In this chapter you will study ways to measure a consumer's valuation of a good given the consumer's demand curve for it. The basic logic is as follows: The height of the demand curve measures how much the consumer is willing to pay for the last unit of the good purchased—the willingness to pay for the marginal unit. Therefore the sum of the willingnesses-to-pay for each unit gives us the total willingness to pay for the consumption of the good.

In geometric terms, the total willingness to pay to consume some amount of the good is just the area under the demand curve up to that amount. This area is called **gross consumer's surplus** or **total benefit** of the consumption of the good. If the consumer has to pay some amount in order to purchase the good, then we must subtract this expenditure in order to calculate the **(net) consumer's surplus**.

When the utility function takes the quasilinear form, $u(x) + m$, the area under the demand curve measures $u(x)$, and the area under the demand curve minus the expenditure on the other good measures $u(x) + m$. Thus in this case, consumer's surplus serves as an exact measure of utility, and the change in consumer's surplus is a monetary measure of a change in utility.

If the utility function has a different form, consumer's surplus will not be an exact measure of utility, but it will often be a good approximation. However, if we want more exact measures, we can use the ideas of the **compensating variation** and the **equivalent variation.**

Recall that the compensating variation is the amount of extra income that the consumer would need at the *new* prices to be as well off as she was facing the old prices; the equivalent variation is the amount of money that it would be necessary to take away from the consumer at the old prices to make her as well off as she would be, facing the new prices. Although different in general, the change in consumer's surplus and the compensating and equivalent variations will be the same if preferences are quasilinear.

In this chapter you will practice:

- Calculating consumer's surplus and the change in consumer's surplus
- Calculating compensating and equivalent variations

Suppose that the inverse demand curve is given by $P(q) = 100 - 10q$ and that the consumer currently has 5 units of the good. How much money would you have to pay him to compensate him for reducing his consumption of the good to zero?

Answer: The inverse demand curve has a height of 100 when $q = 0$ and a height of 50 when $q = 5$. The area under the demand curve is a trapezoid with a base of 5 and heights of 100 and 50. We can calculate the area of this trapezoid by applying the formula

$$\text{Area of a trapezoid} = \text{base} \times \frac{1}{2}(\text{height}_1 + \text{height}_2).$$

In this case we have $A = 5 \times \frac{1}{2}(100 + 50) = \$375$.

Suppose now that the consumer is purchasing the 5 units at a price of \$50 per unit. If you require him to reduce his purchases to zero, how much money would be necessary to compensate him?

In this case, we saw above that his gross benefits decline by \$375. On the other hand, he has to spend $5 \times 50 = \$250$ less. The decline in *net* surplus is therefore \$125.

Suppose that a consumer has a utility function $u(x_1, x_2) = x_1 + x_2$. Initially the consumer faces prices $(1, 2)$ and has income 10. If the prices change to $(4, 2)$, calculate the compensating and equivalent variations.

Answer: Since the two goods are perfect substitutes, the consumer will initially consume the bundle $(10, 0)$ and get a utility of 10. After the prices change, she will consume the bundle $(0, 5)$ and get a utility of 5. After the price change she would need \$20 to get a utility of 10; therefore the compensating variation is $20 - 10 = 10$. Before the price change, she would need an income of 5 to get a utility of 5. Therefore the equivalent variation is $10 - 5 = 5$.

14.1 (0) Sir Plus consumes mead, and his demand function for tankards of mead is given by $D(p) = 100 - p$, where p is the price of mead in shillings.

(a) If the price of mead is 50 shillings per tankard, how many tankards of mead will he consume?____________________.

(b) How much gross consumer's surplus does he get from this consumption?____________________.

(c) How much money does he spend on mead?____________________.

(d) What is his net consumer's surplus from mead consumption?____________________.

14.2 (0) Here is the table of reservation prices for apartments taken from Chapter 1:

Person	= A	B	C	D	E	F	G	H
Price	= 40	25	30	35	10	18	15	5

(a) If the equilibrium rent for an apartment turns out to be \$20, which consumers will get apartments?____________________.

(b) If the equilibrium rent for an apartment turns out to be \$20, what is the consumer's (net) surplus generated in this market for person A? ________ For person B?____________________.

(c) If the equilibrium rent is \$20, what is the total net consumers' surplus generated in the market?____________________.

(d) If the equilibrium rent is \$20, what is the total gross consumers' surplus in the market?____________________.

(e) If the rent declines to \$19, how much does the gross surplus increase? ________

(f) If the rent declines to \$19, how much does the net surplus increase? ________

14.3 (0) Quasimodo consumes earplugs and other things. His utility function for earplugs x and money to spend on other goods y is given by

$$u(x, y) = 100x - \frac{x^2}{2} + y.$$

(a) What kind of utility function does Quasimodo have?____________________.

(b) What is his inverse demand curve for earplugs?____________________.

(c) If the price of earplugs is \$50, how many earplugs will he consume? ____________________.

(d) If the price of earplugs is \$80, how many earplugs will he consume? ____________________.

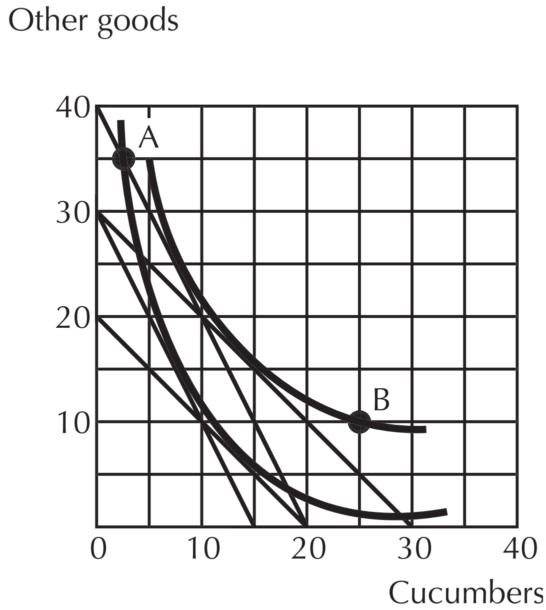
(e) Suppose that Quasimodo has \$4,000 in total to spend a month. What is his total utility for earplugs and money to spend on other things if the price of earplugs is \$50?____________________.

(f) What is his total utility for earplugs and other things if the price of earplugs is \$80?____.

(g) Utility decreases by __________ when the price changes from \$50 to \$80.

(h) What is the change in (net) consumer's surplus when the price changes from \$50 to \$80?____.

14.4 (2) In the graph below, you see a representation of Sarah Gamp's indifference curves between cucumbers and other goods. Suppose that the reference price of cucumbers and the reference price of "other goods" are both 1.



(a) What is the minimum amount of money that Sarah would need in order to purchase a bundle that is indifferent to A?____________________.

(b) What is the minimum amount of money that Sarah would need in order to purchase a bundle that is indifferent to B?____________________.

(c) Suppose that the reference price for cucumbers is 2 and the reference price for other goods is 1. How much money does she need in order to purchase a bundle that is indifferent to bundle A?____________________.

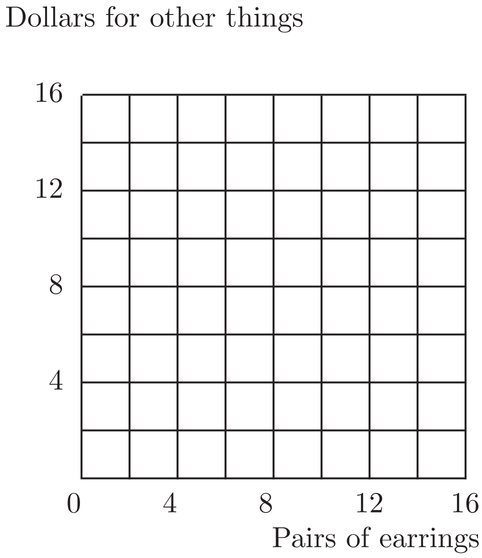
(d) What is the minimum amount of money that Sarah would need to purchase a bundle that is indifferent to B using these new prices?________.

____

(e) No matter what prices Sarah faces, the amount of money she needs to purchase a bundle indifferent to A must be (higher, lower) than the amount she needs to purchase a bundle indifferent to B.______________.

14.5 (2) Bernice's preferences can be represented by $u(x, y) = \min\{x, y\}$, where x is pairs of earrings and y is dollars to spend on other things. She faces prices $(p_x, p_y) = (2, 1)$ and her income is 12.

(a) Draw in pencil on the graph below some of Bernice's indifference curves and her budget constraint. Her optimal bundle is ________ pairs of earrings and ________ dollars to spend on other things.



(b) The price of a pair of earrings rises to \$3 and Bernice's income stays the same. Using blue ink, draw her new budget constraint on the graph above. Her new optimal bundle is ________ pairs of earrings and ________ dollars to spend on other things.

(c) What bundle would Bernice choose if she faced the original prices and had just enough income to reach the new indifference curve? __________ Draw with red ink the budget line that passes through this bundle at the original prices. How much income would Bernice need at the original prices to have this (red) budget line?______________________________.

(d) The maximum amount that Bernice would pay to avoid the price increase is ______ This is the (compensating, equivalent) variation in income.______________________________.

(e) What bundle would Bernice choose if she faced the new prices and had just enough income to reach her original indifference curve? ________ Draw with black ink the budget line that passes through this bundle at the new prices. How much income would Bernice have with this budget?

______________________________.

(f) In order to be as well-off as she was with her original bundle, Bernice's original income would have to rise by ______ This is the (compensating, equivalent) variation in income.______________________________.

14.6 (0) Ulrich likes video games and sausages. In fact, his preferences can be represented by $u(x, y) = \ln(x + 1) + y$ where x is the number of video games he plays and y is the number of dollars that he spends on sausages. Let p_x be the price of a video game and m be his income.

(a) Write an expression that says that Ulrich's marginal rate of substitution equals the price ratio. (Hint: Remember Donald Fribble from Chapter 6?)______________________________.

(b) Since Ulrich has ____________ preferences, you can solve this equation alone to get his demand function for video games, which is ______ ________ His demand function for the dollars to spend on sausages is

______________________________.

(c) Video games cost \$.25 and Ulrich's income is \$10. Then Ulrich demands ________ video games and ________ dollars' worth of sausages. His utility from this bundle is ________ (Round off to two decimal places.)

(d) If we took away all of Ulrich's video games, how much money would he need to have to spend on sausages to be just as well-off as before?

______________________________.

____

(e) Now an amusement tax of $.25 is put on video games and is passed on in full to consumers. With the tax in place, Ulrich demands ________ video game and ______ dollars' worth of sausages. His utility from this bundle is __________ (Round off to two decimal places.)

(f) Now if we took away all of Ulrich's video games, how much money would he have to have to spend on sausages to be just as well-off as with the bundle he purchased after the tax was in place?__________________.

(g) What is the change in Ulrich's consumer surplus due to the tax? __________ How much money did the government collect from Ulrich by means of the tax?__________________.

14.7 (1) Lolita, an intelligent and charming Holstein cow, consumes only two goods, cow feed (made of ground corn and oats) and hay. Her preferences are represented by the utility function $U(x, y) = x - x^2/2 + y$, where x is her consumption of cow feed and y is her consumption of hay. Lolita has been instructed in the mysteries of budgets and optimization and always maximizes her utility subject to her budget constraint. Lolita has an income of $\$m$ that she is allowed to spend as she wishes on cow feed and hay. The price of hay is always $1, and the price of cow feed will be denoted by p, where $0 < p \leq 1$.

(a) Write Lolita's inverse demand function for cow feed. (Hint: Lolita's utility function is quasilinear. When y is the numeraire and the price of x is p, the inverse demand function for someone with quasilinear utility $f(x) + y$ is found by simply setting $p = f'(x)$.)__________________.

(b) If the price of cow feed is p and her income is m, how much hay does Lolita choose? (Hint: The money that she doesn't spend on feed is used to buy hay.)__________________.

(c) Plug these numbers into her utility function to find out the utility level that she enjoys at this price and this income.__________________.

(d) Suppose that Lolita's daily income is $3 and that the price of feed is $.50. What bundle does she buy?______________ What bundle would she buy if the price of cow feed rose to $1?__________________.

(e) How much money would Lolita be willing to pay to avoid having the price of cow feed rise to \$1? ________ This amount is known as the ____________ variation.

(f) Suppose that the price of cow feed rose to \$1. How much extra money would you have to pay Lolita to make her as well-off as she was at the old prices? ________ This amount is known as the ____________ variation. Which is bigger, the compensating or the equivalent variation, or are they the same?________________________________.

(g) At the price \$.50 and income \$3, how much (net) consumer's surplus is Lolita getting?________________________________.

14.8 (2) F. Flintstone has quasilinear preferences and his inverse demand function for Brontosaurus Burgers is $P(b) = 30 - 2b$. Mr. Flintstone is currently consuming 10 burgers at a price of 10 dollars.

(a) How much money would he be willing to pay to have this amount rather than no burgers at all? ________ What is his level of (net) consumer's surplus?________________________________.

(b) The town of Bedrock, the only supplier of Brontosaurus Burgers, decides to raise the price from \$10 a burger to \$14 a burger. What is Mr. Flintstone's change in consumer's surplus?________________________________

____

____.

14.9 (1) Karl Kapitalist is willing to produce $p/2 - 20$ chairs at every price, $p > 40$. At prices below 40, he will produce nothing. If the price of chairs is \$100, Karl will produce ________ chairs. At this price, how much is his producer's surplus?________________________________.

14.10 (2) Ms. Q. Moto loves to ring the church bells for up to 10 hours a day. Where m is expenditure on other goods, and x is hours of bell ringing, her utility is $u(m, x) = m + 3x$ for $x \leq 10$. If $x > 10$, she develops painful blisters and is worse off than if she didn't ring the bells. Her income is equal to \$100 and the sexton allows her to ring the bell for 10 hours.

____

(a) Due to complaints from the villagers, the sexton has decided to restrict Ms. Moto to 5 hours of bell ringing per day. This is bad news for Ms. Moto. In fact she regards it as just as bad as losing _______ dollars of income.

(b) The sexton relents and offers to let her ring the bells as much as she likes so long as she pays $2 per hour for the privilege. How much ringing does she do now? ______________ This tax on her activities is as bad as a loss of how much income?____.

(c) The villagers continue to complain. The sexton raises the price of bell ringing to $4 an hour. How much ringing does she do now? __________ _______ This tax, as compared to the situation in which she could ring the bells for free, is as bad as a loss of how much income?_____________.

Some problems in this chapter will ask you to construct the market demand curve from individual demand curves. The market demand at any given price is simply the sum of the individual demands at that price. The key thing to remember in going from individual demands to the market demand is to *add quantities.* Graphically, you sum the individual demands horizontally to get the market demand. The market demand curve will have a kink in it whenever the market price is high enough that some individual demand becomes zero.

Sometimes you will need to find a consumer's reservation price for a good. Recall that the reservation price is the price that makes the consumer indifferent between having the good at that price and not having the good at all. Mathematically, the reservation price p^* satisfies $u(0, m) = u(1, m - p^*)$, where m is income and the quantity of the other good is measured in dollars.

Finally, some of the problems ask you to calculate price and/or income elasticities of demand. These problems are especially easy if you know a little calculus. If the demand function is $D(p)$, and you want to calculate the price elasticity of demand when the price is p, you only need to calculate $dD(p)/dp$ and multiply it by p/q.

15.0 Warm Up Exercise. (Calculating elasticities.) Here are some drills on price elasticities. For each demand function, find an expression for the price elasticity of demand. The answer will typically be a function of the price, p. As an example, consider the linear demand curve, $D(p) = 30 - 6p$. Then $dD(p)/dp = -6$ and $p/q = p/(30 - 6p)$, so the price elasticity of demand is $-6p/(30 - 6p)$.

(a) $D(p) = 60 - p$.____________________________________.

(b) $D(p) = a - bp$.____________________________________.

(c) $D(p) = 40p^{-2}$.____________________________________.

(d) $D(p) = Ap^{-b}$.____________________________________.

(e) $D(p) = (p + 3)^{-2}$.__________________________________.

(f) $D(p) = (p + a)^{-b}$. ______________________.

15.1 (0) In Gas Pump, South Dakota, there are two kinds of consumers, Buick owners and Dodge owners. Every Buick owner has a demand function for gasoline $D_B(p) = 20 - 5p$ for $p \leq 4$ and $D_B(p) = 0$ if $p > 4$. Every Dodge owner has a demand function $D_D(p) = 15 - 3p$ for $p \leq 5$ and $D_D(p) = 0$ for $p > 5$. (Quantities are measured in gallons per week and price is measured in dollars.) Suppose that Gas Pump has 150 consumers, 100 Buick owners, and 50 Dodge owners.

(a) If the price is \$3, what is the total amount demanded by each individual Buick Owner? ________ And by each individual Dodge owner?

______________________.

(b) What is the total amount demanded by all Buick owners? __________

What is the total amount demanded by all Dodge owners? __________.

(c) What is the total amount demanded by all consumers in Gas Pump at a price of 3? ______________________.

(d) On the graph below, use blue ink to draw the demand curve representing the total demand by Buick owners. Use black ink to draw the demand curve representing total demand by Dodge owners. Use red ink to draw the market demand curve for the whole town.

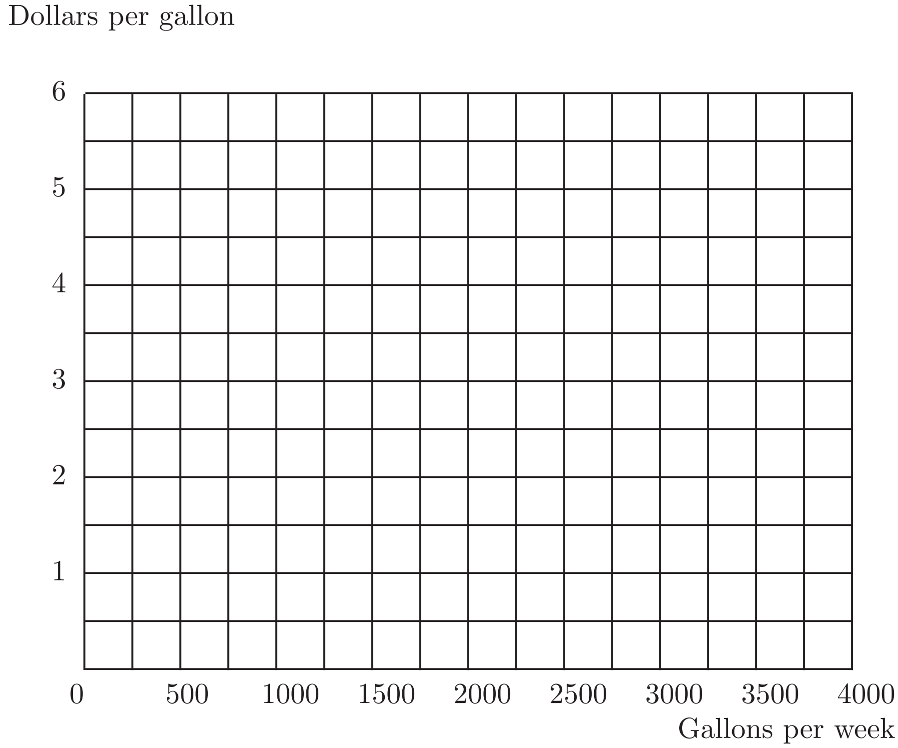
(e) At what prices does the market demand curve have kinks? __________

______________________.

(f) When the price of gasoline is \$1 per gallon, how much does weekly demand fall when price rises by 10 cents? ______________________.

(g) When the price of gasoline is \$4.50 per gallon, how much does weekly demand fall when price rises by 10 cents? ______________________.

(h) When the price of gasoline is \$10 per gallon, how much does weekly demand fall when price rises by 10 cents? ______________________.



15.2 (0) For each of the following demand curves, compute the inverse demand curve.

(a) $D(p) = \max\{10 - 2p, 0\}$. ______________________.

(b) $D(p) = 100/\sqrt{p}$. ______________________.

(c) $\ln D(p) = 10 - 4p$. ______________________.

(d) $\ln D(p) = \ln 20 - 2 \ln p$. ______________________.

15.3 (0) The demand function of dog breeders for electric dog polishers is $q_b = \max\{200 - p, 0\}$, and the demand function of pet owners for electric dog polishers is $q_o = \max\{90 - 4p, 0\}$.

(a) At price p, what is the price elasticity of dog breeders' demand for electric dog polishers? ______________ What is the price elasticity of pet owners' demand? ______________________.

(b) At what price is the dog breeders' elasticity equal to -1? __________

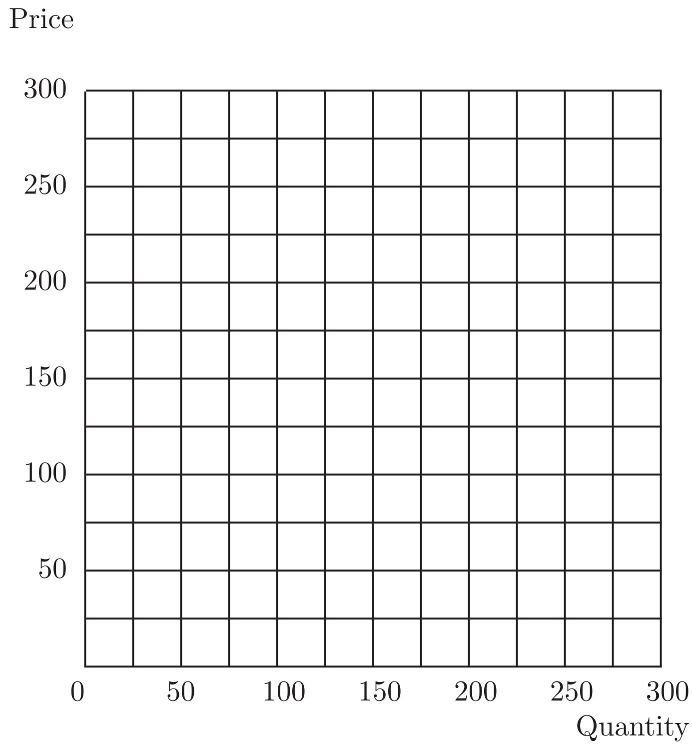
At what price is the pet owners' elasticity equal to -1?__________.

(c) On the graph below, draw the dog breeders' demand curve in blue ink, the pet owners' demand curve in red ink, and the market demand curve in pencil.

(d) Find a nonzero price at which there is positive total demand for dog polishers and at which there is a kink in the demand curve. __________

What is the market demand function for prices below the kink? __________ What is the market demand function for prices above the kink?__________.

(e) Where on the market demand curve is the price elasticity equal to -1? __________ At what price will the revenue from the sale of electric dog polishers be maximized? __________ If the goal of the sellers is to maximize revenue, will electric dog polishers be sold to breeders only, to pet owners only, or to both?__________.



15.4 (0) The demand for kitty litter, in pounds, is $\ln D(p) = 1{,}000 - p + \ln m$, where p is the price of kitty litter and m is income.

(a) What is the price elasticity of demand for kitty litter when $p = 2$ and $m = 500$? ________ When $p = 3$ and $m = 500$? ________ When $p = 4$ and $m = 1{,}500$? ________.

(b) What is the income elasticity of demand for kitty litter when $p = 2$ and $m = 500$? ________ When $p = 2$ and $m = 1{,}000$? ________ When $p = 3$ and $m = 1{,}500$? ________.

(c) What is the price elasticity of demand when price is p and income is m? ________ The income elasticity of demand? ________.

15.5 (0) The demand function for drangles is $q(p) = (p+1)^{-2}$.

(a) What is the price elasticity of demand at price p? ________.

(b) At what price is the price elasticity of demand for drangles equal to -1? ________.

(c) Write an expression for total revenue from the sale of drangles as a function of their price. ________ Use calculus to find the revenue-maximizing price. Don't forget to check the second-order condition. ________.

(d) Suppose that the demand function for drangles takes the more general form $q(p) = (p+a)^{-b}$ where $a > 0$ and $b > 1$. Calculate an expression for the price elasticity of demand at price p. ________ At what price is the price elasticity of demand equal to -1? ________.

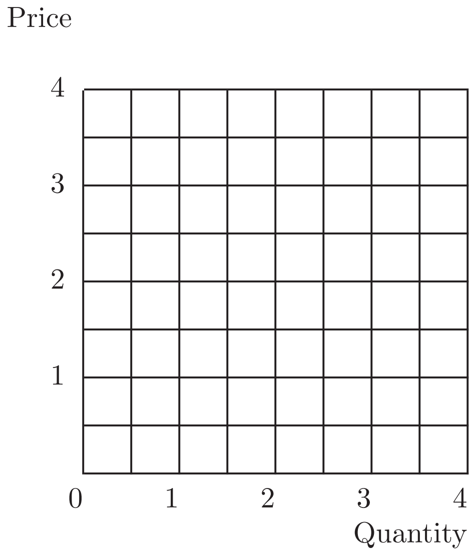
15.6 (0) Ken's utility function is $u_K(x_1, x_2) = x_1 + x_2$ and Barbie's utility function is $u_B(x_1, x_2) = (x_1 + 1)(x_2 + 1)$. A person can buy 1 unit of good 1 or 0 units of good 1. It is impossible for anybody to buy fractional units or to buy more than 1 unit. Either person can buy any quantity of good 2 that he or she can afford at a price of \$1 per unit.

(a) Where m is Barbie's wealth and p_1 is the price of good 1, write an equation that can be solved to find Barbie's reservation price for good 1.

____________________ What is Barbie's reservation price for good 1?

__________________ What is Ken's reservation price for good 1?________.

(b) If Ken and Barbie each have a wealth of 3, plot the market demand curve for good 1.



15.7 (0) The demand function for yo-yos is $D(p, M) = 4 - 2p + \frac{1}{100}M$, where p is the price of yo-yos and M is income. If M is 100 and p is 1,

(a) What is the income elasticity of demand for yo-yos?________________.

(b) What is the price elasticity of demand for yo-yos?__________________.

15.8 (0) If the demand function for zarfs is $P = 10 - Q$,

(a) At what price will total revenue realized from their sale be at a maximum?____.

(b) How many zarfs will be sold at that price?__________________________.

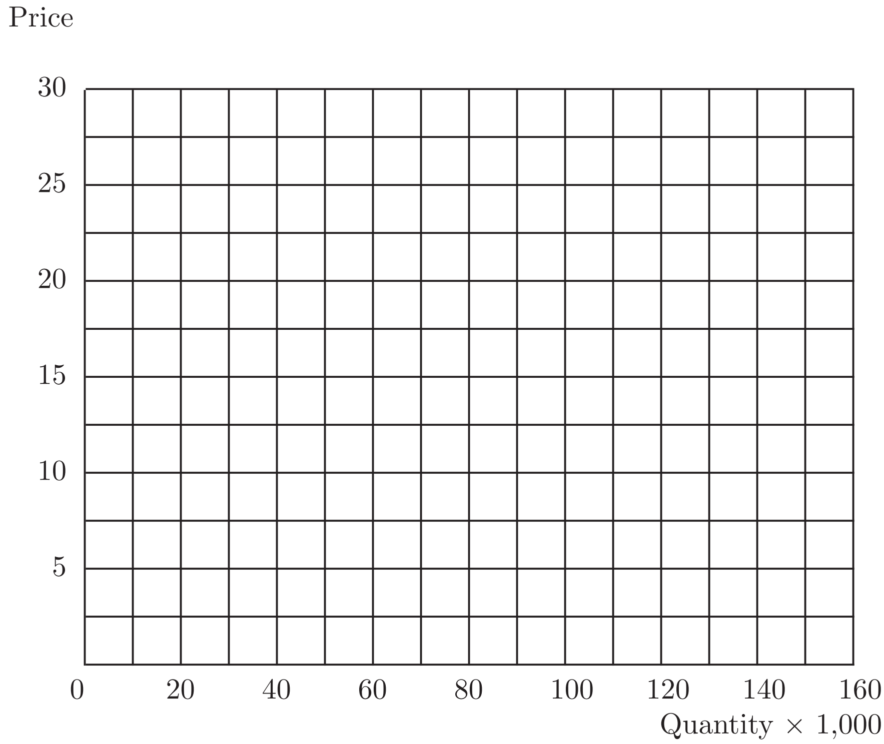
15.9 (0) The demand function for football tickets for a typical game at a large midwestern university is $D(p) = 200,000 - 10,000p$. The university has a clever and avaricious athletic director who sets his ticket prices so as to maximize revenue. The university's football stadium holds 100,000 spectators.

____

(a) Write down the inverse demand function.________________________.

(b) Write expressions for total revenue ____________________ and marginal revenue __________________ as a function of the number of tickets sold.

(c) On the graph below, use blue ink to draw the inverse demand function and use red ink to draw the marginal revenue function. On your graph, also draw a vertical blue line representing the capacity of the stadium.



(d) What price will generate the maximum revenue? __________ What quantity will be sold at this price?________________________________.

(e) At this quantity, what is marginal revenue?_______ At this quantity, what is the price elasticity of demand?__________ Will the stadium be full?____.

(f) A series of winning seasons caused the demand curve for football tickets to shift upward. The new demand function is $q(p) = 300,000 - 10,000p$. What is the new inverse demand function?____.

(g) Write an expression for marginal revenue as a function of output. $MR(q) =$________________ Use red ink to draw the new demand function and use black ink to draw the new marginal revenue function.

(h) Ignoring stadium capacity, what price would generate maximum revenue? __________ What quantity would be sold at this price?____.

(i) As you noticed above, the quantity that would maximize total revenue given the new higher demand curve is greater than the capacity of the stadium. Clever though the athletic director is, he cannot sell seats he hasn't got. He notices that his marginal revenue is positive for any number of seats that he sells up to the capacity of the stadium. Therefore, in order to maximize his revenue, he should sell __________ tickets at a price of ____.

(j) When he does this, his marginal revenue from selling an extra seat is ______ The elasticity of demand for tickets at this price quantity combination is____.

15.10 (0) The athletic director discussed in the last problem is considering the extra revenue he would gain from three proposals to expand the size of the football stadium. Recall that the demand function he is now facing is given by $q(p) = 300,000 - 10,000p$.

(a) How much could the athletic director increase the total revenue per game from ticket sales if he added 1,000 new seats to the stadium's capacity and adjusted the ticket price to maximize his revenue?__________.

(b) How much could he increase the revenue per game by adding 50,000 new seats? _____________ 60,000 new seats? (Hint: The athletic director still wants to maximize revenue.)____________________________________.

____

(c) A zealous alumnus offers to build as large a stadium as the athletic director would like and donate it to the university. There is only one hitch. The athletic director must price his tickets so as to keep the stadium full. If the athletic director wants to maximize his revenue from ticket sales, how large a stadium should he choose?_______________________.