

Problem Set 1: Consumer Theory

Theory

1. Consider two utility functions $u : R_+^K \rightarrow R$ and $v : R_+^K \rightarrow R$, where $v = f(u)$ and $f : R \rightarrow R$ is *strictly increasing*. Find the relation between the Marshallian demand correspondences for u and v . Do the same for the indirect utility functions, Hicksian demand correspondences and expenditure functions.
2. Find the Marshallian and Hicksian demands, indirect utility, and expenditure functions for the following utility functions:
 - (a) (Cobb-Douglas) $u(x_1, \dots, x_n) = x_1^{\alpha_1} \dots x_n^{\alpha_n}$; $\alpha_1, \dots, \alpha_n > 0$
 - (b) (Leontieff, i.e. perfect complements) $u(x_1, \dots, x_n) = \min\{\alpha_1 x_1, \dots, \alpha_n x_n\}$; $\alpha_1, \dots, \alpha_n > 0$
 - (c) (Linear, i.e. perfect substitutes) $u(x_1, \dots, x_n) = \alpha_1 x_1 + \dots + \alpha_n x_n$; $\alpha_1, \dots, \alpha_n > 0$
 - (d) (Constant elasticity of substitution) $u(x_1, \dots, x_n) = [\sum_{i=1}^n a_i (b_i x_i)^\sigma]^{1/\sigma}$; $a_i, b_i > 0$, $\sigma \leq 1$. The elasticity of substitution is equal to $1/(1 - \sigma)$. Argue that: (1) when the elasticity of substitution is ∞ ($\sigma = 1$), preferences are of the perfect substitutes type that we considered in part (c) with $\alpha_i = a_i b_i$; (2) when the elasticity of substitution is 0 ($\sigma \rightarrow -\infty$), then, assuming $a_i = 1$ for all $i = 1, \dots, n$, preferences are of the perfect complements type that we considered in part (b) with $\alpha_i = b_i$; (3) when the elasticity of substitution is 1 ($\sigma = 0$), then, assuming $\sum_{i=1}^n a_i = 1$, preferences are of the Cobb-Douglas type that we considered in part (a) with $\alpha_i = a_i$, implying the restriction that the exponents in the utility function sum up to 1.
3. Let the utility function of a consumer be in a *quasi-linear* form:

$$u(x_1, \dots, x_K) = x_1 + f(x_2, \dots, x_K).$$

- (a) Assuming an interior optimum, show that Marshallian demands are of the form

$$\begin{aligned} x_1(p, w) &= w/p_1 - f(p) \\ x_i(p, w) &= g_i(p), \quad i = 2, \dots, n \end{aligned}$$

for some functions $f(\cdot)$ and $g_2(\cdot), \dots, g_n(\cdot)$. Interpret in terms of wealth effects.

- (b) Assuming an interior optimum, show that Hicksian demands are of the form

$$\begin{aligned} h_1(p, u) &= u - r(p) \\ h_i(p, u) &= s_i(p), \quad i = 2, \dots, n \end{aligned}$$

for some functions $r(\cdot)$ and $s_2(\cdot), \dots, s_n(\cdot)$.

4. Consider a consumer whose utility function is *homothetic*, i.e.

$$u(x) = U[f(x)],$$

where $U(\cdot)$ is strictly increasing and $f(\cdot)$ is homogeneous (of degree k), meaning that there exists $k > 0$ such that

$$f(\alpha x) = \alpha^k f(x) \text{ for all } \alpha \geq 0.$$

(a) Show that Marshallian demands are homogeneous of degree 1 in w , i.e.

$$x_i(p, w) = wx_i(p, 1).$$

Interpret in terms of wealth elasticities.

(b) Show that Hicksian demands are of the form

$$h_i(p, u) = g(u)h_i(p, 1),$$

where $g(\cdot)$ is some strictly increasing function that satisfies $g(1) = 1$.

Applications

Answer each question as “true”, “false”, or “uncertain” and justify your answer.

5. A good can be inferior over all wealth levels.
6. In the two-period consumption model, a lender may switch to being a borrower if the interest rate rises.
7. Libor consumes $q = 500$ half-liters of Pilsner Urquell per year to stay regular. Also assume that his demand for Pilsner Urquell is price elastic. If the price were to fall by Kc 0.5, he would buy 10 more half-liters a year. Then the price is at most Kc 25 per half-liter.
8. Ctibor and Umed have identical well-behaved preferences over leisure and income. They are paid Kc 100 for the first 40 hours of work a week and Kc 150 for each additional hour of TAing for Peter. They choose to work 50 hours per week carefully grading the problem sets and holding flawless TA sessions. Peter proposes to replace the "time and a half for overtime" pay schedule with a constant wage rate of Kc 110 per hour. Then Ctibor and Umed can legitimately claim that this will reduce their income, and Peter can rightly claim that this will make Ctibor and Umed better off.
9. Every day Umed has an endowment of 24 hours and has homothetic and strictly convex preferences over the consumption of leisure and income. He works 8 hours in the present job at a small private company. Then he gets a very lucrative alternative offer from CERGE-EI to work as a micro TA that pays twice as high the wage as his current job. He can choose how many hours to work at CERGE-EI (i.e., he would not necessarily work 8 hours). However, academia is not industry, and it is understood that Umed will be required to spend some time every day in informal discussions with his students regarding the latest problem set and he will not be compensated for that time. He is going to take the CERGE-EI offer only if it makes him strictly better off than the present job. Then if Umed decides to take the CERGE-EI offer, he will earn a higher income.

10. A consumer cares about the consumption of a composite consumption good with the (pre-tax) price equal to unity and leisure. He has a flex-hours job with a constant hourly wage rate w for as many hours as he decides to work. The government needs to raise a positive tax revenue R from this consumer using a flat (constant) tax rate $\tau \in [0, 1]$ on labor income and/or a flat tax rate $t \in [0, 1]$ on consumer expenditure. Under these taxes, the net pay per hour of labor a consumer receives is $(1 - \tau)w$, and the unit of consumption costs $1 + t$. The government designs the tax system optimally, i.e., in a way that maximizes consumer utility subject to the given amount of revenue being collected. Then only the tax on consumer expenditure is used.
11. Ctirad, apart from enjoying leisure from time to time, also consumes two other consumption goods that he finances out of his flexible hour job earnings. Libor, a chief advisor to the Ministry of Finance, realizes government budgetary needs and identifies Ctirad as a suitable source of revenue. Being a Chicago-style public finance economist, he suggests levying a lump-sum tax on Ctirad. The Ministry, however, aware of Ctirad's inequality aversion, is afraid that he will take to the streets protesting the tax. Well aware of the importance of proper framing, the Ministry comes up with an alternative proposal to put an equal proportional consumption tax on both consumption goods. Then, given that all the consumed goods are taxed at the same rate, the two proposals are equivalent in terms of the deadweight loss that they generate.