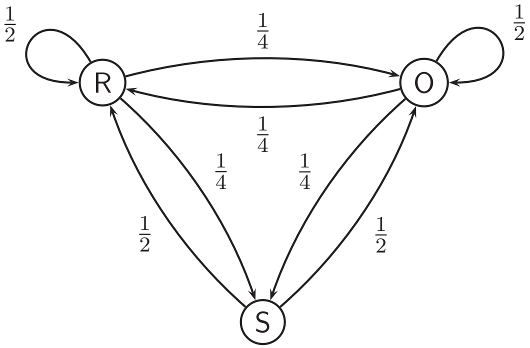
Example: Weather transitions



where

R is rain,

O is overcast, and

S is sunshine.

Represented as a transition matrix

		$t+1$		
		R	O	S
	R	0.50	0.25	0.25
t	O	0.25	0.50	0.25
	S	0.50	0.50	0.00

Such a square array is called *the matrix of transition probabilities*, or *the transition matrix*.

We denote the probability that, given the chain is in state i today, it will be in state j n days from now $p_{ij}^{(n)}$.

What is the probability that it will be overcast in two days if it is overcast today?

Represented as a transition matrix

The weather today is known to be overcast. This can represented by the following vector:

$$\mathbf{x}^{(0)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

The weather tomorrow (one day from now) can be predicted by

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)}\Pi = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.25 & 0.50 & 0.25 \\ 0.50 & 0.50 & 0.00 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.50 & 0.25 \end{bmatrix}$$

The weather two days from now can be predicted by

$$\mathbf{x}^{(2)} = \mathbf{x}^{(1)}\Pi = \begin{bmatrix} 0.25 & 0.50 & 0.25 \end{bmatrix} \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.25 & 0.50 & 0.25 \\ 0.50 & 0.50 & 0.00 \end{bmatrix} = \begin{bmatrix} 0.3750 \\ 0.4375 \\ 0.1875 \end{bmatrix}'$$

cont'd

The weather n days from now can be predicted by

$$\mathbf{x}^{(n)} = \mathbf{x}^{(0)}\Pi^n = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.25 & 0.50 & 0.25 \\ 0.50 & 0.50 & 0.00 \end{bmatrix}^n$$

and in the limit

$$\lim_{n\to\infty} \mathbf{x}^{(n)} = \lim_{n\to\infty} \mathbf{x}^{(0)}\Pi^n$$
$$= \lim_{n\to\infty} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.25 & 0.50 & 0.25 \\ 0.50 & 0.50 & 0.00 \end{bmatrix}^n = \begin{bmatrix} 0.4 & 0.4 & 0.2 \end{bmatrix}$$