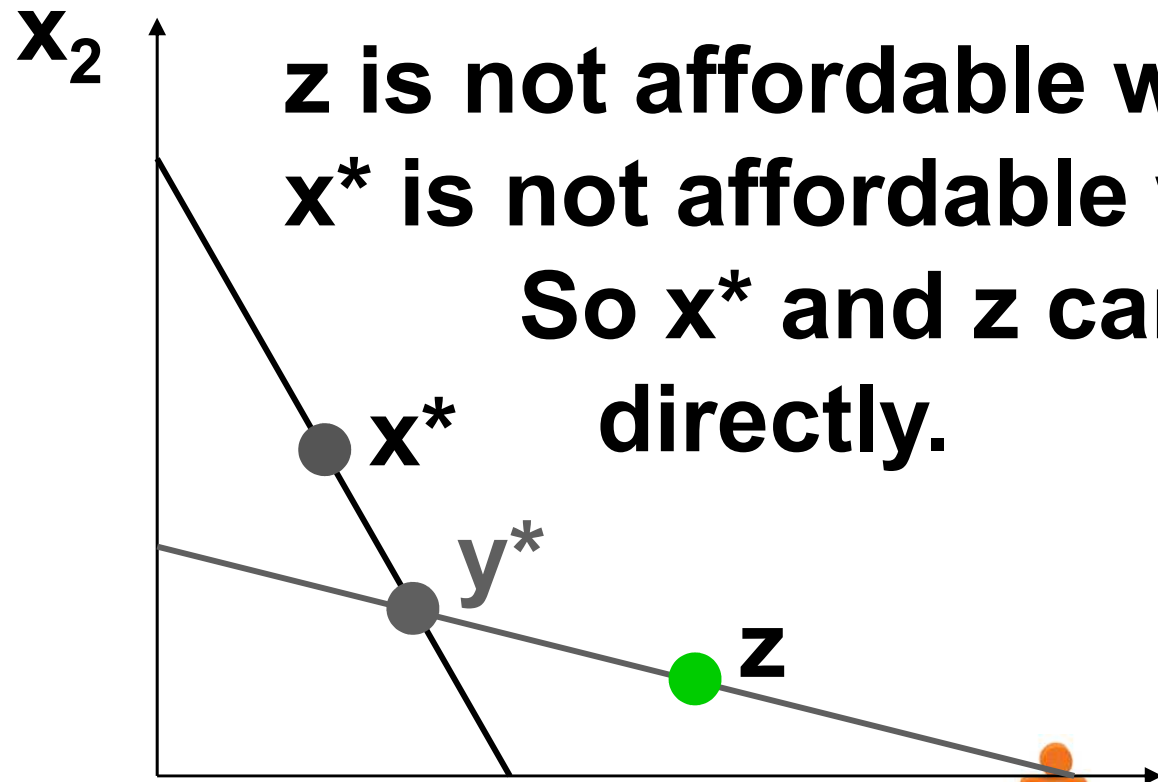
# Indirect Preference Revelation



# Indirect Preference Revelation



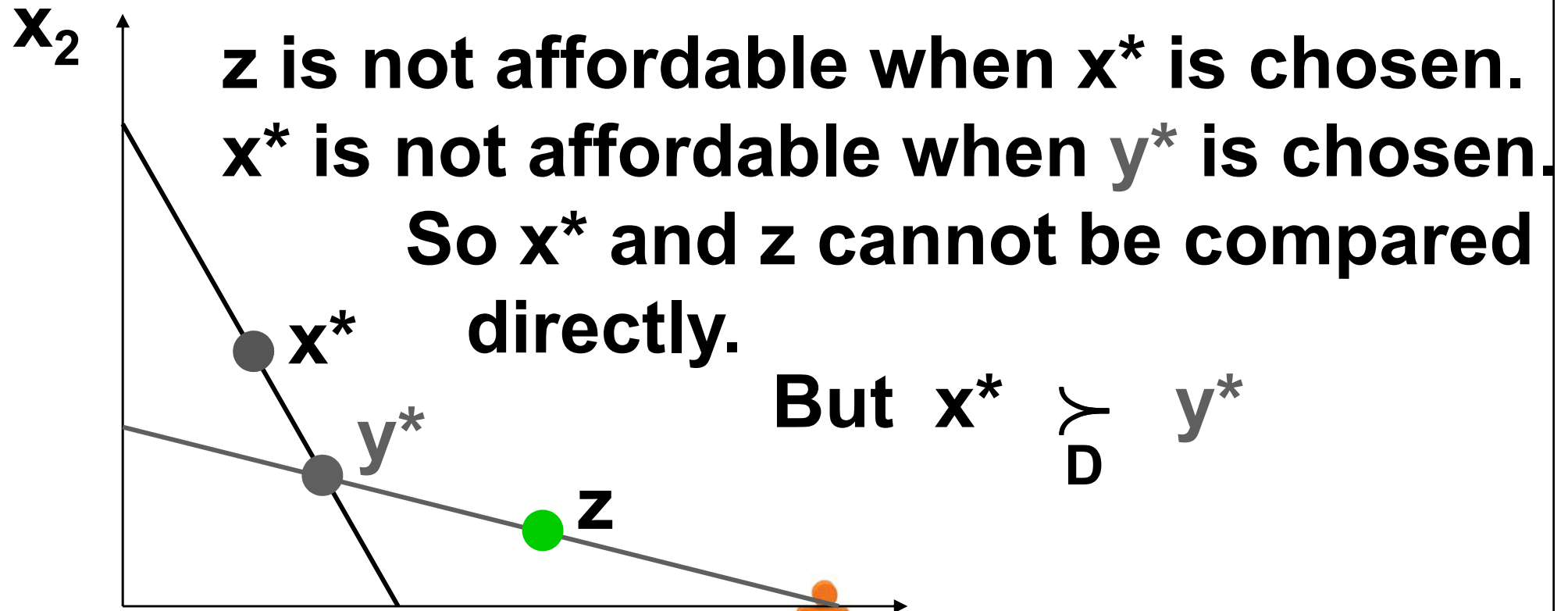
# Indirect Preference Revelation



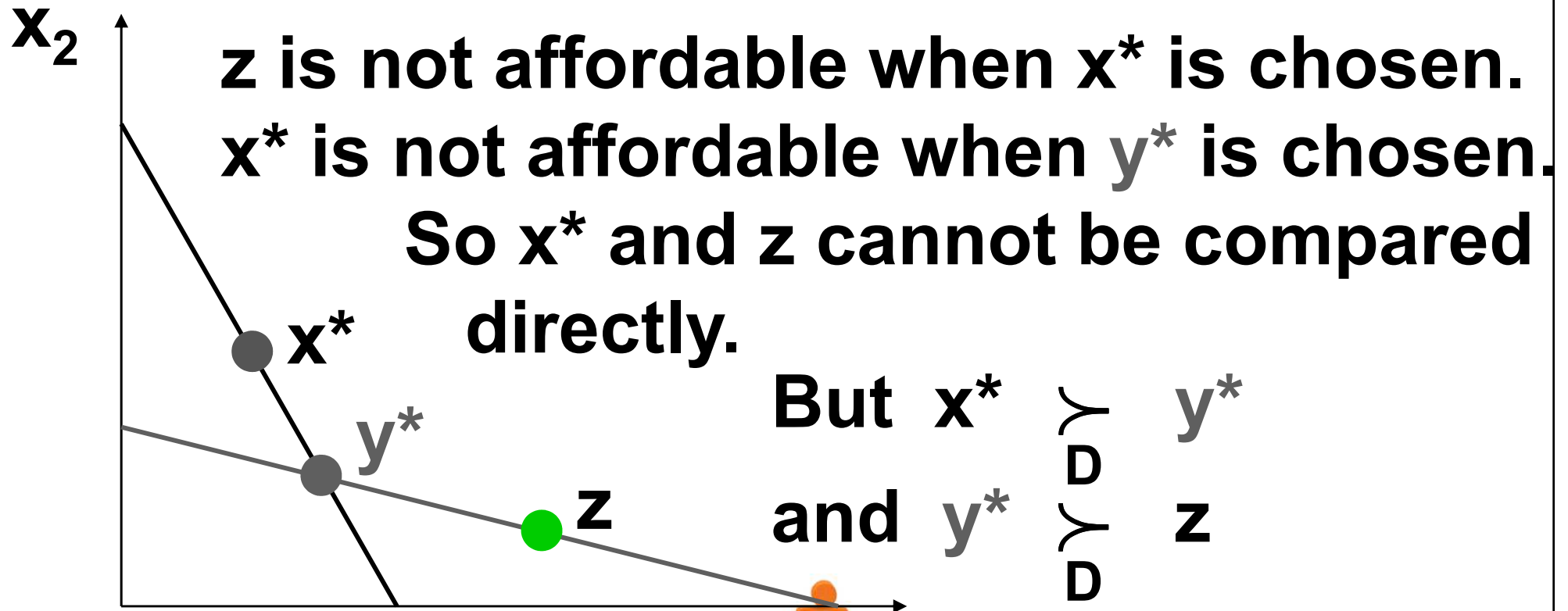
**$z$  is not affordable when  $x^*$  is chosen.**  
 **$x^*$  is not affordable when  $y^*$  is chosen.**  
**So  $x^*$  and  $z$  cannot be compared directly.**



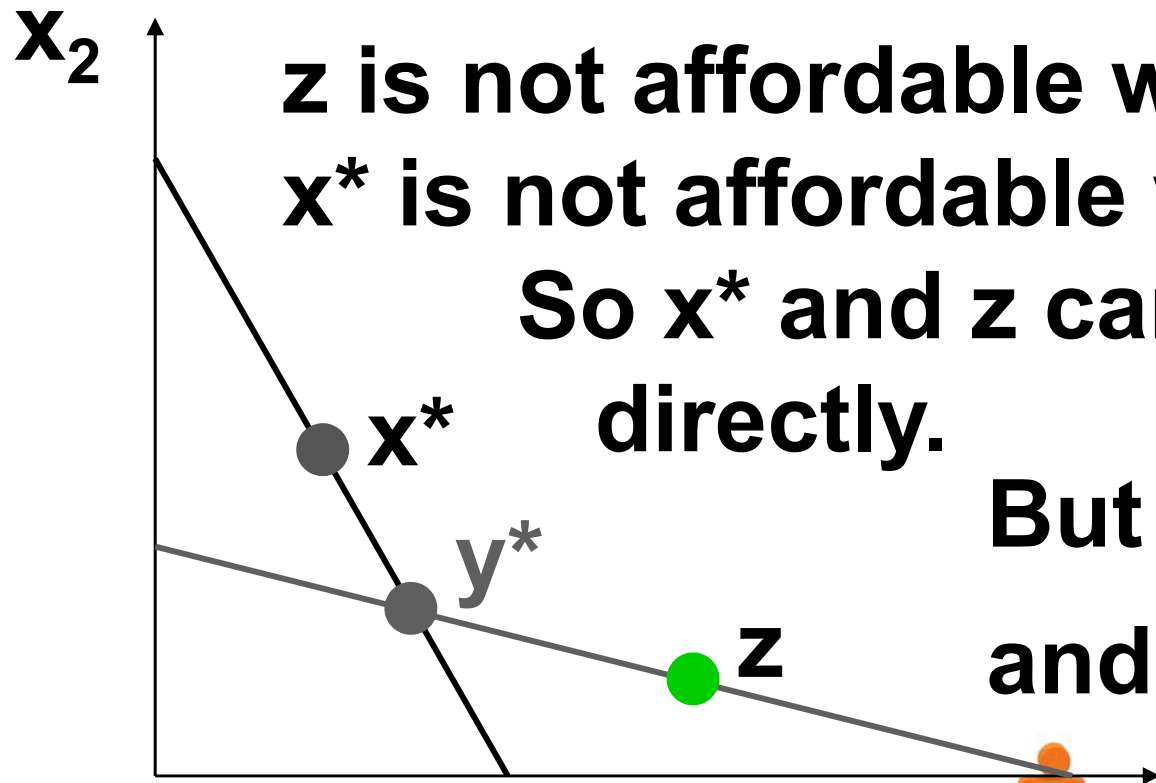
# Indirect Preference Revelation



# Indirect Preference Revelation



# Indirect Preference Revelation



**z is not affordable when  $x^*$  is chosen.  
 $x^*$  is not affordable when  $y^*$  is chosen.  
 So  $x^*$  and z cannot be compared directly.**

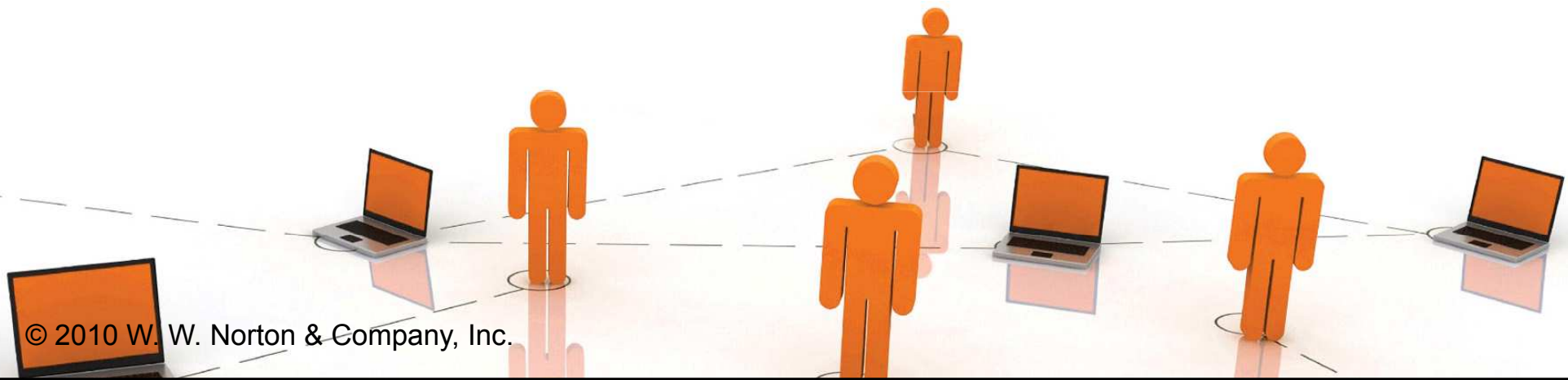
**But  $x^* \succ y^*$   
 and  $y^* \succ z$   
 so  $x^* \succ z$ .**





# Two Axioms of Revealed Preference

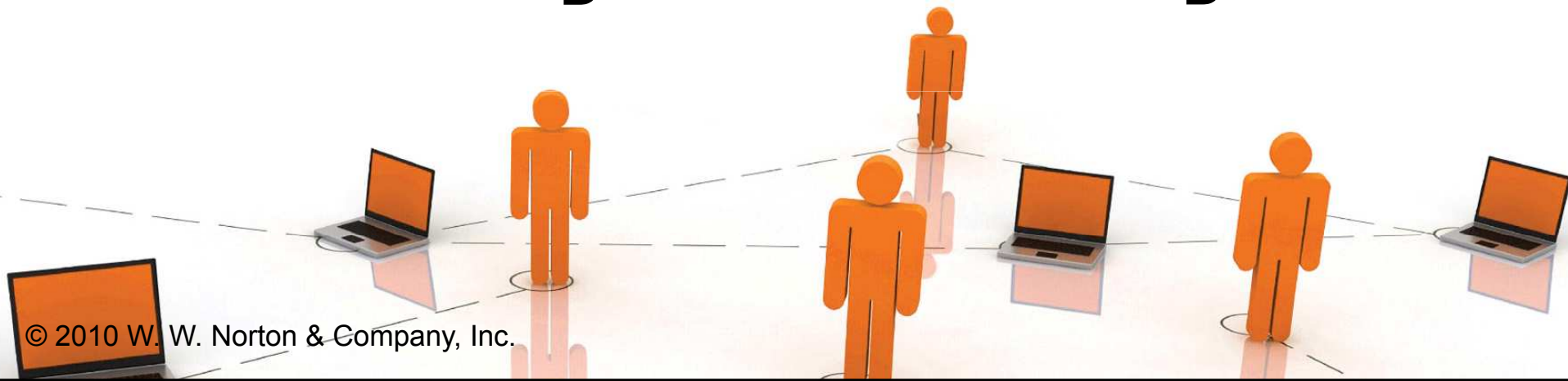
- ◆ **To apply revealed preference analysis, choices must satisfy two criteria -- the Weak and the Strong Axioms of Revealed Preference.**



# The Weak Axiom of Revealed Preference (WARP)

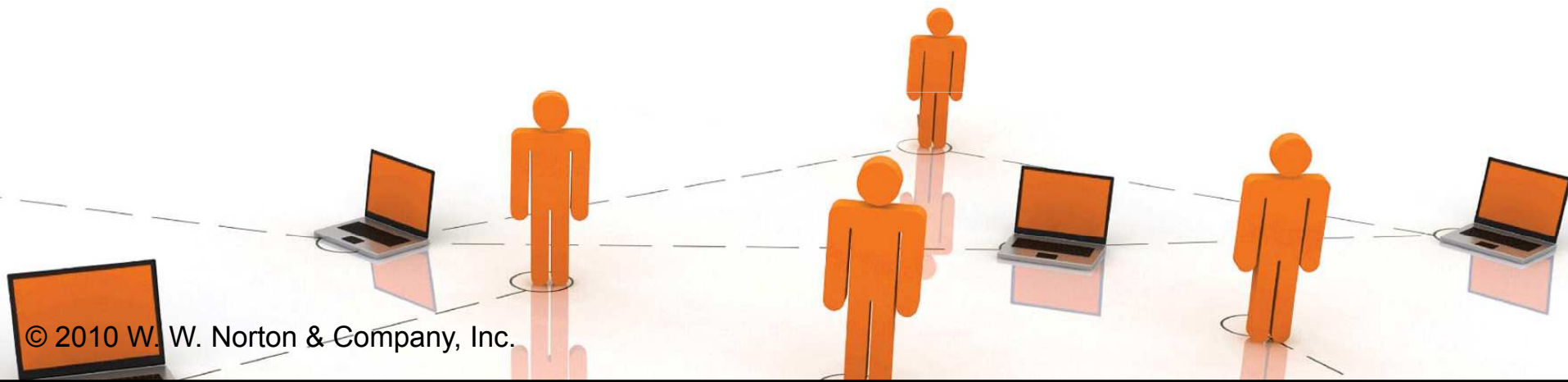
- ◆ If the bundle  $x$  is revealed directly as preferred to the bundle  $y$  then it is never the case that  $y$  is revealed directly as preferred to  $x$ ; *i.e.*

$$x \succ_D y \longrightarrow \text{not } (y \succ_D x).$$



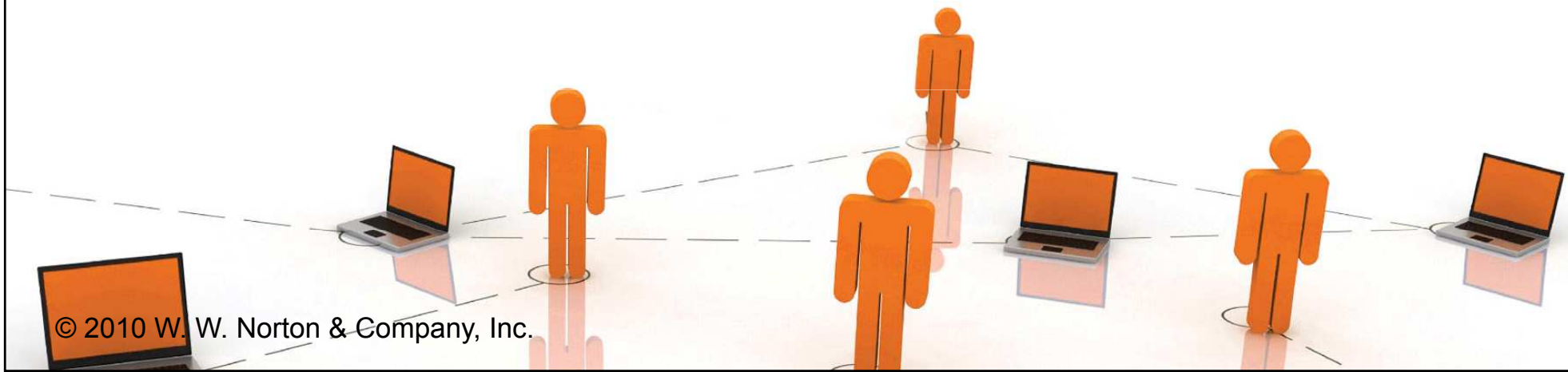
# The Weak Axiom of Revealed Preference (WARP)

- ◆ Choice data which violate the WARP are inconsistent with economic rationality.
- ◆ The WARP is a **necessary** condition for applying economic rationality to explain observed choices.

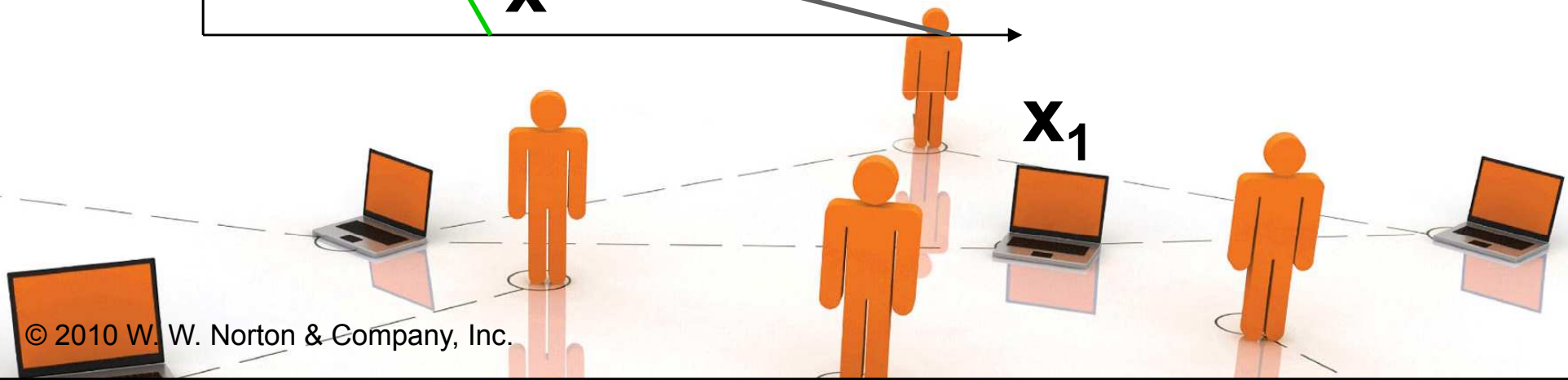
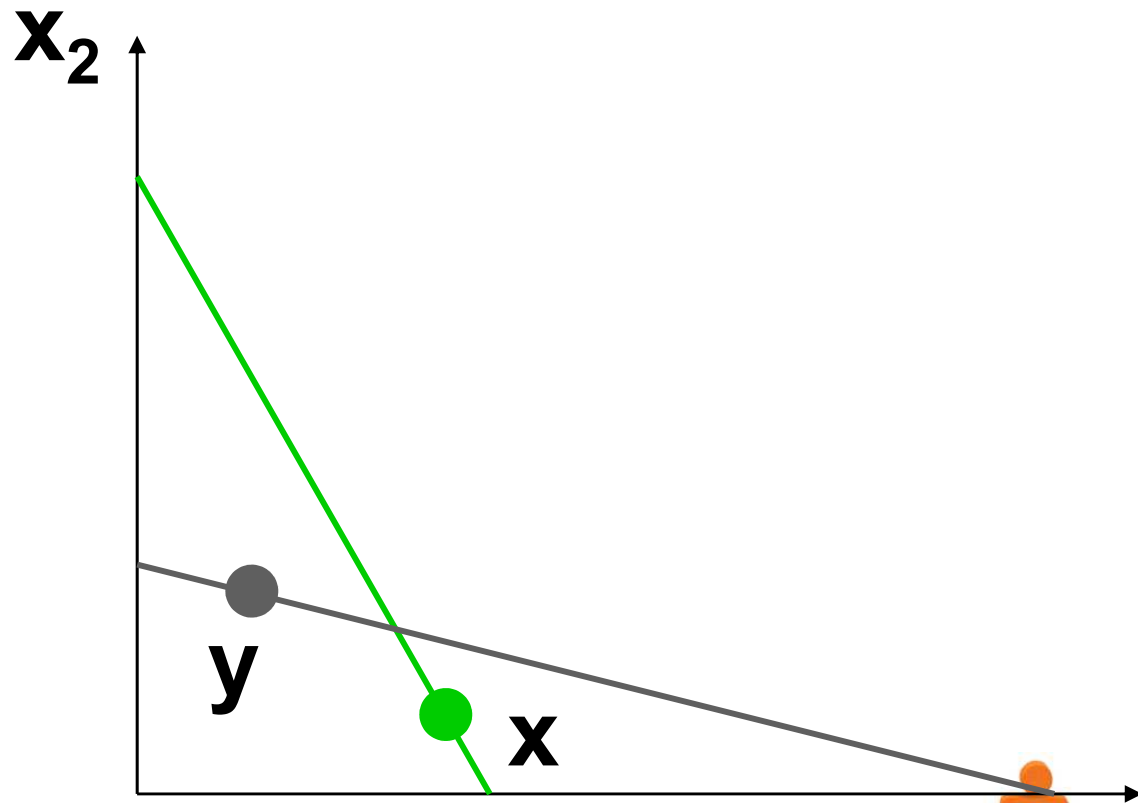


# The Weak Axiom of Revealed Preference (WARP)

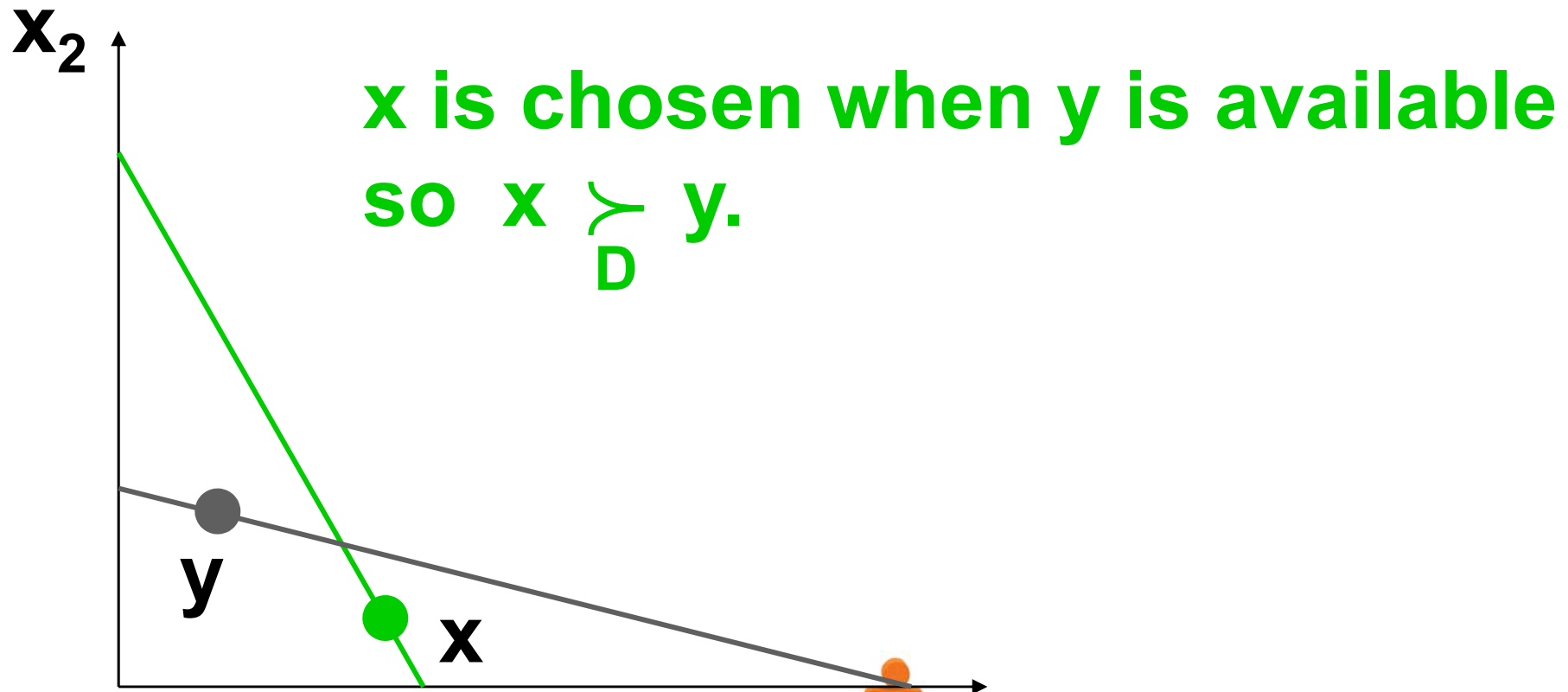
◆ **What choice data violate the WARP?**



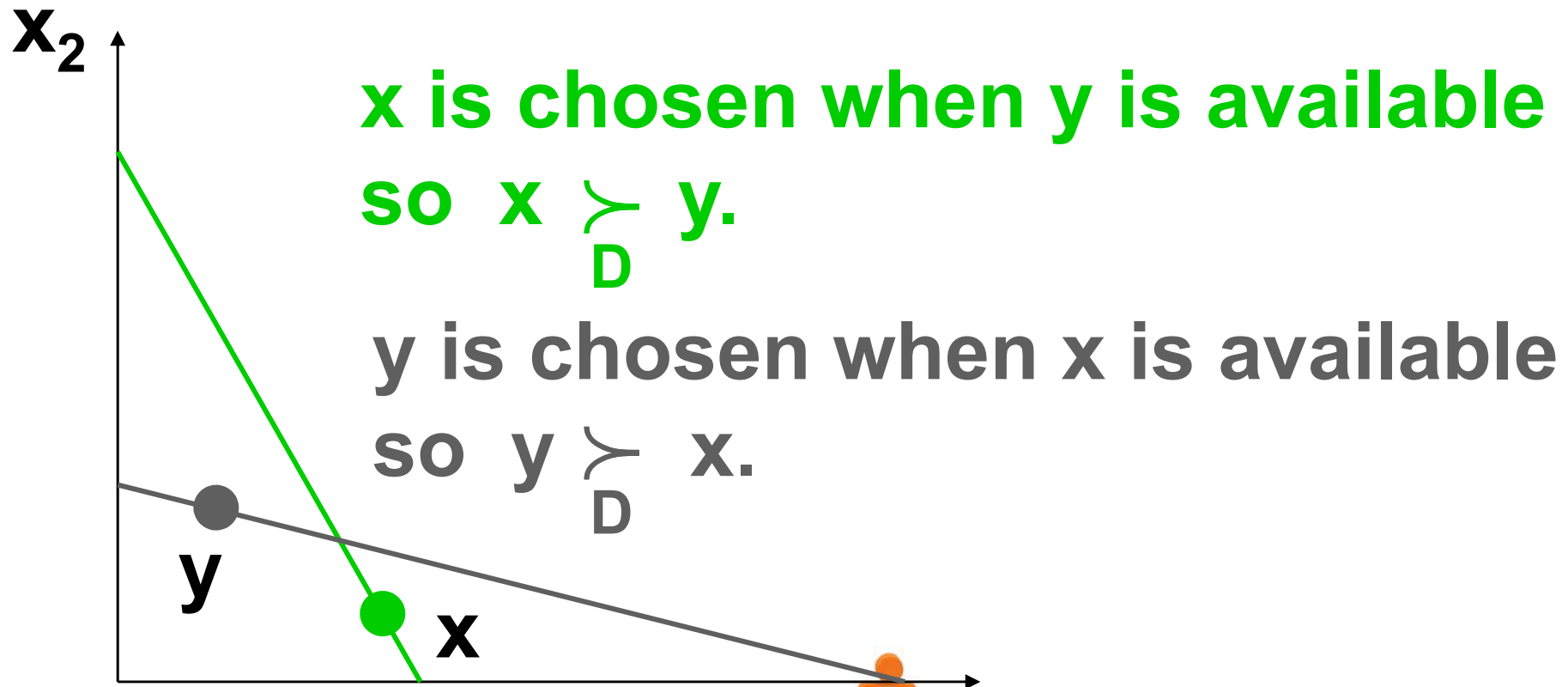
# The Weak Axiom of Revealed Preference (WARP)



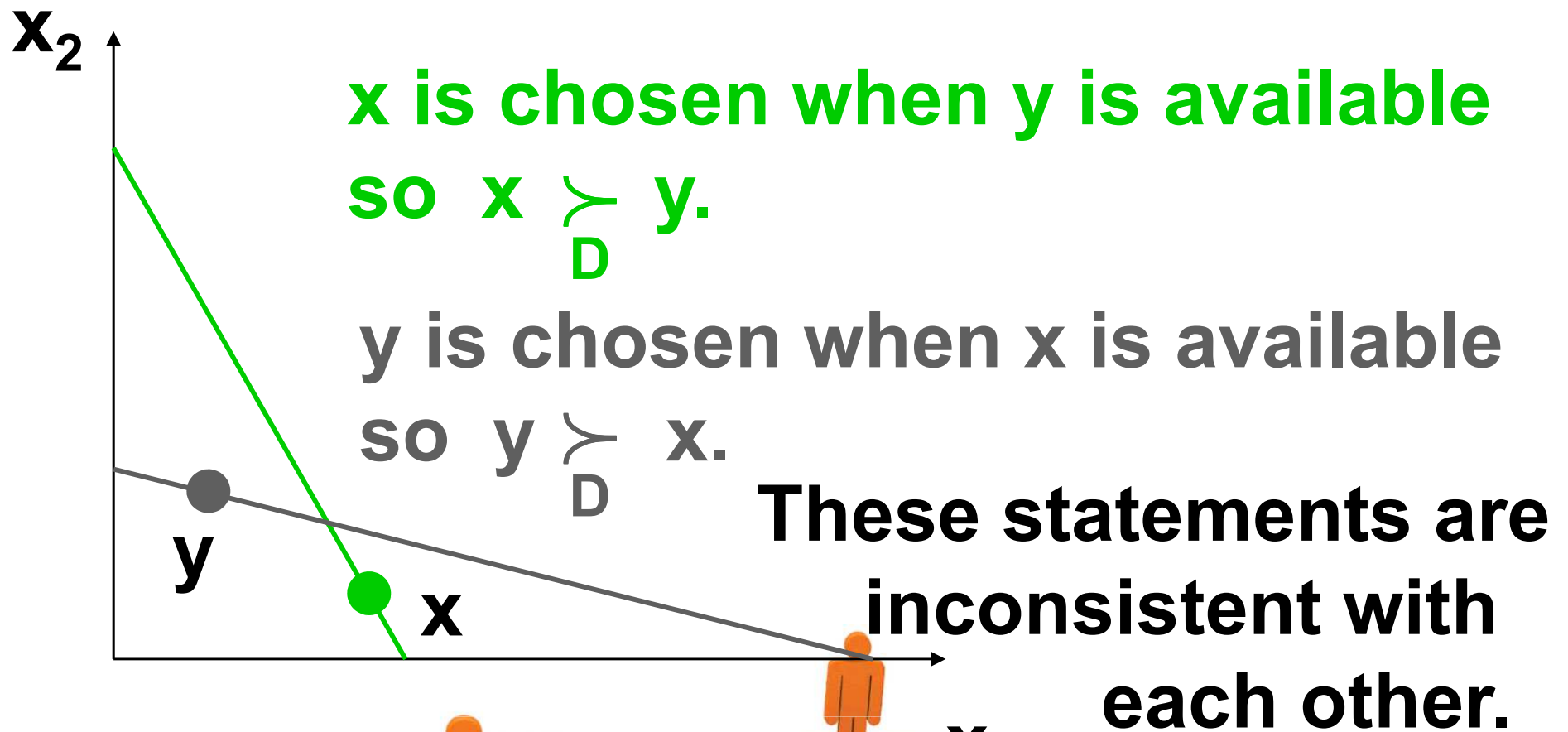
# The Weak Axiom of Revealed Preference (WARP)



# The Weak Axiom of Revealed Preference (WARP)



# The Weak Axiom of Revealed Preference (WARP)





# Checking if Data Violate the WARP

◆ A consumer makes the following choices:

– At prices  $(p_1, p_2) = (\$2, \$2)$  the choice was  $(x_1, x_2) = (10, 1)$ .

– At  $(p_1, p_2) = (\$2, \$1)$  the choice was  $(x_1, x_2) = (5, 5)$ .

– At  $(p_1, p_2) = (\$1, \$2)$  the choice was  $(x_1, x_2) = (5, 4)$ .

◆ Is the WARP violated by these data?

# Checking if Data Violate the WARP

<b>Choices Prices</b>	<b>(10, 1)</b>	<b>(5, 5)</b>	<b>(5, 4)</b>
<b>(\$2, \$2)</b>	\$22	\$20	\$18
<b>(\$2, \$1)</b>	\$21	\$15	\$14
<b>(\$1, \$2)</b>	\$12	\$15	\$13



# Checking if Data Violate the WARP

Choices Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

**Red numbers are costs of chosen bundles.**

# Checking if Data Violate the WARP

Choices Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
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Circles surround affordable bundles that were not chosen.

# Checking if Data Violate the WARP

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**Circles surround affordable bundles that were not chosen.**

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Choices Prices	(10, 1)	(5, 5)	(5, 4)
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Circles surround affordable bundles that were not chosen.

# Checking if Data Violate the WARP

Choices / Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

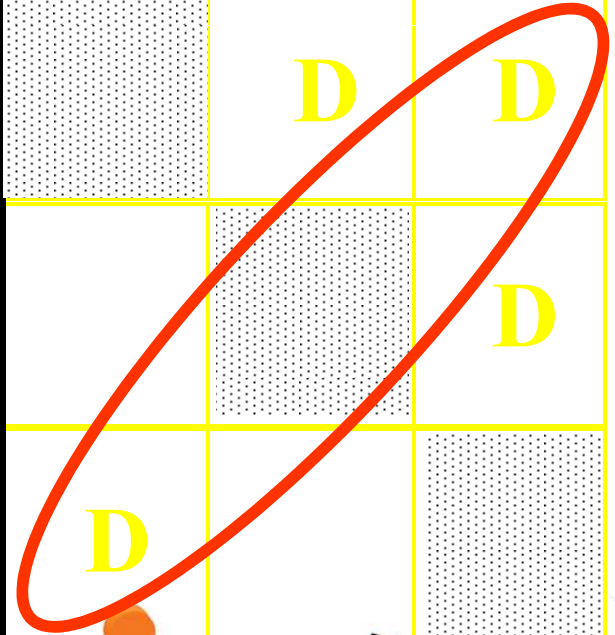
	(10, 1)	(5, 5)	(5, 4)
(10, 1)	D	D	D
(5, 5)		D	D
(5, 4)	D		D

# Checking if Data Violate the WARP

Choices / Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

	(10, 1)	(5, 5)	(5, 4)
(10, 1)	D	D	D
(5, 5)		D	D
(5, 4)			D



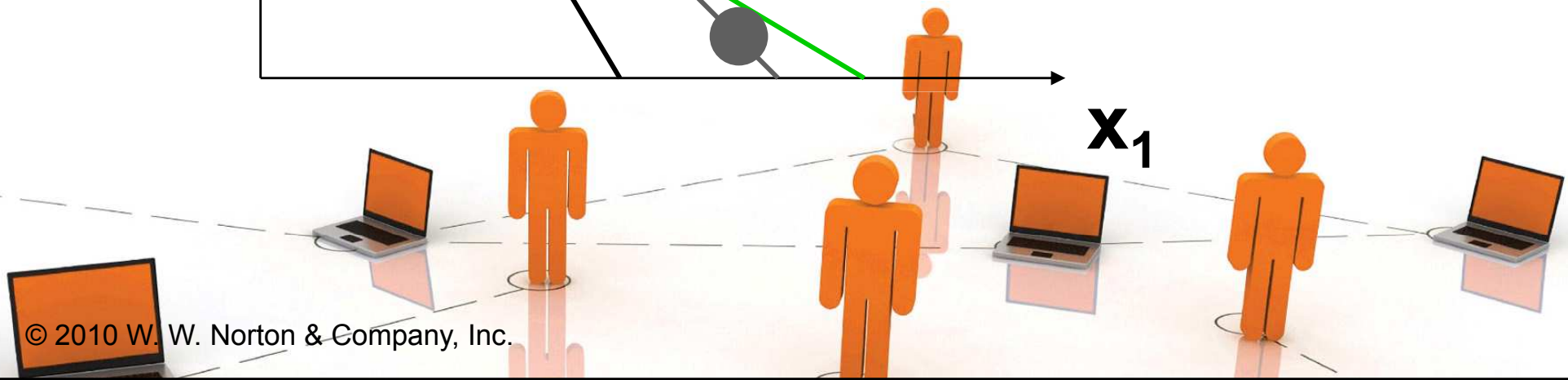
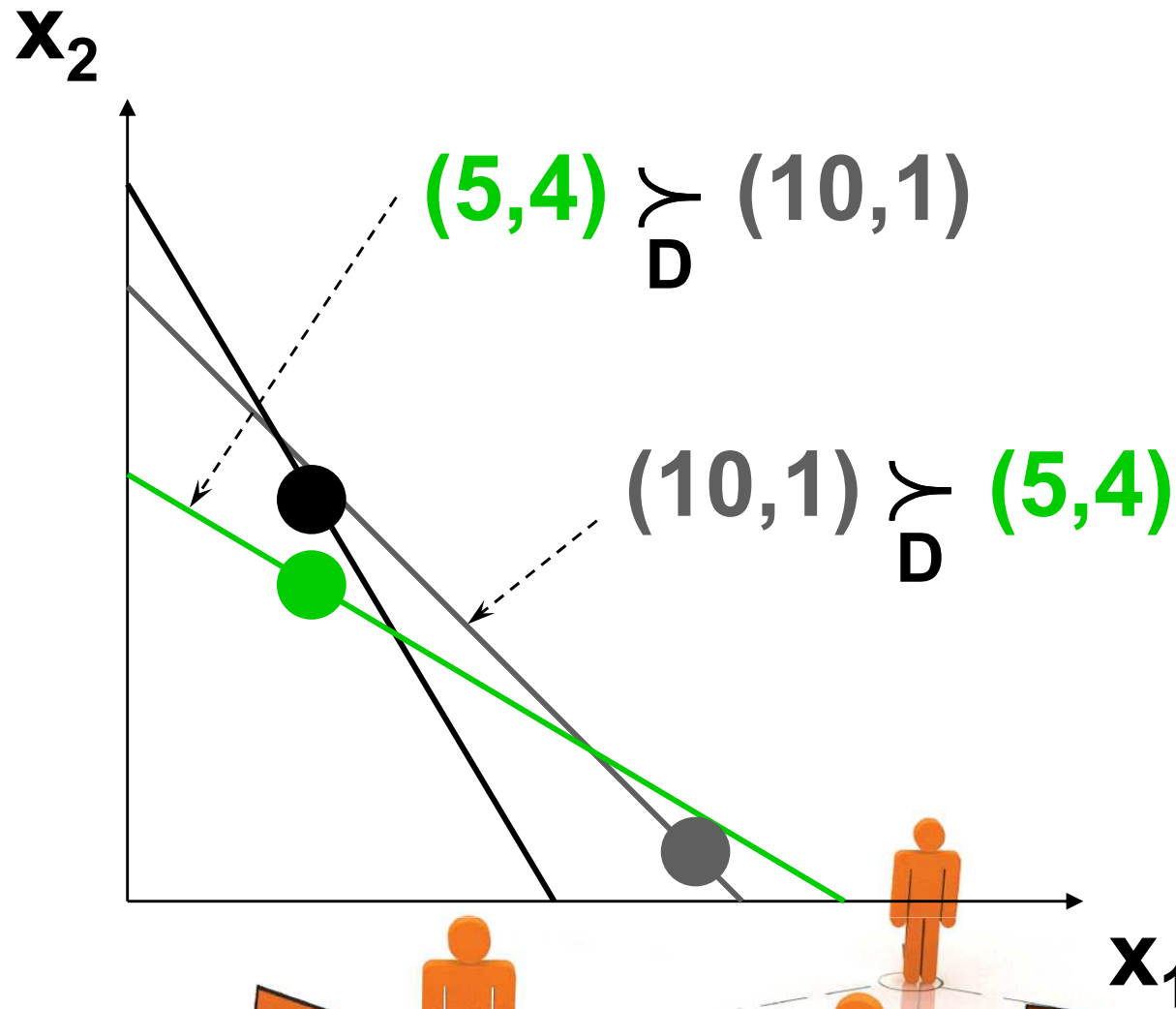


# Checking if Data Violate the WARP

**(10,1) is directly revealed preferred to (5,4), but (5,4) is directly revealed preferred to (10,1), so the WARP is violated by the data.**

	(10, 1)	(5, 5)	(5, 4)
(10, 1)		D	D
(5, 5)			D
(5, 4)	D		

# Checking if Data Violate the WARP



# The Strong Axiom of Revealed Preference (SARP)

- ◆ If the bundle  $x$  is revealed (directly or indirectly) as preferred to the bundle  $y$  and  $x \neq y$ , then it is never the case that the  $y$  is revealed (directly or indirectly) as preferred to  $x$ ; *i.e.*

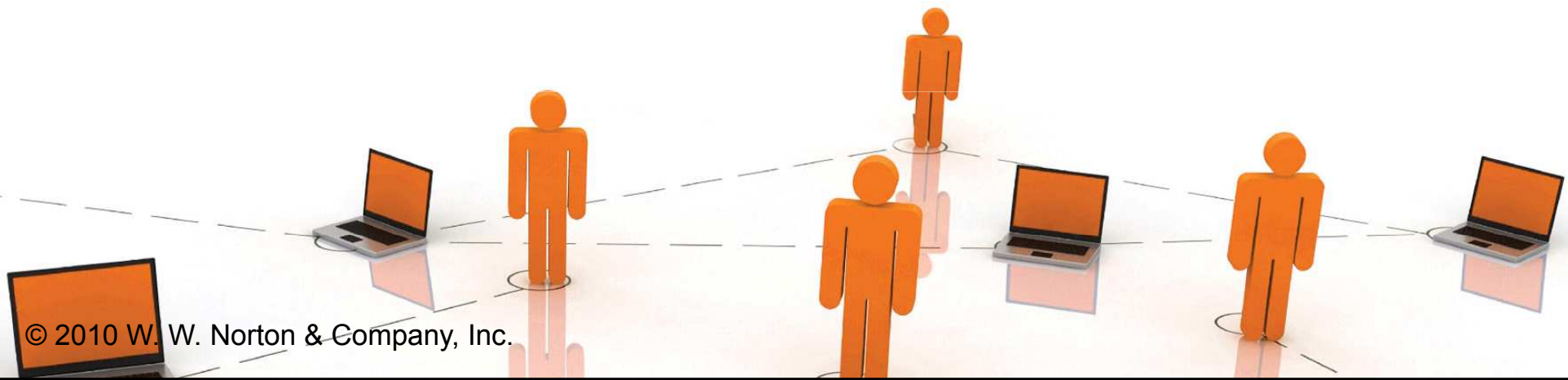
$$x \succ_D y \text{ or } x \succ_I y$$

➔  $\text{not } (y \succ_D x \text{ or } y \succ_I x).$



# The Strong Axiom of Revealed Preference

- ◆ **What choice data would satisfy the WARP but violate the SARP?**



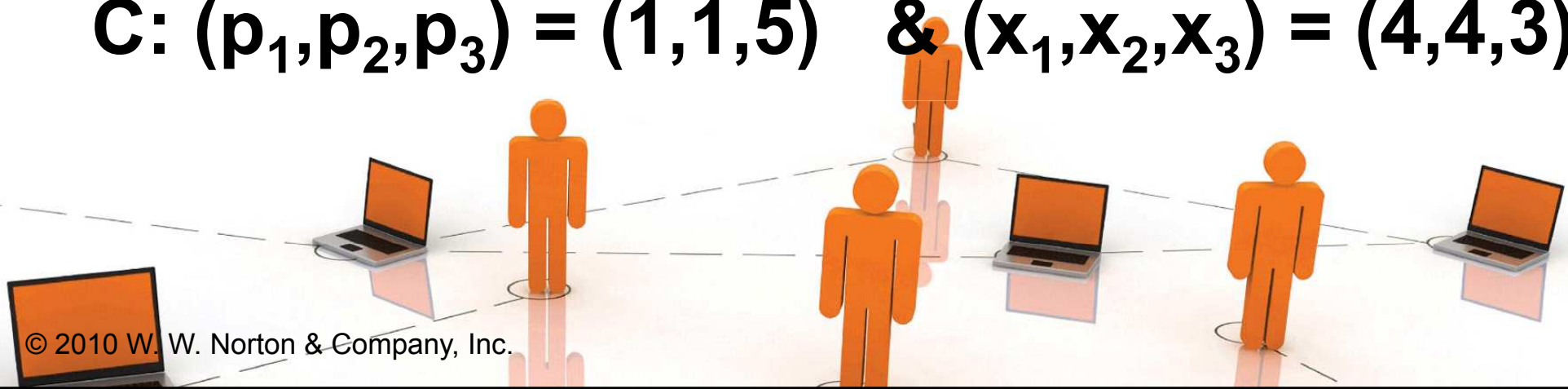
# The Strong Axiom of Revealed Preference

◆ Consider the following data:

$$\mathbf{A}: (p_1, p_2, p_3) = (1, 3, 10) \ \& \ (x_1, x_2, x_3) = (3, 1, 4)$$

$$\mathbf{B}: (p_1, p_2, p_3) = (4, 3, 6) \ \& \ (x_1, x_2, x_3) = (2, 5, 3)$$

$$\mathbf{C}: (p_1, p_2, p_3) = (1, 1, 5) \ \& \ (x_1, x_2, x_3) = (4, 4, 3)$$



# The Strong Axiom of Revealed Preference

**A: (\$1,\$3,\$10)**  
**(3,1,4).**

**B: (\$4,\$3,\$6)**  
**(2,5,3).**

**C: (\$1,\$1,\$5)**  
**(4,4,3).**

<b>Choice Prices</b>	<b>A</b>	<b>B</b>	<b>C</b>
<b>A</b>	<b>\$46</b>	<b>\$47</b>	<b>\$46</b>
<b>B</b>	<b>\$39</b>	<b>\$41</b>	<b>\$46</b>
<b>C</b>	<b>\$24</b>	<b>\$22</b>	<b>\$23</b>

# The Strong Axiom of Revealed Preference

<b>Choices Prices</b>	<b>A</b>	<b>B</b>	<b>C</b>
<b>A</b>	<b>\$46</b>	\$47	\$46
<b>B</b>	\$39	<b>\$41</b>	\$46
<b>C</b>	\$24	\$22	<b>\$23</b>

# The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation A,  
bundle A is  
directly revealed  
preferred to  
bundle C;

$$A \underset{D}{\succ} C.$$



# The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation B,  
bundle B is  
directly revealed  
preferred to  
bundle A;

$$B \underset{D}{\succ} A.$$

# The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation C,  
bundle C is  
directly revealed  
preferred to  
bundle B;

$$C \succ_D B.$$

# The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

	A	B	C
A			D
B	D		
C		D	

# The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

	A	B	C
A			D
B	D		
C		D	

**The data do not violate the WARP.**

# The Strong Axiom of Revealed Preference

We have that

$A \succ_D C$ ,  $B \succ_D A$  and  $C \succ_D B$

so, by transitivity,

$A \succ_D B$ ,  $B \succ_D C$  and  $C \succ_D A$ .

	A	B	C
A			D
B	D		
C		D	

The data do not violate the WARP but ...

# The Strong Axiom of Revealed Preference

We have that

$A \succ_D C$ ,  $B \succ_D A$  and  $C \succ_D B$

so, by transitivity,

$A \succ_I B$ ,  $B \succ_I C$  and  $C \succ_I A$ .

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

# The Strong Axiom of Revealed Preference

$B \succ_D A$  is inconsistent

with  $A \succ_I B$ .

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

# The Strong Axiom of Revealed Preference

$A \succ_D C$  is inconsistent  
with  $C \succ_I A$ .

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...



# The Strong Axiom of Revealed Preference

$C \succ_D B$  is inconsistent  
with  $B \succ_I C$ .

	A	B	C
A		I	D
B	D		I
C	I	D	

The table illustrates revealed preferences between bundles A, B, and C. The diagonal cells are shaded gray. Red dashed lines and a red oval highlight specific revealed preference relationships: A ≻<sub>D</sub> B (indicated by a dashed line from A to B), B ≻<sub>I</sub> C (indicated by a dashed line from B to C), and C ≻<sub>D</sub> B (indicated by a red oval around the path from C to B to A).

The data do not violate the WARP but ...

# The Strong Axiom of Revealed Preference

**The data do not violate the WARP but there are 3 violations of the SARP.**

	A	B	C
A		I	D
B	D		I
C	I	D	

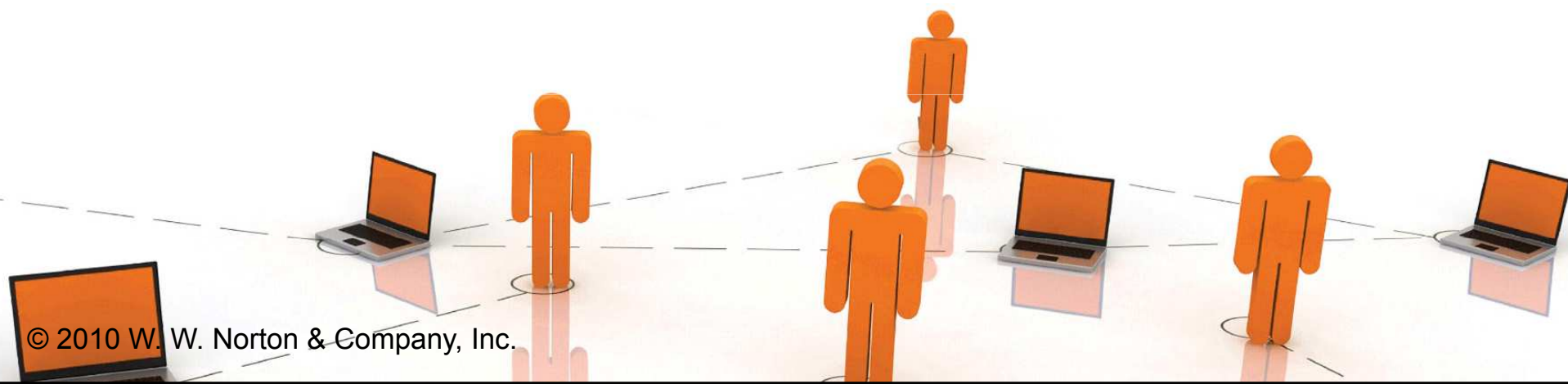
# The Strong Axiom of Revealed Preference

- ◆ That the observed choice data satisfy the **SARP** is a condition necessary and sufficient for there to be a well-behaved preference relation that “rationalizes” the data.
- ◆ So our 3 data cannot be rationalized by a well-behaved preference relation.



# Recovering Indifference Curves

- ◆ **Suppose we have the choice data satisfy the SARP.**
- ◆ **Then we can discover approximately where are the consumer's indifference curves.**
- ◆ **How?**



# Recovering Indifference Curves

## ◆ Suppose we observe:

A:  $(p_1, p_2) = (\$1, \$1)$  &  $(x_1, x_2) = (15, 15)$

B:  $(p_1, p_2) = (\$2, \$1)$  &  $(x_1, x_2) = (10, 20)$

C:  $(p_1, p_2) = (\$1, \$2)$  &  $(x_1, x_2) = (20, 10)$

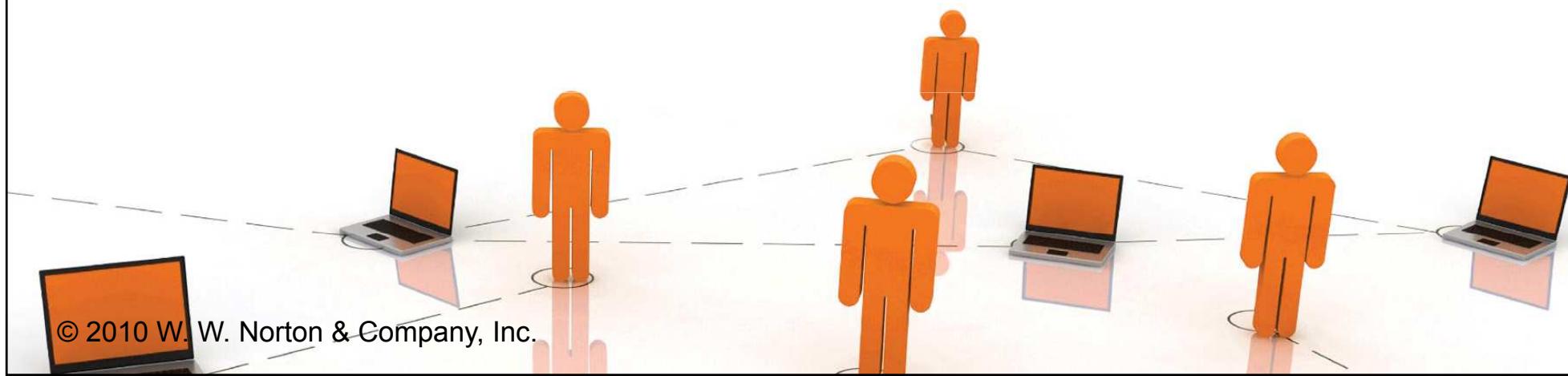
D:  $(p_1, p_2) = (\$2, \$5)$  &  $(x_1, x_2) = (30, 12)$

E:  $(p_1, p_2) = (\$5, \$2)$  &  $(x_1, x_2) = (12, 30)$ .

## ◆ Where lies the indifference curve containing the bundle A = (15, 15)?

# Recovering Indifference Curves

- ◆ **The table showing direct preference revelations is:**



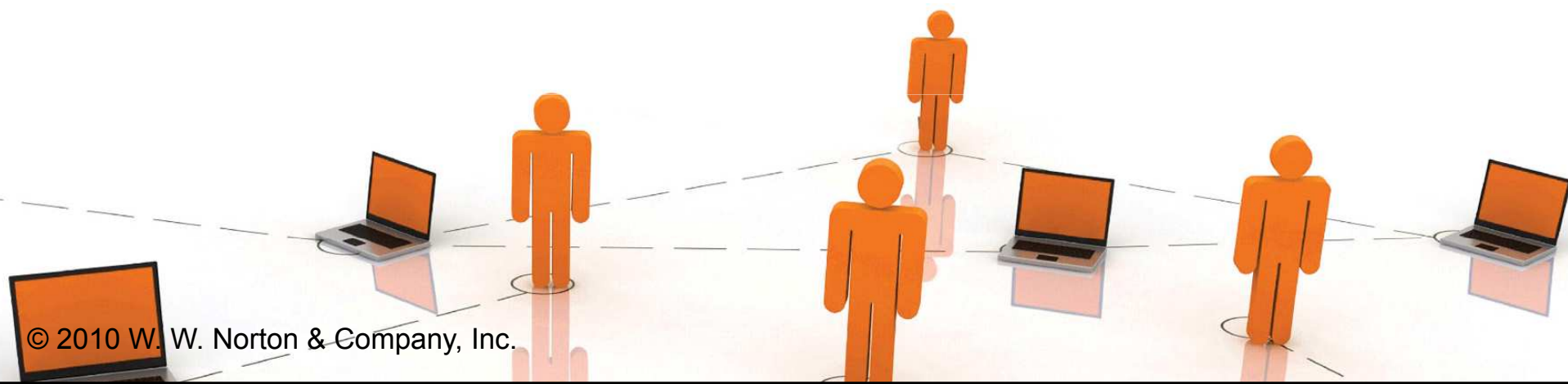
# Recovering Indifference Curves

	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

**Direct revelations only; the WARP is not violated by the data.**

# Recovering Indifference Curves

- ◆ **Indirect preference revelations add no extra information, so the table showing both direct and indirect preference revelations is the same as the table showing only the direct preference revelations:**





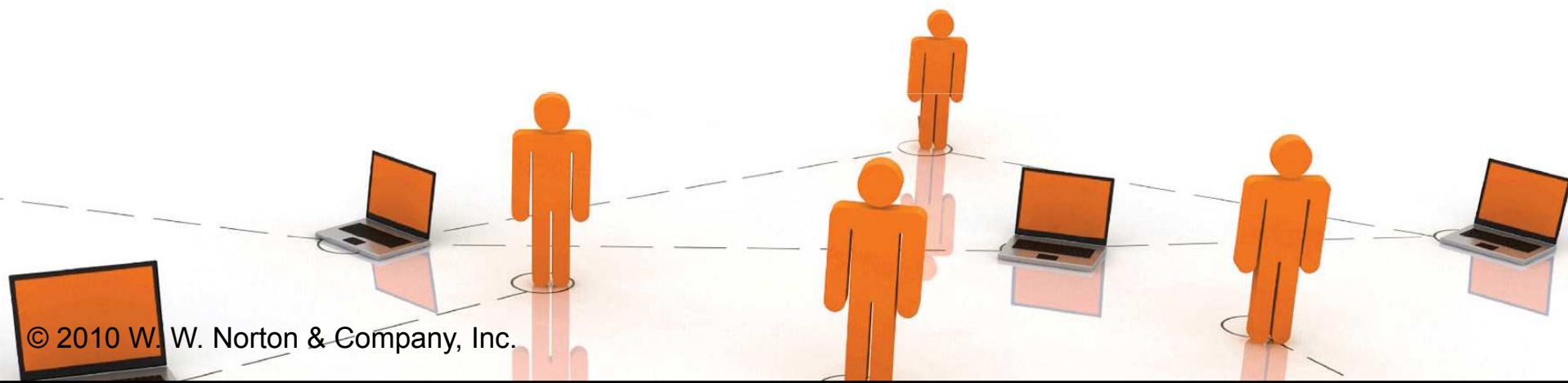
# Recovering Indifference Curves

	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

**Both direct and indirect revelations; neither WARP nor SARP are violated by the data.**

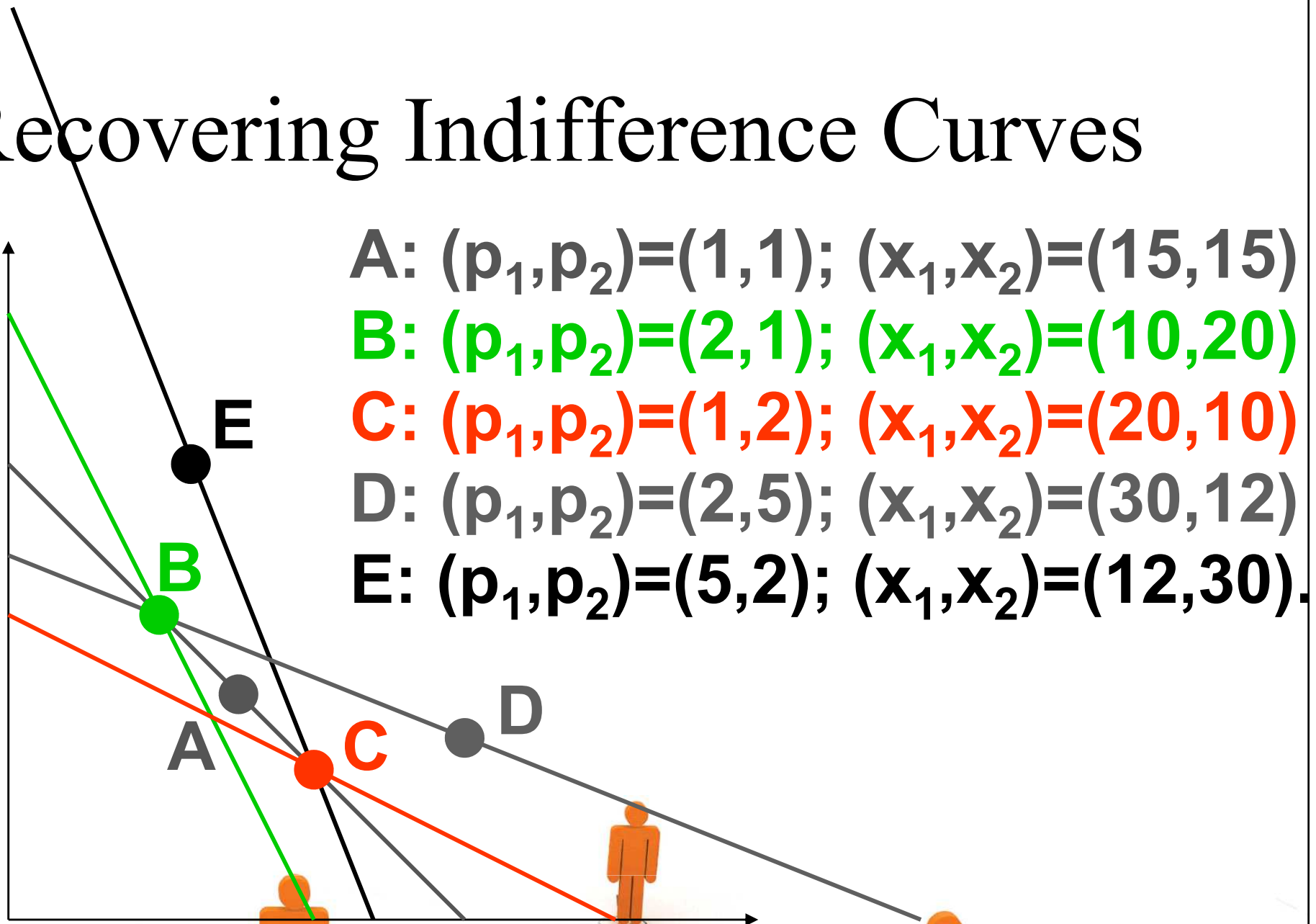
# Recovering Indifference Curves

- ◆ **Since the choices satisfy the SARP, there is a well-behaved preference relation that “rationalizes” the choices.**



# Recovering Indifference Curves

$x_2$



**A:**  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

**B:**  $(p_1, p_2) = (2, 1)$ ;  $(x_1, x_2) = (10, 20)$

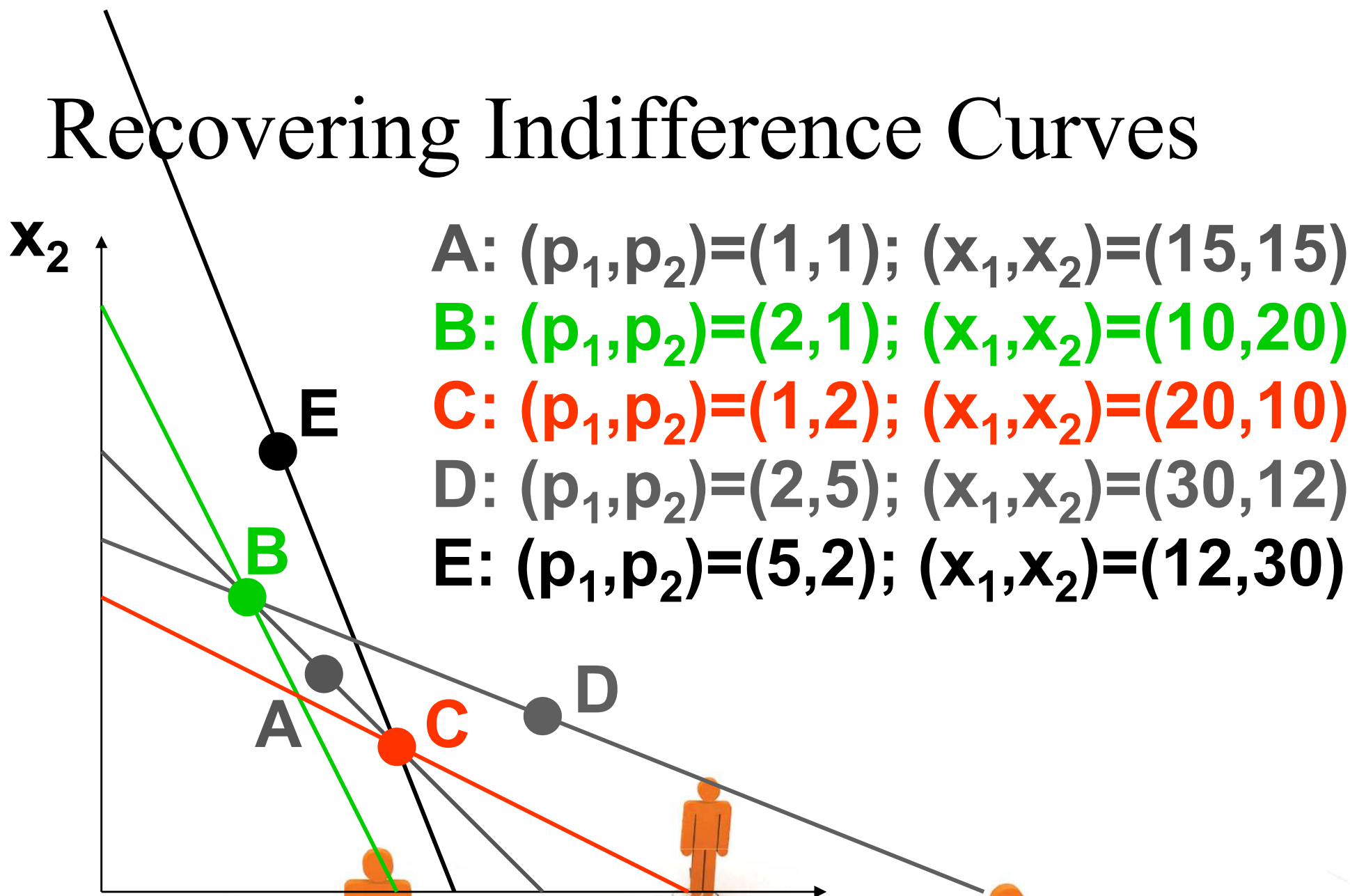
**C:**  $(p_1, p_2) = (1, 2)$ ;  $(x_1, x_2) = (20, 10)$

**D:**  $(p_1, p_2) = (2, 5)$ ;  $(x_1, x_2) = (30, 12)$

**E:**  $(p_1, p_2) = (5, 2)$ ;  $(x_1, x_2) = (12, 30)$



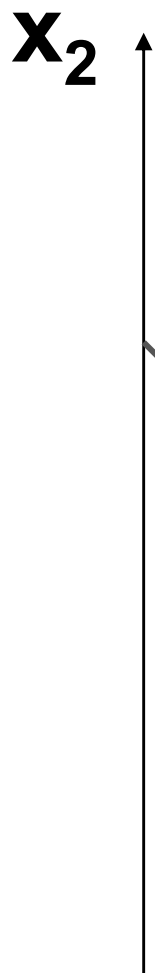
# Recovering Indifference Curves



Begin with bundles revealed to be less preferred than bundle A.

# Recovering Indifference Curves

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$ .

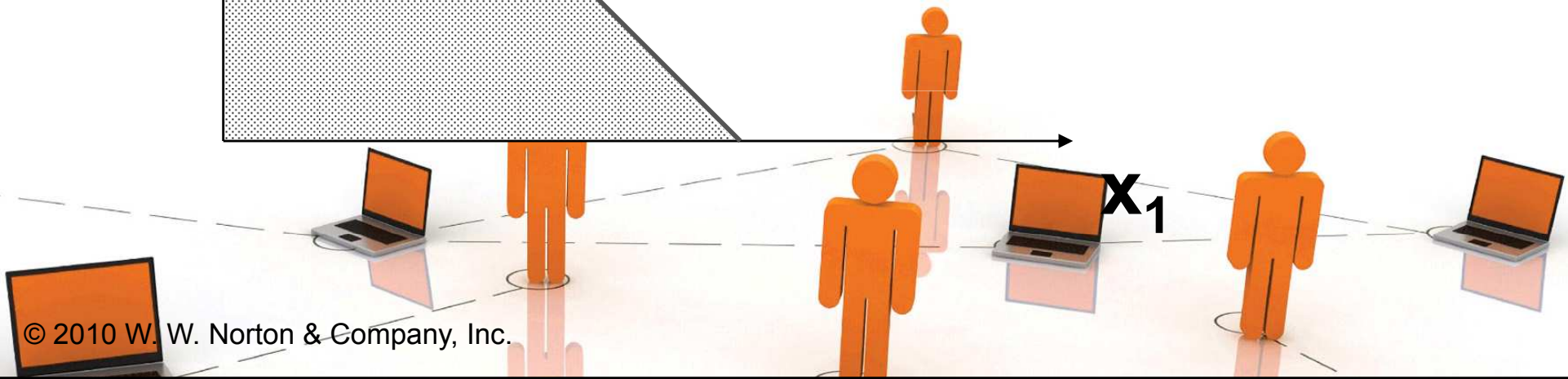
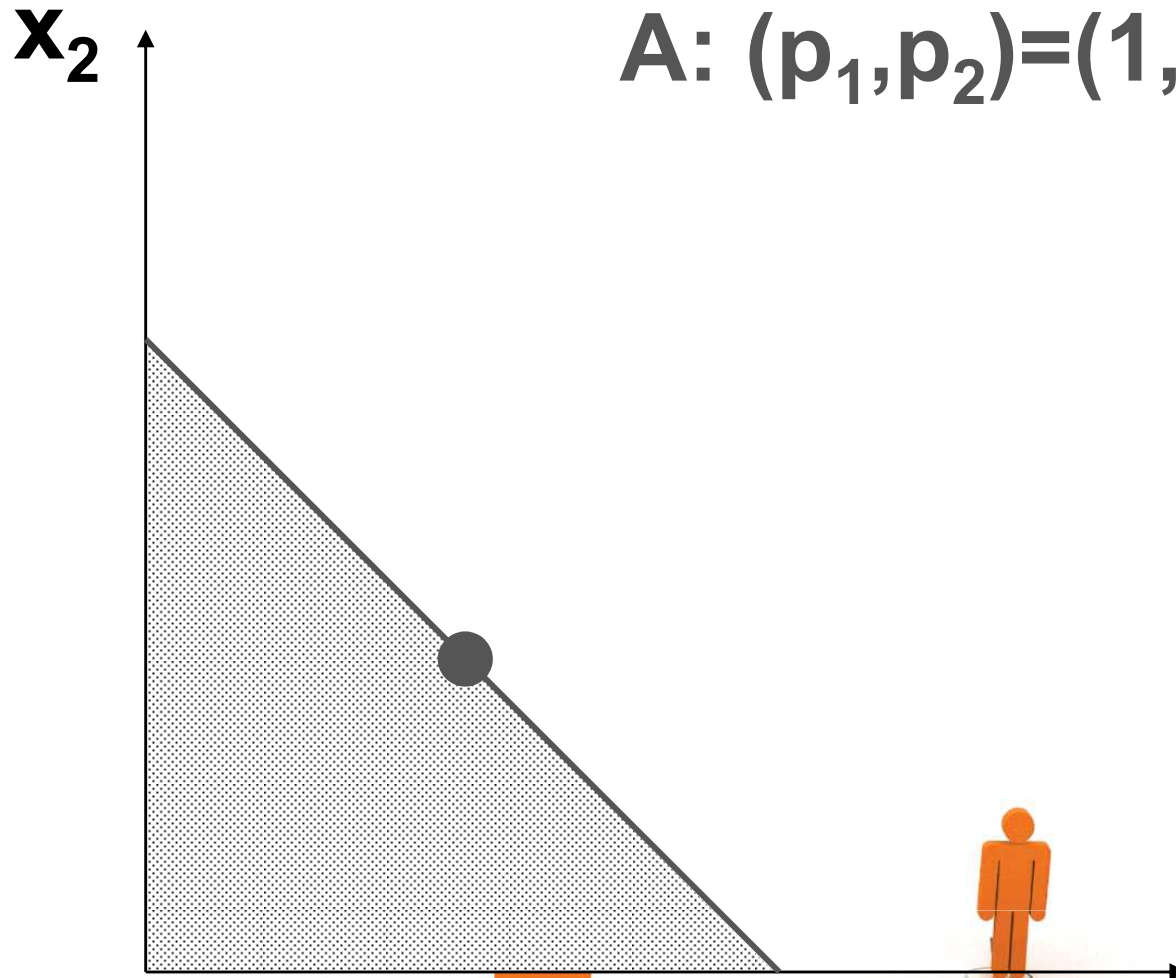


A

$x_1$

# Recovering Indifference Curves

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$ .

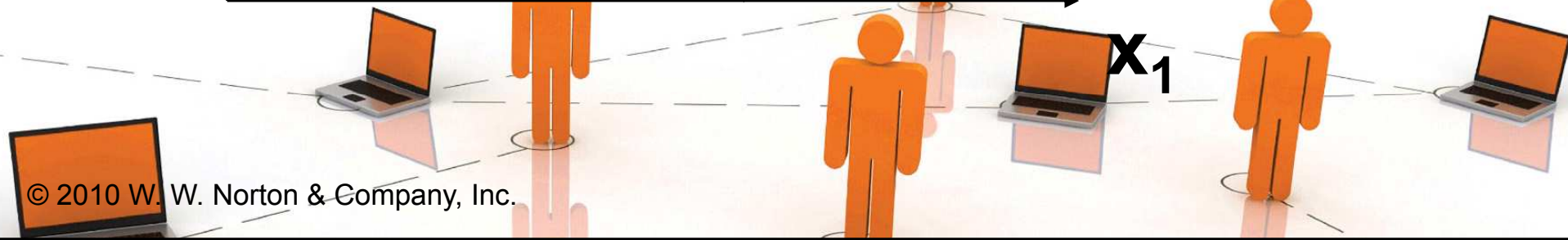
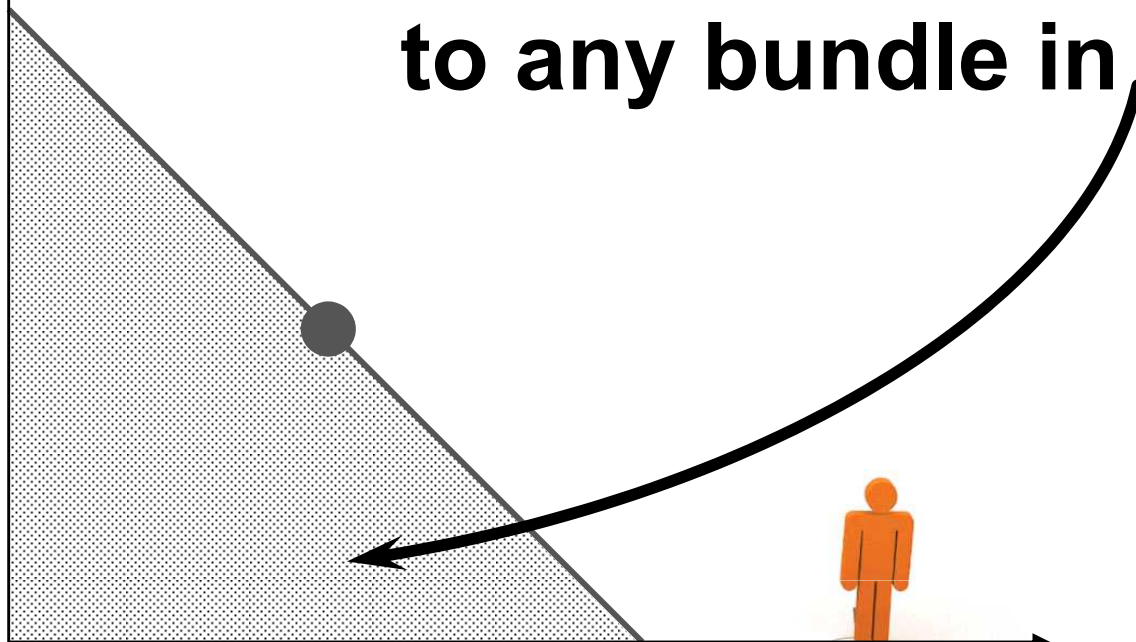


# Recovering Indifference Curves

$x_2$

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$ .

**A is directly revealed preferred to any bundle in**

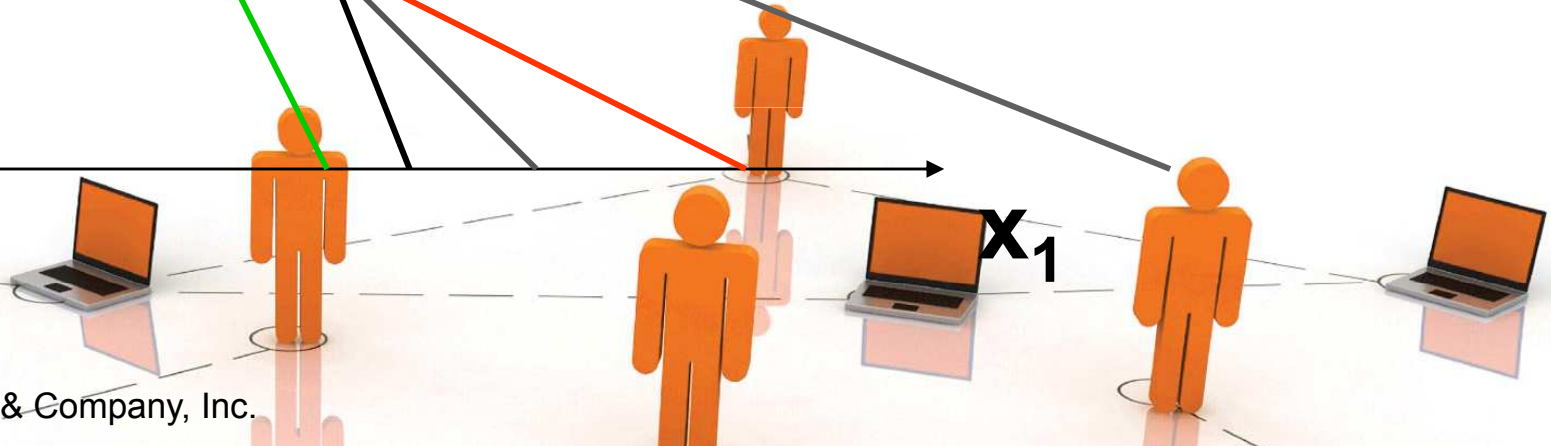
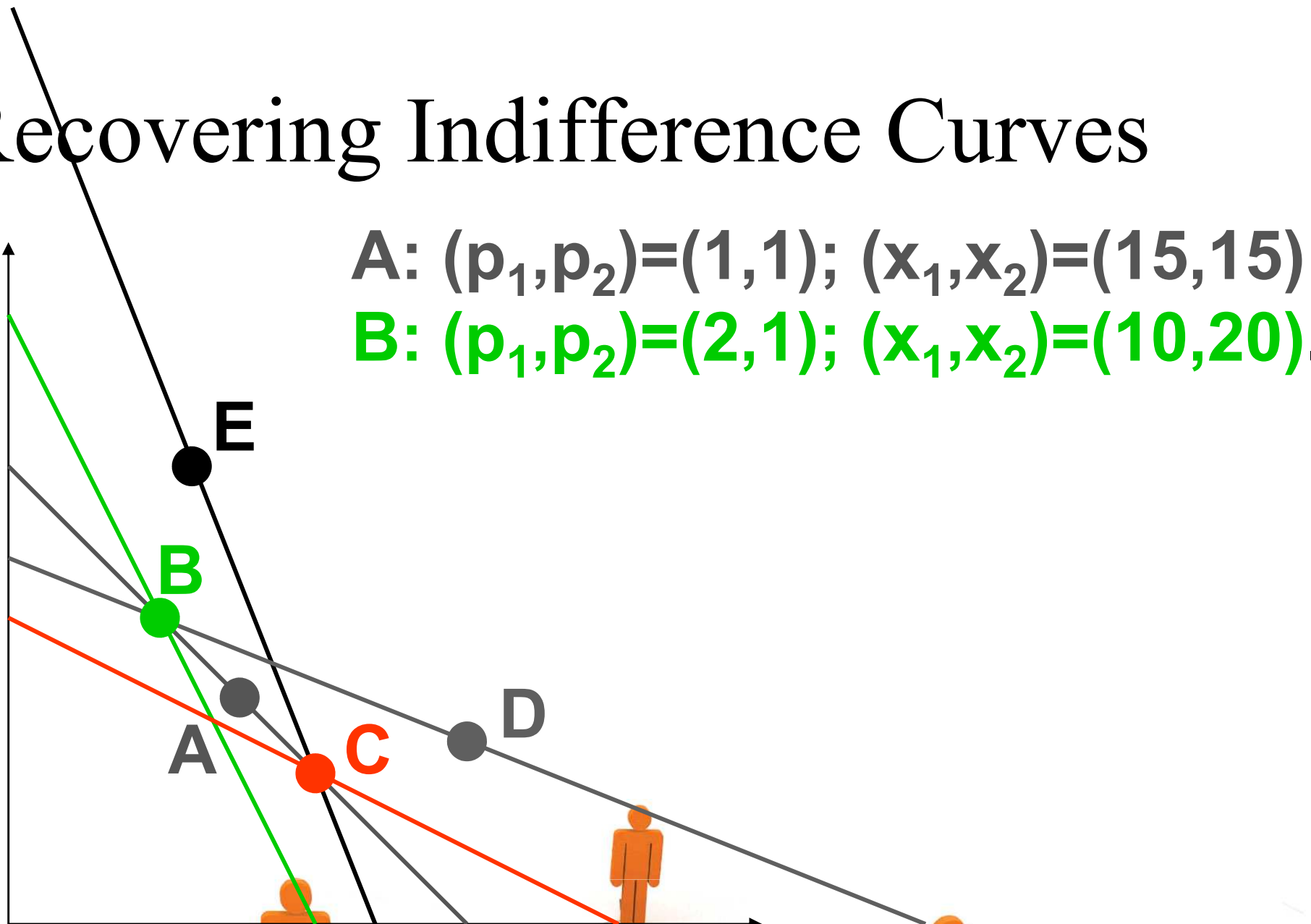


# Recovering Indifference Curves

$x_2$

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

B:  $(p_1, p_2) = (2, 1)$ ;  $(x_1, x_2) = (10, 20)$





# Recovering Indifference Curves

$x_2$

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

B:  $(p_1, p_2) = (2, 1)$ ;  $(x_1, x_2) = (10, 20)$

B

A

$x_1$

# Recovering Indifference Curves

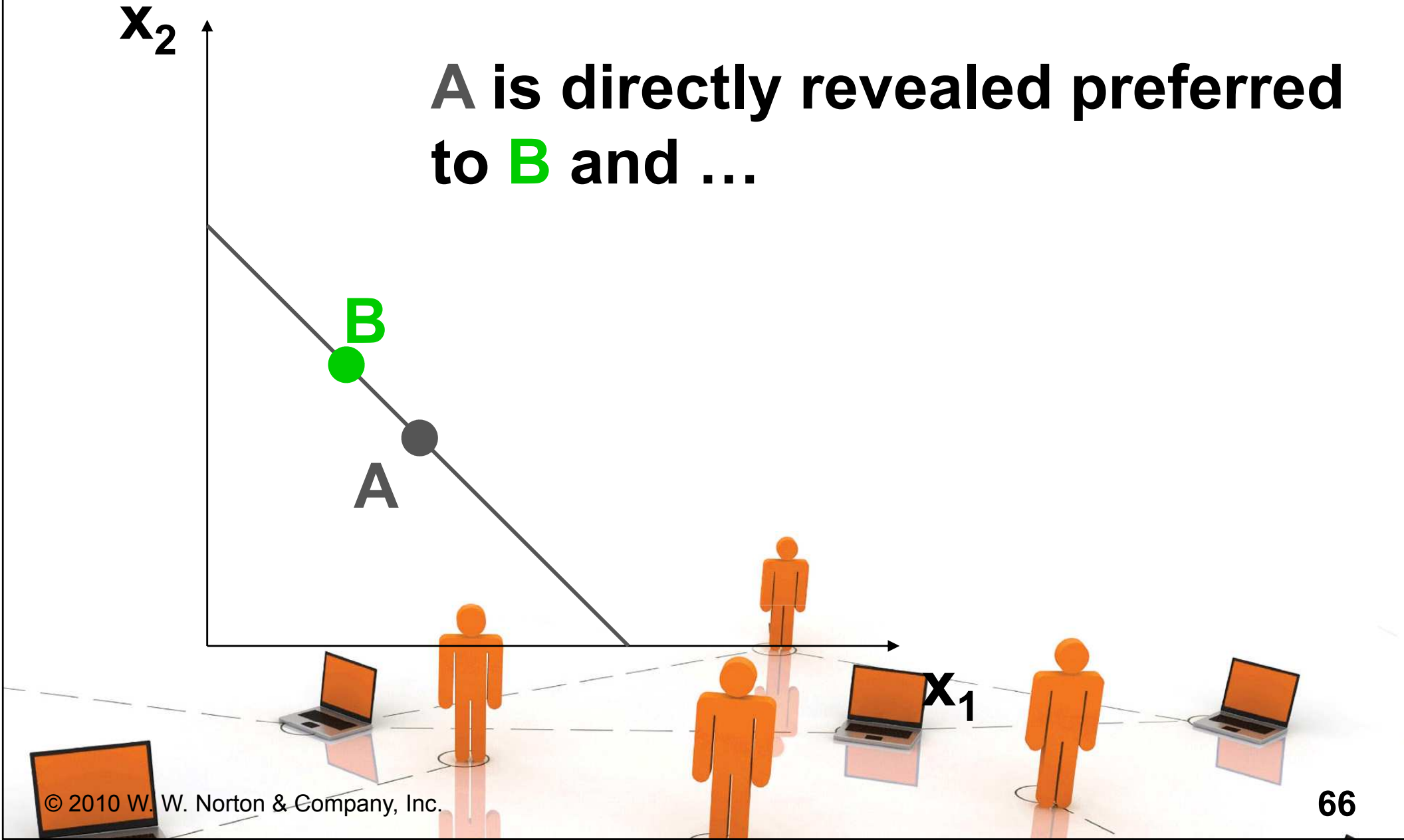
$x_2$

**A is directly revealed preferred to B and ...**

**B**

**A**

$x_1$



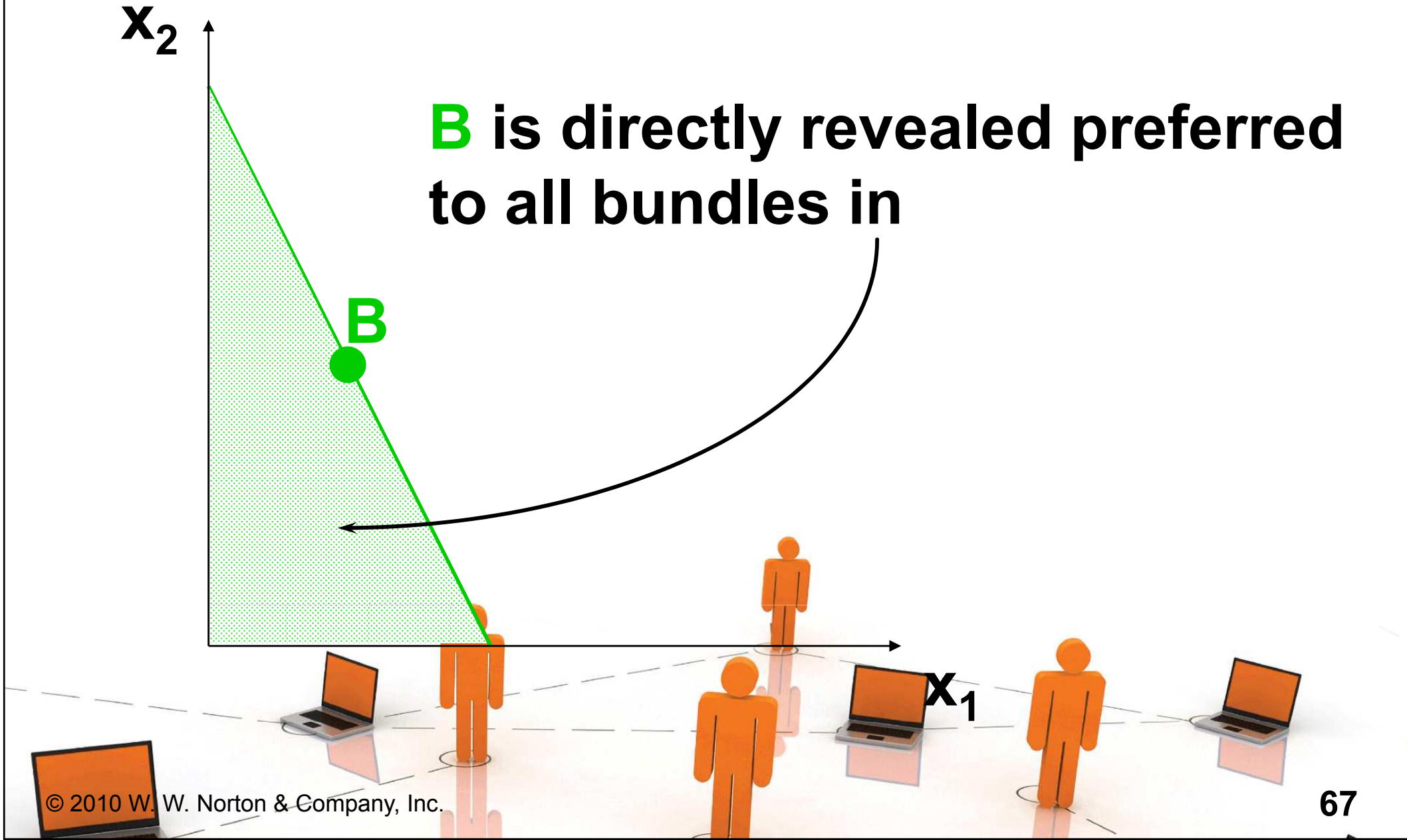
# Recovering Indifference Curves

$x_2$

**B** is directly revealed preferred to all bundles in

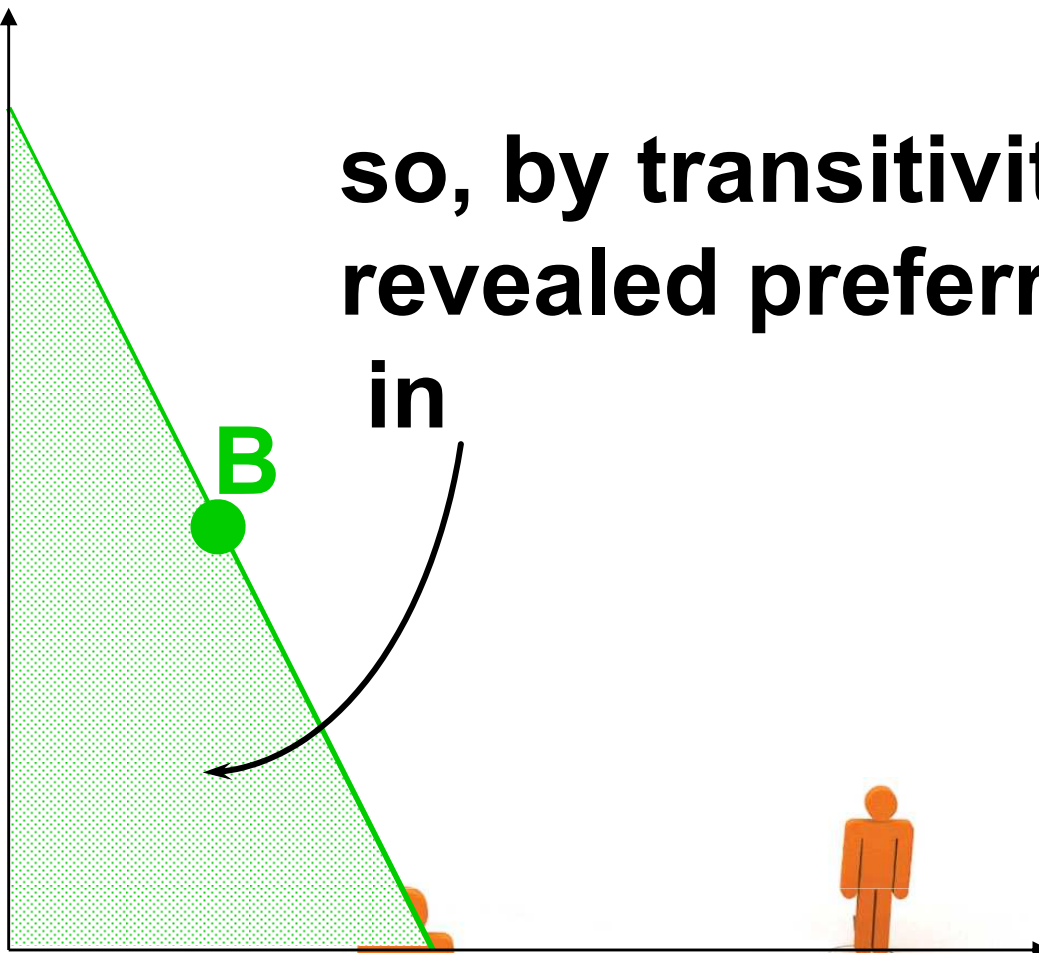
**B**

$x_1$



# Recovering Indifference Curves

$x_2$



so, by transitivity, A is indirectly revealed preferred to all bundles

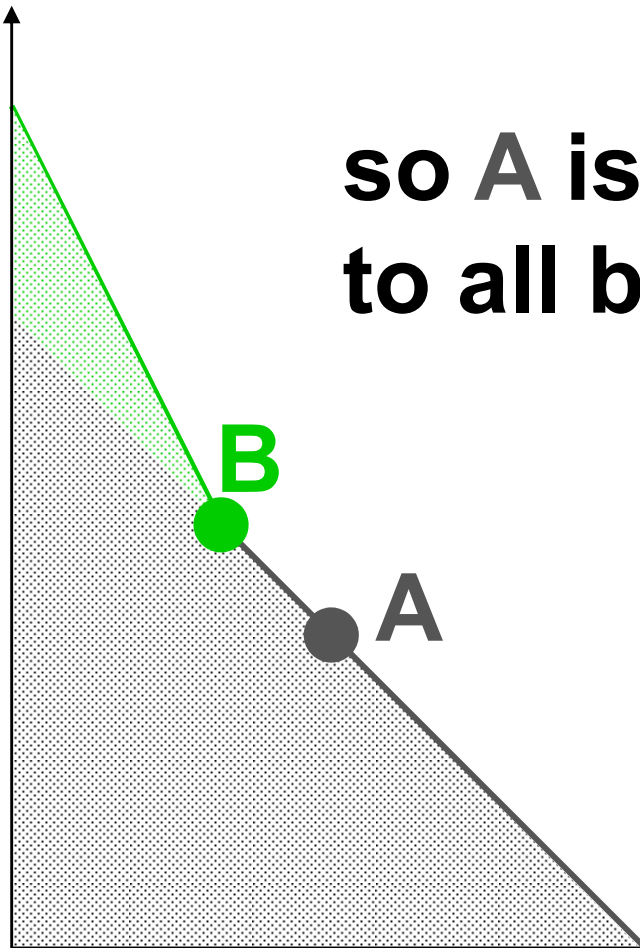
in

B

$x_1$

# Recovering Indifference Curves

$x_2$



so  $A$  is now revealed preferred to all bundles in the union.



$x_1$

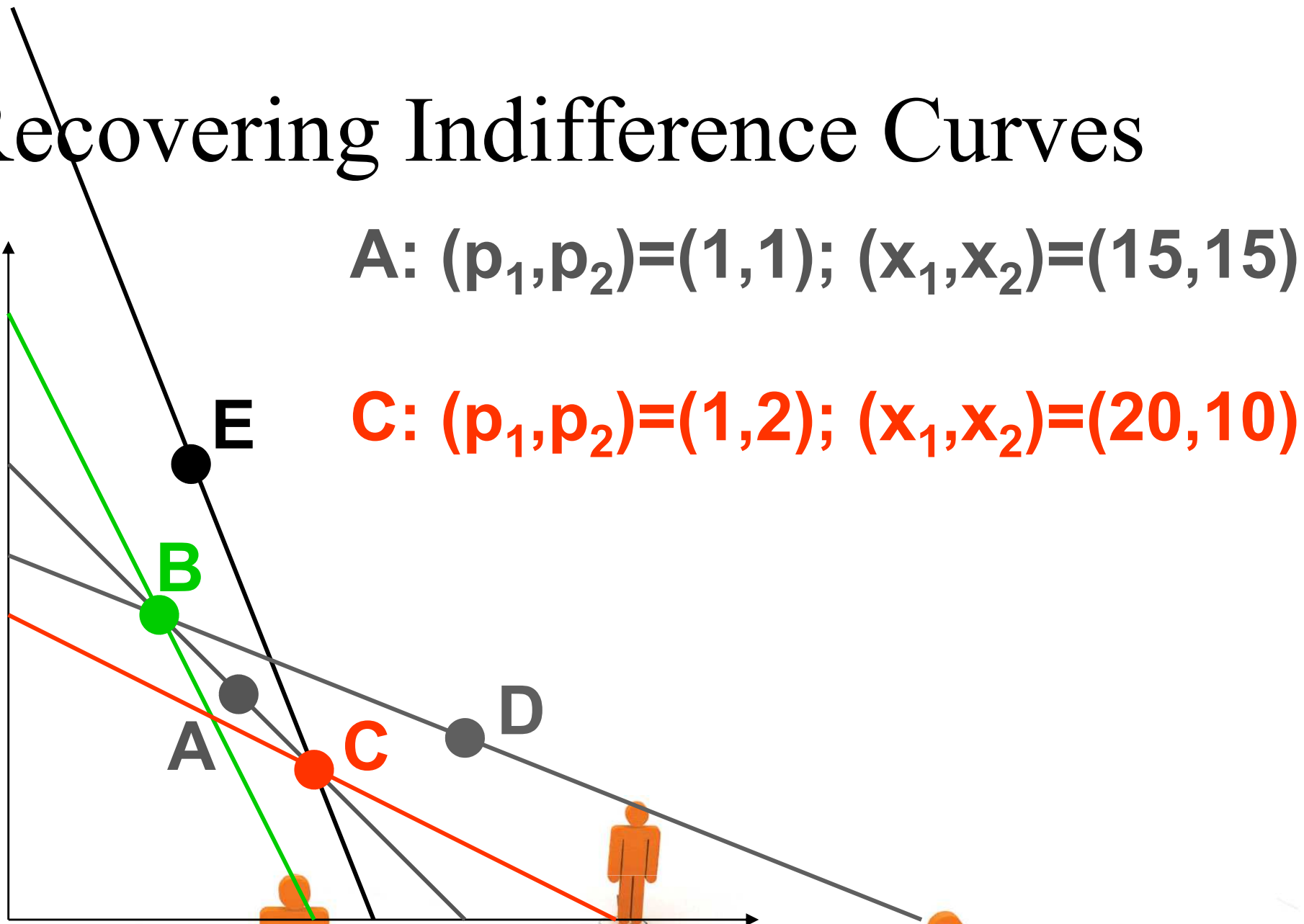


# Recovering Indifference Curves

$x_2$

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

C:  $(p_1, p_2) = (1, 2)$ ;  $(x_1, x_2) = (20, 10)$



# Recovering Indifference Curves

$x_2$

A:  $(p_1, p_2) = (1, 1); (x_1, x_2) = (15, 15)$

C:  $(p_1, p_2) = (1, 2); (x_1, x_2) = (20, 10)$

A

C

$x_1$

# Recovering Indifference Curves

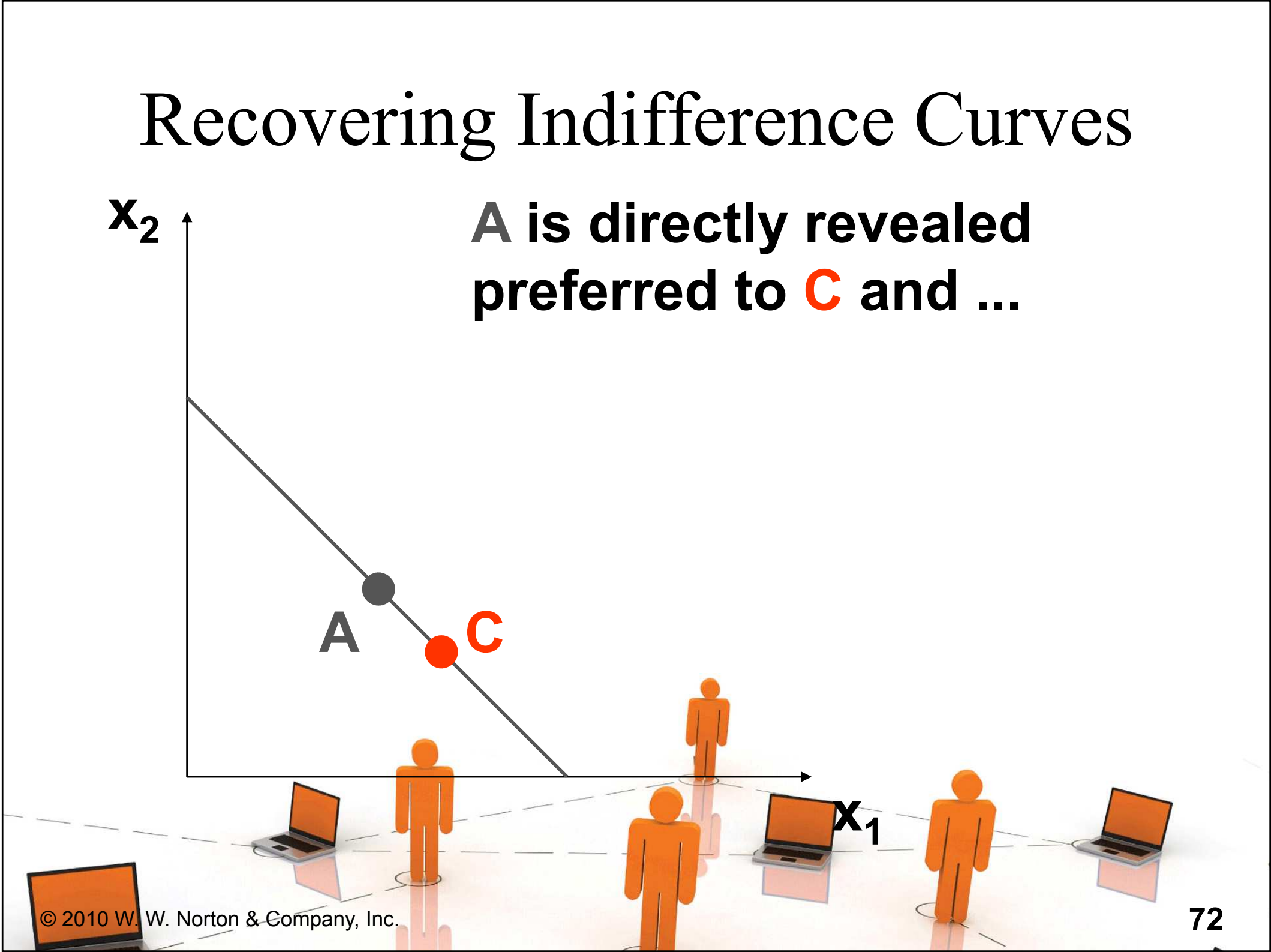
$x_2$

**A is directly revealed preferred to C and ...**

A

C

$x_1$

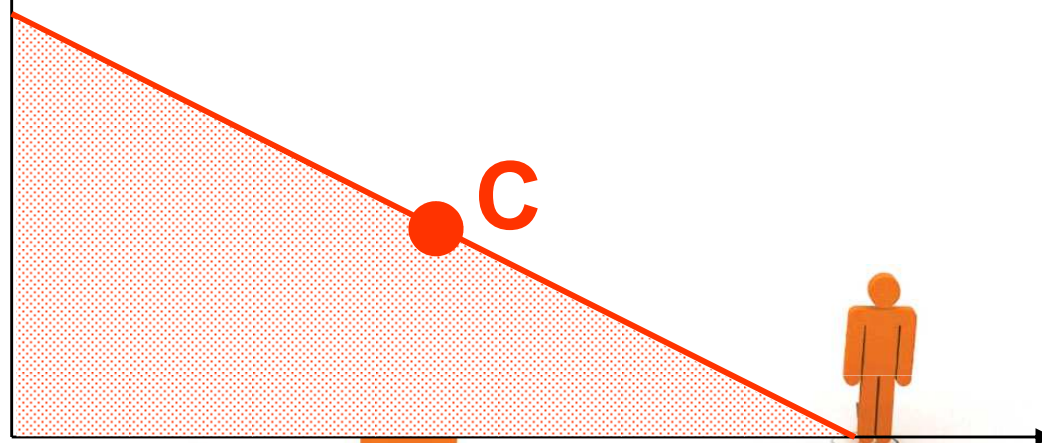




# Recovering Indifference Curves

$x_2$

**C** is directly revealed preferred to all bundles in

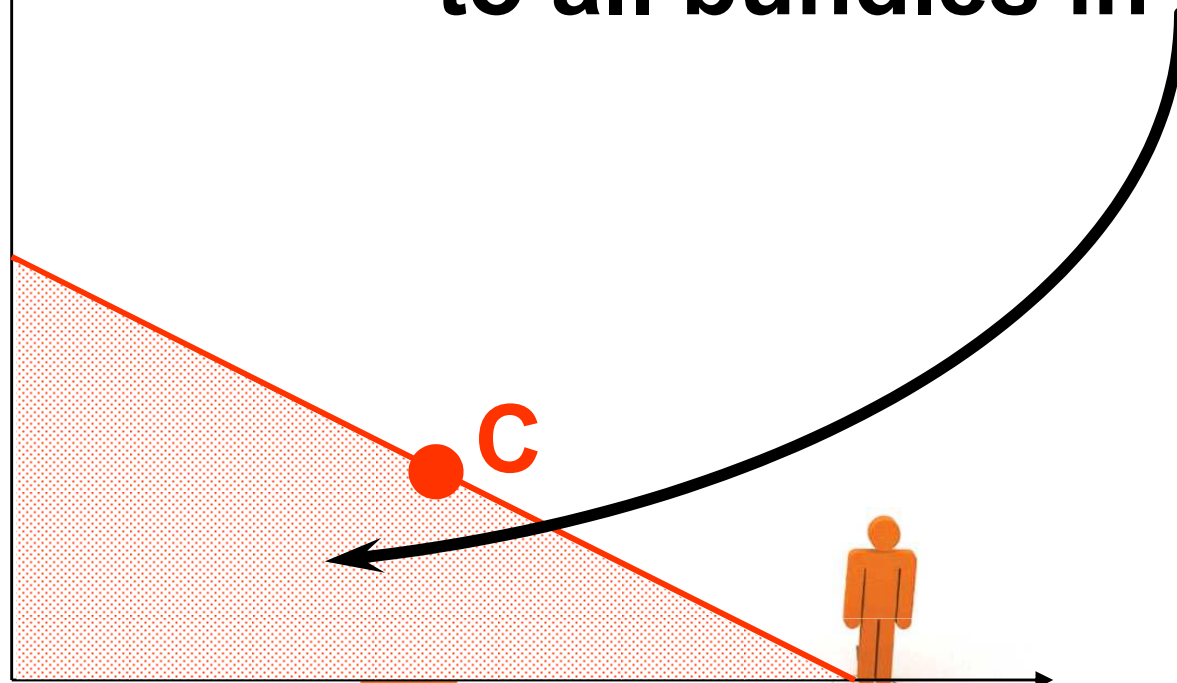


$x_1$

# Recovering Indifference Curves

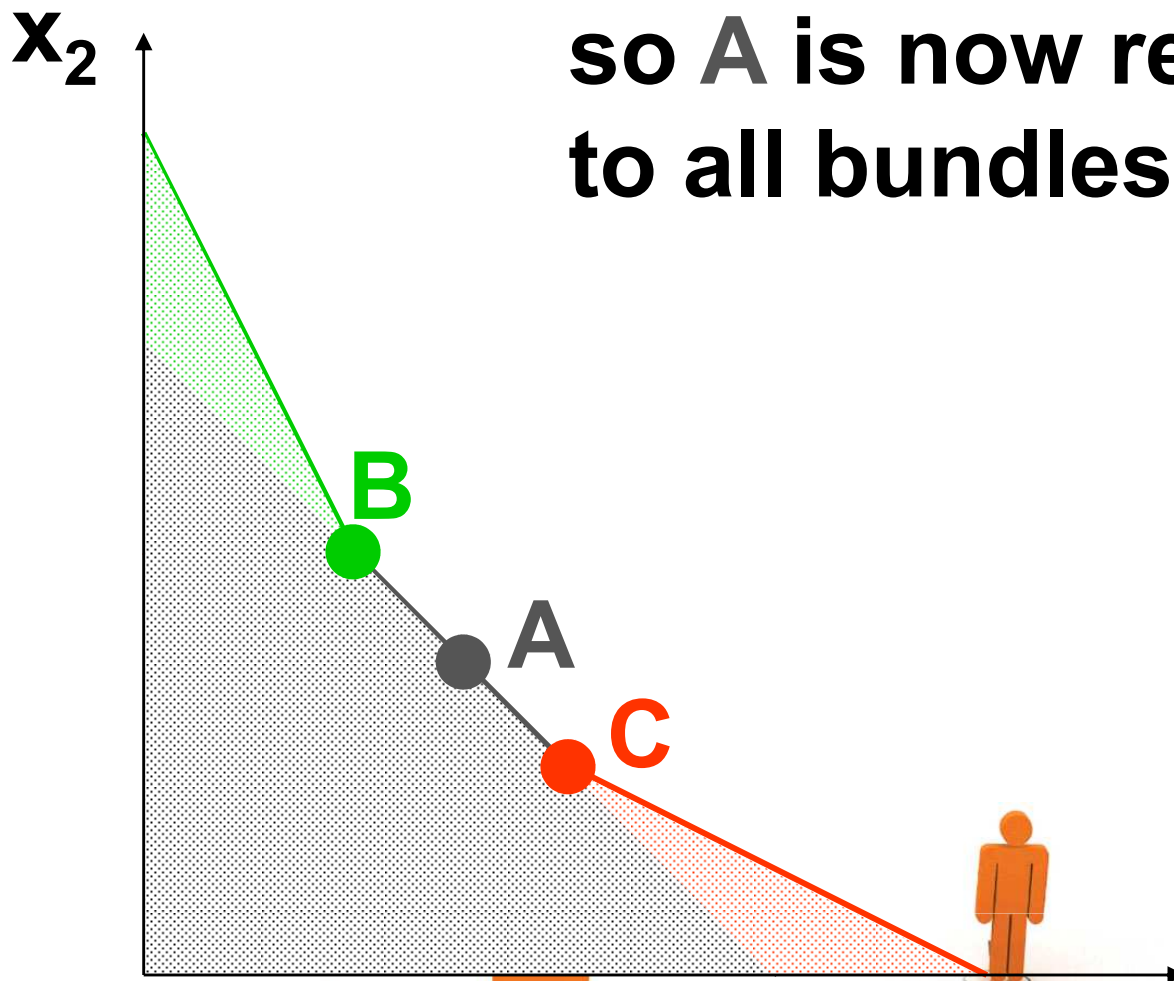
$x_2$

so, by transitivity, **A** is indirectly revealed preferred to all bundles in

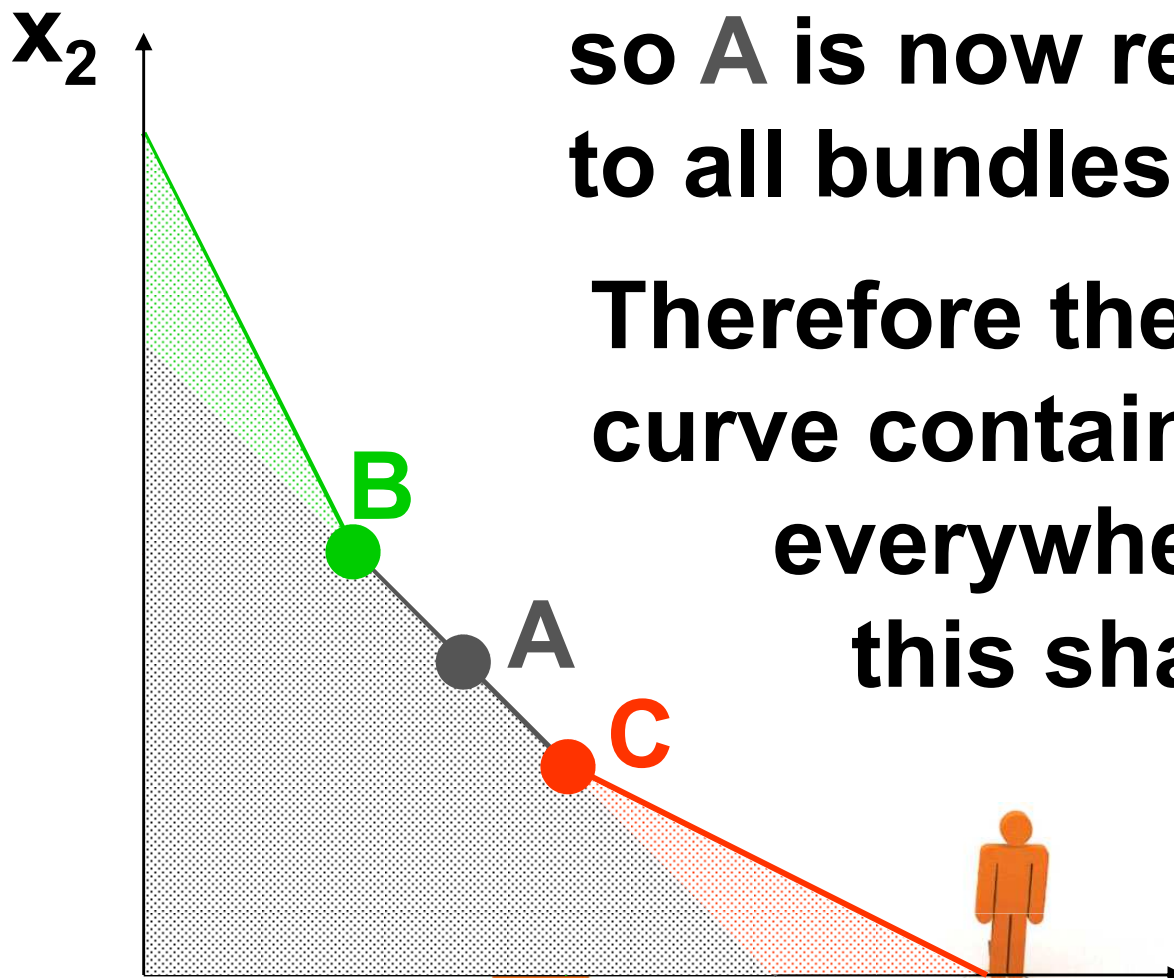


# Recovering Indifference Curves

so  $A$  is now revealed preferred to all bundles in the union.



# Recovering Indifference Curves

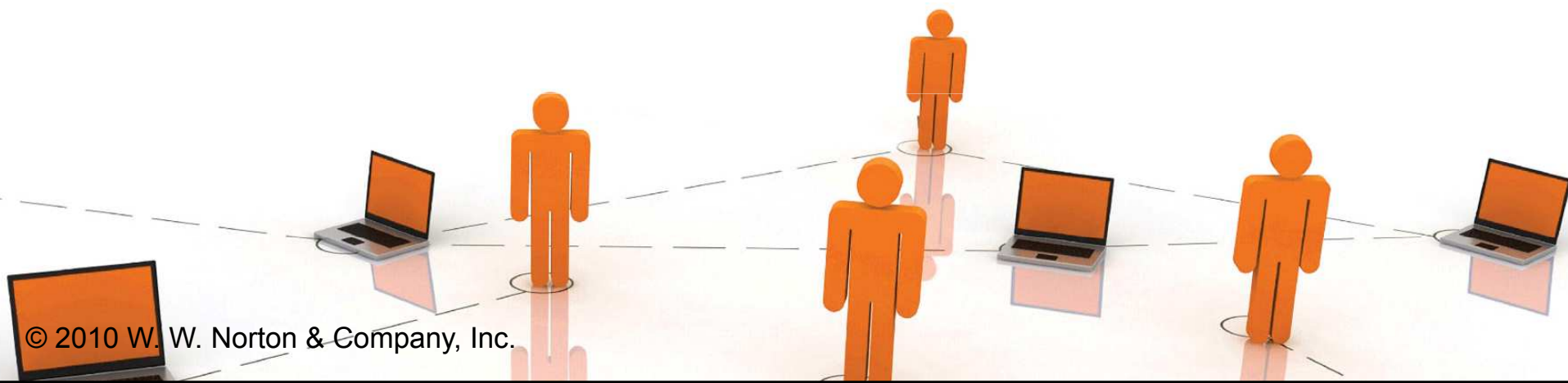


so  $A$  is now revealed preferred to all bundles in the union.

Therefore the indifference curve containing  $A$  must lie everywhere else above this shaded set.

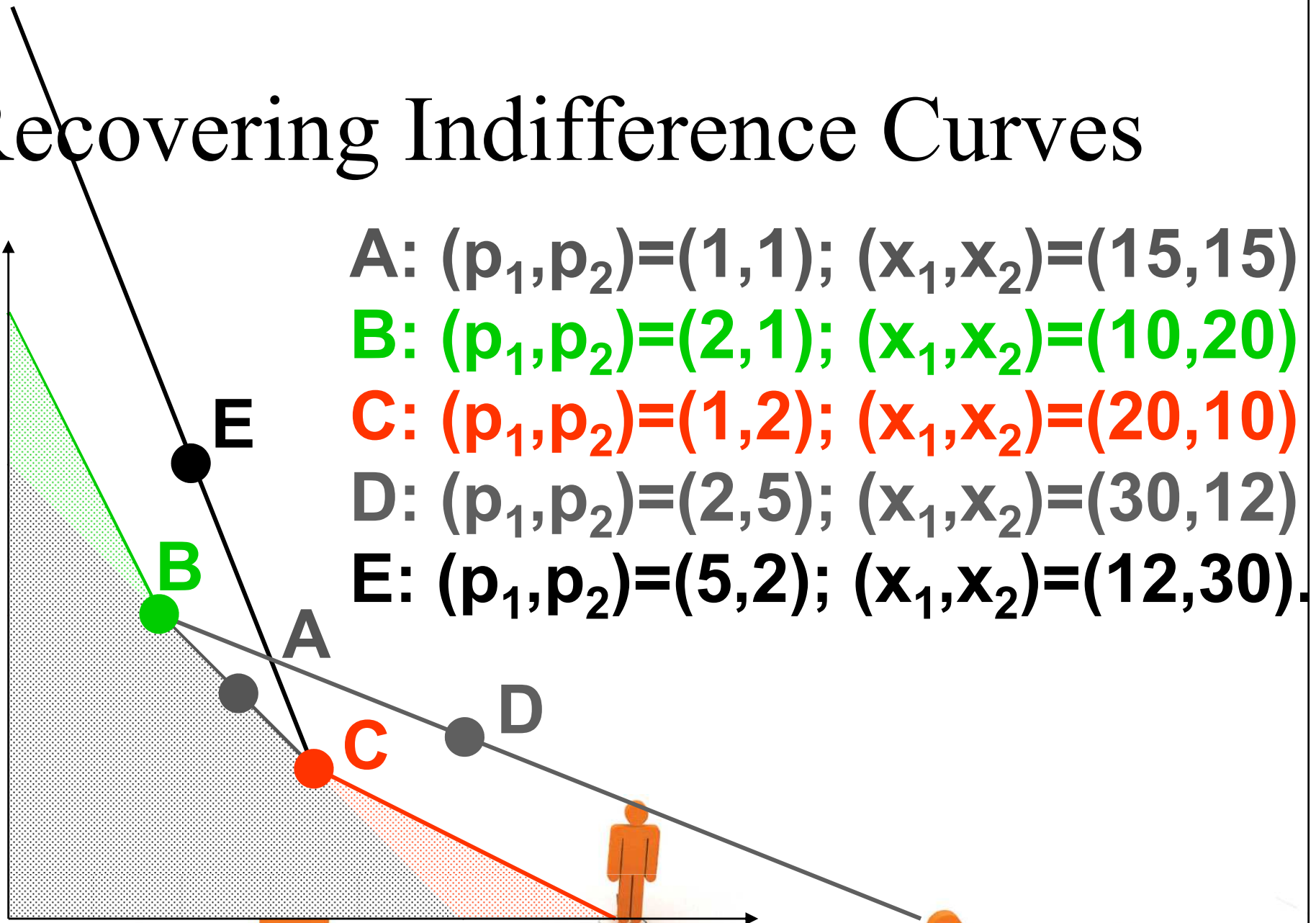
# Recovering Indifference Curves

- ◆ **Now, what about the bundles revealed as more preferred than A?**



# Recovering Indifference Curves

$x_2$



A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

B:  $(p_1, p_2) = (2, 1)$ ;  $(x_1, x_2) = (10, 20)$

C:  $(p_1, p_2) = (1, 2)$ ;  $(x_1, x_2) = (20, 10)$

D:  $(p_1, p_2) = (2, 5)$ ;  $(x_1, x_2) = (30, 12)$

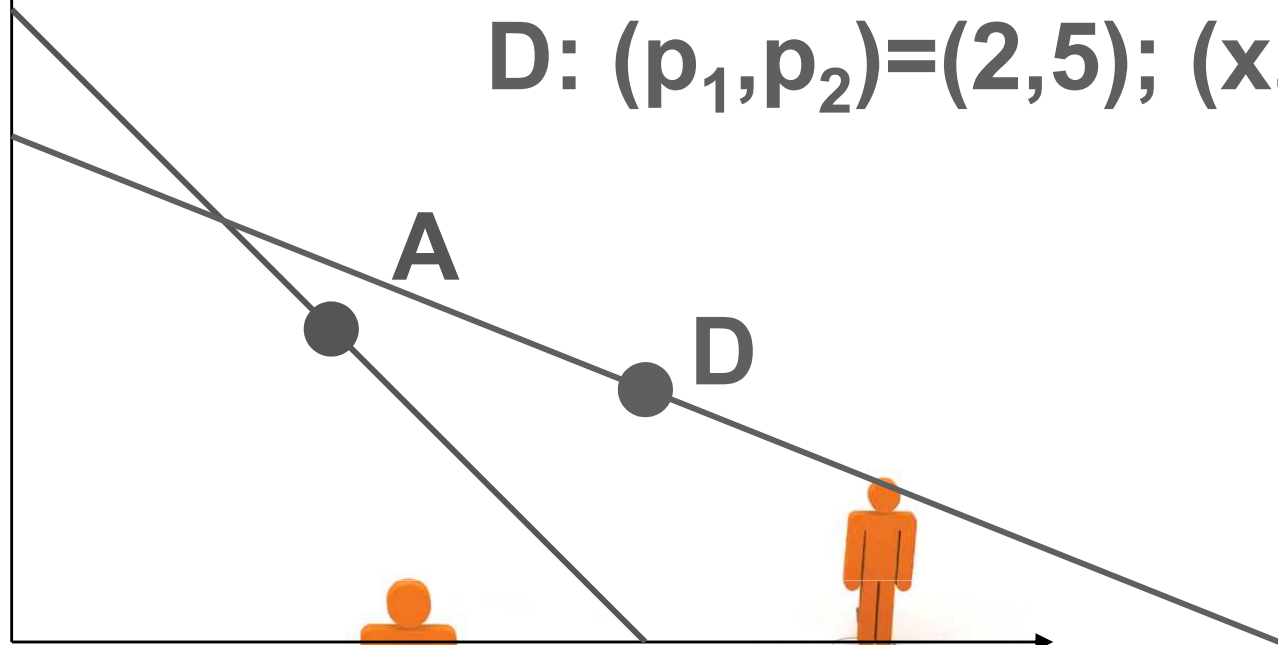
E:  $(p_1, p_2) = (5, 2)$ ;  $(x_1, x_2) = (12, 30)$

# Recovering Indifference Curves

$x_2$

A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

D:  $(p_1, p_2) = (2, 5)$ ;  $(x_1, x_2) = (30, 12)$



A

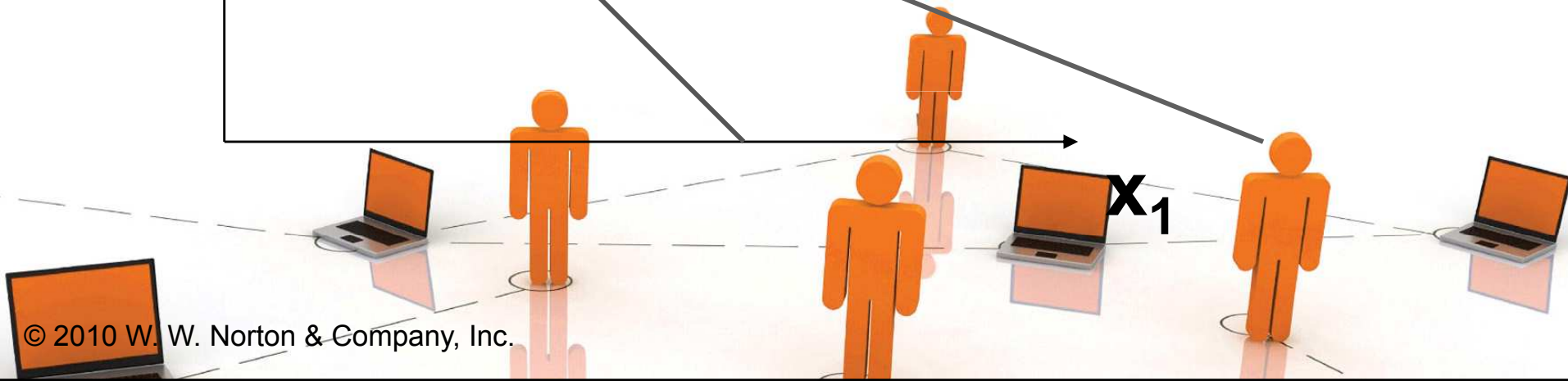
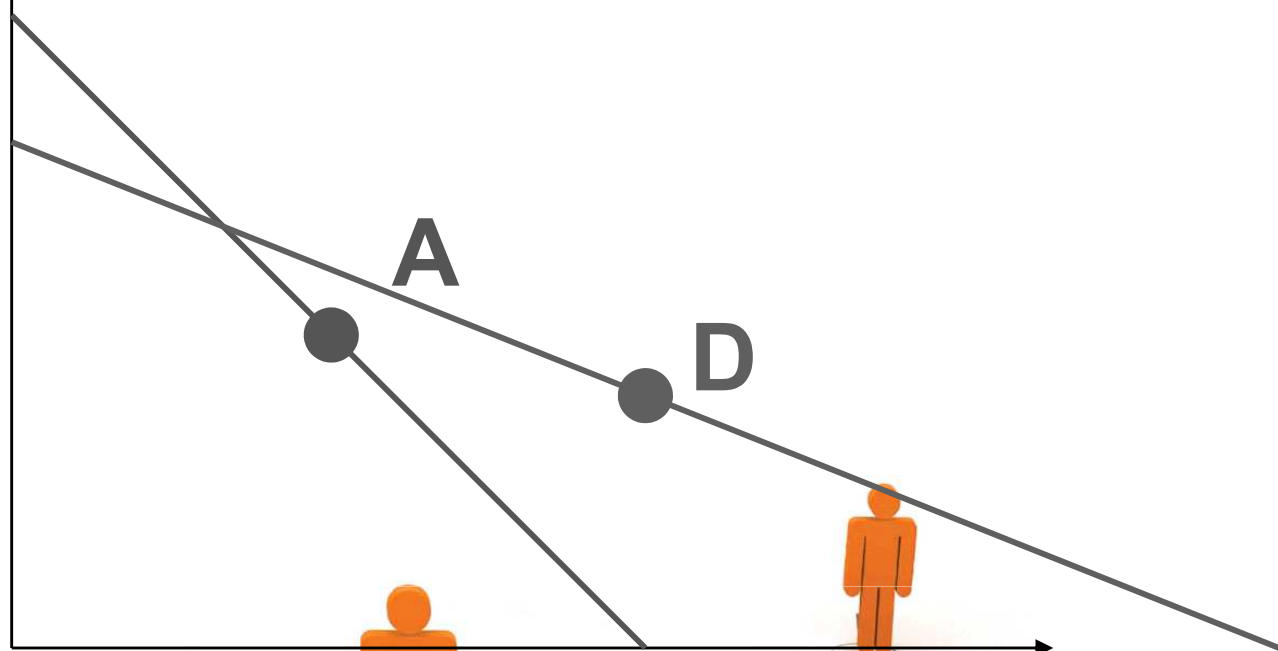
D

$x_1$

# Recovering Indifference Curves

$x_2$

**D is directly revealed preferred to A.**



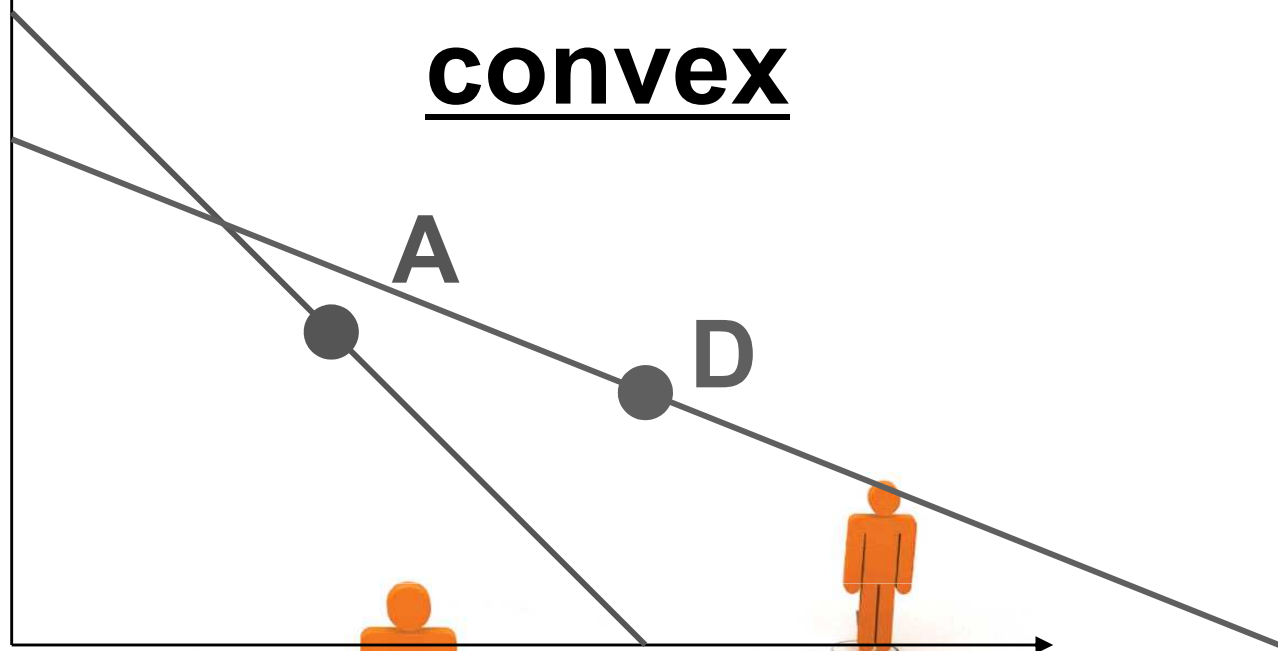


# Recovering Indifference Curves

$x_2$

**D is directly revealed preferred to A.**

**Well-behaved preferences are convex**



# Recovering Indifference Curves



**D is directly revealed preferred to A.**

**Well-behaved preferences are convex so all bundles on the line between A and D are preferred to A also.**



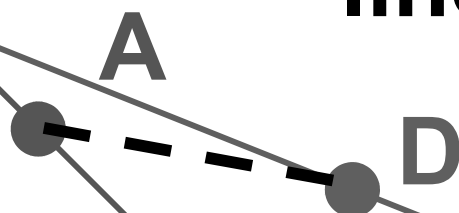
# Recovering Indifference Curves

$x_2$

**D is directly revealed preferred to A.**

**Well-behaved preferences are convex so all bundles on the line between A and D are preferred to A also.**

**As well, ...**



$x_1$



# Recovering Indifference Curves

$x_2$

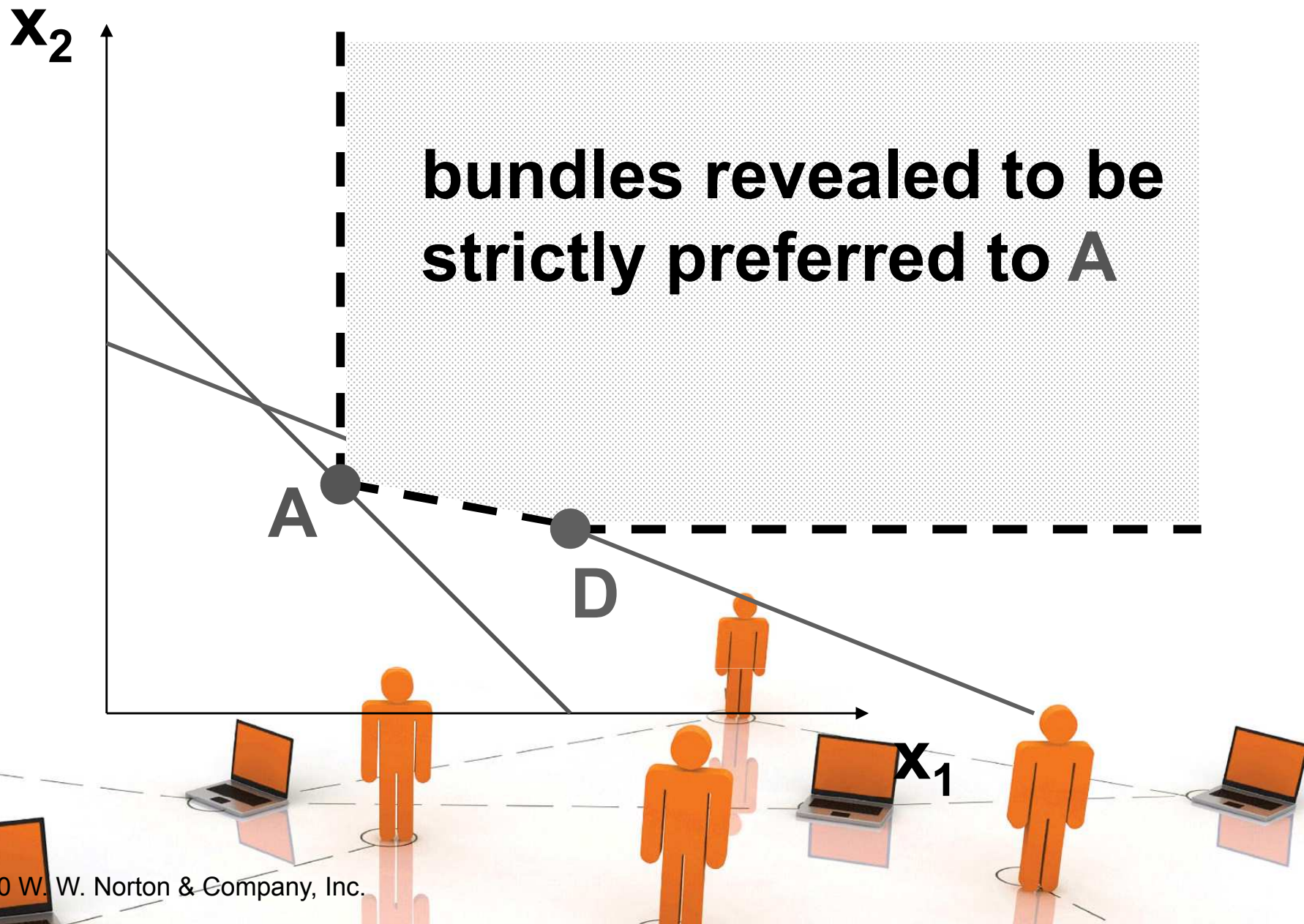
**all bundles containing the same amount of commodity 2 and more of commodity 1 than D are preferred to D and therefore are preferred to A also.**

A

D

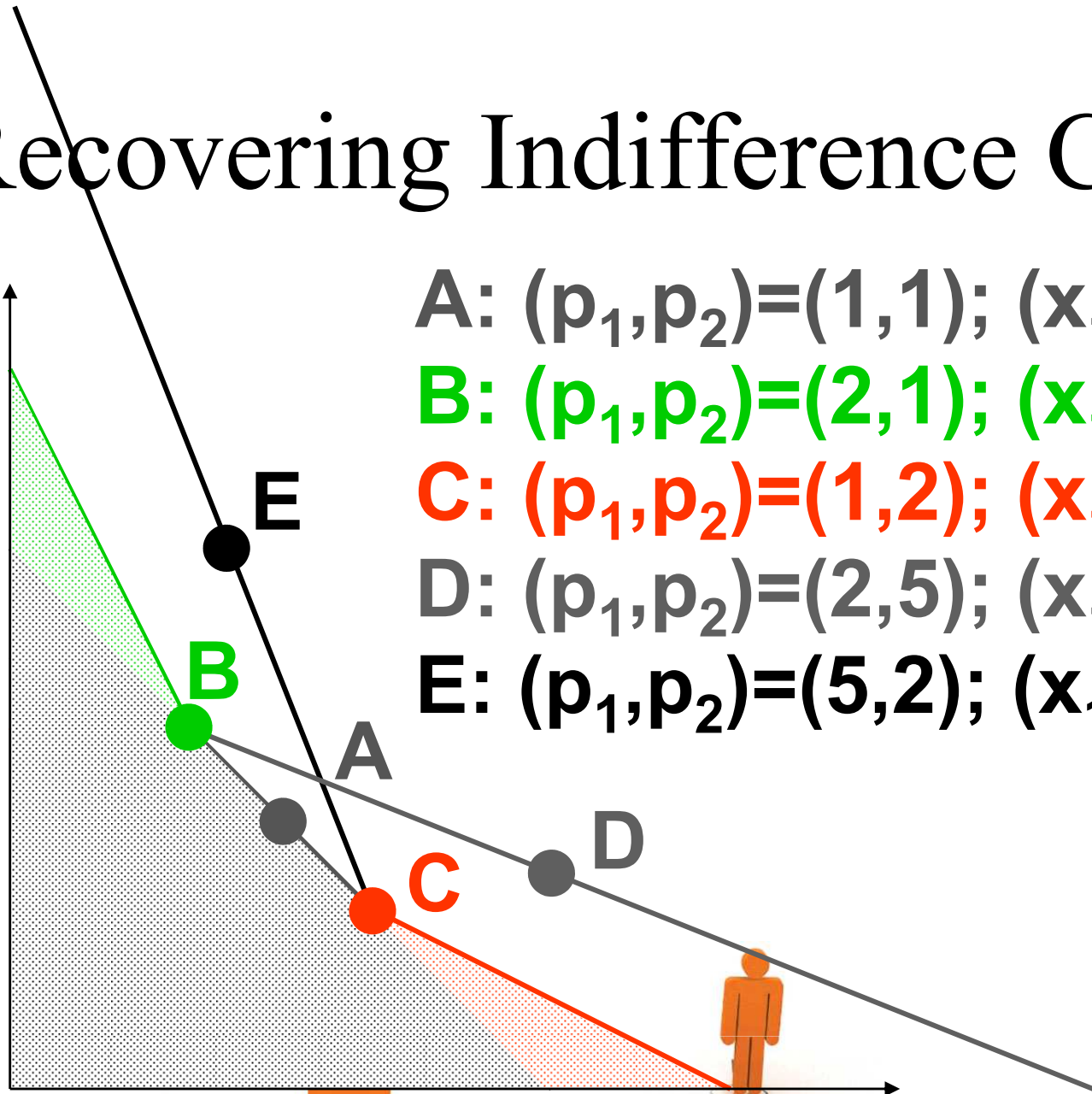
$x_1$

# Recovering Indifference Curves



# Recovering Indifference Curves

$x_2$



A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

B:  $(p_1, p_2) = (2, 1)$ ;  $(x_1, x_2) = (10, 20)$

C:  $(p_1, p_2) = (1, 2)$ ;  $(x_1, x_2) = (20, 10)$

D:  $(p_1, p_2) = (2, 5)$ ;  $(x_1, x_2) = (30, 12)$

E:  $(p_1, p_2) = (5, 2)$ ;  $(x_1, x_2) = (12, 30)$

# Recovering Indifference Curves

$x_2$

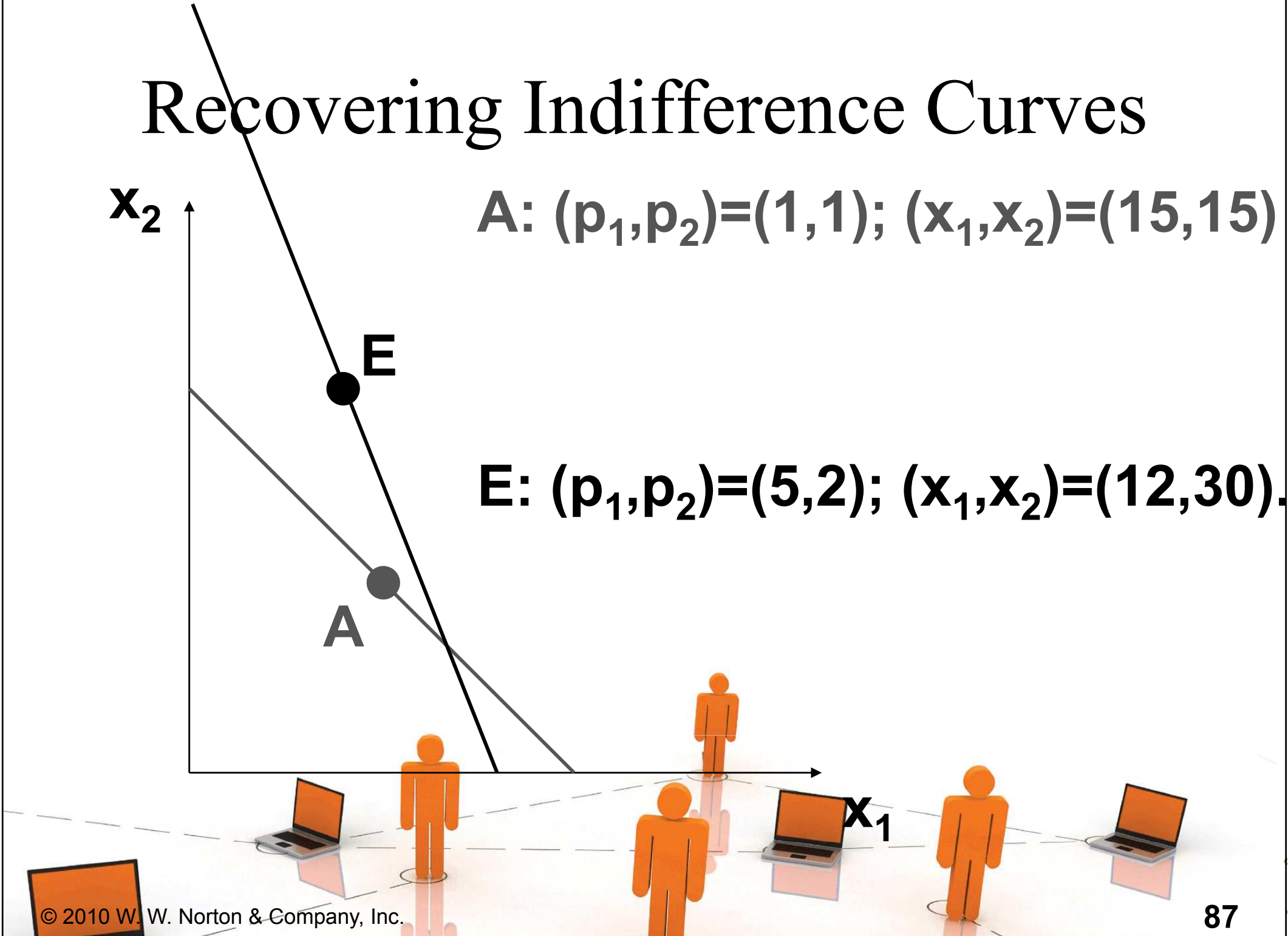
A:  $(p_1, p_2) = (1, 1)$ ;  $(x_1, x_2) = (15, 15)$

E

E:  $(p_1, p_2) = (5, 2)$ ;  $(x_1, x_2) = (12, 30)$

A

$x_1$



# Recovering Indifference Curves

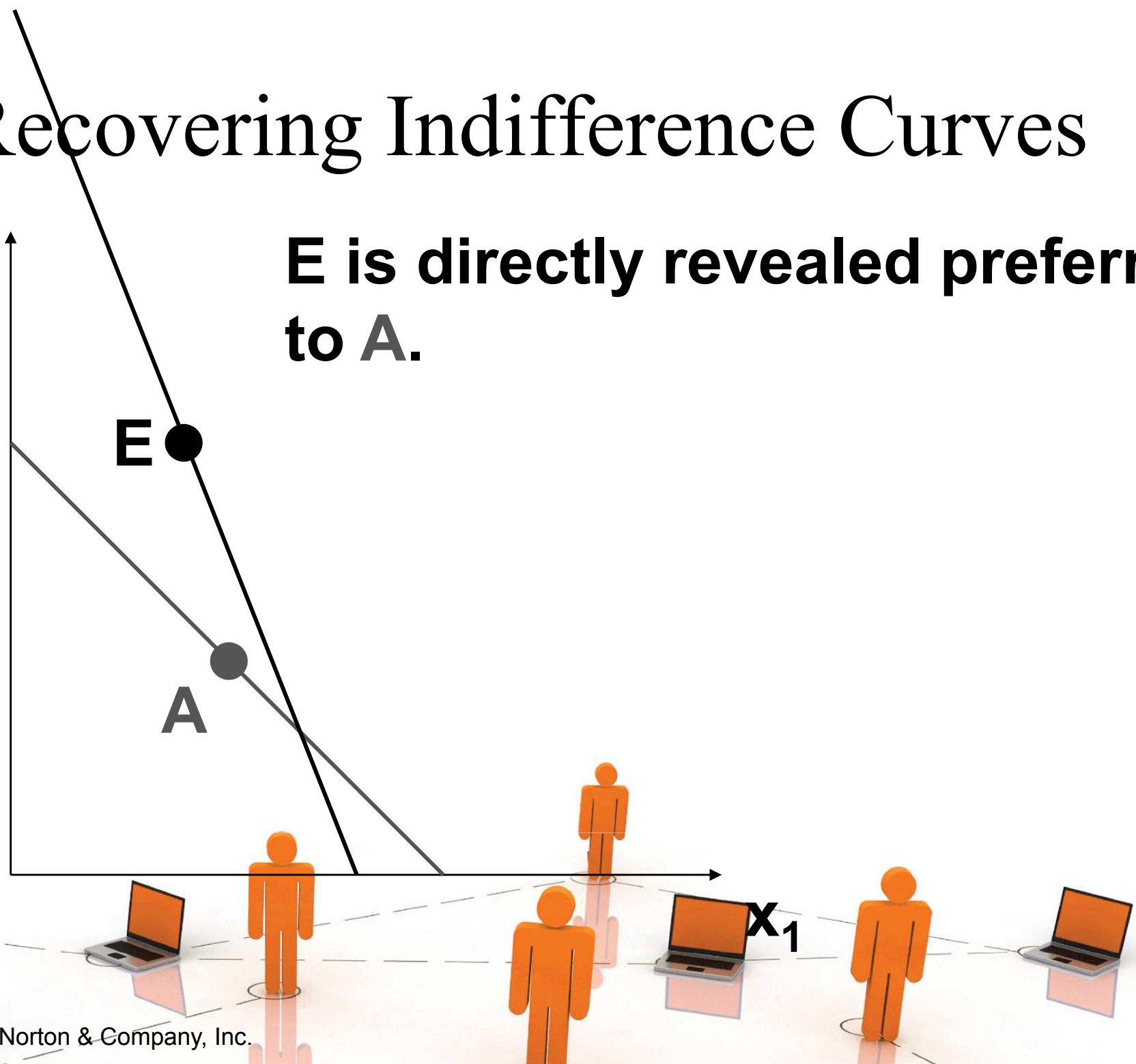
$x_2$

**E is directly revealed preferred to A.**

**E**

**A**

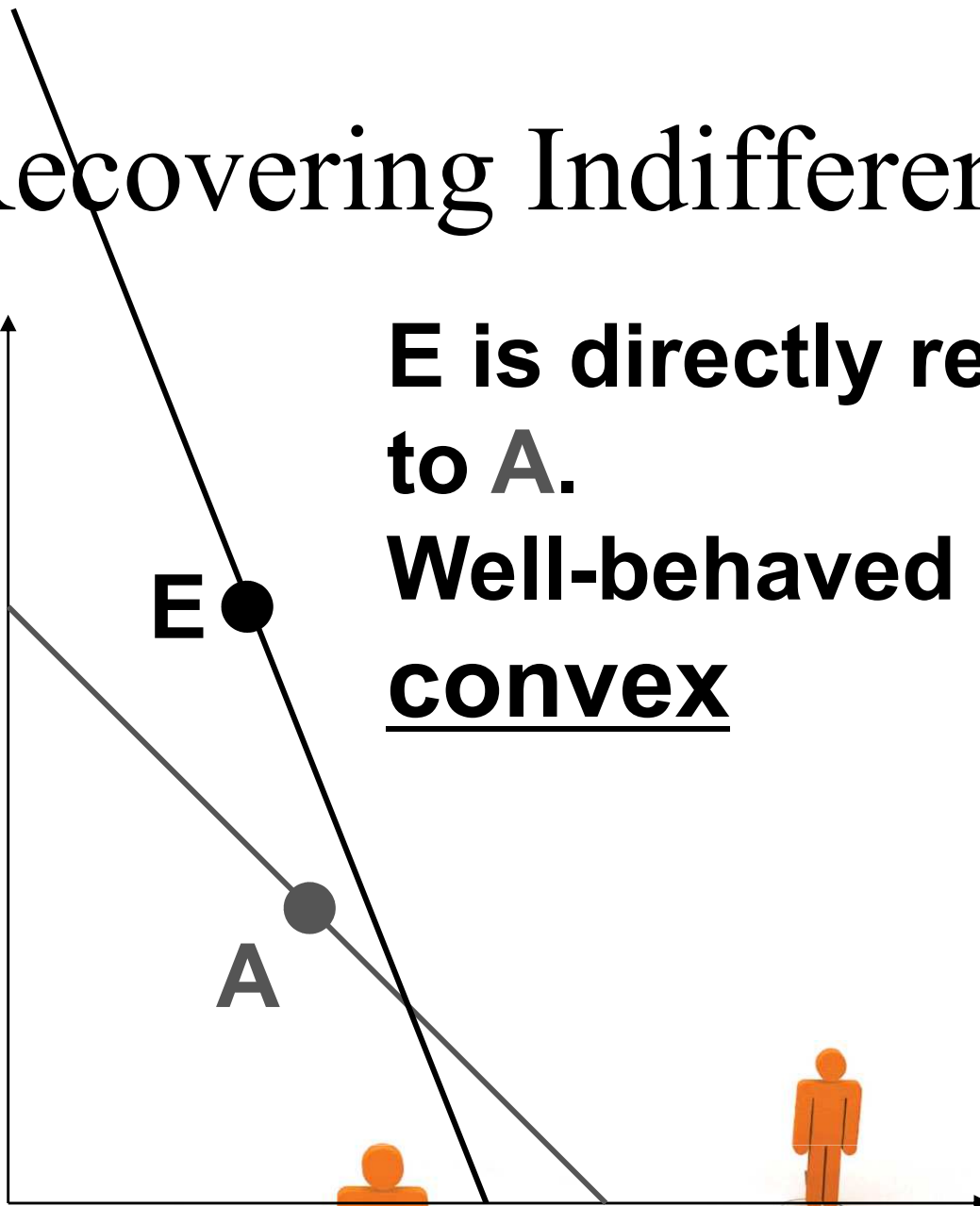
$x_1$





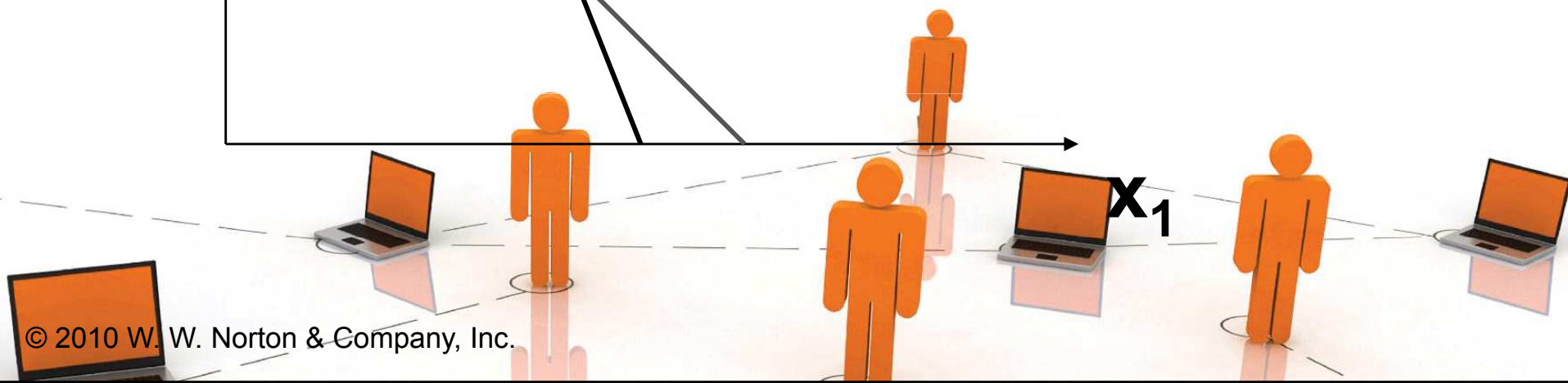
# Recovering Indifference Curves

$x_2$



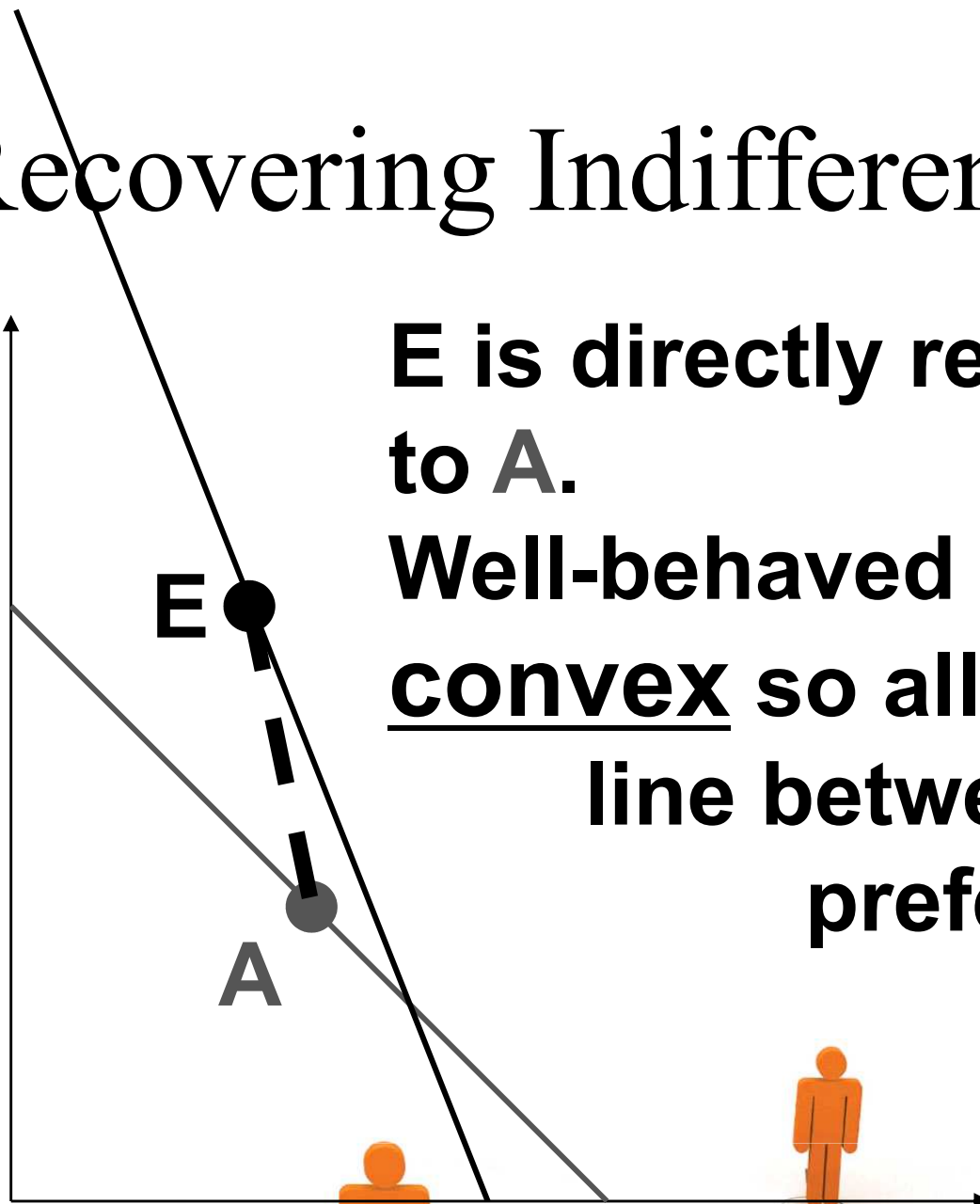
**E is directly revealed preferred to A.**

**Well-behaved preferences are convex**



# Recovering Indifference Curves

$x_2$



**E is directly revealed preferred to A.**

**Well-behaved preferences are convex so all bundles on the line between A and E are preferred to A also.**

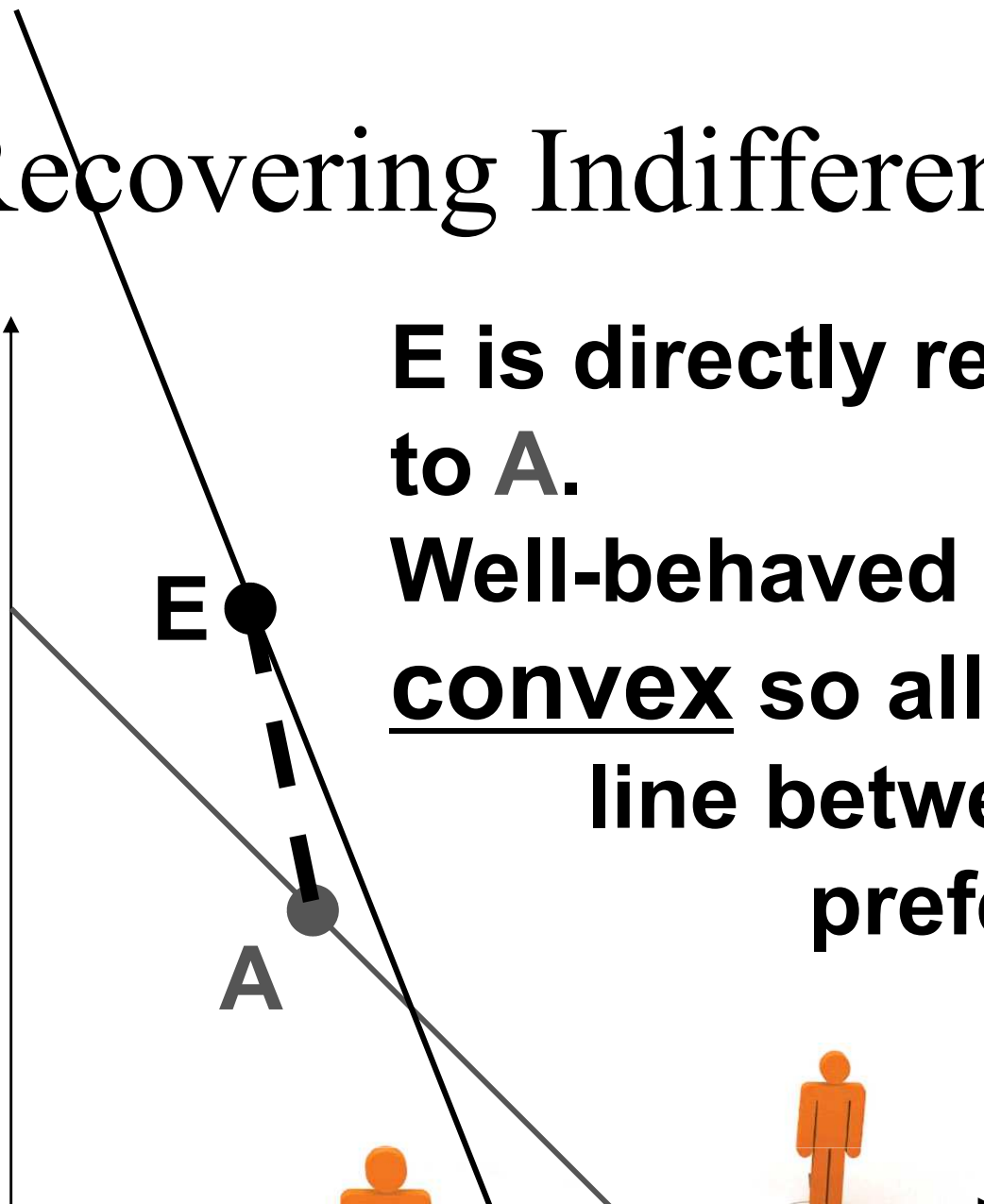


$x_1$



# Recovering Indifference Curves

$x_2$



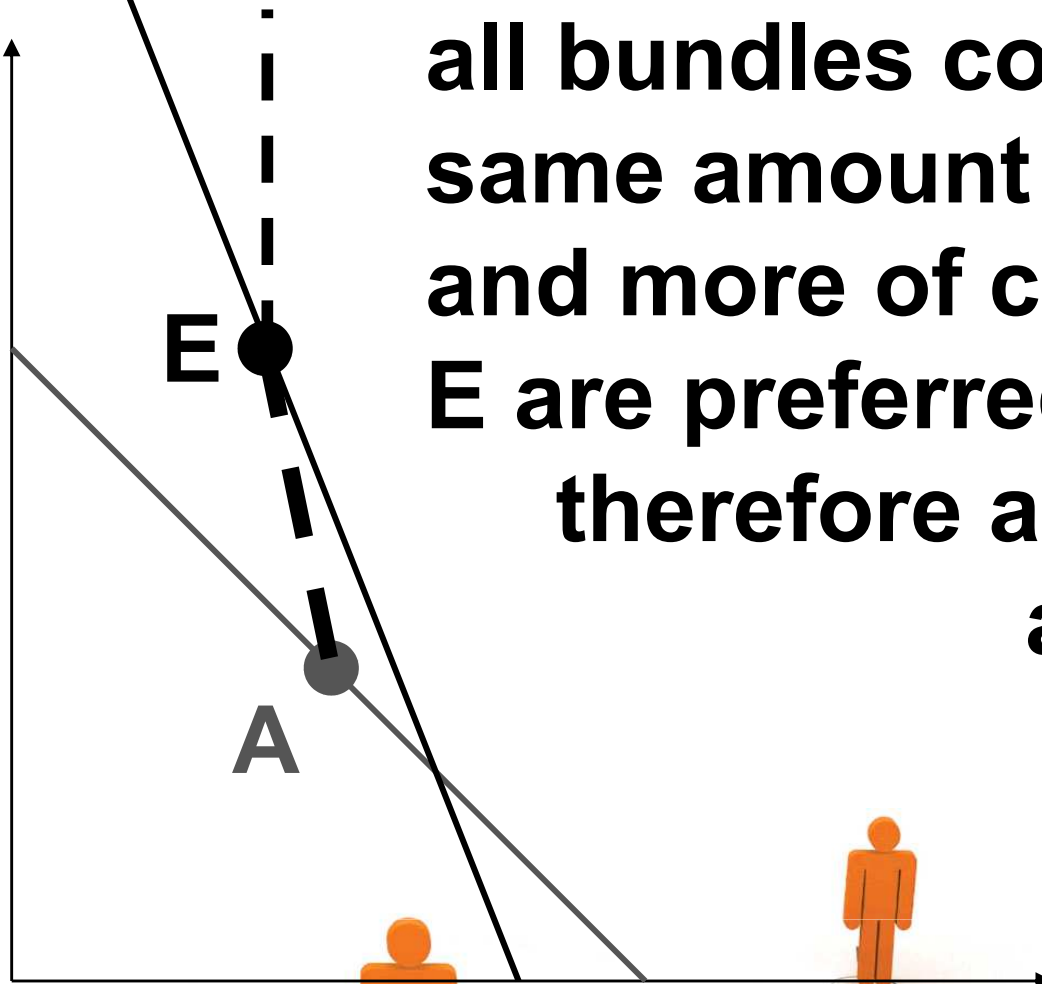
**E is directly revealed preferred to A.**

**Well-behaved preferences are convex so all bundles on the line between A and E are preferred to A also.**

**As well, ...**

# Recovering Indifference Curves

$x_2$



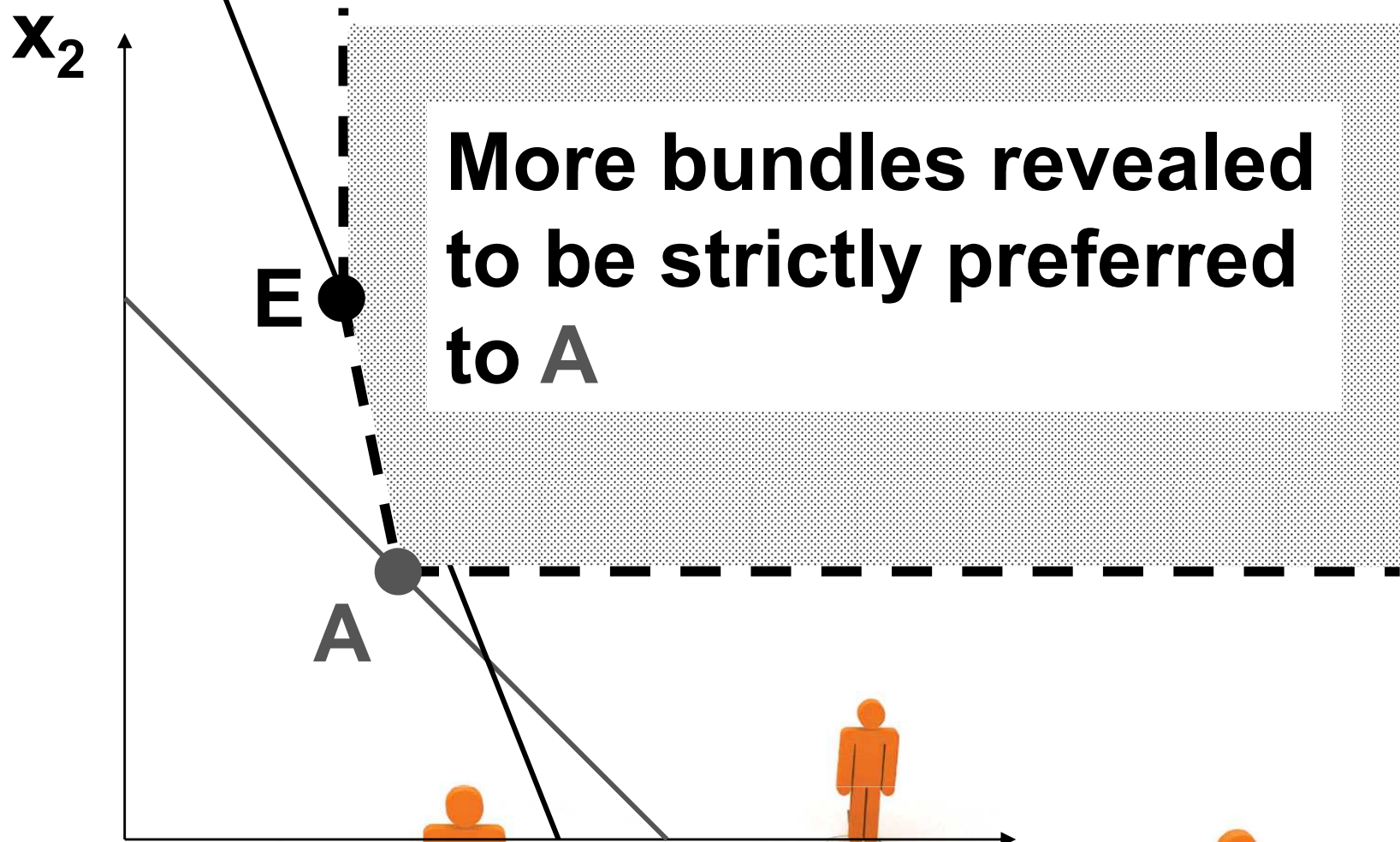
**all bundles containing the same amount of commodity 1 and more of commodity 2 than E are preferred to E and therefore are preferred to A also.**



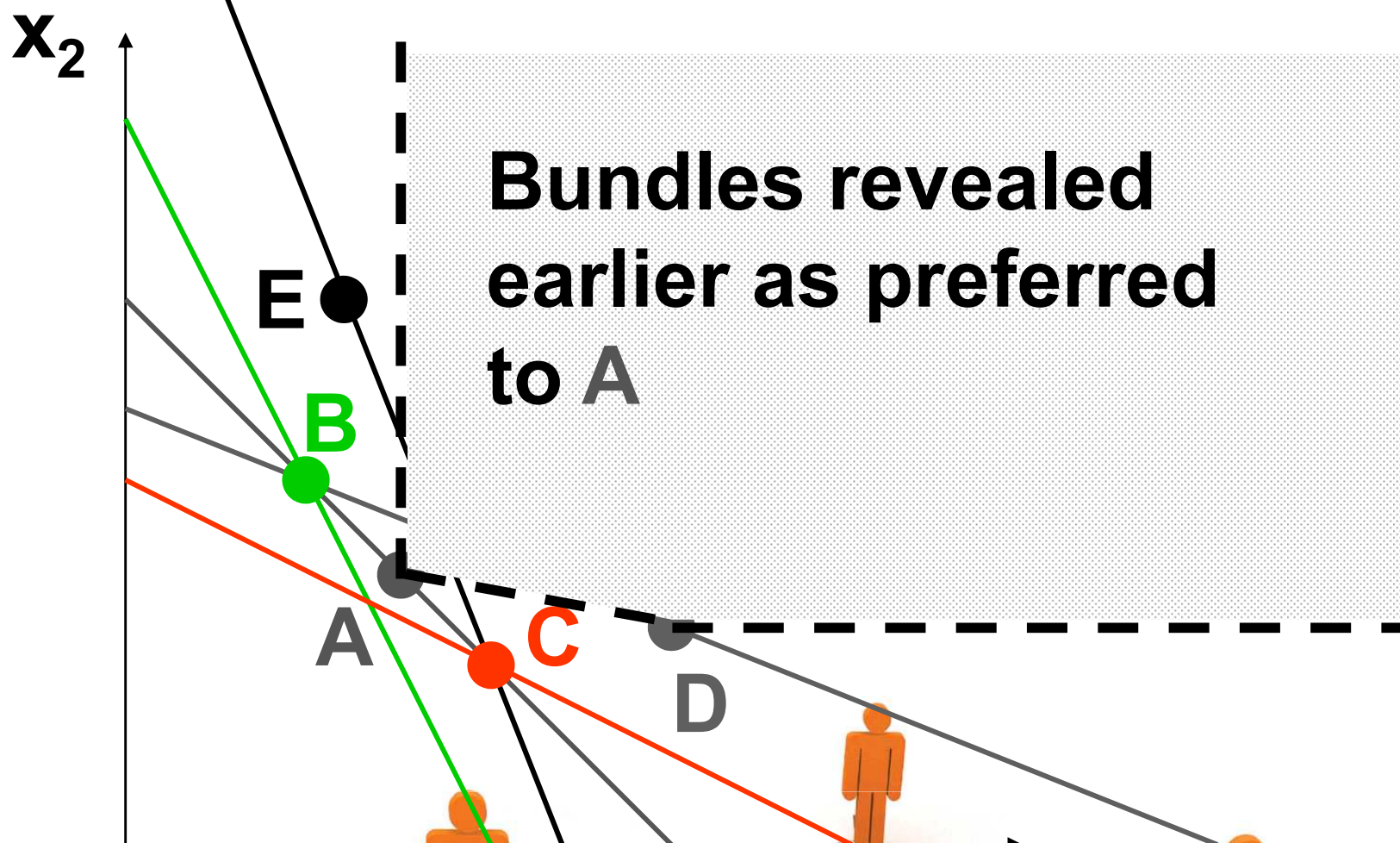
$x_1$



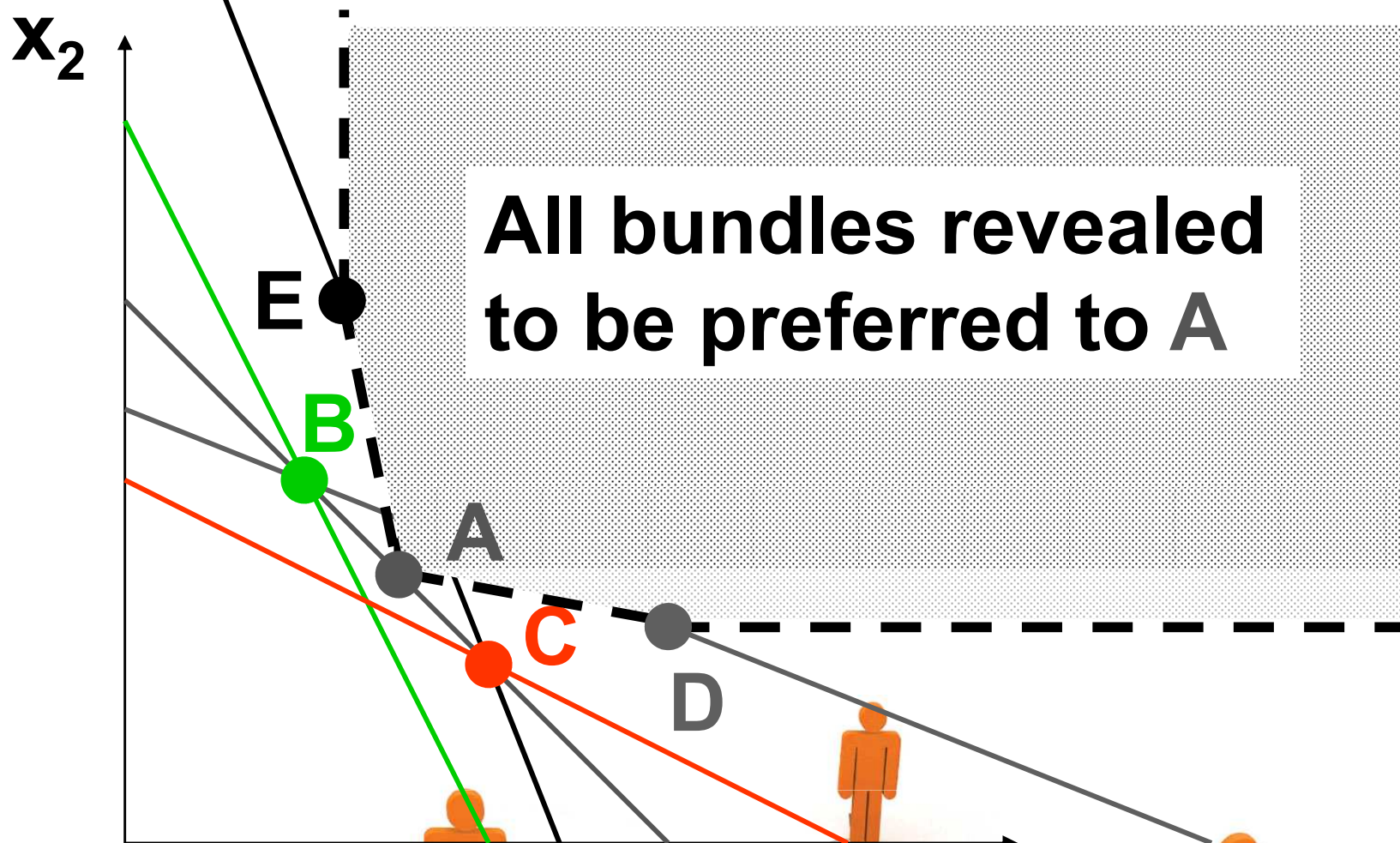
# Recovering Indifference Curves



# Recovering Indifference Curves



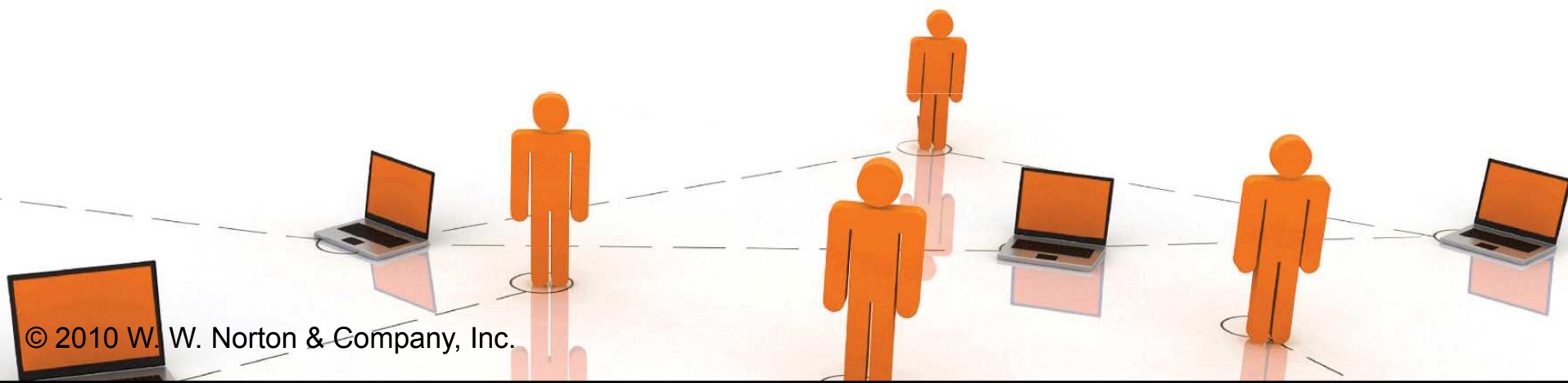
# Recovering Indifference Curves



All bundles revealed to be preferred to A

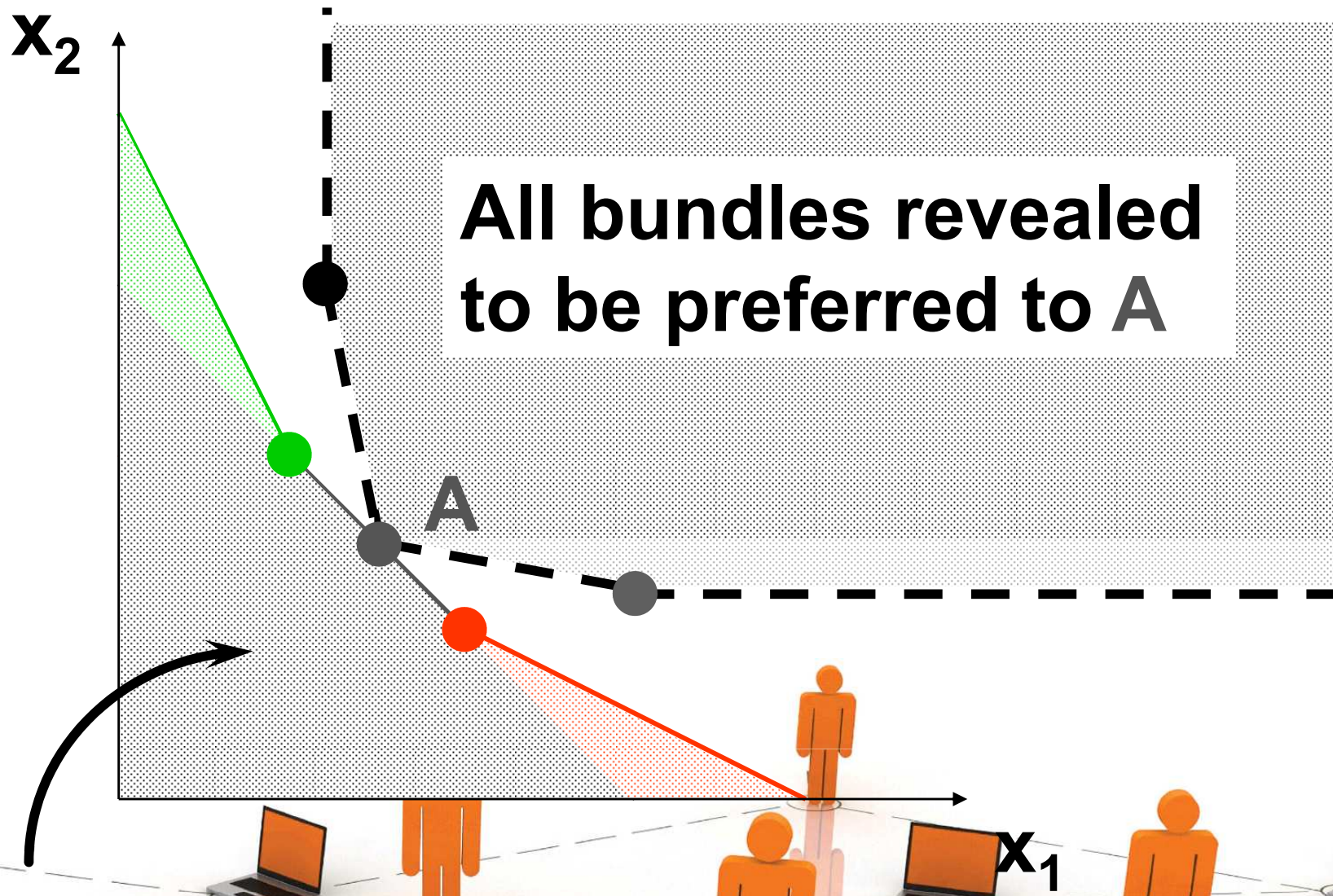
# Recovering Indifference Curves

- ◆ **Now we have upper and lower bounds on where the indifference curve containing bundle A may lie.**



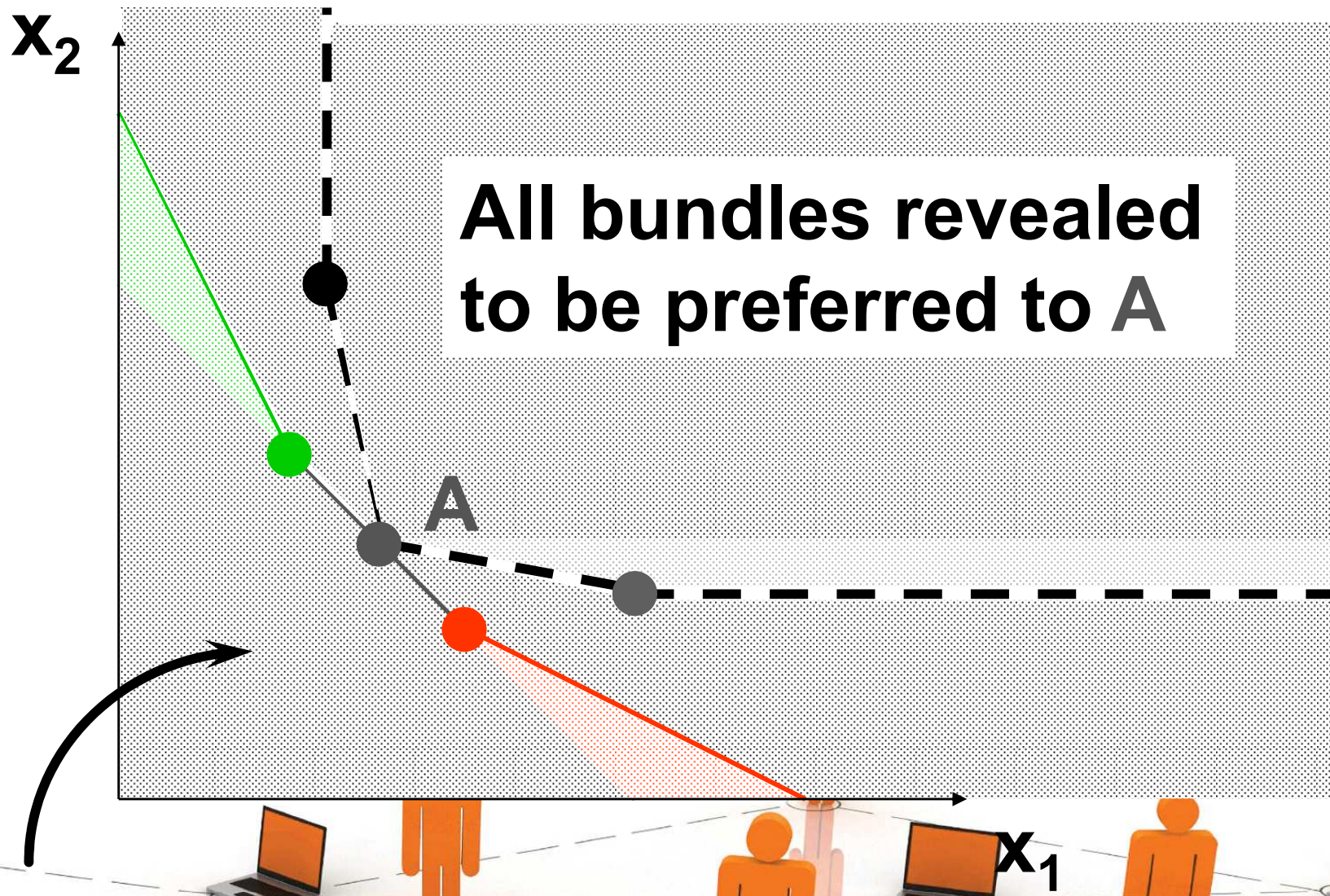


# Recovering Indifference Curves



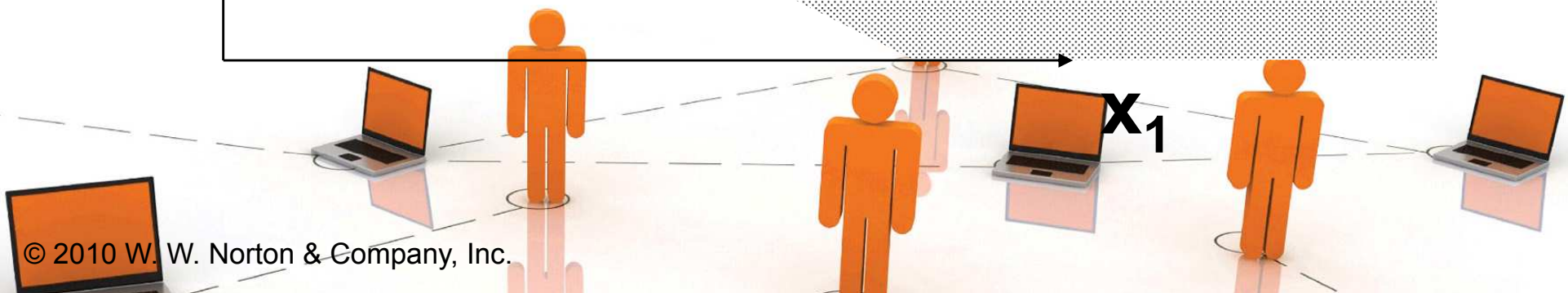
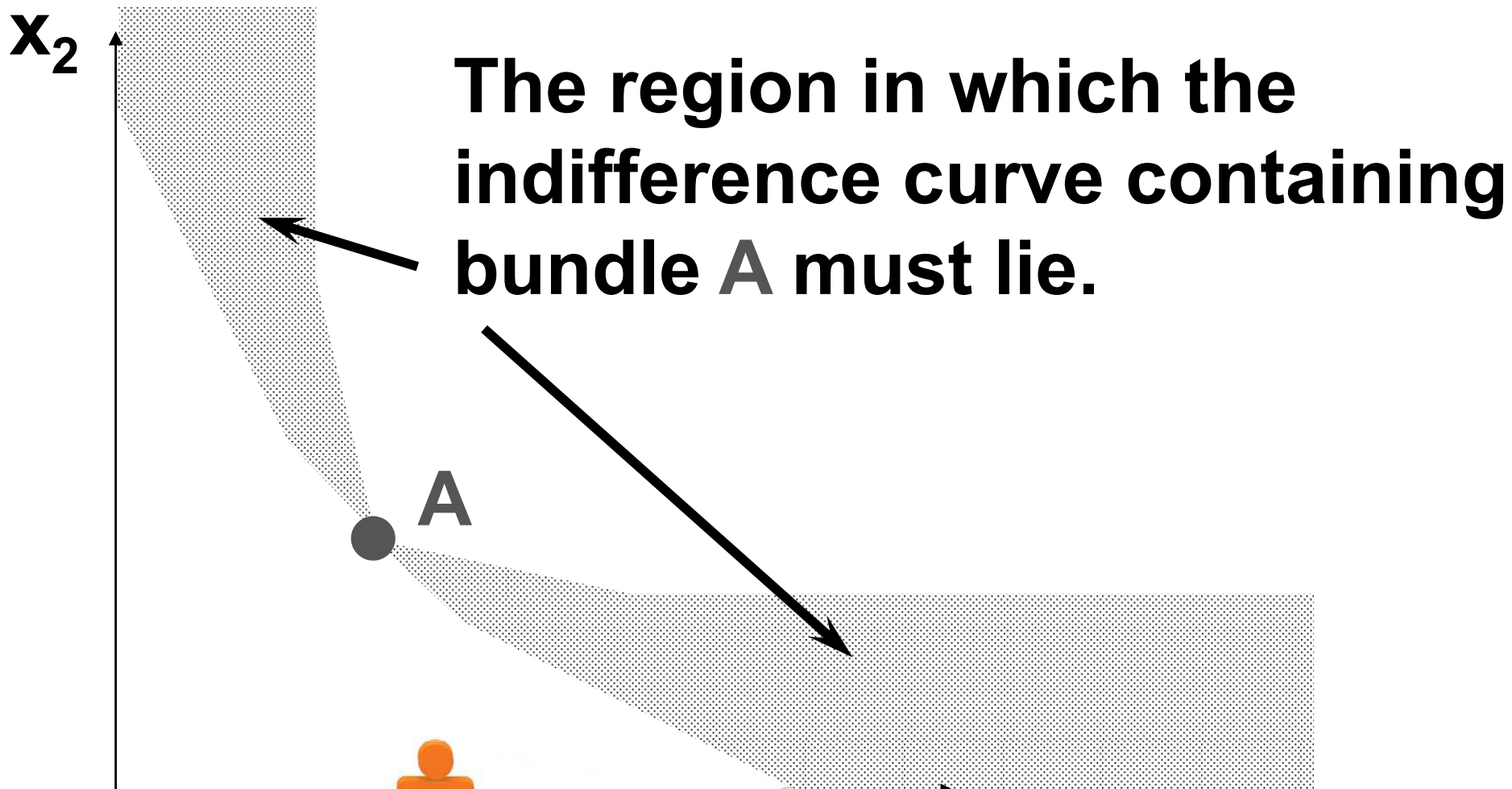
**All bundles revealed to be less preferred to A**

# Recovering Indifference Curves



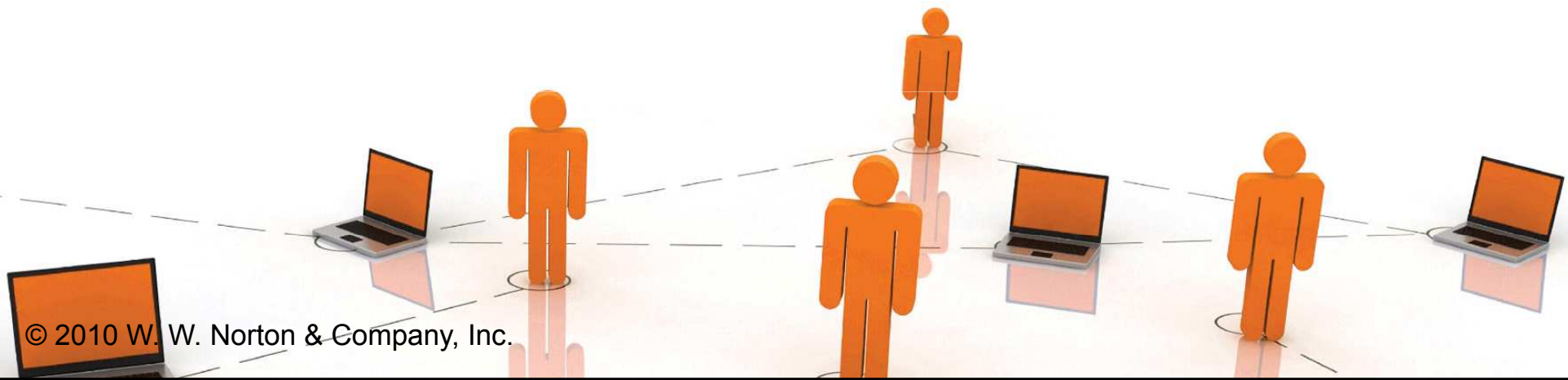
**All bundles revealed to be less preferred to A**

# Recovering Indifference Curves



# Index Numbers

- ◆ **Over time, many prices change. Are consumers better or worse off “overall” as a consequence?**
- ◆ **Index numbers give approximate answers to such questions.**



# Index Numbers

- ◆ **Two basic types of indices**
  - price indices, and
  - quantity indices
- ◆ **Each index compares expenditures in a base period and in a current period by taking the ratio of expenditures.**



# Quantity Index Numbers

- ◆ A quantity index is a price-weighted average of quantities demanded; *i.e.*

$$I_q = \frac{p_1 x_1^t + p_2 x_2^t}{p_1 x_1^b + p_2 x_2^b}$$

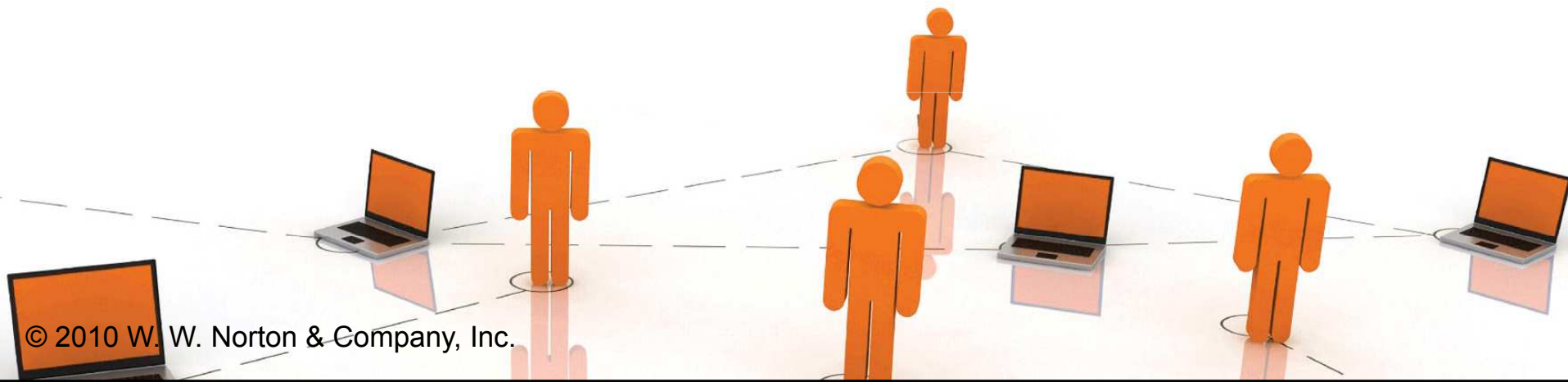
- ◆  $(p_1, p_2)$  can be base period prices  $(p_1^b, p_2^b)$  or current period prices  $(p_1^t, p_2^t)$ .



# Quantity Index Numbers

- ◆ If  $(p_1, p_2) = (p_1^b, p_2^b)$  then we have the Laspeyres quantity index;

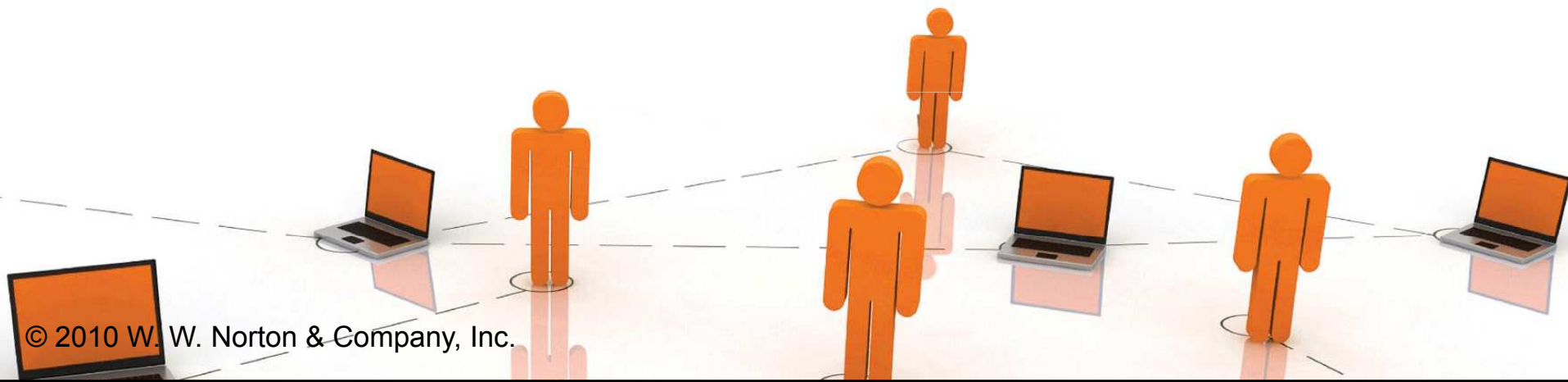
$$L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$



# Quantity Index Numbers

- ◆ If  $(p_1, p_2) = (p_1^t, p_2^t)$  then we have the Paasche quantity index;

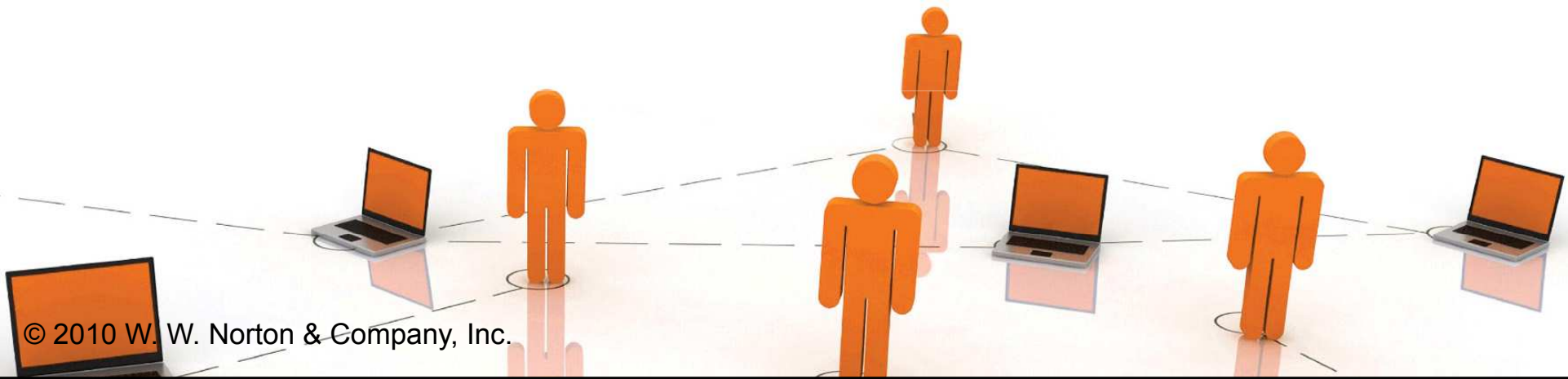
$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$





# Quantity Index Numbers

- ◆ **How can quantity indices be used to make statements about changes in welfare?**



# Quantity Index Numbers

◆ If  $L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b} < 1$  then

$$p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

**so consumers overall were better off in the base period than they are now in the current period.**



# Quantity Index Numbers

◆ If  $P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1$  then

$$p_1^t x_1^t + p_2^t x_2^t > p_1^t x_1^b + p_2^t x_2^b$$

**so consumers overall are better off in the current period than in the base period.**



# Price Index Numbers

- ◆ A price index is a quantity-weighted average of prices; *i.e.*

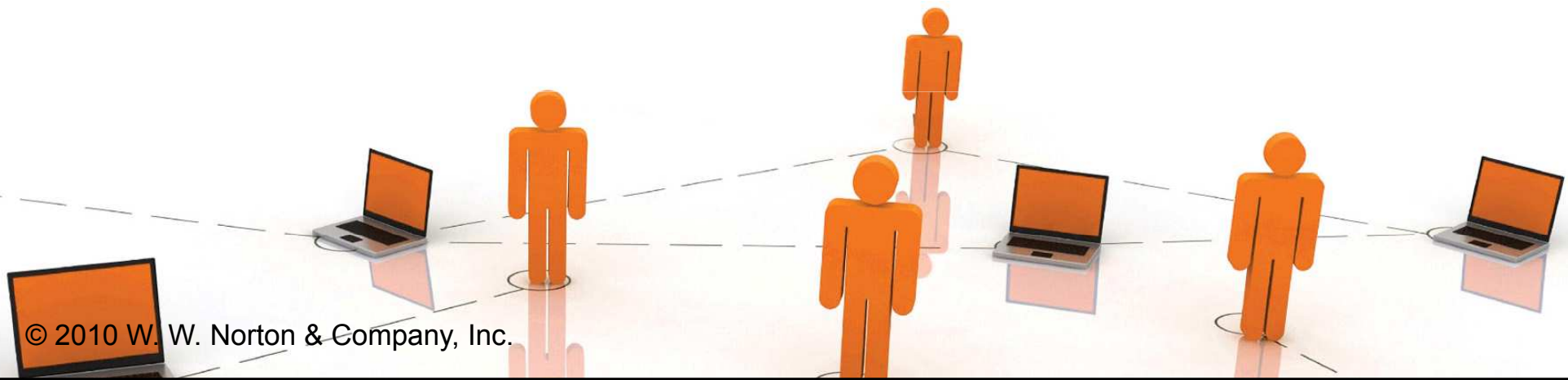
$$I_p = \frac{p_1^t x_1 + p_2^t x_2}{p_1^b x_1 + p_2^b x_2}$$

- ◆  $(x_1, x_2)$  can be the base period bundle  $(x_1^b, x_2^b)$  or else the current period bundle  $(x_1^t, x_2^t)$ .

# Price Index Numbers

- ◆ If  $(x_1, x_2) = (x_1^b, x_2^b)$  then we have the Laspeyres price index;

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b}$$



# Price Index Numbers

- ◆ If  $(x_1, x_2) = (x_1^t, x_2^t)$  then we have the Paasche price index;

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}$$



# Price Index Numbers

- ◆ How can price indices be used to make statements about changes in welfare?
- ◆ Define the expenditure ratio

$$M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$



# Price Index Numbers

◆ If

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b} < \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M$$

then

$$p_1^t x_1^b + p_2^t x_2^b < p_1^t x_1^t + p_2^t x_2^t$$

**so consumers overall are better off in the current period.**





# Price Index Numbers

◆ But, if

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t} > \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M$$

then

$$p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

so consumers overall were better off in the base period.

# Full Indexation?

- ◆ **Changes in price indices are sometimes used to adjust wage rates or transfer payments. This is called “indexation”.**
- ◆ **“Full indexation” occurs when the wages or payments are increased at the same rate as the price index being used to measure the aggregate inflation rate.**



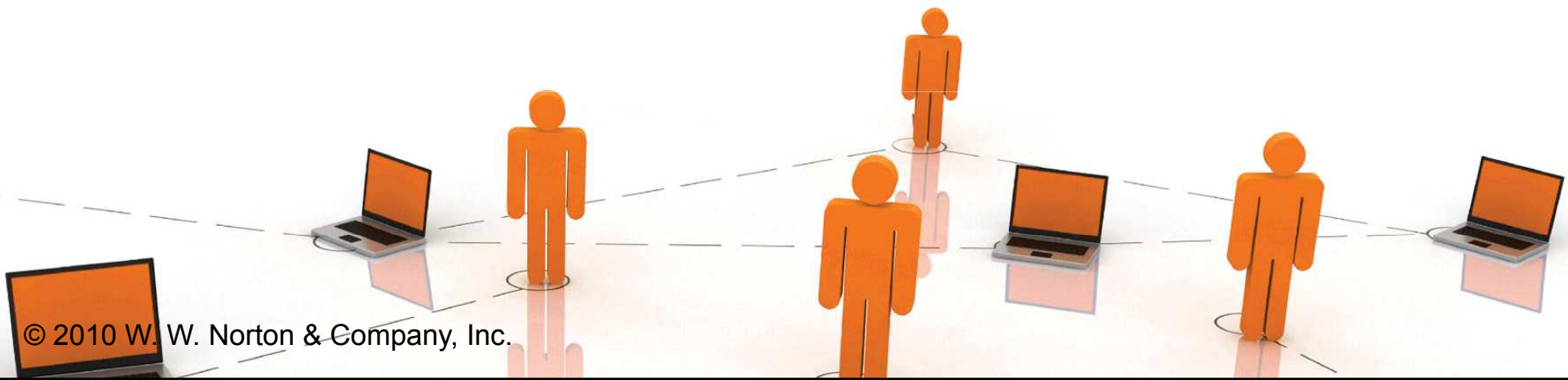
# Full Indexation?

- ◆ **Since prices do not all increase at the same rate, relative prices change along with the “general price level”.**
- ◆ **A common proposal is to index fully Social Security payments, with the intention of preserving for the elderly the “purchasing power” of these payments.**



# Full Indexation?

- ◆ **The usual price index proposed for indexation is the Paasche quantity index (the Consumers' Price Index).**
- ◆ **What will be the consequence?**



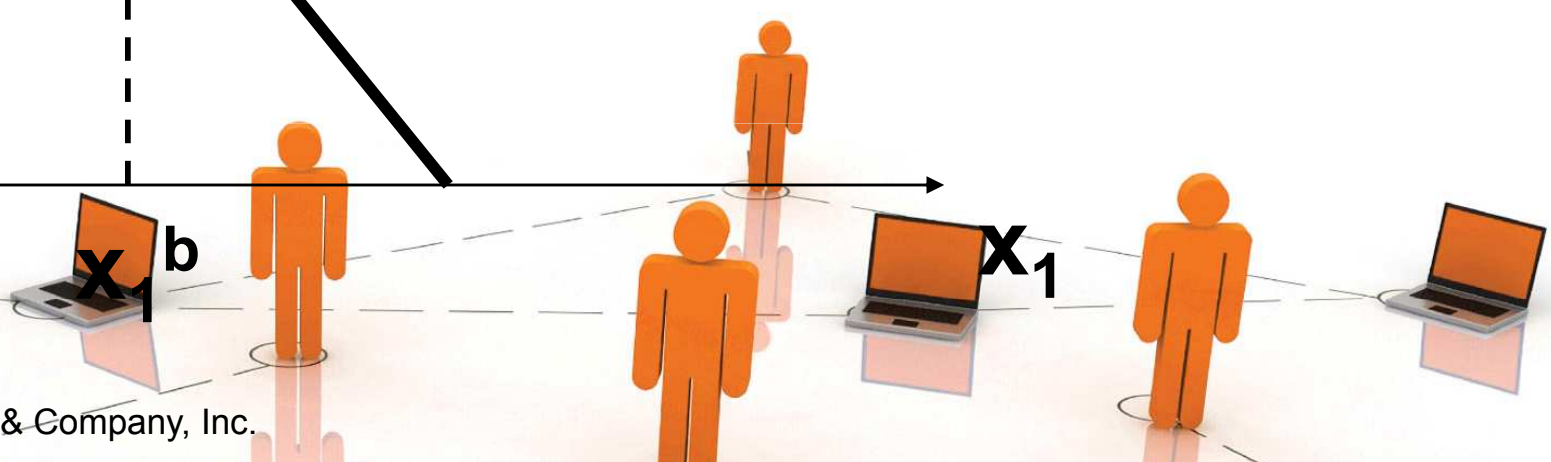
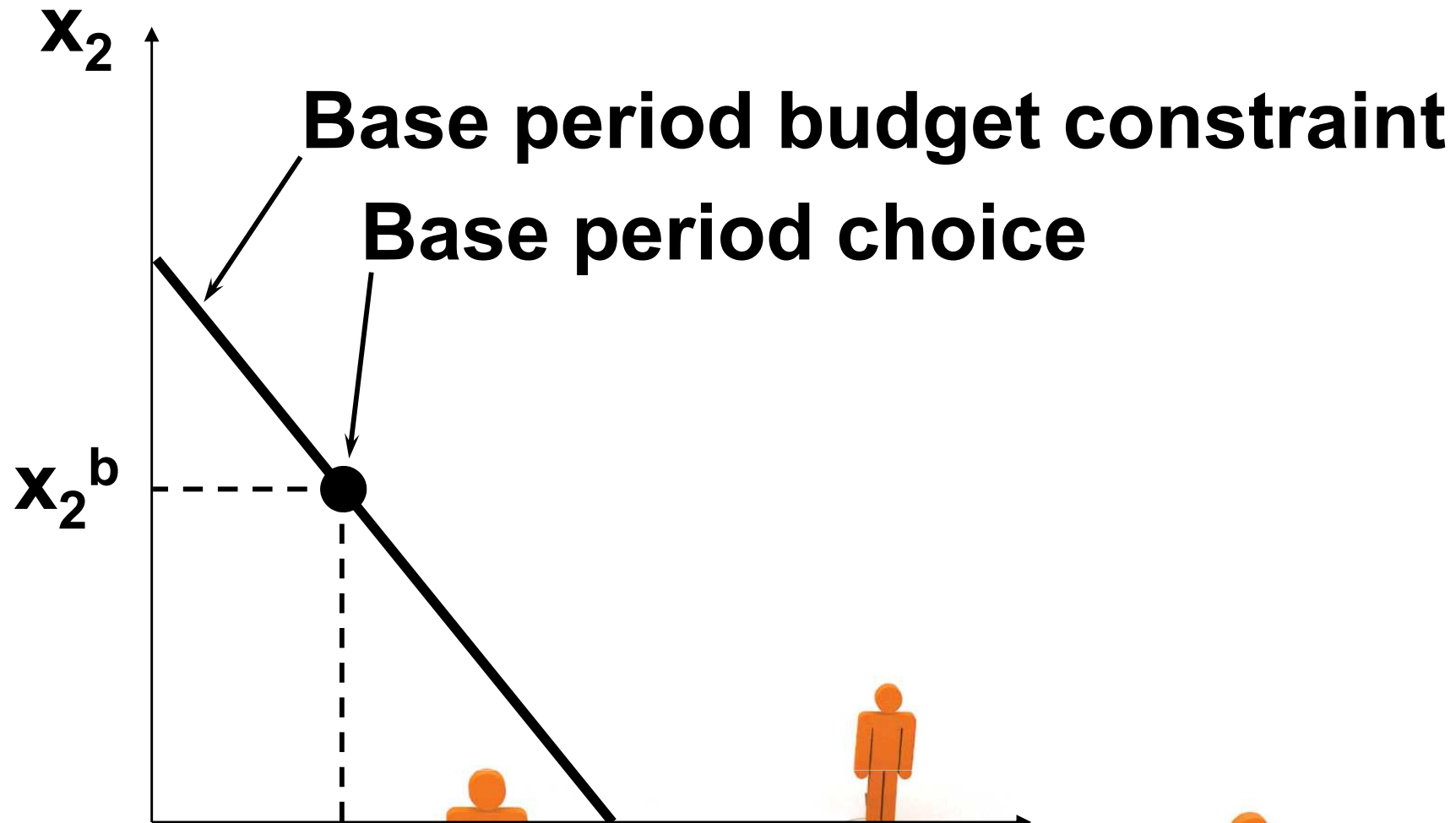
# Full Indexation?

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

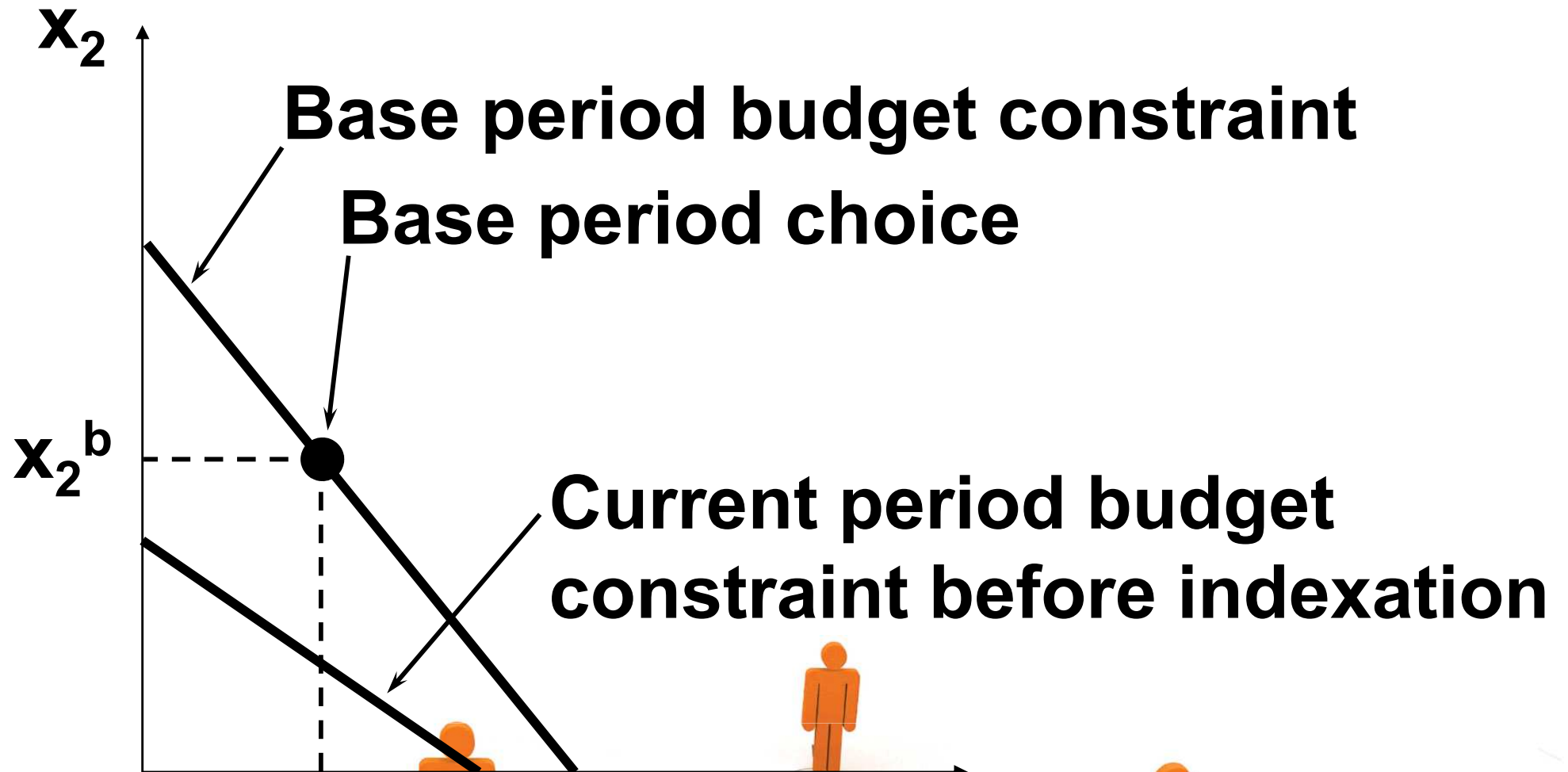
**Notice that this index uses current period prices to weight both base and current period consumptions.**



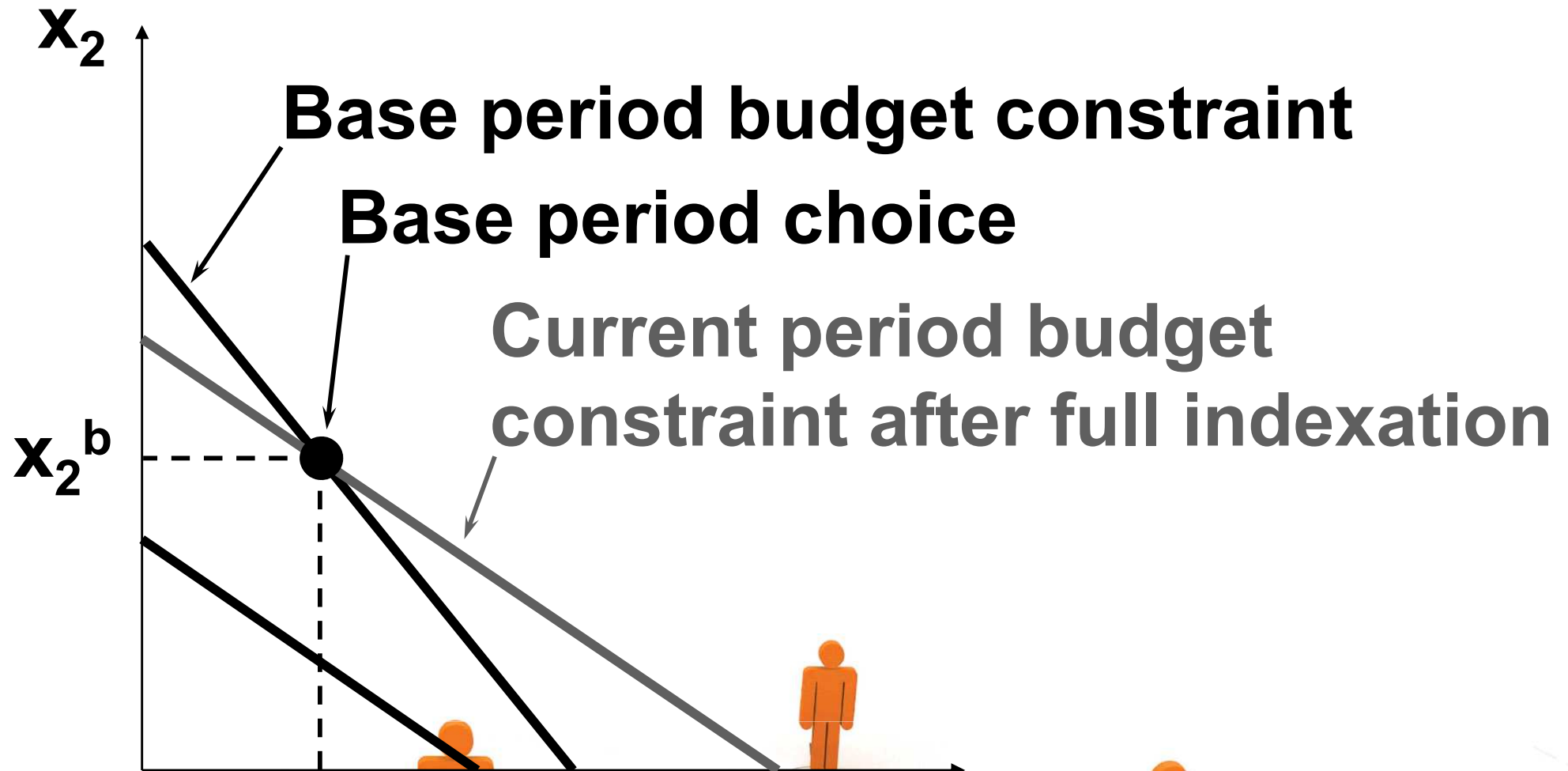
# Full Indexation?



# Full Indexation?

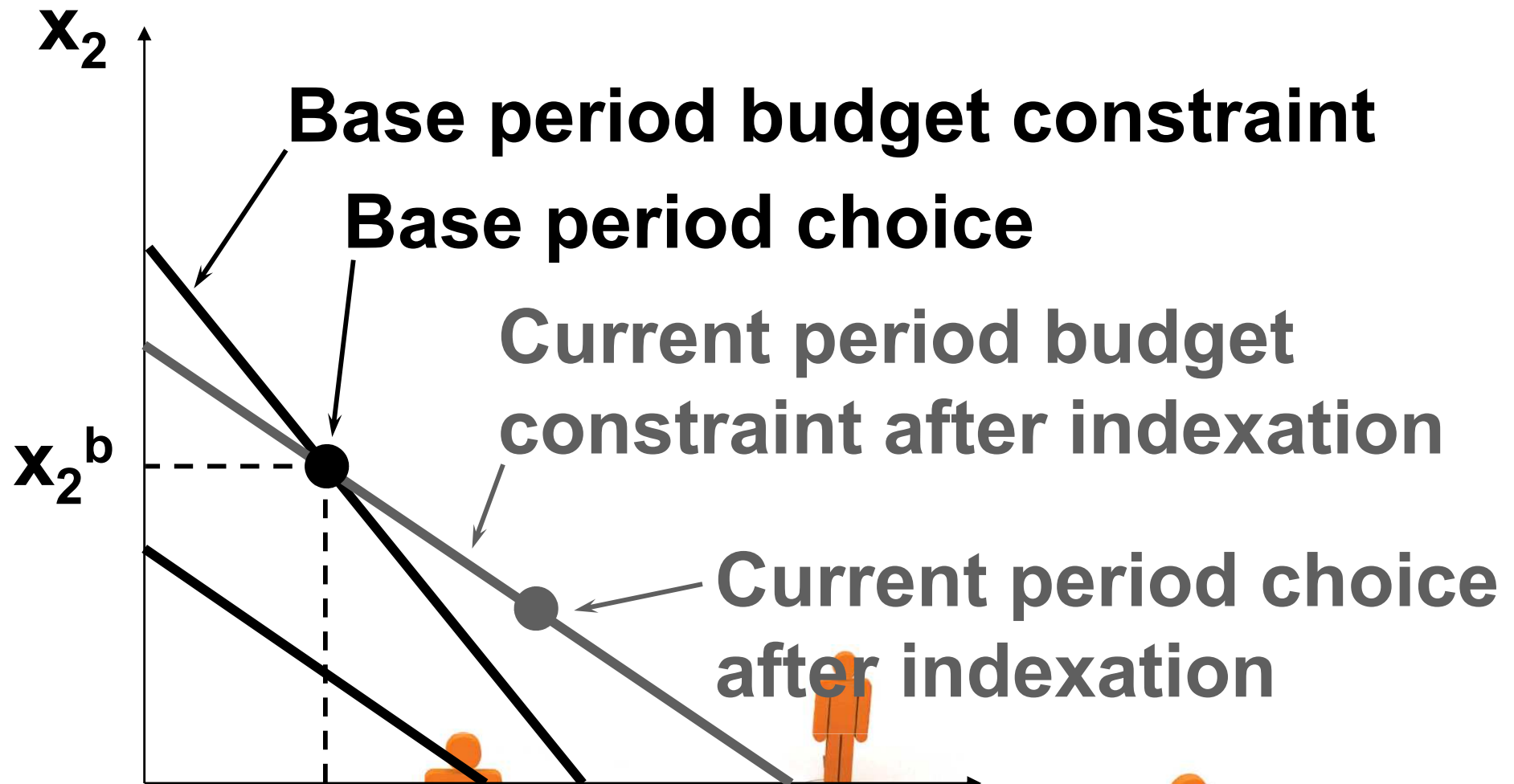


# Full Indexation?

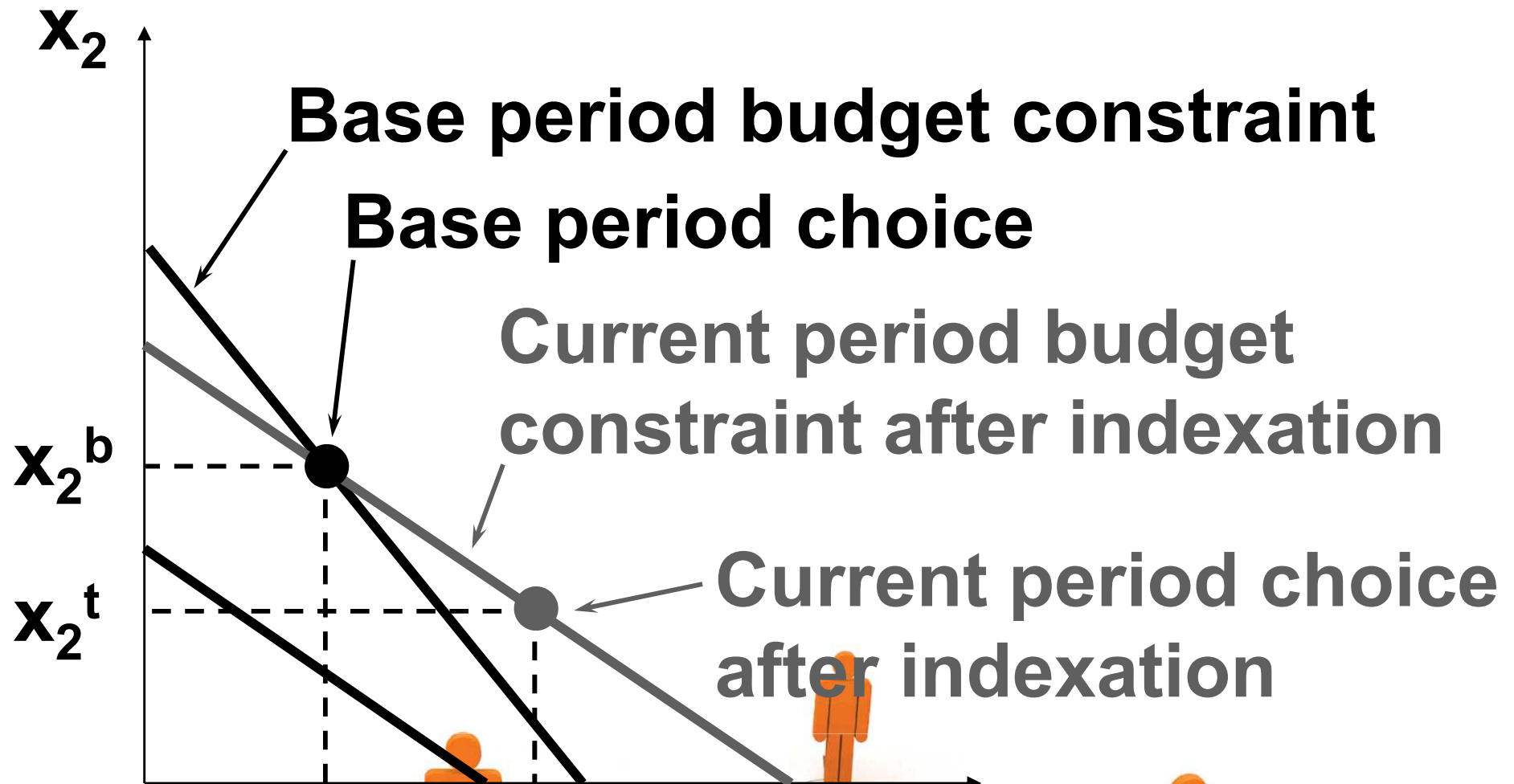




# Full Indexation?



# Full Indexation?



**Base period budget constraint**

**Base period choice**

**Current period budget constraint after indexation**

**Current period choice after indexation**

$x_2^b$

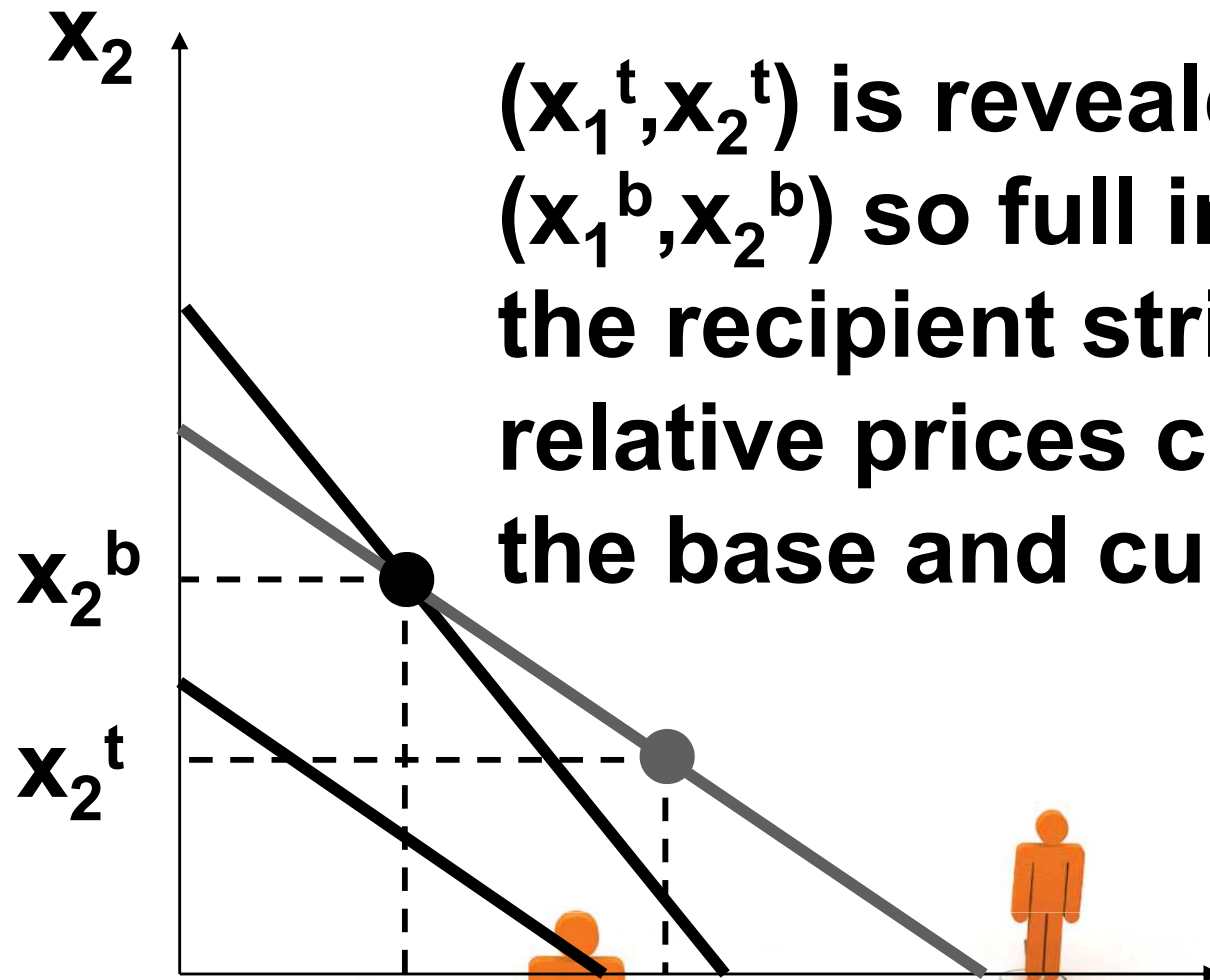
$x_2^t$

$x_1^b$

$x_1^t$

$x_1$

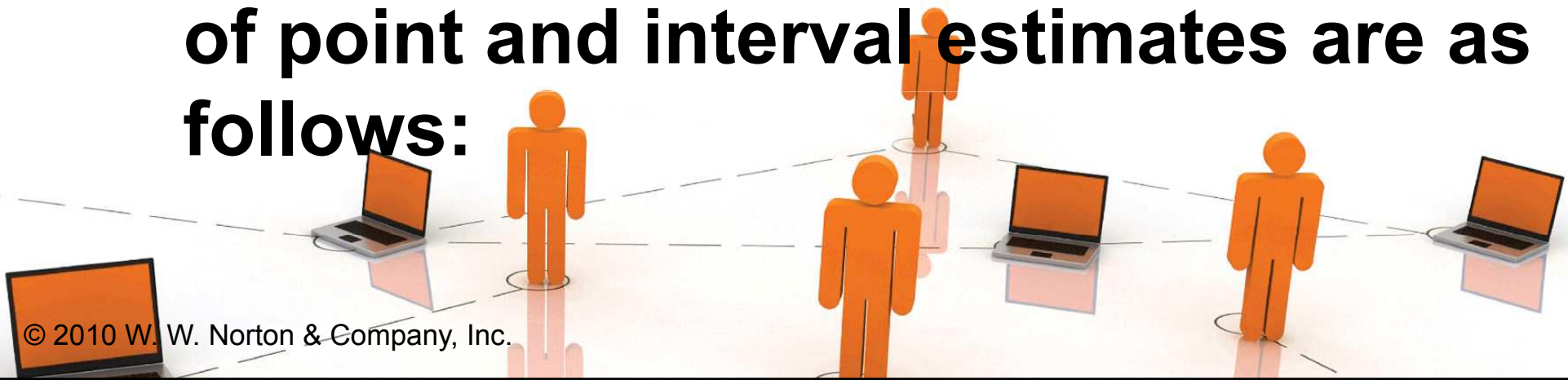
# Full Indexation?



$(x_1^t, x_2^t)$  is revealed preferred to  $(x_1^b, x_2^b)$  so full indexation makes the recipient strictly better off if relative prices change between the base and current periods.

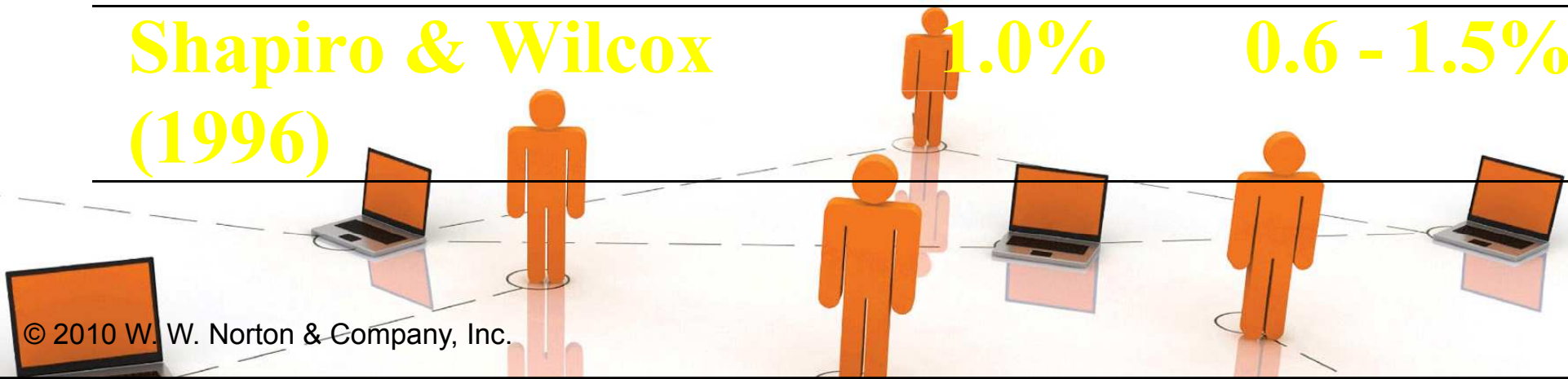
# Full Indexation?

- ◆ So how large is this “bias” in the US CPI?
- ◆ A table of recent estimates of the bias is given in the *Journal of Economic Perspectives*, Volume 10, No. 4, p. 160 (1996). Some of this list of point and interval estimates are as follows:



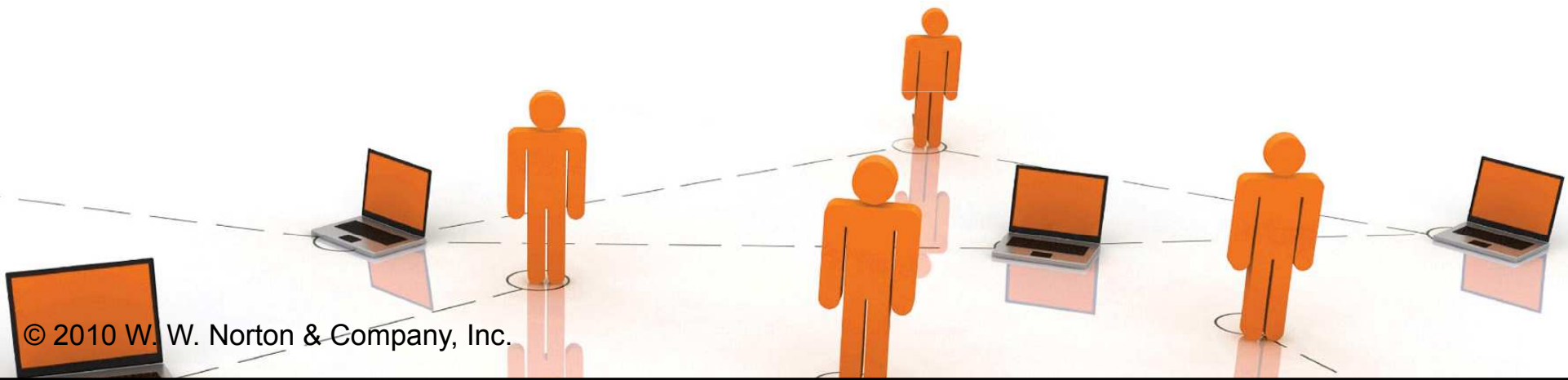
# Full Indexation?

Author	Point Est.	Int. Est.
Adv. Commission to Study the CPI (1995)	1.0%	0.7 - 2.0%
Congressional Budget Office (1995)		0.2 - 0.8%
Alan Greenspan (1995)		0.5 - 1.5%
Shapiro & Wilcox (1996)	1.0%	0.6 - 1.5%



# Full Indexation?

- ◆ **So suppose a social security recipient gained by 1% per year for 20 years.**
- ◆ **Q: How large would the bias have become at the end of the period?**



# Full Indexation?

- ◆ So suppose a social security recipient gained by 1% per year for 20 years.
- ◆ Q: How large would the bias have become at the end of the period?
- ◆ A:  $(1 + 0.01)^{20} = 1.01^{20} = 1.22$  so after 20 years social security payments would be about 22% “too large”.

