An Introduction to Portfolio Management

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- Creation of an optimal portfolio
  - Not only combination a lot of individual securities with described risk-return characteristics
- Reaction among investments

Portfolio theory - assumptions

- An investor want to maximize the return from your investments for a given level of risk

Risk Aversion

Given a choice between two assets with equal rates of return, most investors will select the asset with the lower level of risk.

Evidence That Investors are Risk Averse

- Many investors purchase insurance for: Life, Automobile, Health, and Disability Income.
  - The purchaser trades known costs for unknown risk of loss
- Yield on bonds increases with risk classifications from AAA to AA to A….

Not all investors are risk averse

Not everybody buys insurance for everything

Friedman and Savage:

Risk preference may have to do with amount of money involved - risking small amounts, but insuring large losses

Basis assumption

Positive relationship between expected return and expected risk

Definition of Risk

1. Uncertainty of future outcomes

or

2. Probability of an adverse outcome

Markowitz Portfolio Theory

- In early 1960's, investment community talked about risk
  - No specific measure for them
- Quantifies risk variable
- Model of Harry Markowitz
- Derives the expected rate of return for a portfolio of assets and an expected risk measure
- Shows that the variance of the rate of return is a meaningful measure of portfolio risk
- Derives the formula for computing the variance of a portfolio, showing how to effectively diversify a portfolio

Assumptions of Markowitz Portfolio Theory

1. Investors consider each investment alternative as being presented by a probability distribution of expected returns over some holding period.

Assumptions of Markowitz Portfolio Theory

2. Investors minimize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.

Assumptions of Markowitz Portfolio Theory

3. Investors estimate the risk of the portfolio on the basis of the variability of expected returns.

Assumptions of Markowitz Portfolio Theory

4. Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance (or standard deviation) of returns only.

Assumptions of Markowitz Portfolio Theory

5. For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected returns, investors prefer less risk to more risk.

Markowitz Portfolio Theory

Using these five assumptions, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

Alternative Measures of Risk

- Variance or standard deviation of expected return
  - Dispersion of returns around the expected value
    - Larger variance greater dispersion and greater uncertainity of future returns
- Range of returns
  - A larger range of expected returns, from lowest ro the highest return, means greater uncertainty and risk regarding future expected returns
- Returns below expectations
  - Semivariance – a measure that only considers deviations below the mean
    - Computed expected returns below zero
  - These measures of risk implicitly assume that investors want to minimize the damage from returns less than some target rate

- Variance or standard deviation
  - This measure is intuitive
  - It is correct and widely recognized risk measure
  - Used in most of the theoretical asset pricing models

Expected Rates of Return

- For an individual asset - sum of the potential returns multiplied with the corresponding probability of the returns
- For a portfolio of assets - weighted average of the expected rates of return for the individual investments in the portfolio

Computation of Expected Return for an Individual Risky Investment

Exhibit 7.1

Probability	Possible Rate of Return (Percent)	Expected Return (Percent)
0.25	0.08	0.0200
0.25	0.10	0.0250
0.25	0.12	0.0300
0.25	0.14	0.0350
		E(R) = 0.1100

Computation of the Expected Return for a Portfolio of Risky Assets

Weight (W_i) (Percent of Portfolio)	**Expected Security Return (R_i)**	**Expected Portfolio Return (W_i X R_i)**
0.20	0.10	0.0200
0.30	0.11	0.0330
0.30	0.12	0.0360
0.20	0.13	0.0260
		$E(R_{por\ i}) =$ 0.1150

$$E(R_{por\ i}) = \sum_{i=1}^{n} W_i R_i$$

where :

W_i = the percent of the portfolio in asset i

$E(R_i)$ = the expected rate of return for asset i

Exhibit 7.2

Variance (Standard Deviation) of Returns for an Individual Investment

Standard deviation is the square root of the variance

Variance is a measure of the variation of possible rates of return R_i, from the expected rate of return $[E(R_i)]$

Variance (Standard Deviation) of Returns for an Individual Investment

$$\text{Variance}\ (\sigma^2) = \sum_{i=1}^{n} [R_i - E(R_i)]^2 P_i$$

where P_i is the probability of the possible rate of return, R_i

Variance (Standard Deviation) of Returns for an Individual Investment

Standard Deviation

$$(\sigma) = \sqrt{\sum_{i=1}^{n} [R_i - E(R_i)]^2 P_i}$$

Variance (Standard Deviation) of Returns for an Individual Investment

Exhibit 7.3

Possible Rate of Return (R_i)	Expected Return $E(R_i)$	$R_i - E(R_i)$	$[R_i - E(R_i)]^2$	P_i	$[R_i - E(R_i)]^2 P_i$
0.08	0.11	0.03	0.0009	0.25	0.000225
0.10	0.11	0.01	0.0001	0.25	0.000025
0.12	0.11	0.01	0.0001	0.25	0.000025
0.14	0.11	0.03	0.0009	0.25	0.000225
					0.000500

Variance (σ^2) = .0050

Standard Deviation (σ) = .02236

Variance (Standard Deviation) of Returns for a Portfolio

Exhibit 7.4

Computation of Monthly Rates of Return

Date	Closing Price	Dividend	Return (%)	Closing Price	Dividend	Return (%)
Dec.00	60.938			45.688		
Jan.01	58.000		-4.82%	48.200		5.50%
Feb.01	53.030		-8.57%	42.500		-11.83%
Mar.01	45.160	0.18	-14.50%	43.100	0.04	1.51%
Apr.01	46.190		2.28%	47.100		9.28%
May.01	47.400		2.62%	49.290		4.65%
Jun.01	45.000	0.18	-4.68%	47.240	0.04	-4.08%
Jul.01	44.600		-0.89%	50.370		6.63%
Aug.01	48.670		9.13%	45.950	0.04	-8.70%
Sep.01	46.850	0.18	-3.37%	38.370		-16.50%
Oct.01	47.880		2.20%	38.230		-0.36%
Nov.01	46.960	0.18	-1.55%	46.650	0.05	22.16%
Dec.01	47.150		0.40%	51.010		9.35%
		E(RCoca-Cola)=	-1.81%		E(Rhome Depot)==	1.47%

Covariance of Returns

- A measure of the degree to which two variables "move together" relative to their individual mean values over time
- In portfolio analysis
  - Concerned with the covariances of rates of return rather than prices
    - Positive – tend to move in the same direction relative to individual means
    - Negative – tend to move in different directions

Covariance of Returns

For two assets, i and j, the covariance of rates of return is defined as:

$Cov_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$

COMPUTATION OF STANDARD DEVIATION OF RETURNS FOR COCA-COLA AND HOME DEPOT: 2001

	Coca-Cola		Home Depot	
Date	$R_i - E(R_i)$	$[R_i - E(R_i)]^2$	$R_j - E(R_j)$	$[R_j - E(R_j)]^2$
Jan-01	–3.01	9.05	4.03	16.26
Feb-01	–6.76	45.65	–13.29	176.69
Mar-01	–12.69	161.01	0.04	0.00
Apr-01	4.09	16.75	7.81	61.06
May-01	4.43	19.64	3.18	10.13
Jun-01	–2.87	8.24	–5.54	30.74
Jul-01	0.92	0.85	5.16	26.61
Aug-01	10.94	119.64	–10.16	103.28
Sep-01	–1.56	2.42	–17.96	322.67
Oct-01	4.01	16.09	–1.83	3.36
Nov-01	0.27	0.07	20.69	428.01
Dec-01	2.22	4.92	7.88	62.08
		Sum = 404.34		Sum = 1240.90
	Variance_j = 404.34/12 =	33.69	Variance_j = 240.90/12 =	103.41
	$\text{Standard Deviation}_j = (33.69)^{1/2} =$	5.80	$\text{Standard Deviation}_j = (103.41)^{1/2} =$	10.17

Covariance and Correlation

- The correlation coefficient is obtained by standardizing (dividing) the covariance by the product of the individual standard deviations

Covariance and Correlation

Correlation coefficient varies from -1 to +1

$$r_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j}$$

where:

r_{ij} = the correlation coefficient of returns

σ_i = the standard deviation of R_{it}

σ_j = the standard deviation of R_{jt}

Correlation Coefficient

- It can vary only in the range +1 to -1.
  - A value of +1 would indicate perfect positive correlation. This means that returns for the two assets move together in a completely linear manner.
  - A value of –1 would indicate perfect correlation. This means that the returns for two assets have the same percentage movement, but in opposite directions

Portfolio Standard Deviation Formula

$$\sigma_{port} = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j Cov_{ij}}$$

where:

σ_{port} = the standard deviation of the portfolio

W_i = the weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio

σ_i^2 = the variance of rates of return for asset i

Cov_{ij} = the covariance between the rates of return for assets i and j,

where $Cov_{ij} = r_{ij}\sigma_i \sigma_j$

Portfolio Standard Deviation Calculation

- Any asset of a portfolio may be described by two characteristics:
  - The expected rate of return
  - The expected standard deviations of returns
- The correlation, measured by covariance, affects the portfolio standard deviation
- Low correlation reduces portfolio risk while not affecting the expected return

Combining Stocks with Different Returns and Risk

Asset	$E(R_i)$	W_i	σ^2_i	σ_i
1	.10	.50	.0049	.07
2	.20	.50	.0100	.10

Case	Correlation Coefficient	Covariance
a	+1.00	.0070
b	+0.50	.0035
c	0.00	.0000
d	-0.50	-.0035
e	-1.00	-.0070

Combining Stocks with Different Returns and Risk

- Assets may differ in expected rates of return and individual standard deviations
- Negative correlation reduces portfolio risk
- Combining two assets with -1.0 correlation reduces the portfolio standard deviation to zero only when individual standard deviations are equal

Constant Correlation with Changing Weights

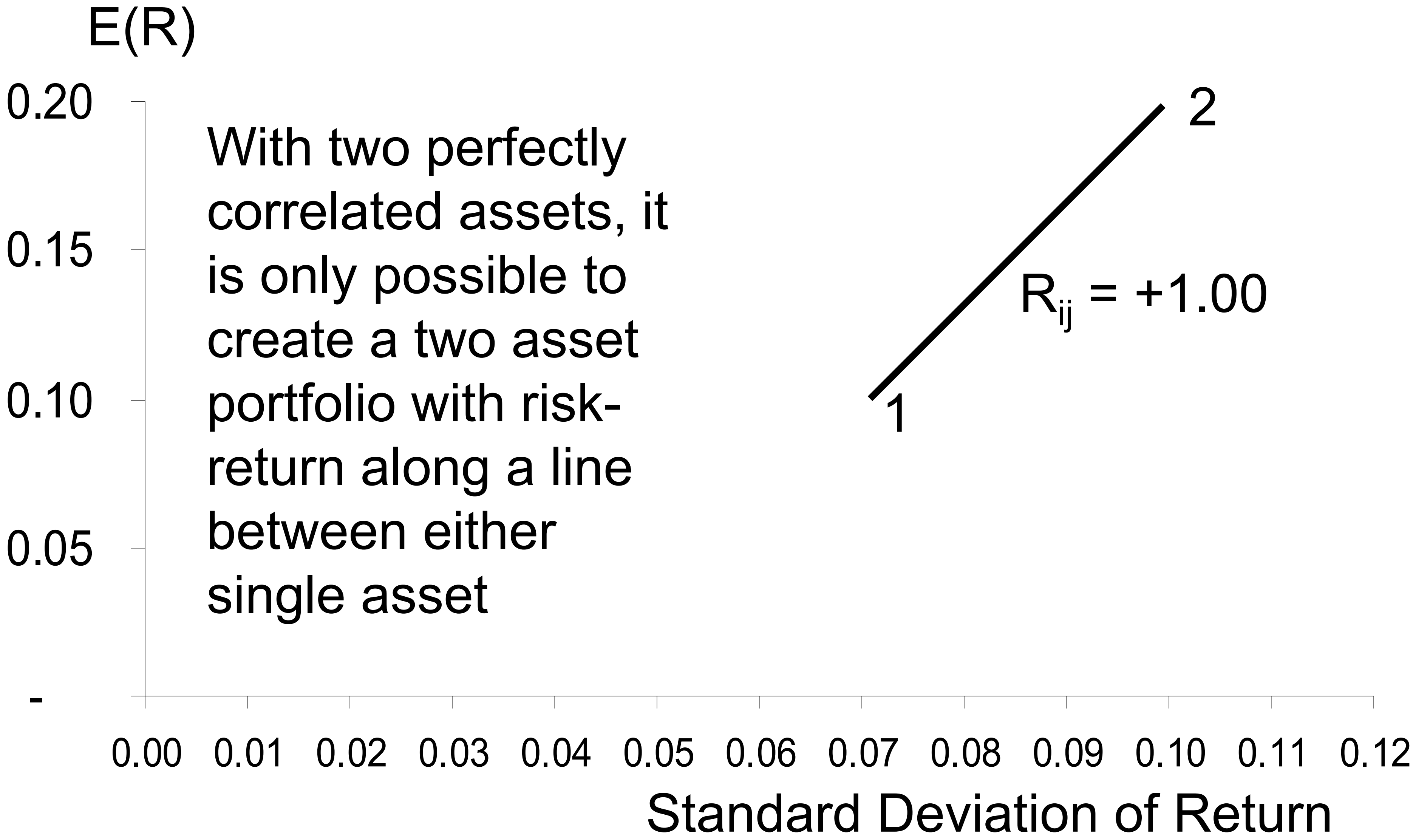
Asset	$E(R_i)$	
1	.10	$r_{ij} = 0.00$
2	.20	

Case	$\mathbf{W_1}$	$\mathbf{W^2}$	$\mathbf{E(R_i)}$
f	0.00	1.00	0.20
g	0.20	0.80	0.18
h	0.40	0.60	0.16
i	0.50	0.50	0.15
j	0.60	0.40	0.14
k	0.80	0.20	0.12
l	1.00	0.00	0.10

Constant Correlation with Changing Weights

Case	W_1	W_2	$E(R_i)$	E(☞$_{port}$)
f	0.00	1.00	0.20	0.1000
g	0.20	0.80	0.18	0.0812
h	0.40	0.60	0.16	0.0662
i	0.50	0.50	0.15	0.0610
j	0.60	0.40	0.14	0.0580
k	0.80	0.20	0.12	0.0595
l	1.00	0.00	0.10	0.0700

Portfolio Risk-Return Plots for Different Weights



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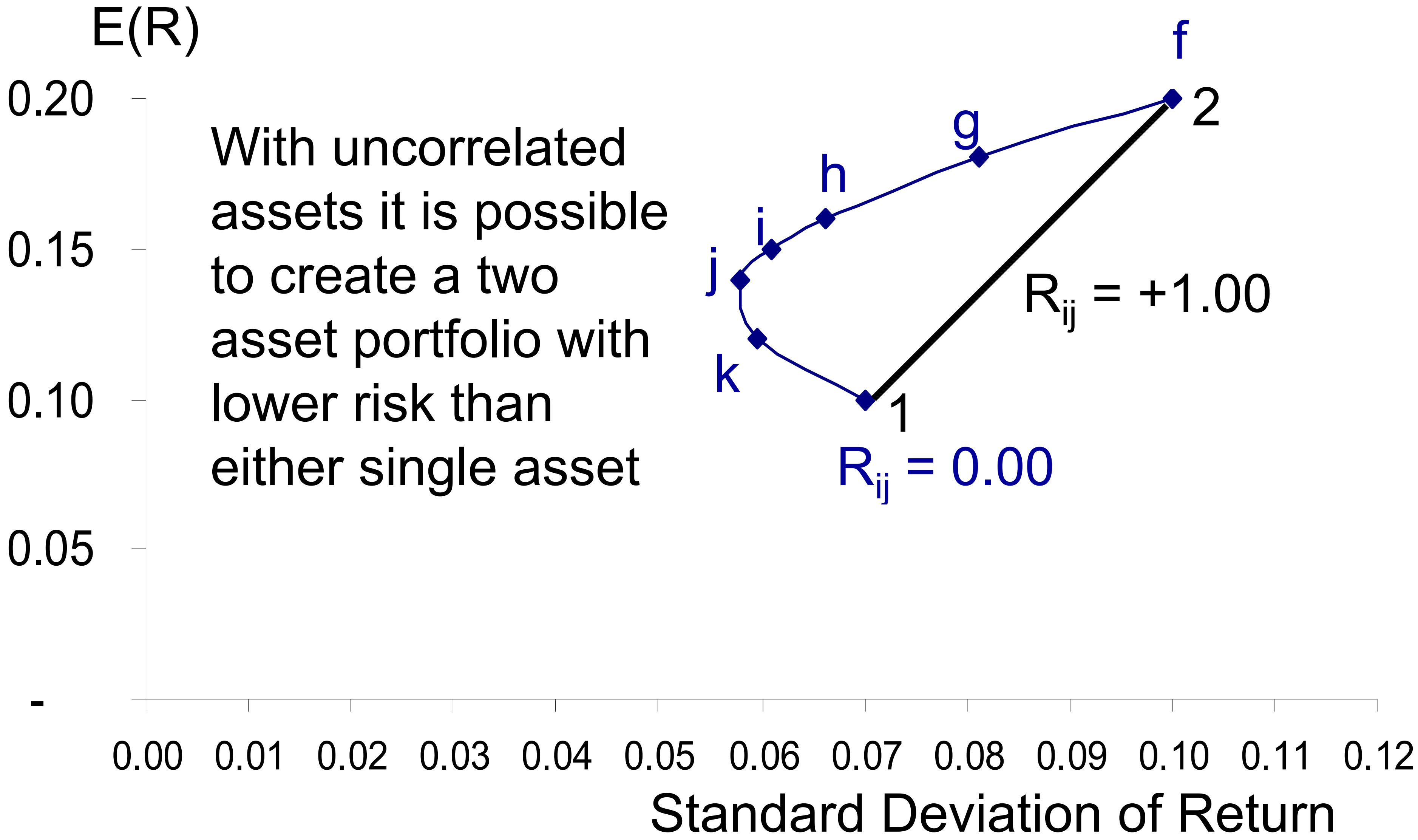

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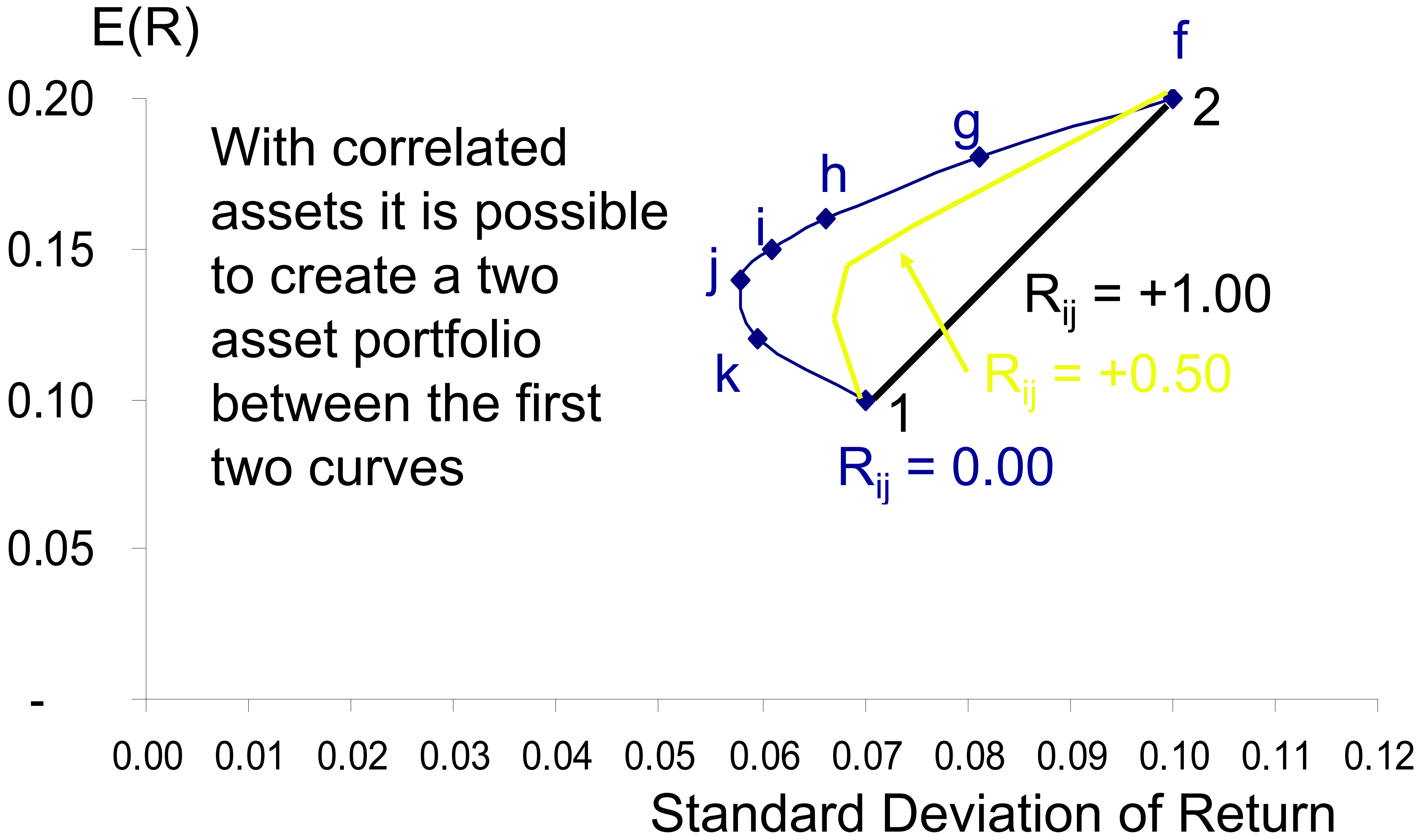

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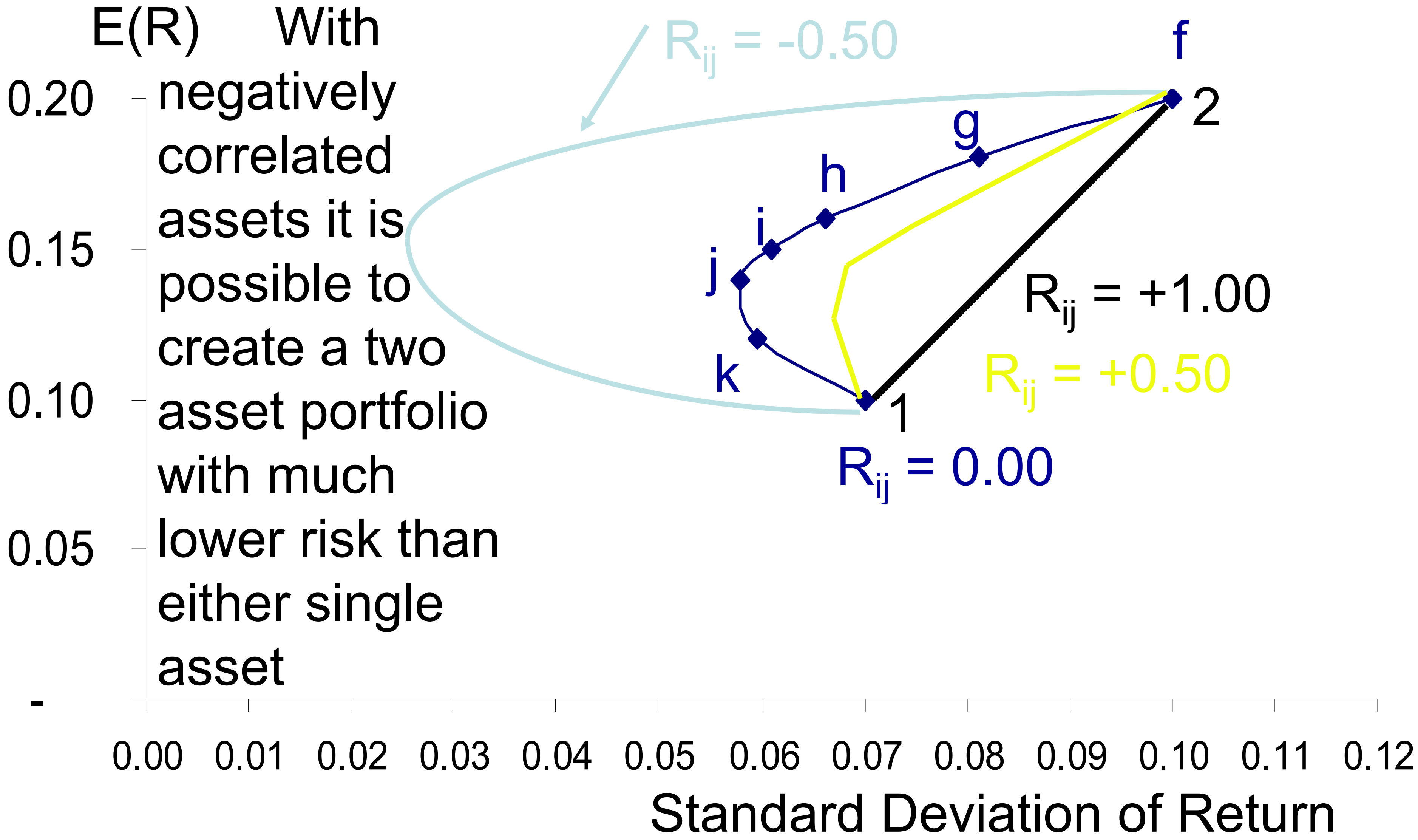
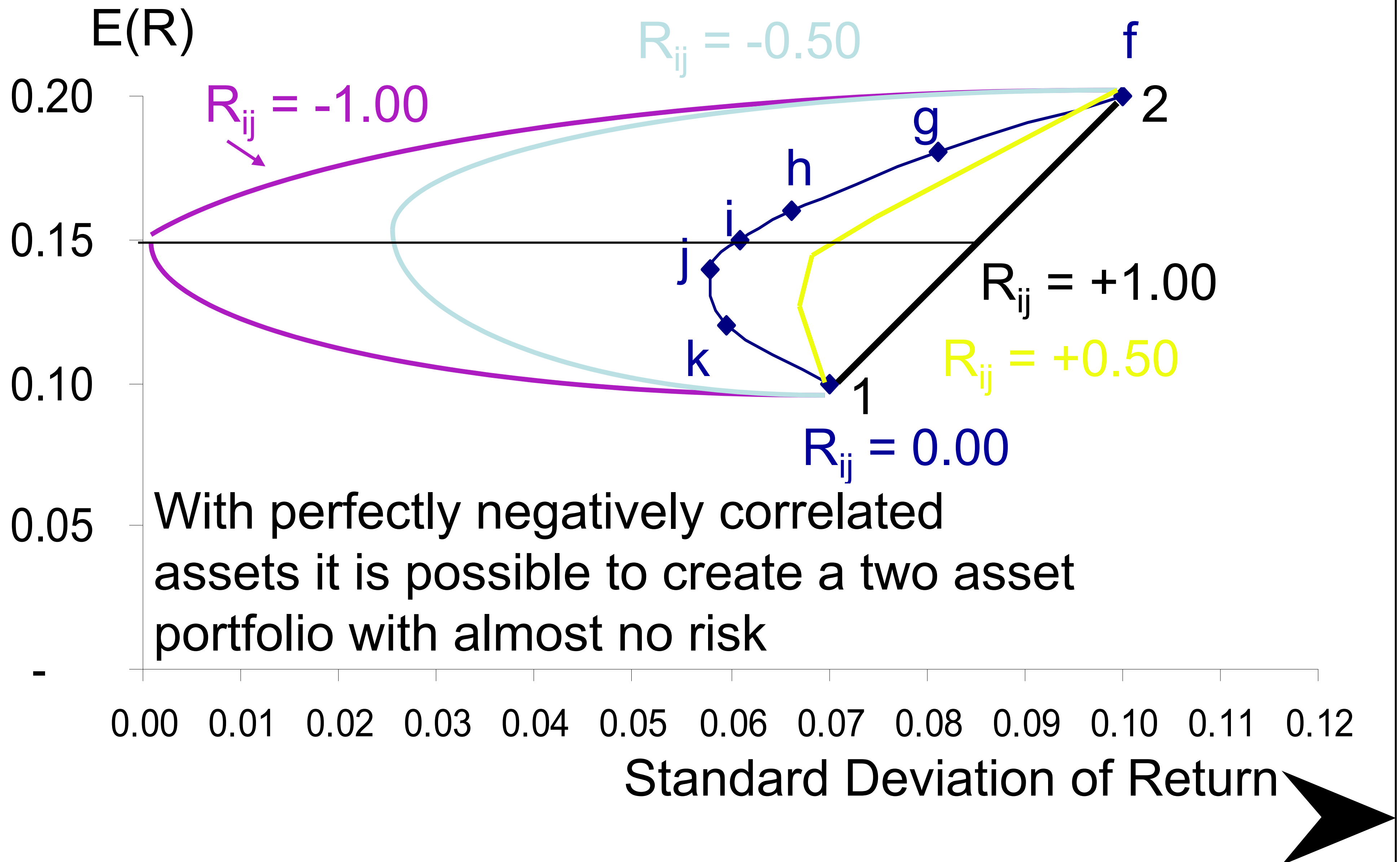


Portfolio Risk-Return Plots for Different Weights

Exhibit 7.13



Estimation Issues

- Results of portfolio allocation depend on accurate statistical inputs
- Estimates of
  - Expected returns
  - Standard deviation
  - Correlation coefficient
    - Among entire set of assets
    - With 100 assets, 4,950 correlation estimates
- Estimation risk refers to potential errors

Estimation Issues

- With assumption that stock returns can be described by a single market model, the number of correlations required reduces to the number of assets
- Single index market model:

$$R_i = a_i + b_i R_m + \varepsilon_i$$

b_i = the slope coefficient that relates the returns for security i to the returns for the aggregate stock market

R_m = the returns for the aggregate stock market

Estimation Issues

If all the securities are similarly related to the market and a b_i derived for each one, it can be shown that the correlation coefficient between two securities i and j is given as:

$$r_{ij} = b_i b_j \frac{\sigma_m^2}{\sigma_i \sigma_j}$$

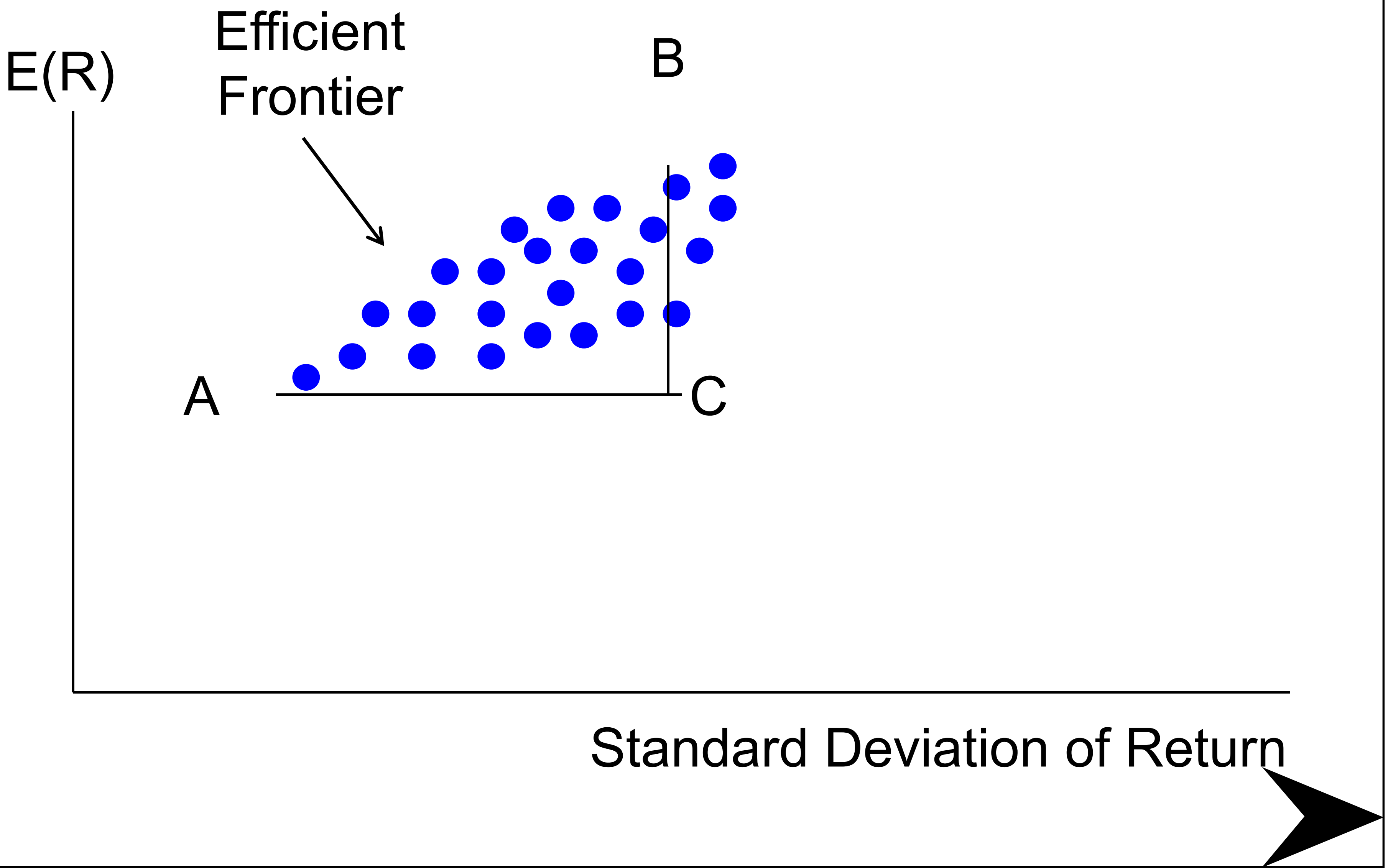
where σ_m^2 = the variance of returns for the aggregate stock market

The Efficient Frontier

- The efficient frontier represents that set of portfolios with the maximum rate of return for every given level of risk, or the minimum risk for every level of return
- Frontier will be portfolios of investments rather than individual securities
  - Exceptions being the asset with the highest return and the asset with the lowest risk

Efficient Frontier for Alternative Portfolios

Exhibit 7.15



The Efficient Frontier and Investor Utility

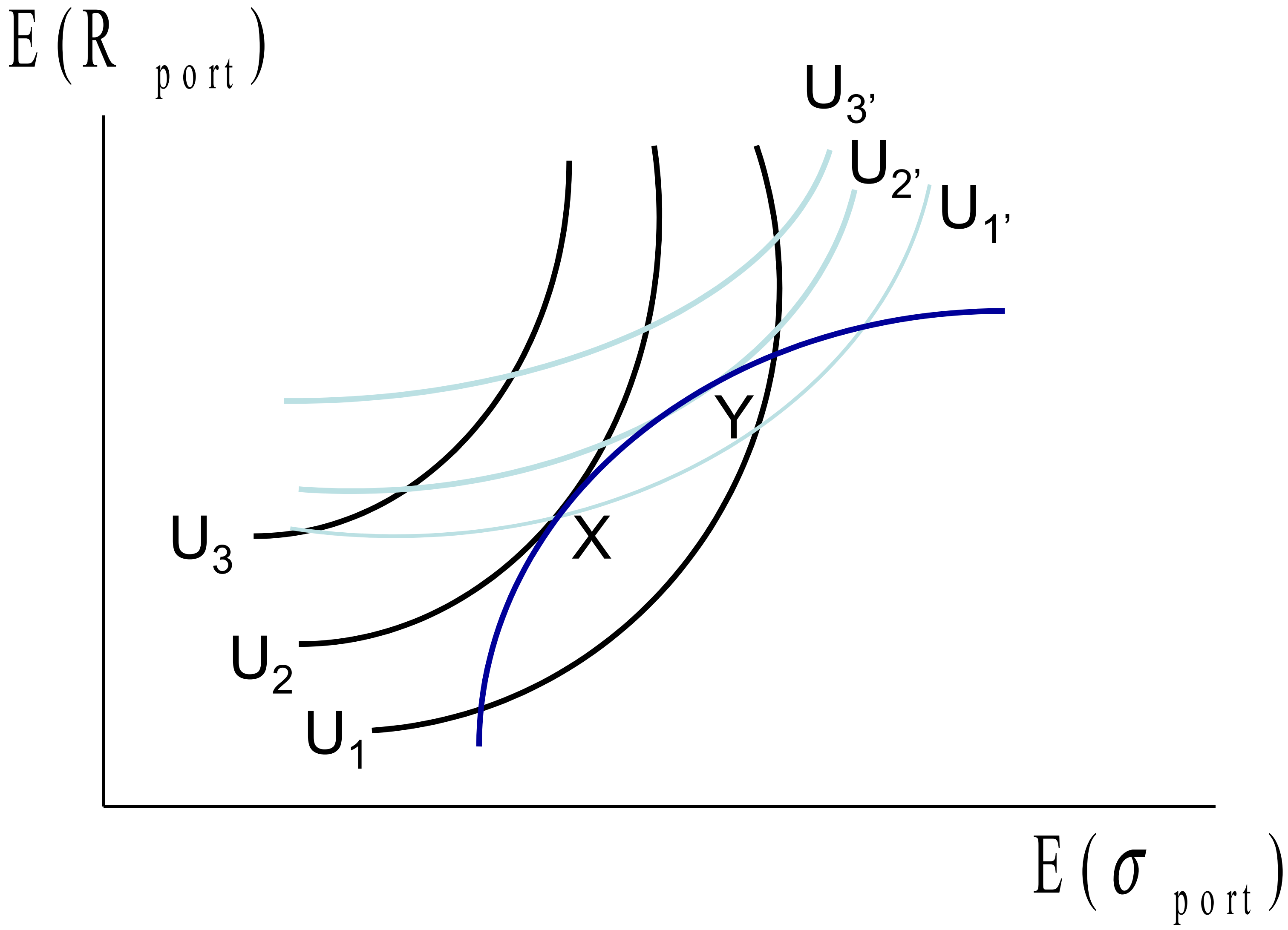
- An individual investor's utility curve specifies the trade-offs he is willing to make between expected return and risk
- The slope of the efficient frontier curve decreases steadily as you move upward
- These two interactions will determine the particular portfolio selected by an individual investor

The Efficient Frontier and Investor Utility

- The optimal portfolio has the highest utility for a given investor
- It lies at the point of tangency between the efficient frontier and the utility curve with the highest possible utility

Selecting an Optimal Risky Portfolio

Exhibit 7.16



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Future topics
Chapter 8

- Capital Market Theory
- Capital Asset Pricing Model
- Beta
- Expected Return and Risk
- Arbitrage Pricing Theory