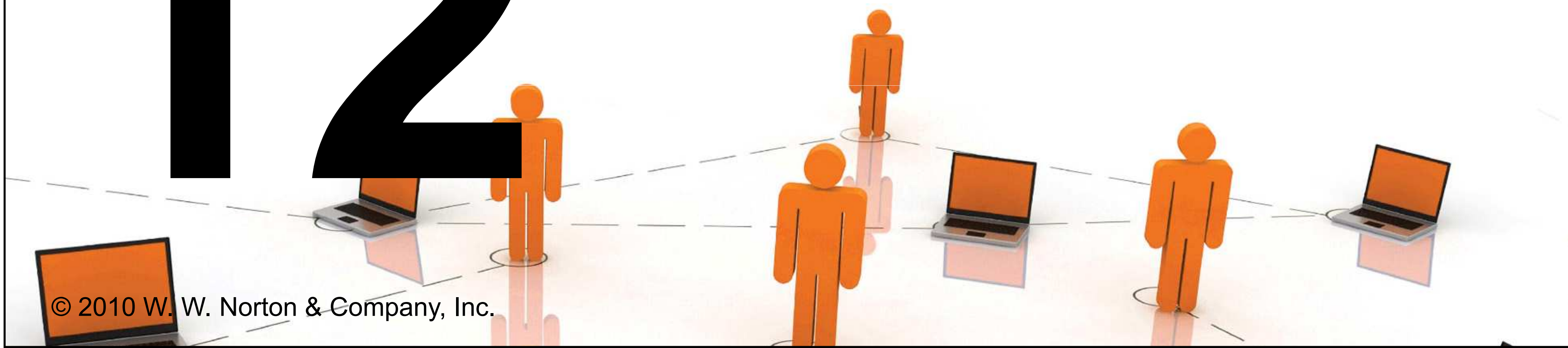
8TH EDITION

INTERMEDIATE MICROECONOMICS

HAL R. VARIAN

12 Uncertainty



Uncertainty is Pervasive

- **What is uncertain in economic systems?**
  - **tomorrow's prices**
  - **future wealth**
  - **future availability of commodities**
  - **present and future actions of other people.**

Uncertainty is Pervasive

- **What are rational responses to uncertainty?**
  - **buying insurance (health, life, auto)**
  - **a portfolio of contingent consumption goods.**

States of Nature

- Possible states of Nature:
  - "car accident" (a)
  - "no car accident" (na).
- Accident occurs with probability π_a, does not with probability π_{na} ;

$$\pi_a + \pi_{na} = 1.$$

- Accident causes a loss of \$L.

Contingencies

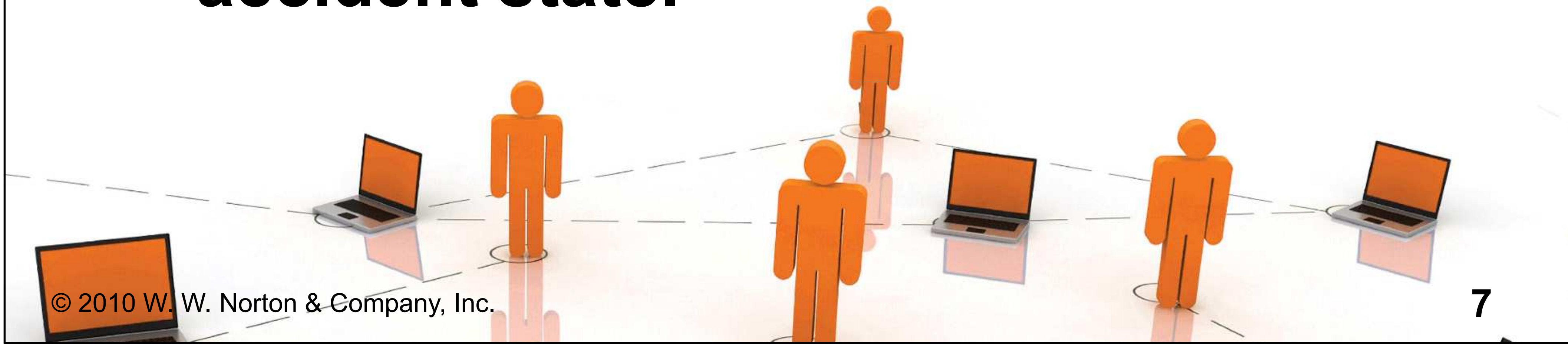
- A contract implemented only when a particular state of Nature occurs is state-contingent.
- E.g. the insurer pays only if there is an accident.

Contingencies

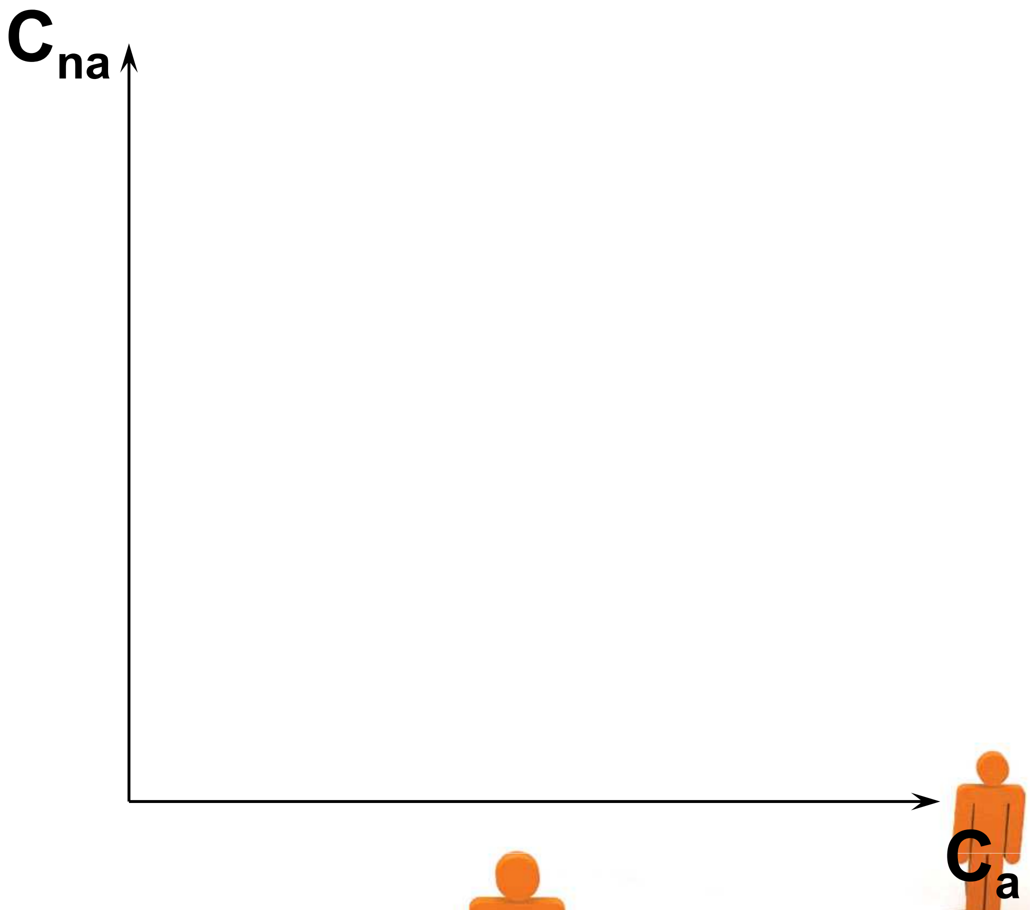
- **A state-contingent consumption plan is implemented only when a particular state of Nature occurs.**
- **E.g. take a vacation only if there is no accident.**

State-Contingent Budget Constraints

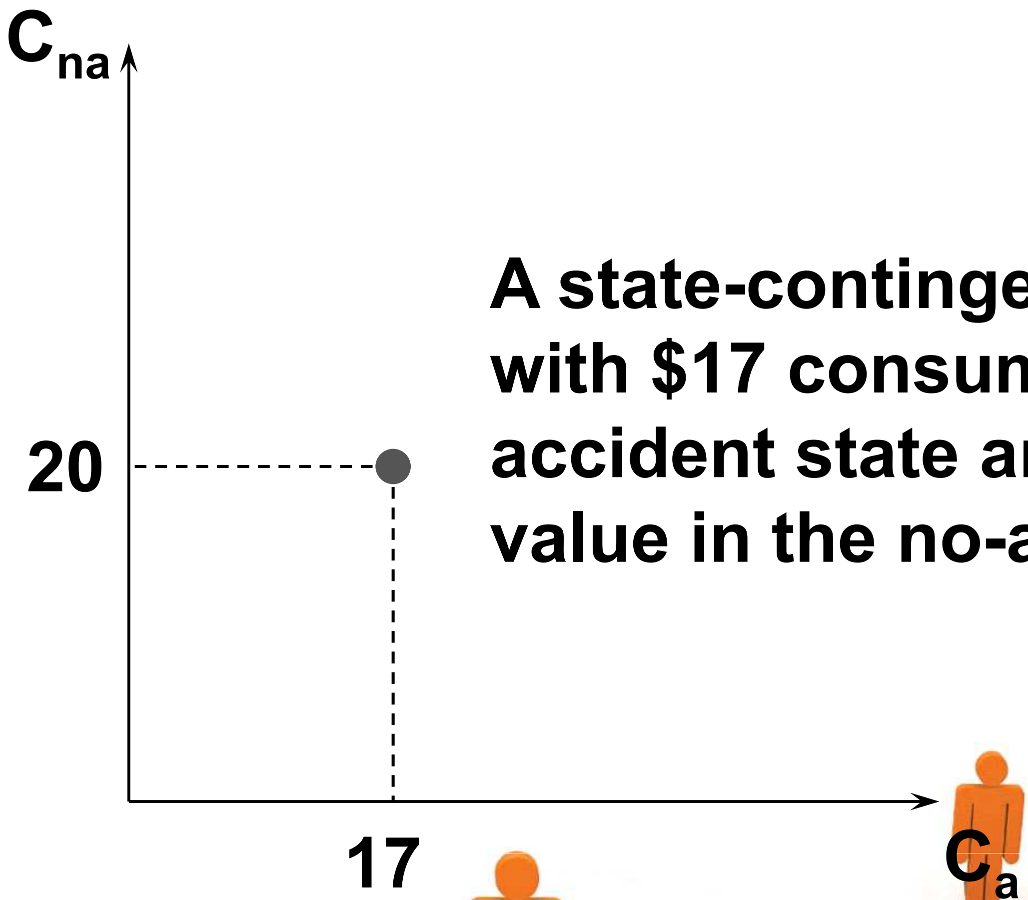
- **Each $1 of accident insurance costs γ.**
- **Consumer has $m of wealth.**
- **C_{na} is consumption value in the no-accident state.**
- **C_a is consumption value in the accident state.**



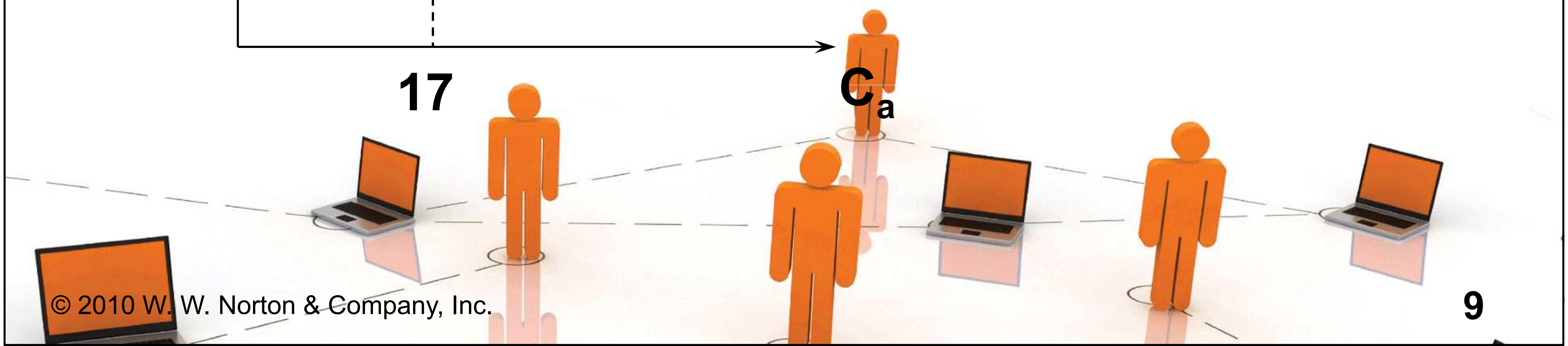
State-Contingent Budget Constraints



State-Contingent Budget Constraints

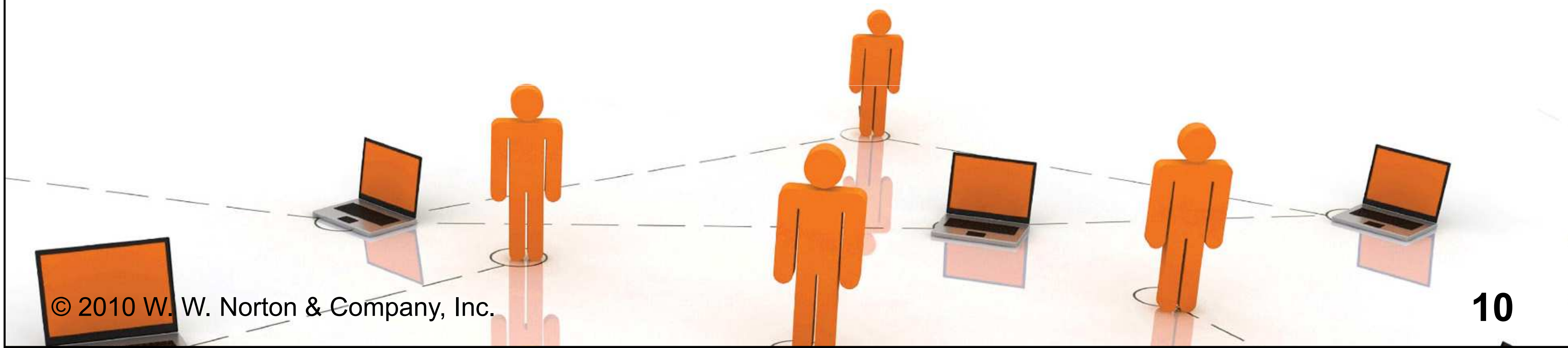


A state-contingent consumption with \$17 consumption value in the accident state and \$20 consumption value in the no-accident state.

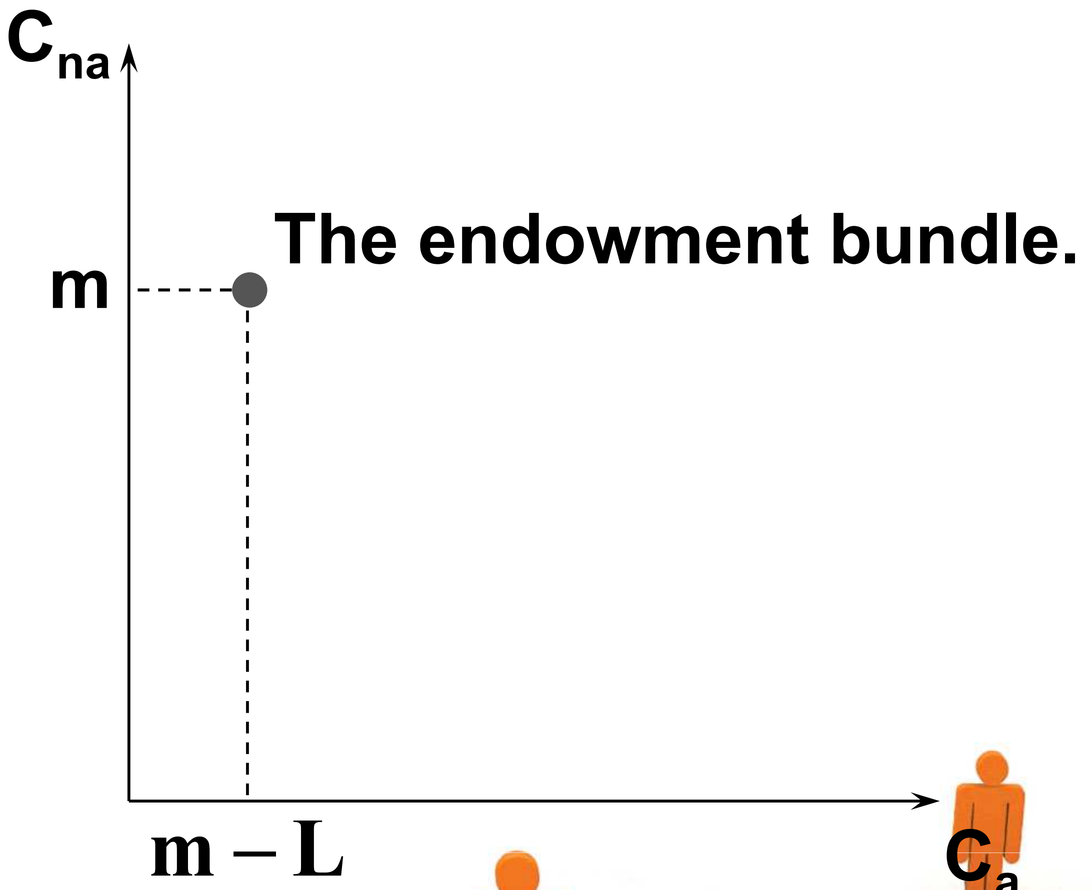


State-Contingent Budget Constraints

- **Without insurance,**
- $\mathbf{C_a = m - L}$
- $\mathbf{C_{na} = m.}$



State-Contingent Budget Constraints



State-Contingent Budget Constraints

- **Buy \$K of accident insurance.**
- $C_{na} = m - \gamma K.$
- $C_a = m - L - \gamma K + K = m - L + (1 - \gamma)K.$

State-Contingent Budget Constraints

- Buy \$K of accident insurance.
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- So $K = (C_a - m + L)/(1- \gamma)$

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- And $C_{na} = m - \gamma\,(C_a - m + L)/(1- \gamma)$

State-Contingent Budget Constraints

- **Buy \$K of accident insurance.**
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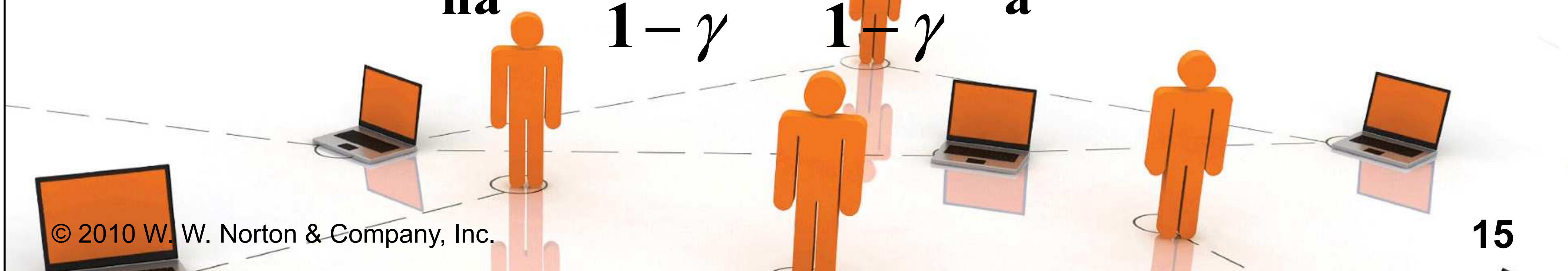
- $C_a = m - L - \gamma K + K = m - L + (1- \gamma)K$.
- So $K = (C_a - m + L)/(1- \gamma)$
- And $C_{na} = m - \gamma\,(C_a - m + L)/(1- \gamma)$
- I.e.

$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$



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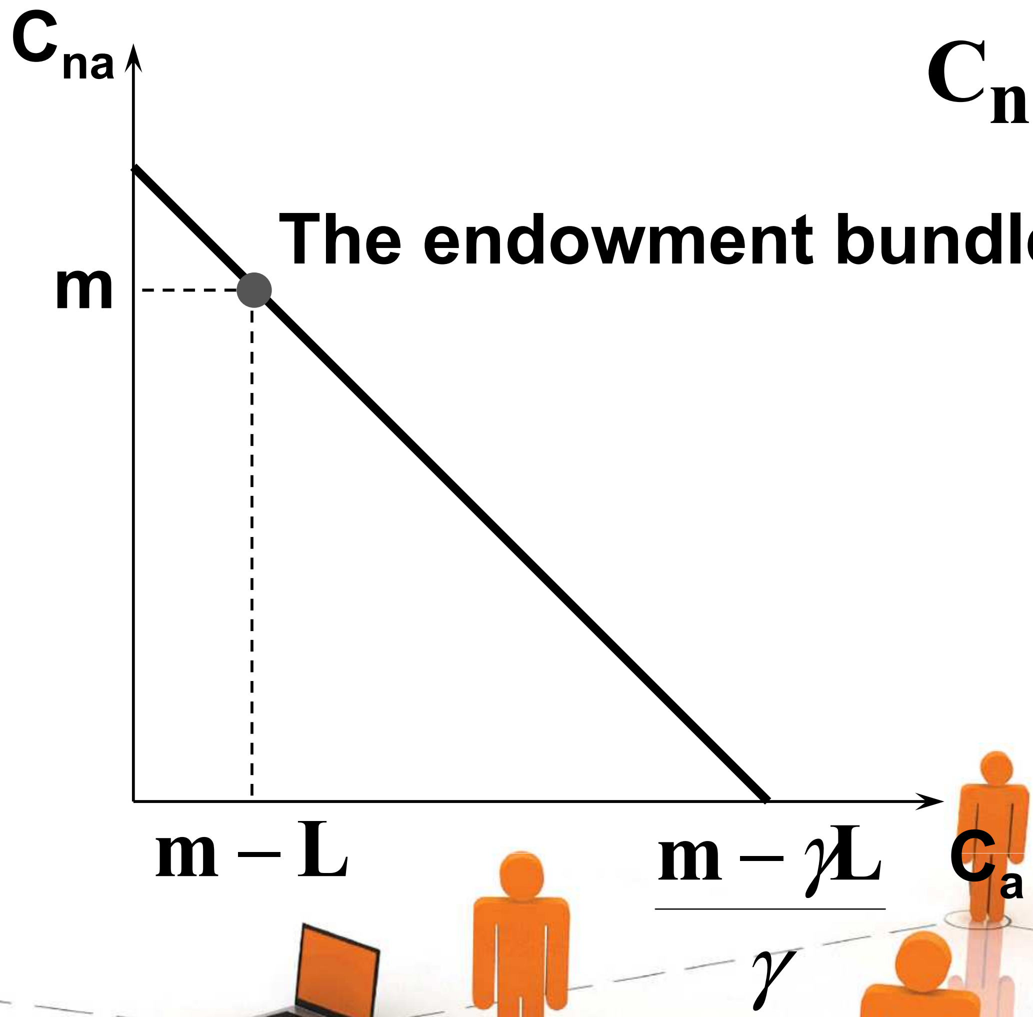

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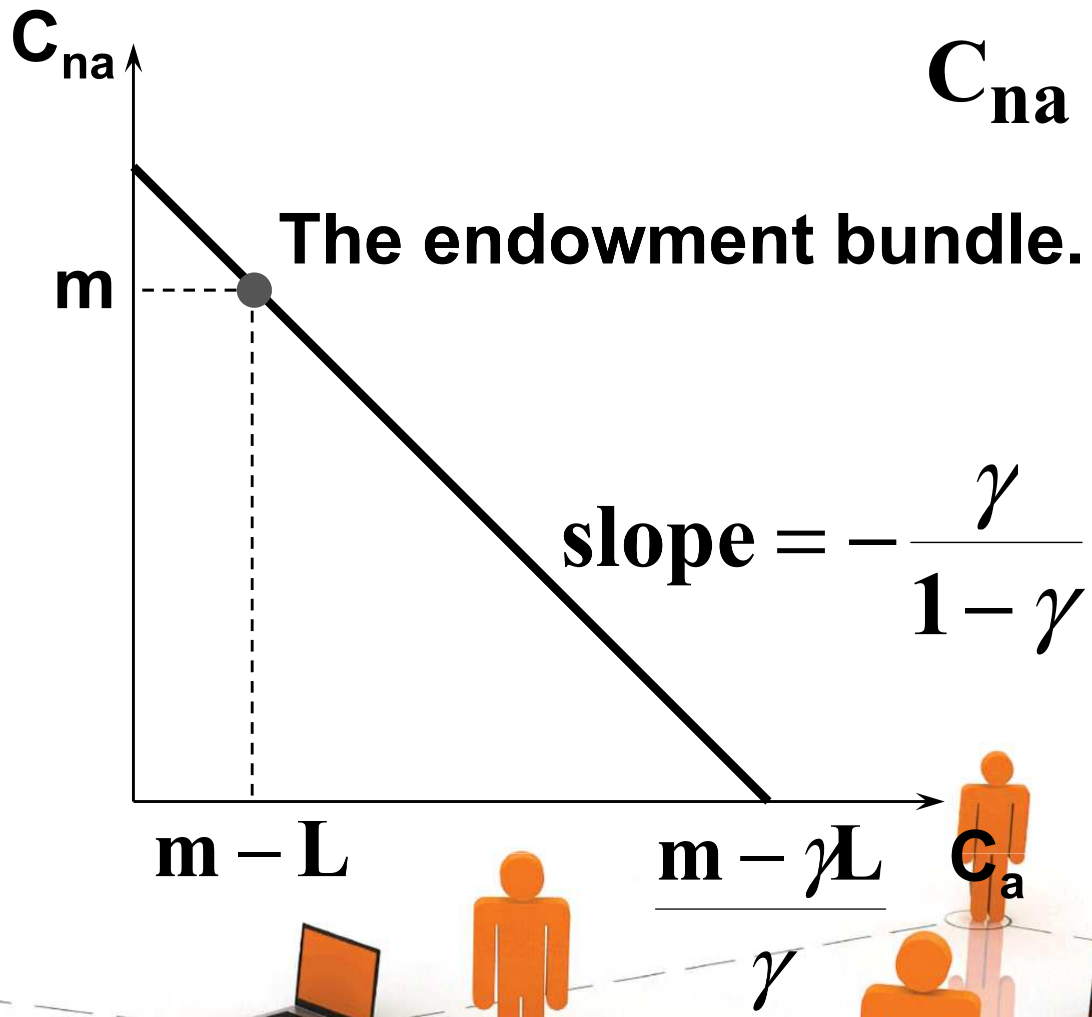

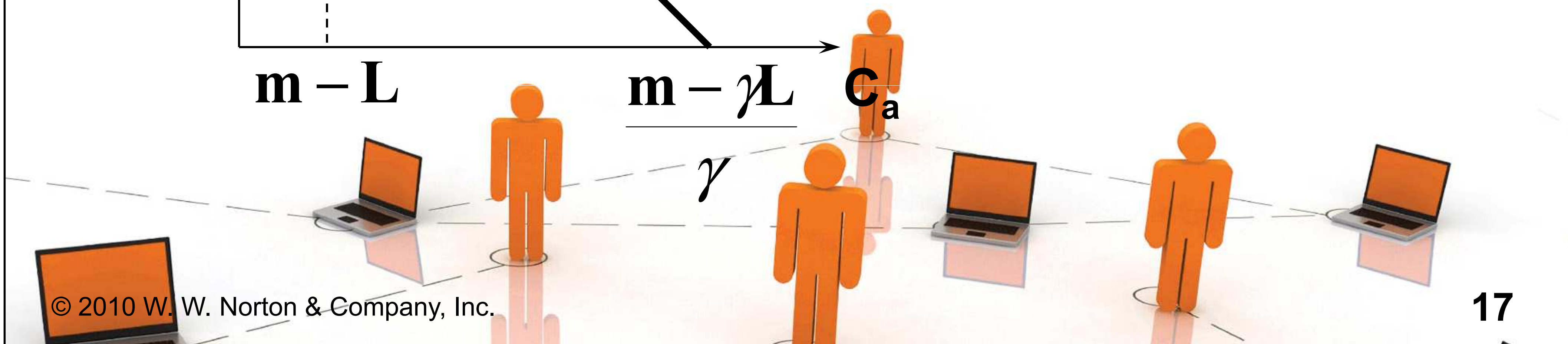

State-Contingent Budget Constraints

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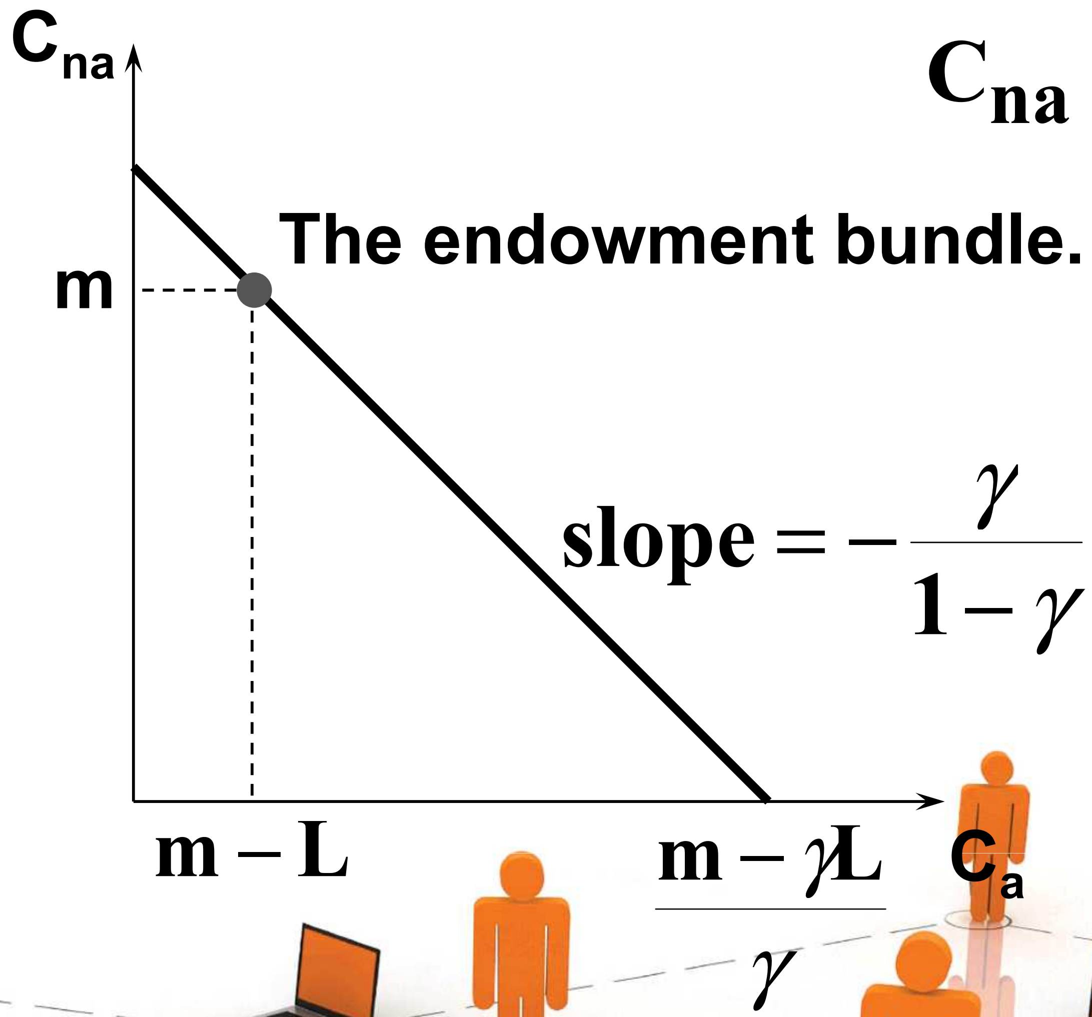


Where is the most preferred state-contingent consumption plan?

Preferences Under Uncertainty

- **Think of a lottery.**
- **Win $90 with probability 1/2 and win $0 with probability 1/2.**
- **U($90) = 12,   U($0) = 2.**
- **Expected utility is**

Preferences Under Uncertainty

- **Think of a lottery.**
- **Win \$90 with probability 1/2 and win \$0 with probability 1/2.**
- **U(\$90) = 12,   U(\$0) = 2.**
- **Expected utility is**

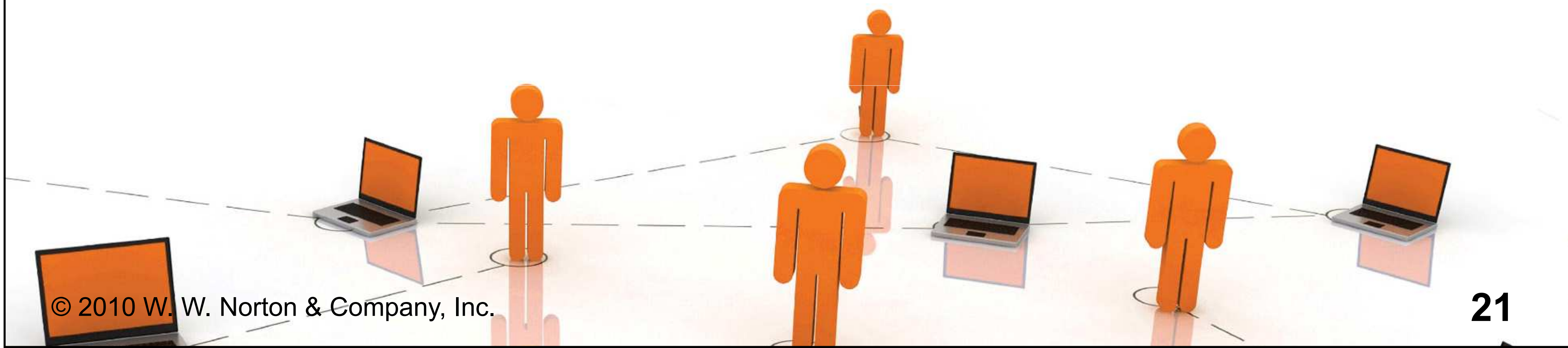
$$\mathrm{EU} = \frac{1}{2} \times \mathrm{U}(\$90) + \frac{1}{2} \times \mathrm{U}(\$0)$$

$$= \frac{1}{2} \times 12 + \frac{1}{2} \times 2 = 7.$$

Preferences Under Uncertainty

- **Think of a lottery.**
- **Win \$90 with probability 1/2 and win \$0 with probability 1/2.**
- **Expected money value of the lottery is**

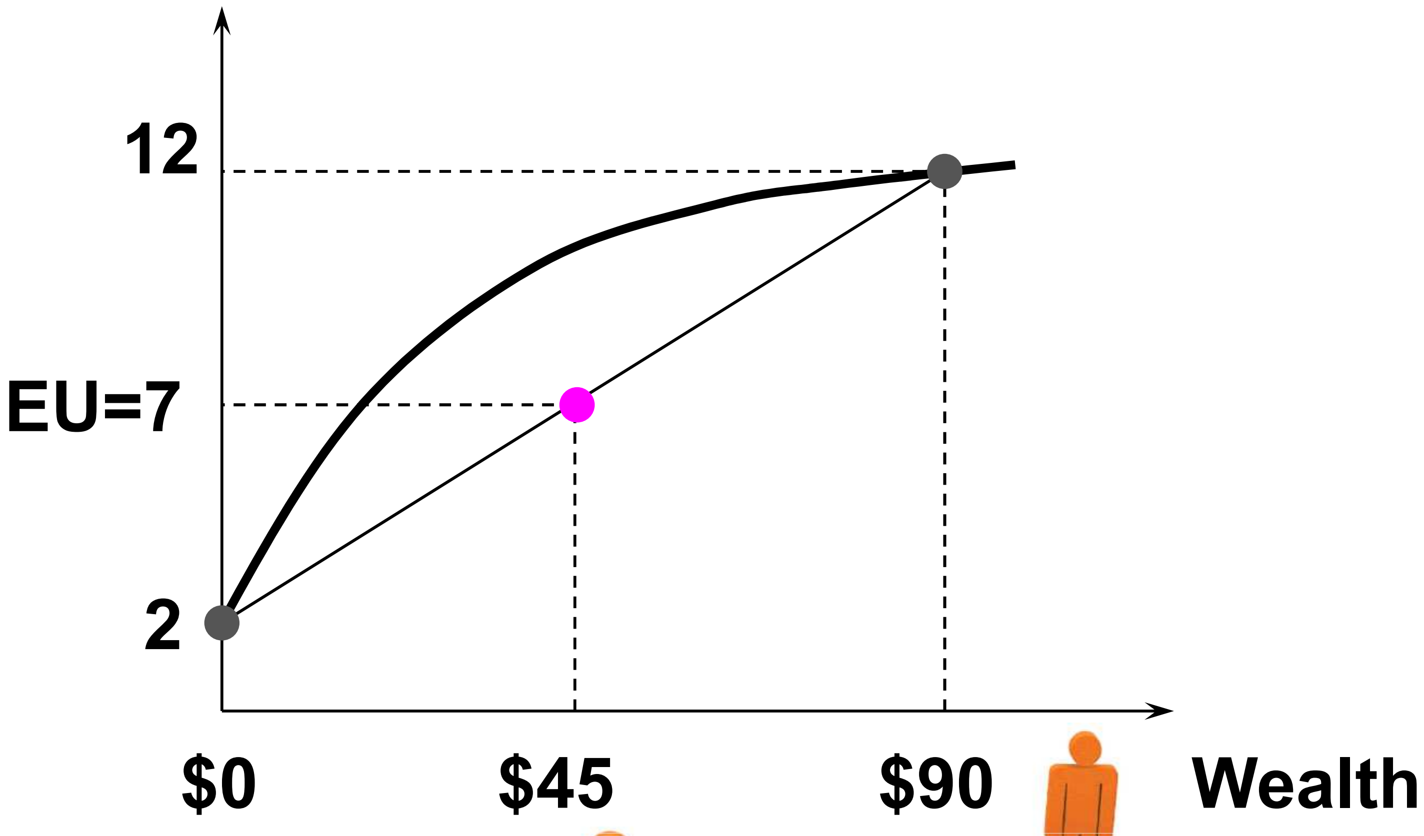
$$\mathbf{EM} = \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{\$90} + \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{\$0} = \$\mathbf{45}.$$



Preferences Under Uncertainty

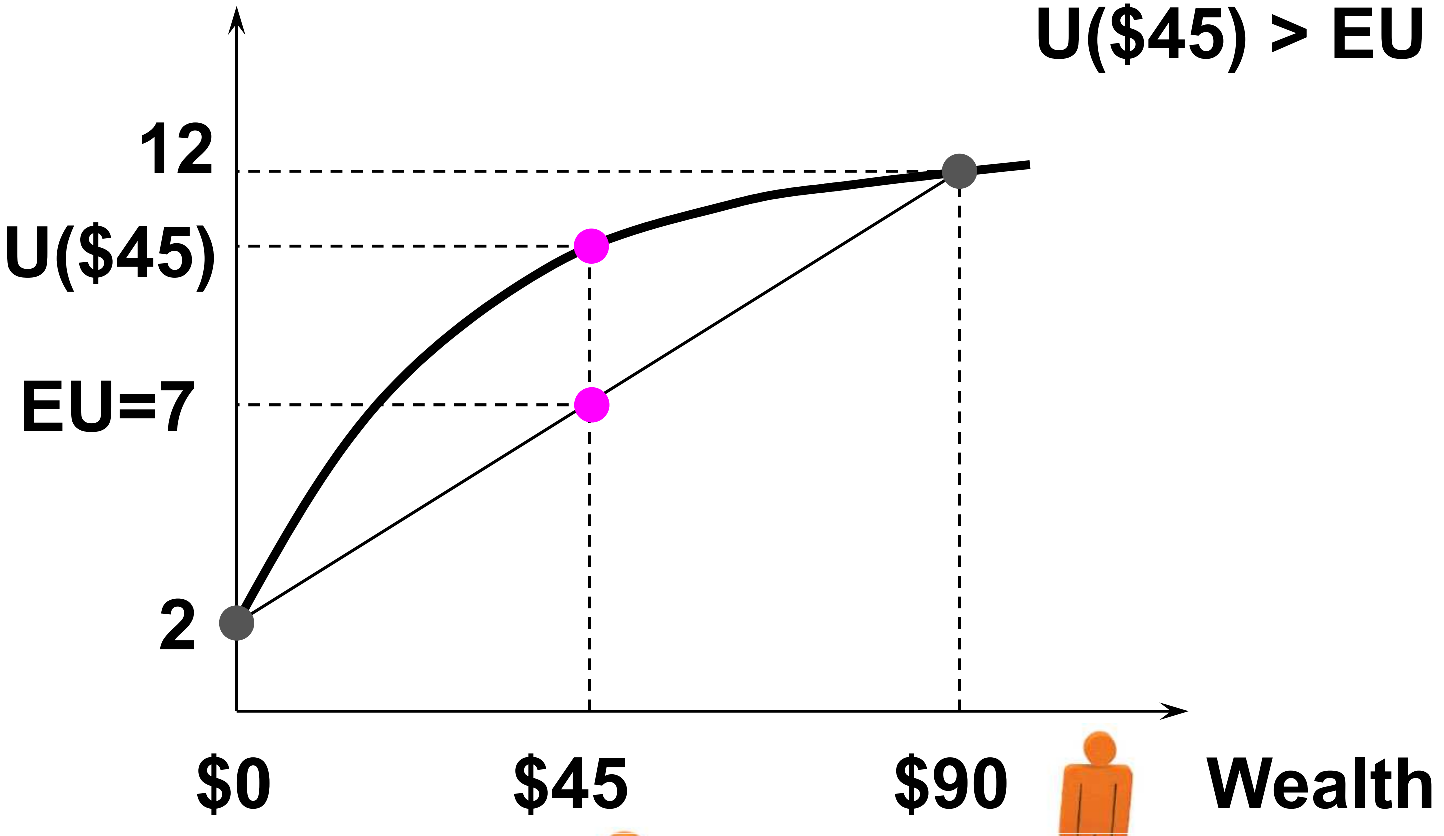
- EU = 7 and EM = \$45.
- U(\$45) > 7 ⇒ \$45 for sure is preferred to the lottery ⇒ risk-aversion.
- U(\$45) < 7 ⇒ the lottery is preferred to \$45 for sure ⇒ risk-loving.
- U(\$45) = 7 ⇒ the lottery is preferred equally to \$45 for sure ⇒ risk-neutrality.

Preferences Under Uncertainty

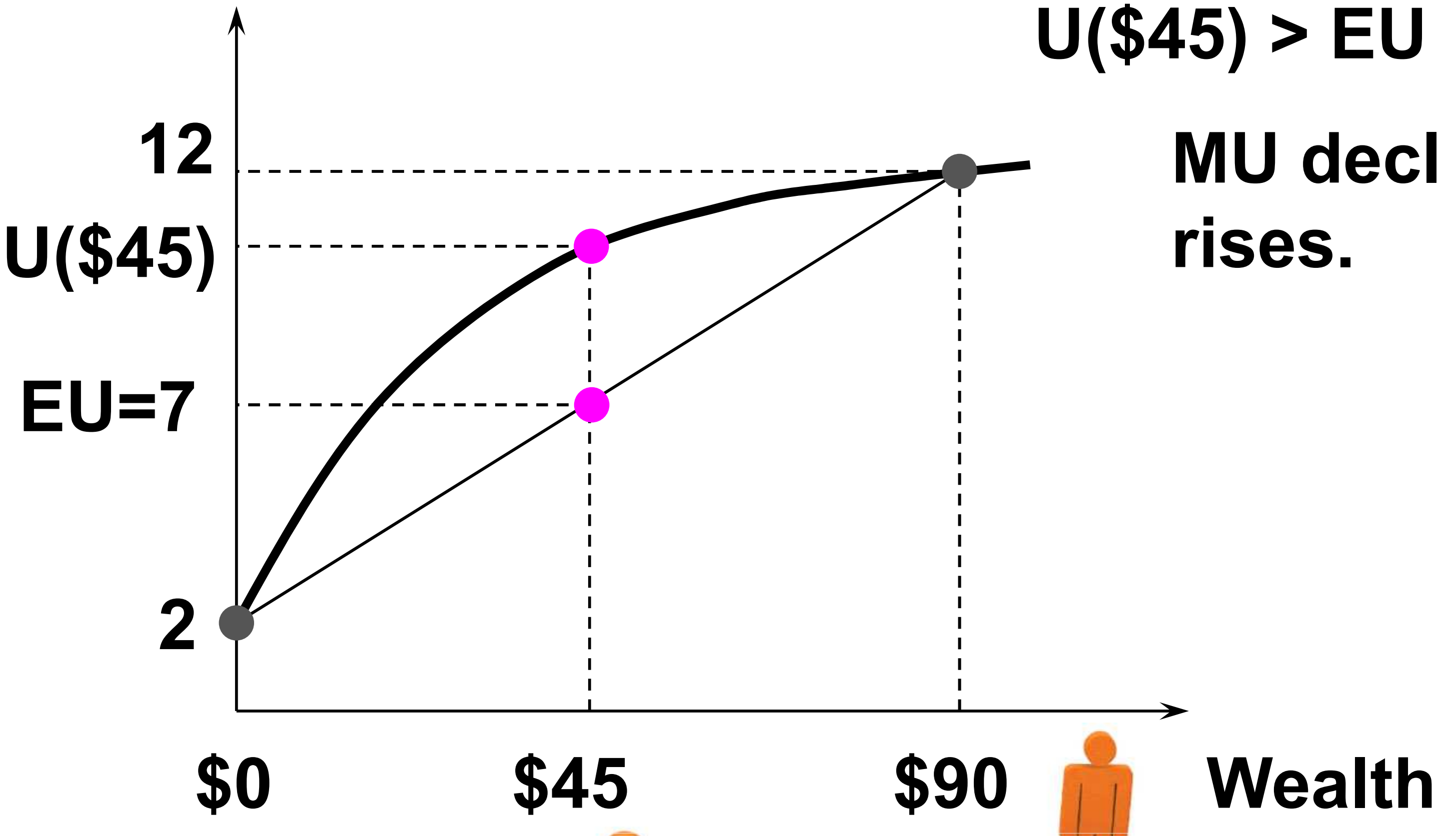


Preferences Under Uncertainty

U($45) > EU ⇒ risk-aversion.

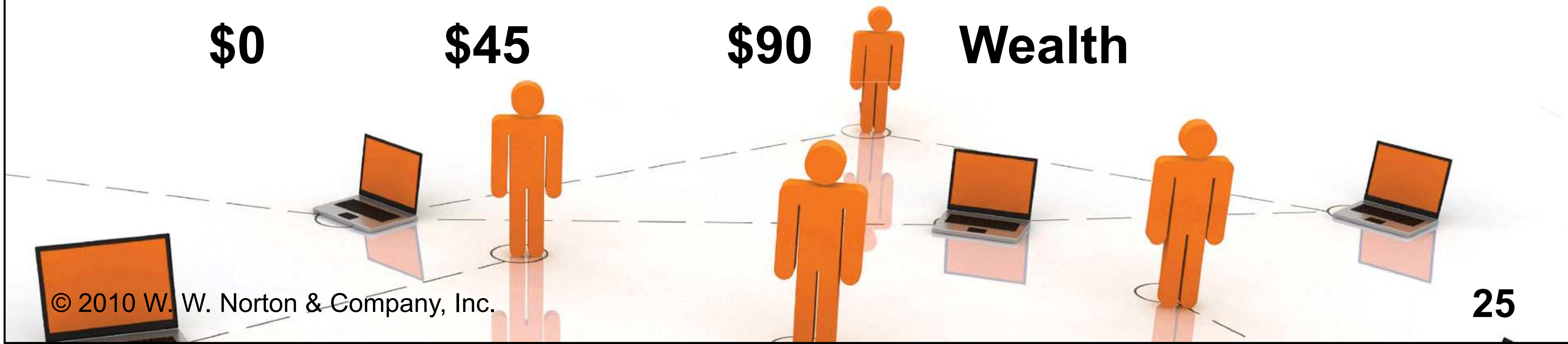


Preferences Under Uncertainty

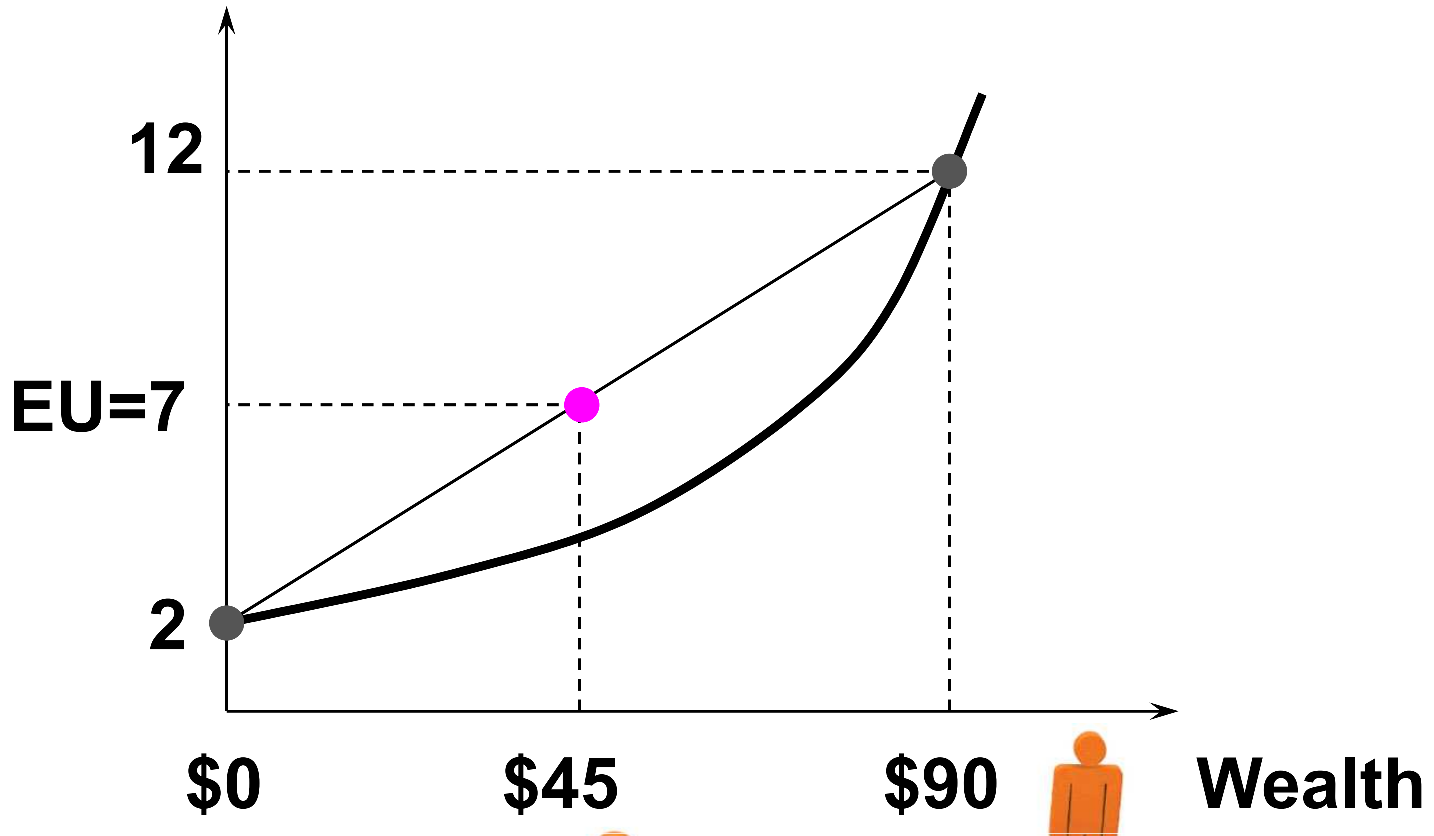


U($45) > EU ⇒ risk-aversion.

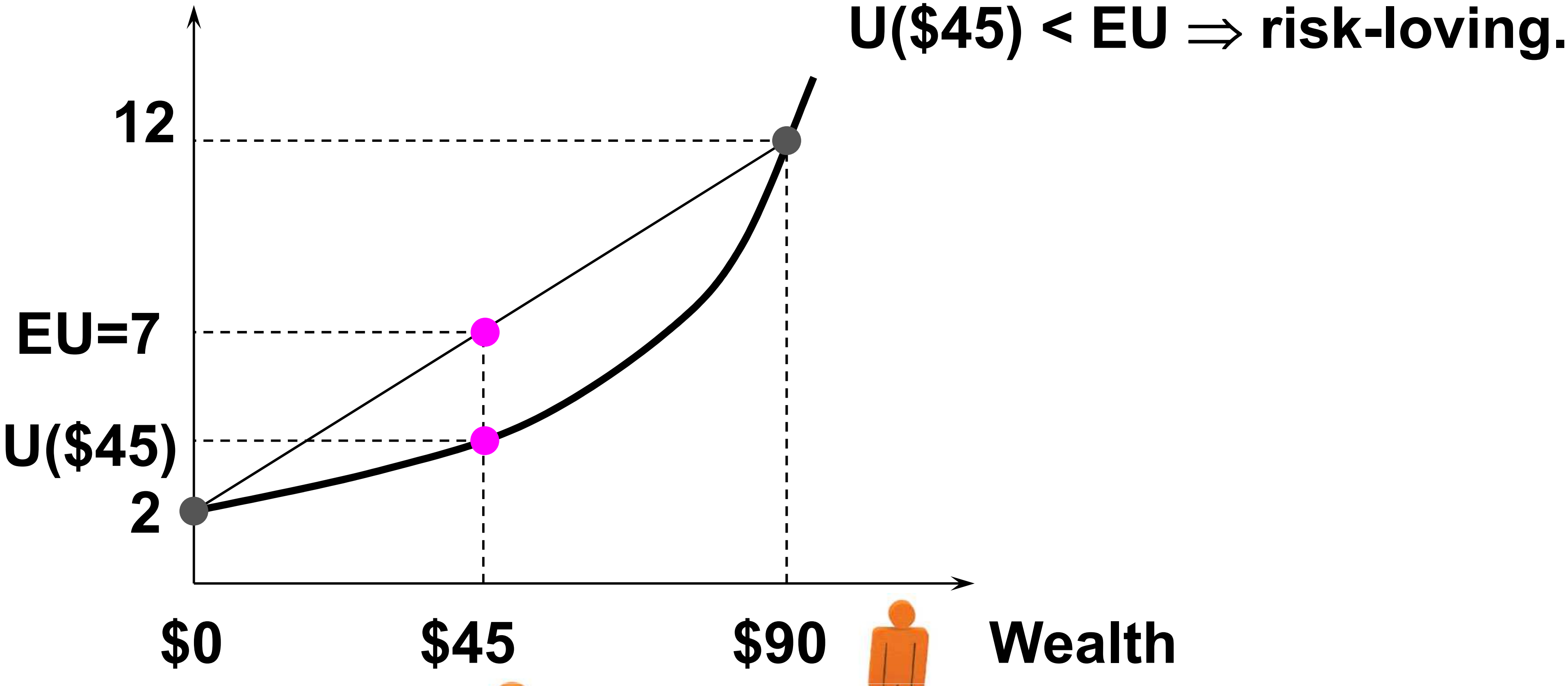
MU declines as wealth rises.



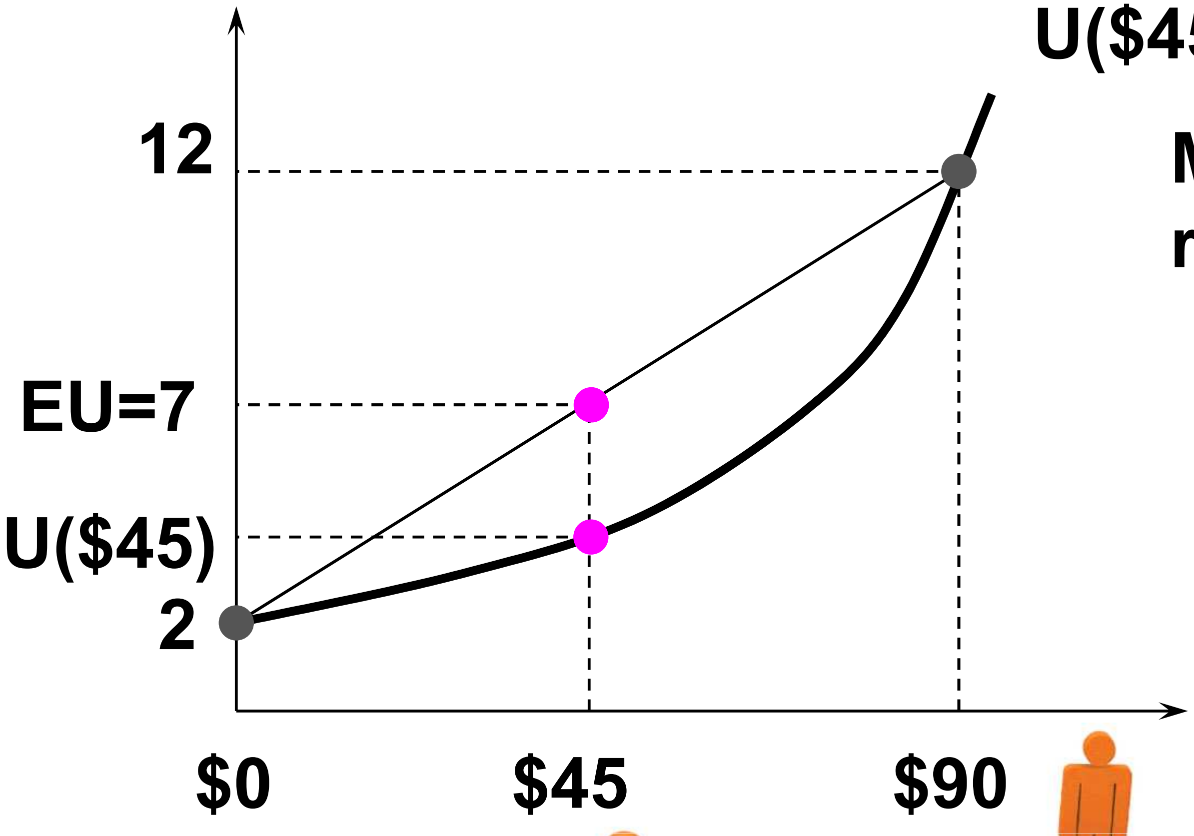
Preferences Under Uncertainty



Preferences Under Uncertainty



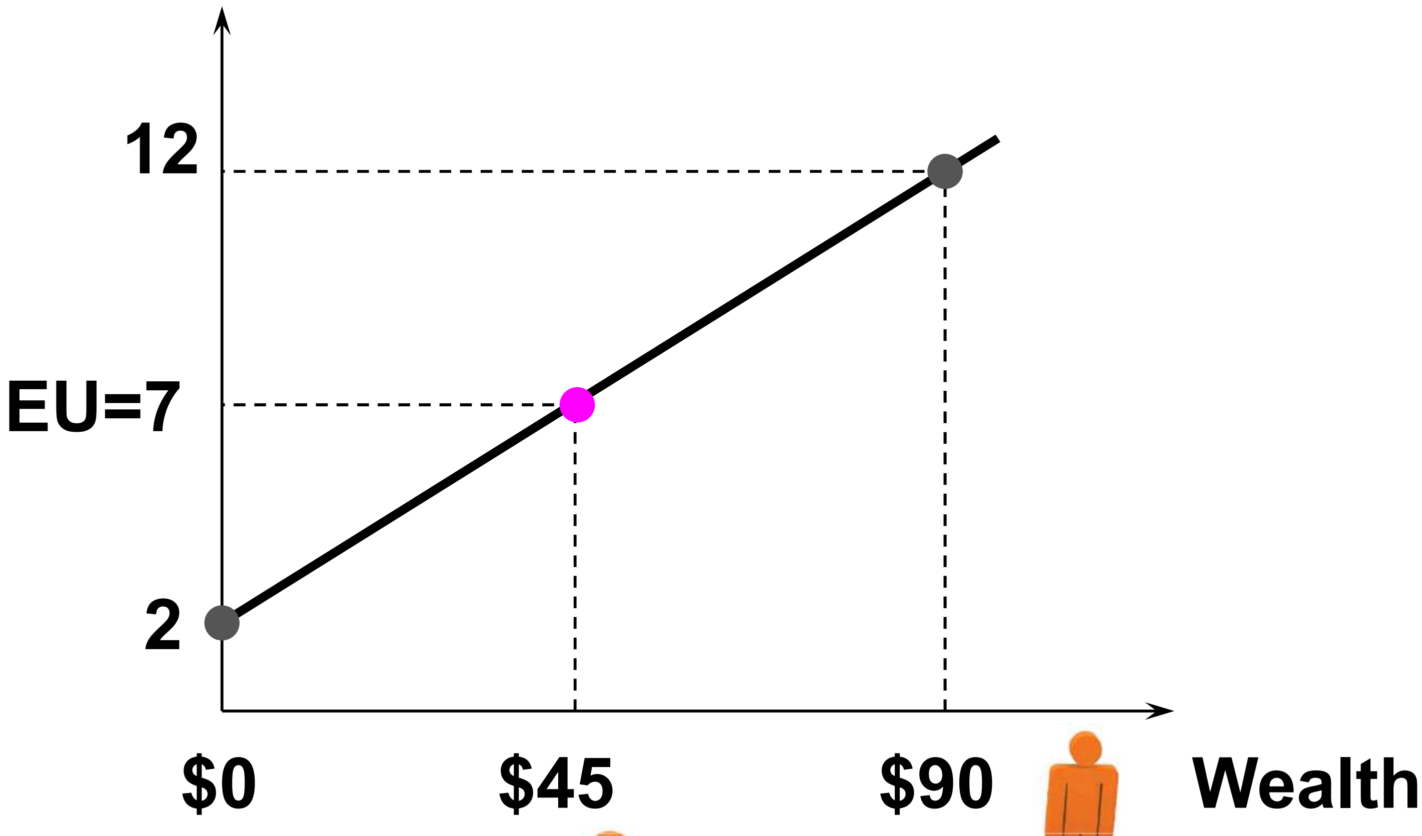
Preferences Under Uncertainty



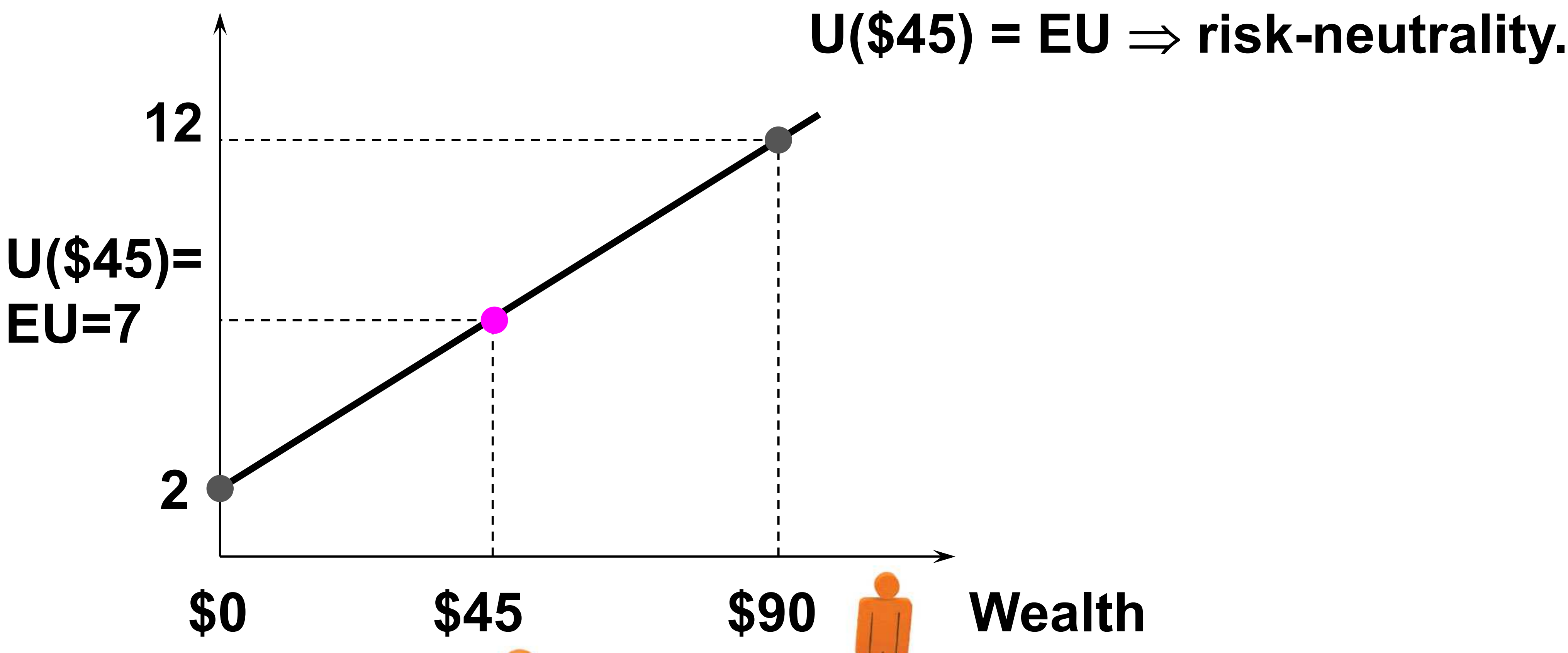
U($45) < EU ⇒ risk-loving.

MU rises as wealth rises.

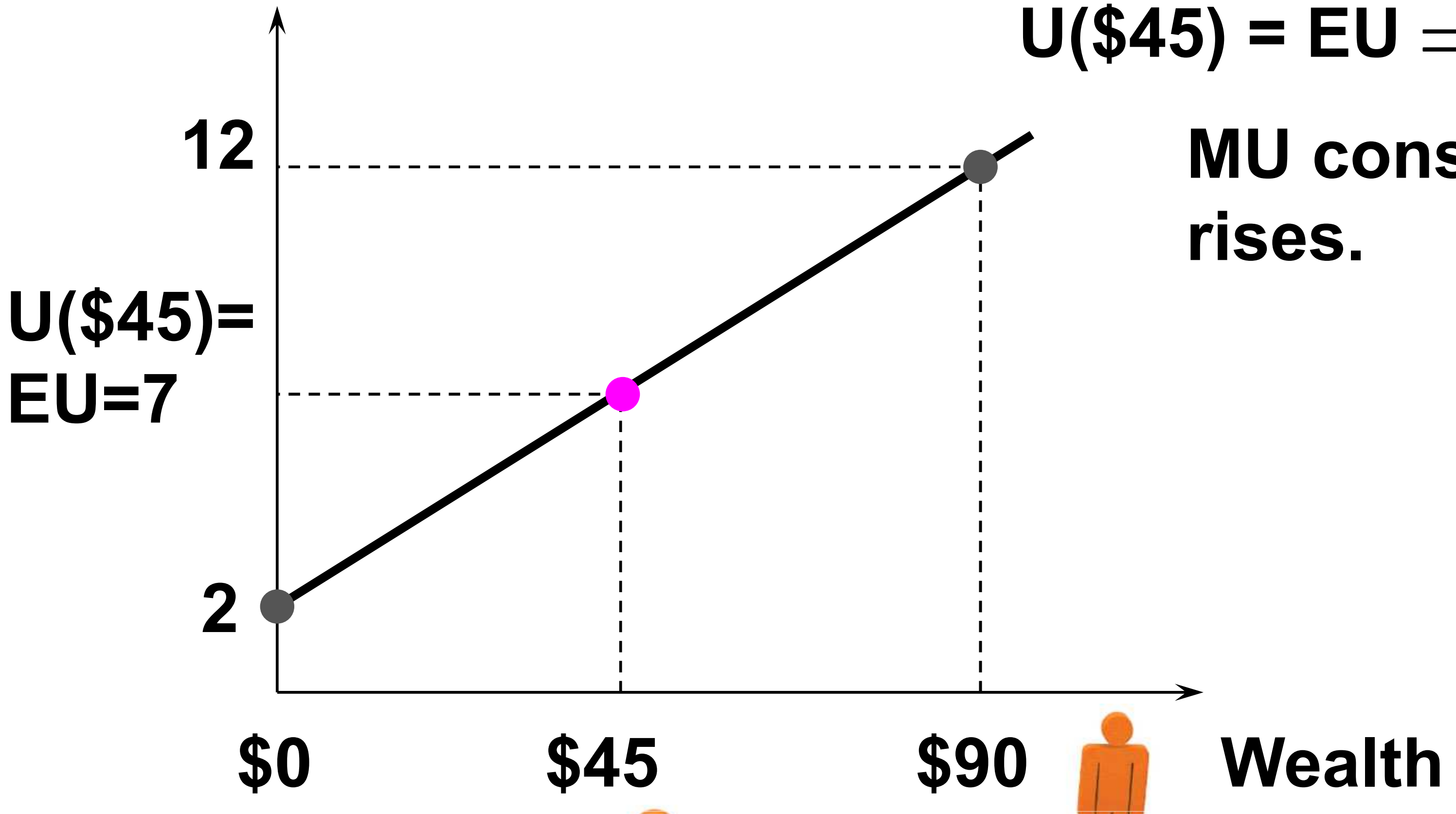
Preferences Under Uncertainty



Preferences Under Uncertainty



Preferences Under Uncertainty



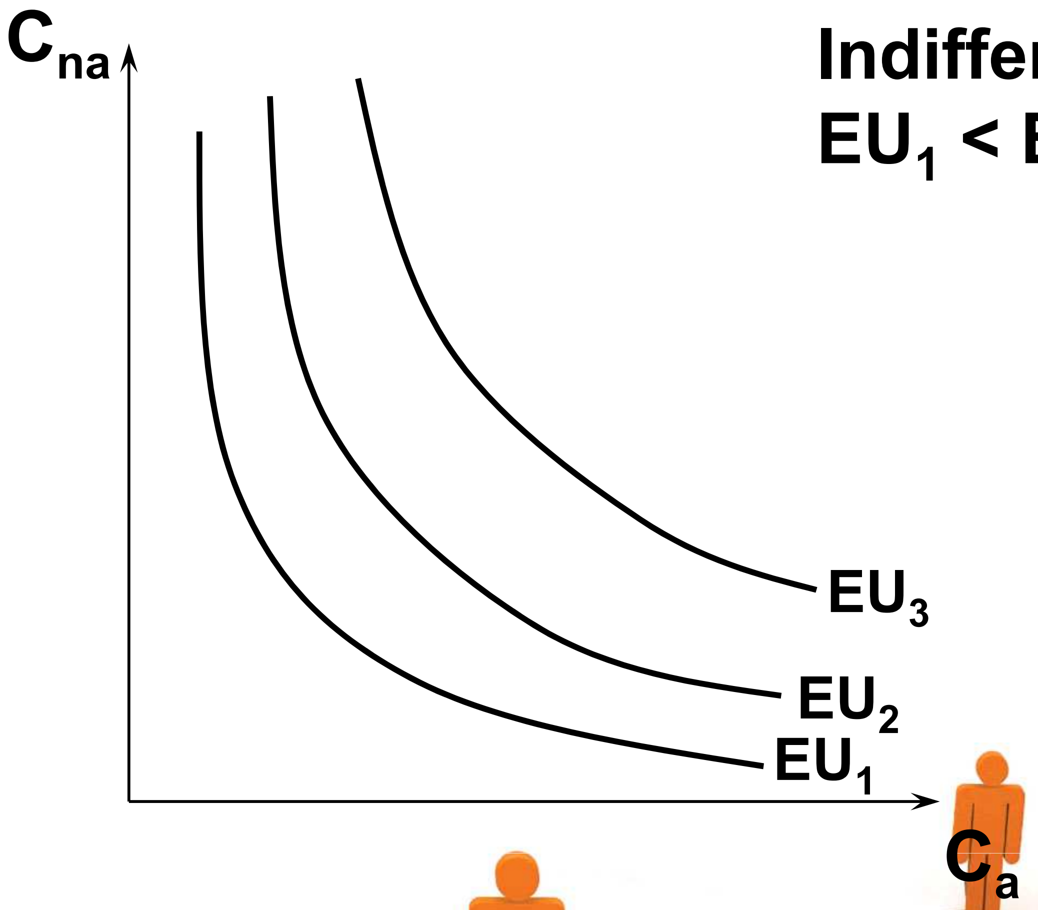
U($45) = EU ⇒ risk-neutrality.

MU constant as wealth rises.

Preferences Under Uncertainty

- **State-contingent consumption plans that give equal expected utility are equally preferred.**

Preferences Under Uncertainty



Preferences Under Uncertainty

- **What is the MRS of an indifference curve?**
- **Get consumption c_1 with prob. π_1 and c_2 with prob. π_2 $(\pi_1 + \pi_2 = 1)$.**
- **$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$.**
- **For constant EU, dEU = 0.**

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

Preferences Under Uncertainty

$$\mathbf{EU = \pi_1 U(c_1) + \pi_2 U(c_2)}$$

$$\mathbf{dEU = \pi_1 MU(c_1) dc_1 + \pi_2 MU(c_2) dc_2}$$

Preferences Under Uncertainty

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

$$dEU = \pi_1 MU(c_1) dc_1 + \pi_2 MU(c_2) dc_2$$

$$dEU = 0 \Rightarrow \pi_1 MU(c_1) dc_1 + \pi_2 MU(c_2) dc_2 = 0$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

$$\mathbf{dEU} = \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\mathbf{dEU} = 0 \Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2 = 0$$

$$\Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 = -\pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

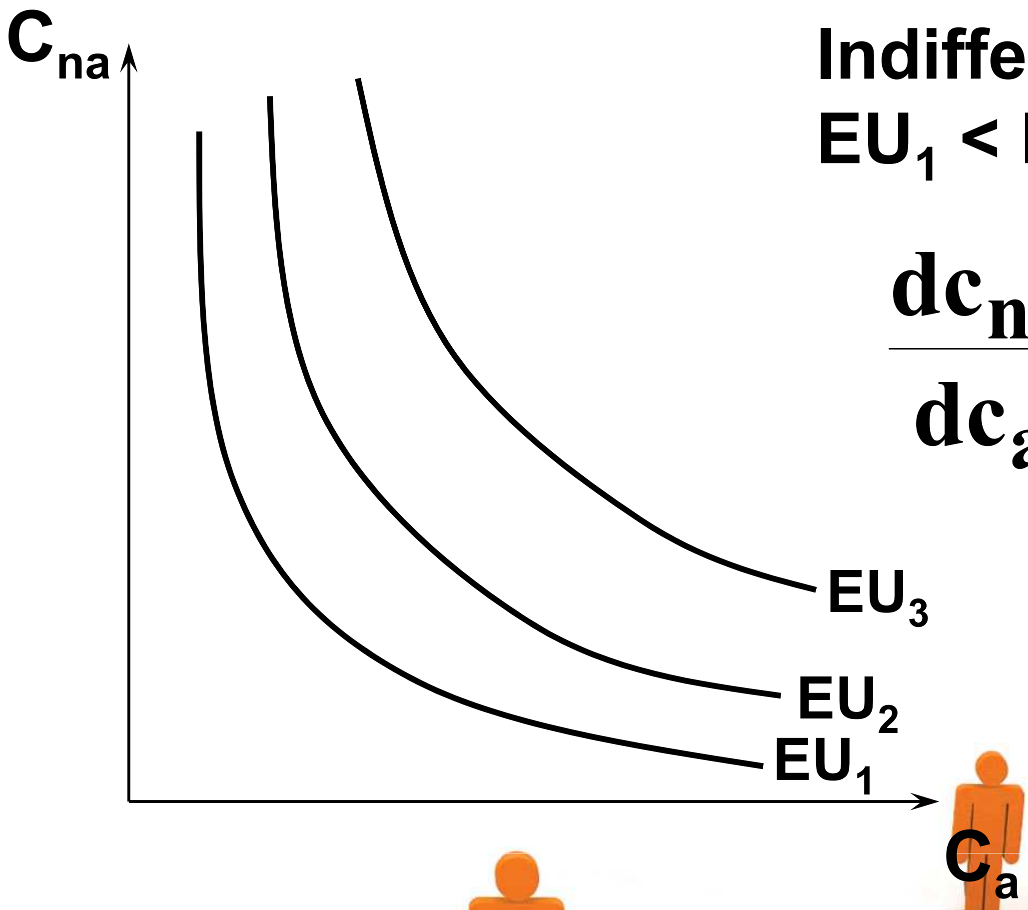
$$\mathbf{dEU} = \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\mathbf{dEU} = 0 \Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2 = 0$$

$$\Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 = -\pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\Rightarrow \frac{\mathbf{dc}_2}{\mathbf{dc}_1} = -\frac{\pi_1 \mathbf{MU}(\mathbf{c}_1)}{\pi_2 \mathbf{MU}(\mathbf{c}_2)}.$$

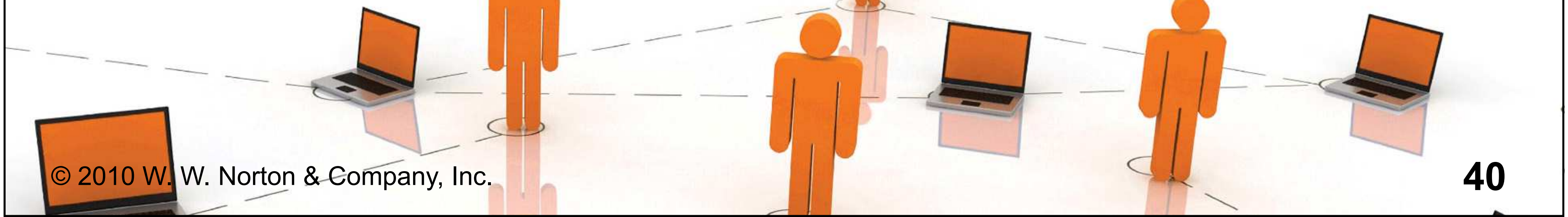
Preferences Under Uncertainty



Indifference curves

$EU_1 < EU_2 < EU_3$

$$\frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

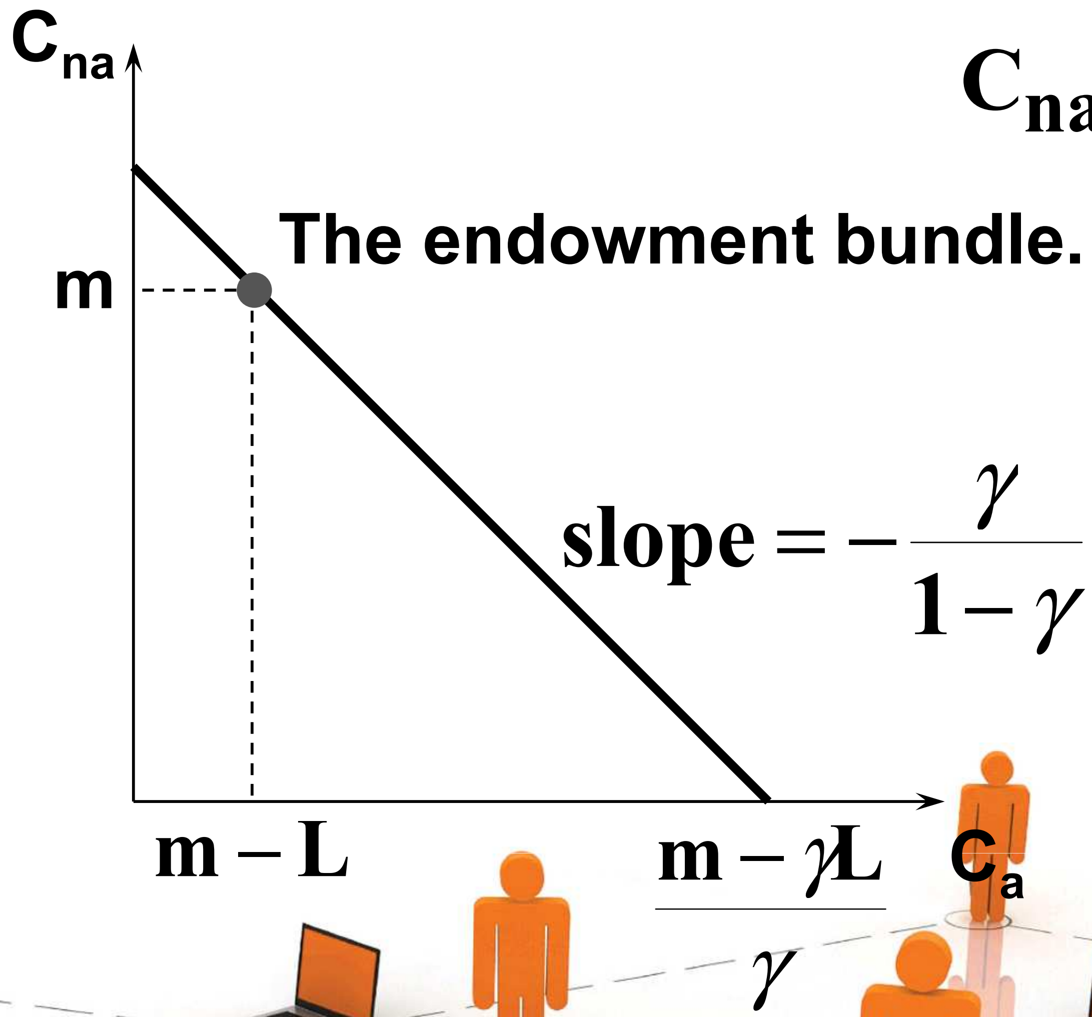


Choice Under Uncertainty

- **Q: How is a rational choice made under uncertainty?**
- **A: Choose the most preferred affordable state-contingent consumption plan.**

State-Contingent Budget Constraints

$$C_{na} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_a$$



Where is the most preferred state-contingent consumption plan?

State-Contingent Budget Constraints

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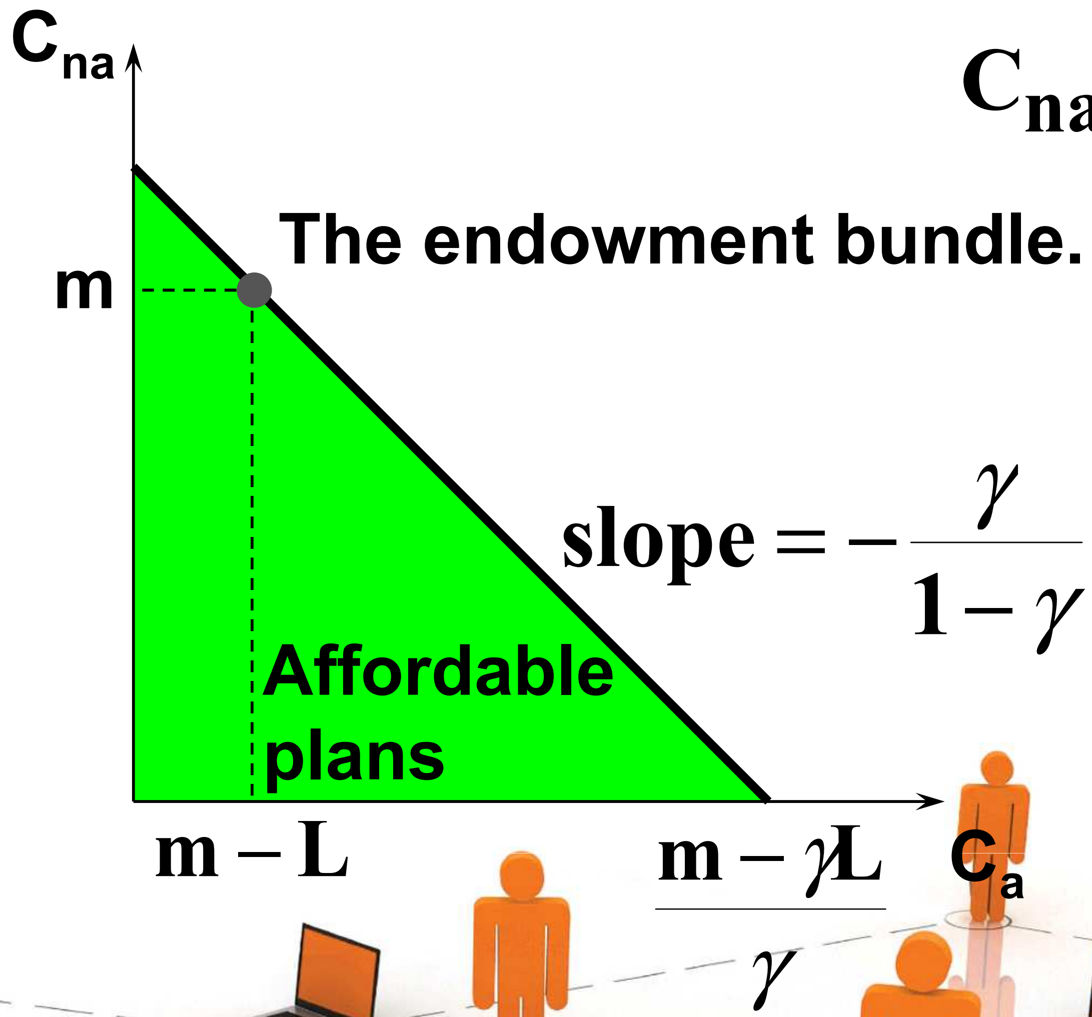
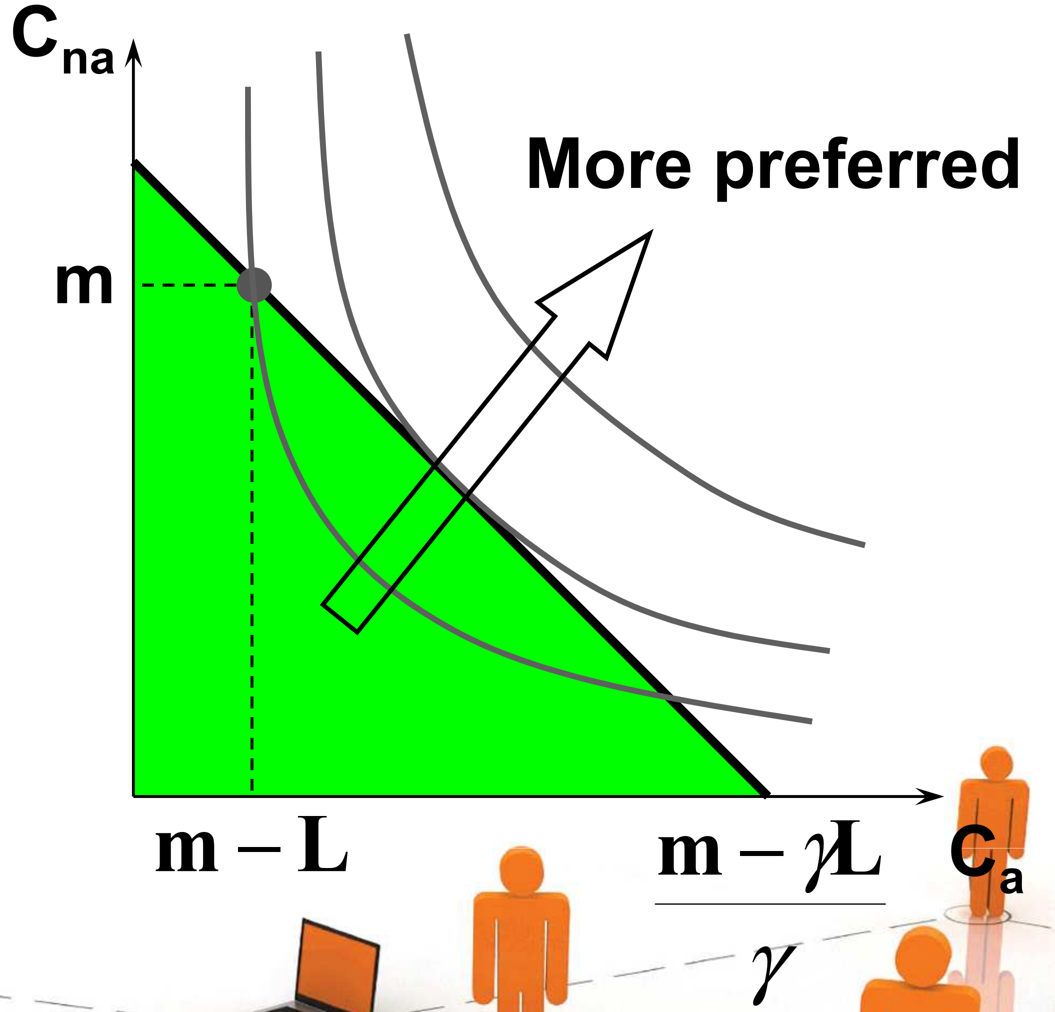


Where is the most preferred state-contingent consumption plan?

State-Contingent Budget Constraints



Where is the most preferred state-contingent consumption plan?

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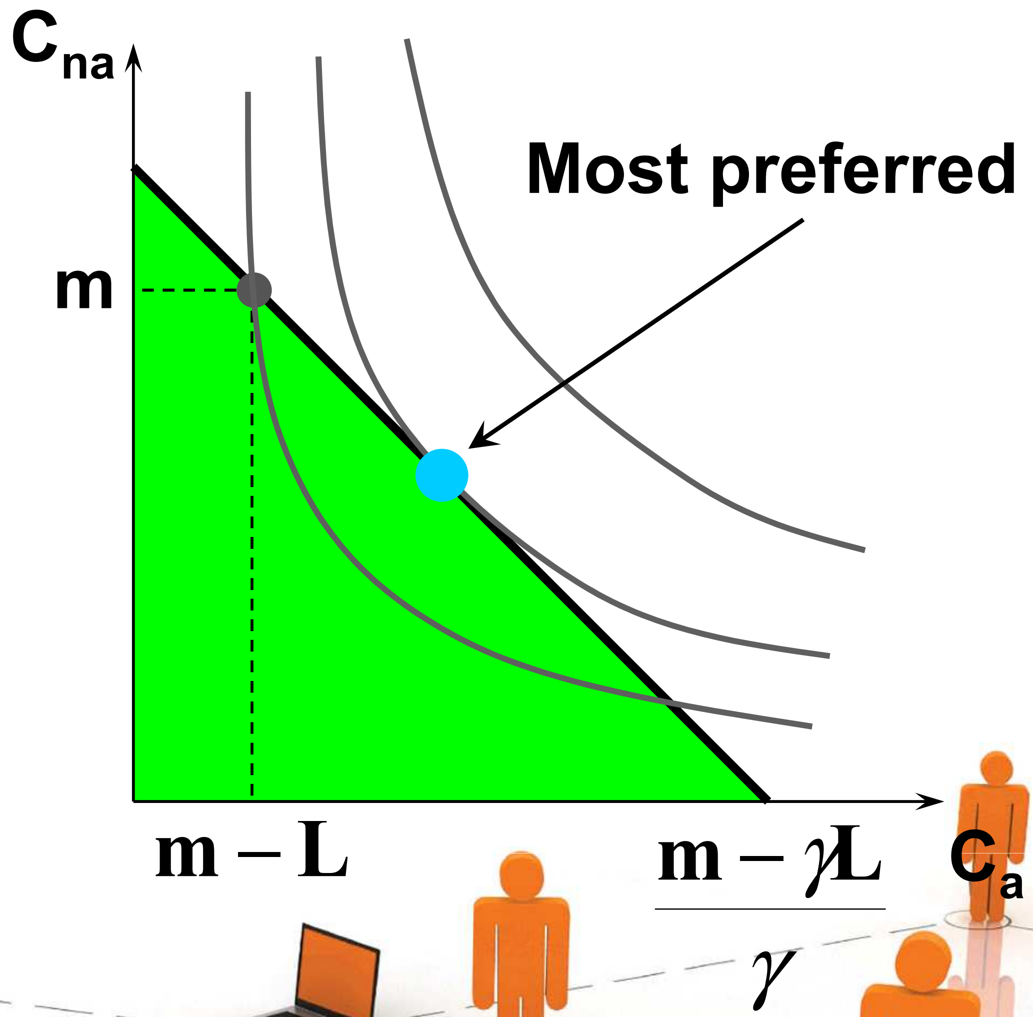

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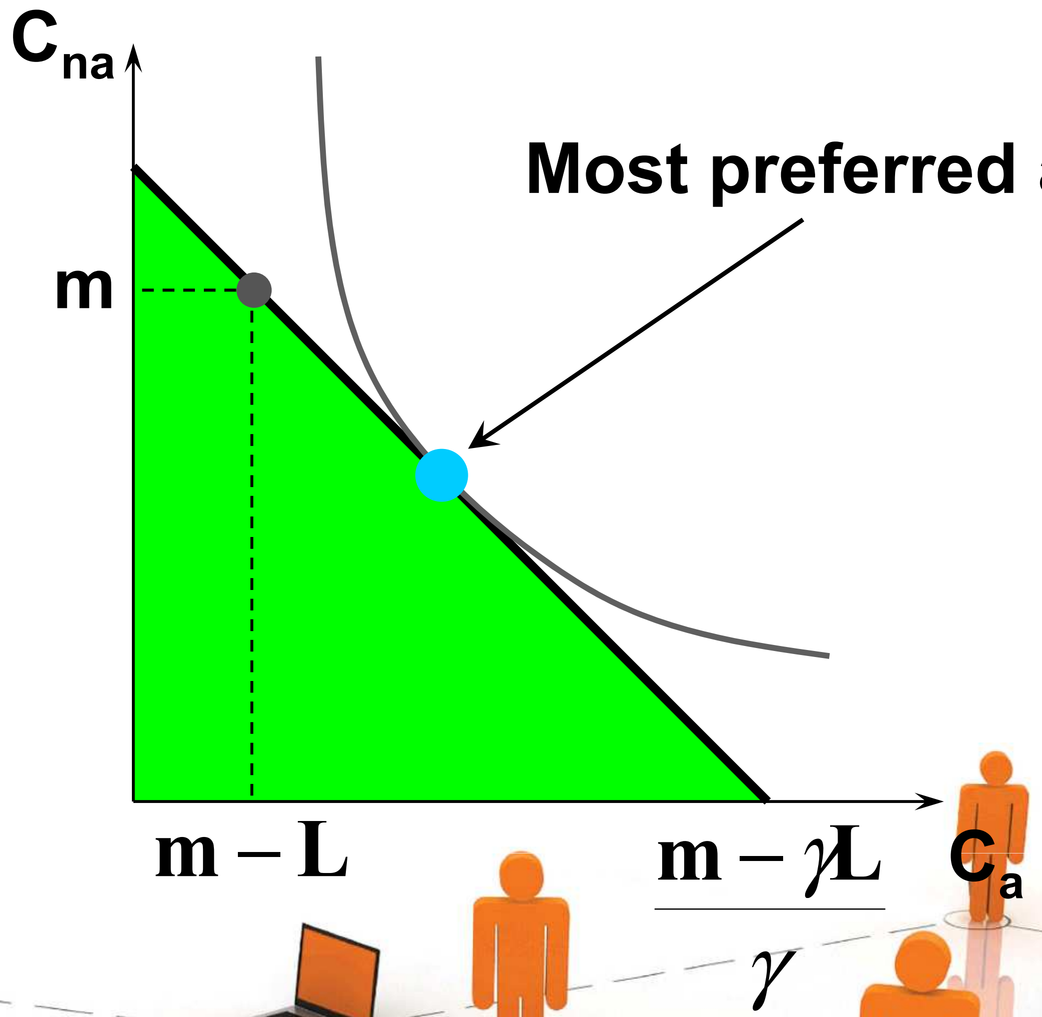

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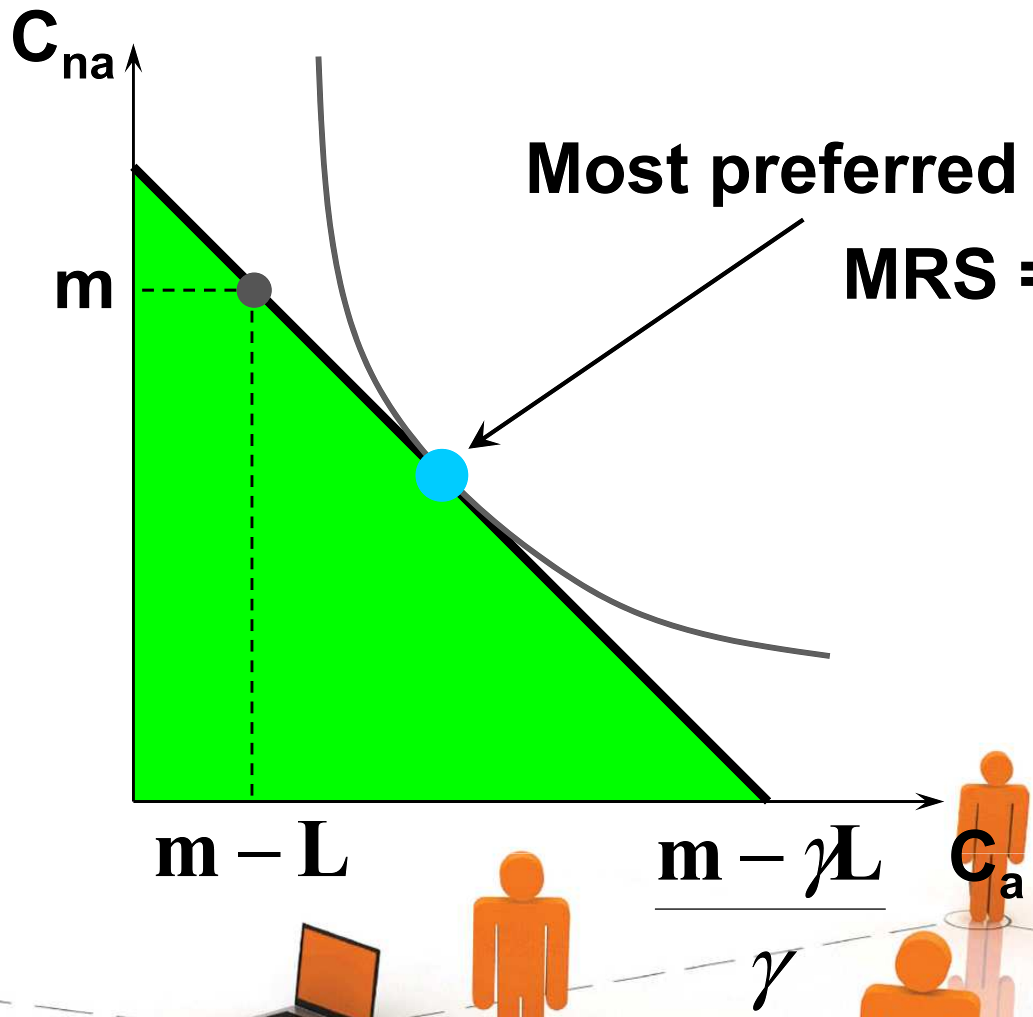

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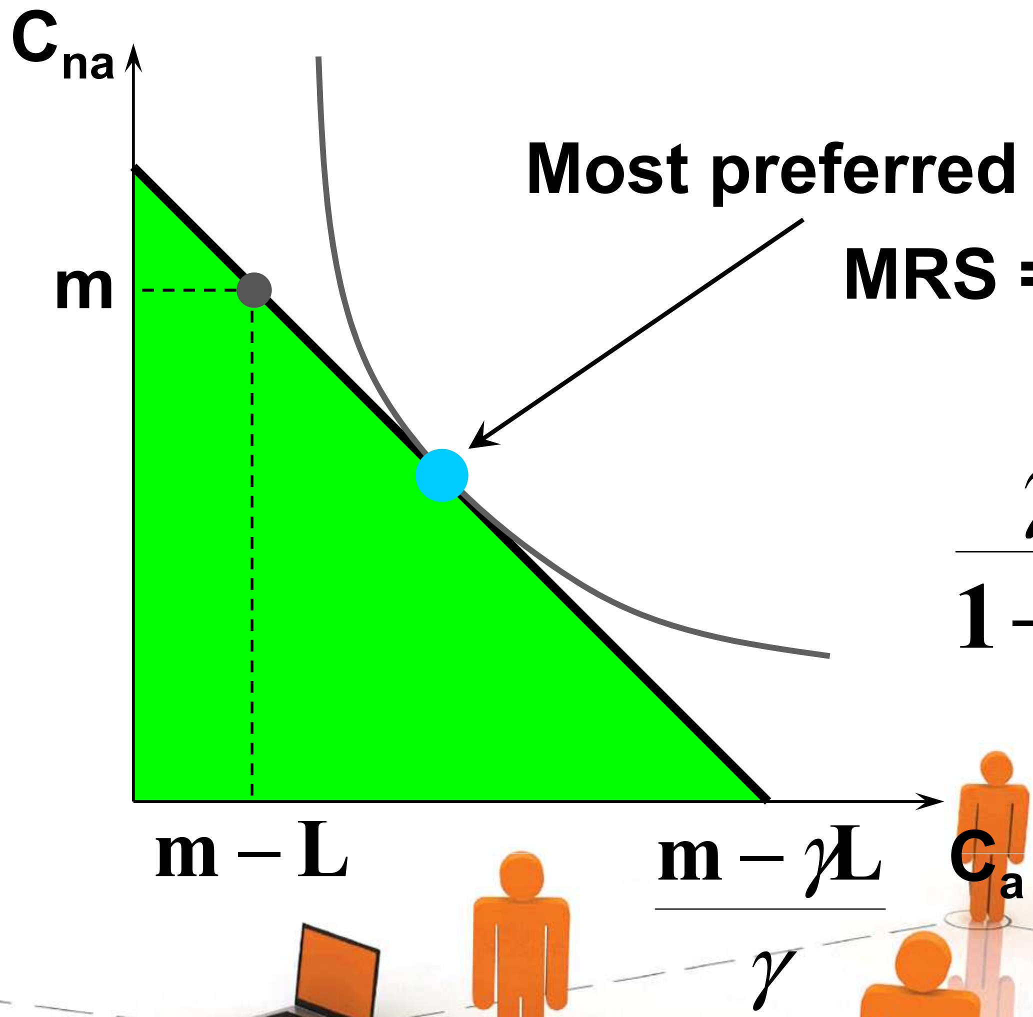

Most preferred affordable plan

MRS = slope of budget constraint; i.e.

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

Competitive Insurance

- **Suppose entry to the insurance industry is free.**
- **Expected economic profit = 0.**
- **I.e. $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K = 0$.**
- **I.e. free entry $\Rightarrow \gamma = \pi_a$.**
- **If price of $1 insurance = accident probability, then insurance is fair.**

Competitive Insurance

- **When insurance is fair, rational insurance choices satisfy**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

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Competitive Insurance

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- **I.e.** $\mathbf{MU}(\mathbf{c_a}) = \mathbf{MU}(\mathbf{c_{na}})$
- **Marginal utility of income must be the same in both states.**

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

$$\mathbf{MU(c_a) = MU(c_{na})}$$

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

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- **Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.**

Competitive Insurance

- How much fair insurance does a risk-averse consumer buy?

$$MU(c_a) = MU(c_{na})$$

- Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.
- Hence $c_a = c_{na}$.

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

$$MU(c_a) = MU(c_{na})$$

- **Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.**
- **Hence** $c_a = c_{na}.$
- **I.e. full-insurance.**

“Unfair” Insurance

- **Suppose insurers make positive expected economic profit.**
- **I.e.** $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K > 0.$

“Unfair” Insurance

- **Suppose insurers make positive expected economic profit.**
- **I.e.** $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K > 0.$
- **Then** $\Rightarrow \gamma > \pi_a \Rightarrow \dfrac{\gamma}{1-\gamma} > \dfrac{\pi_a}{1-\pi_a}.$

“Unfair” Insurance

- **Rational choice requires**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

“Unfair” Insurance

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"Unfair" Insurance

- Rational choice requires

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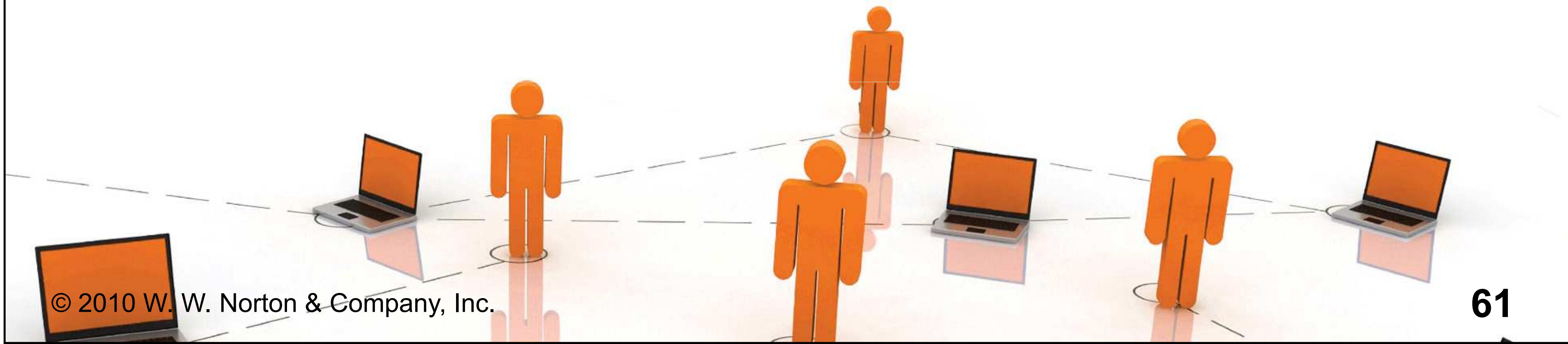
- Since $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $\mathbf{MU(c_a) > MU(c_{na})}$
- Hence $\mathbf{c_a < c_{na}}$ for a risk-averter.



“Unfair” Insurance

- **Rational choice requires**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathrm{MU}(c_a)}{\pi_{na} \mathrm{MU}(c_{na})}$$

- **Since** $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $\mathrm{MU}(c_a) > \mathrm{MU}(c_{na})$

- **Hence** $c_a < c_{na}$ **for a risk-averter.**
- **I.e. a risk-averter buys less than full “unfair” insurance.**

Uncertainty is Pervasive

- **What are rational responses to uncertainty?**
  - **buying insurance (health, life, auto)**
  - **a portfolio of contingent consumption goods.**

Uncertainty is Pervasive

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  - ✓ **– buying insurance (health, life, auto)**
  - ? **– a portfolio of contingent consumption goods.**

Diversification

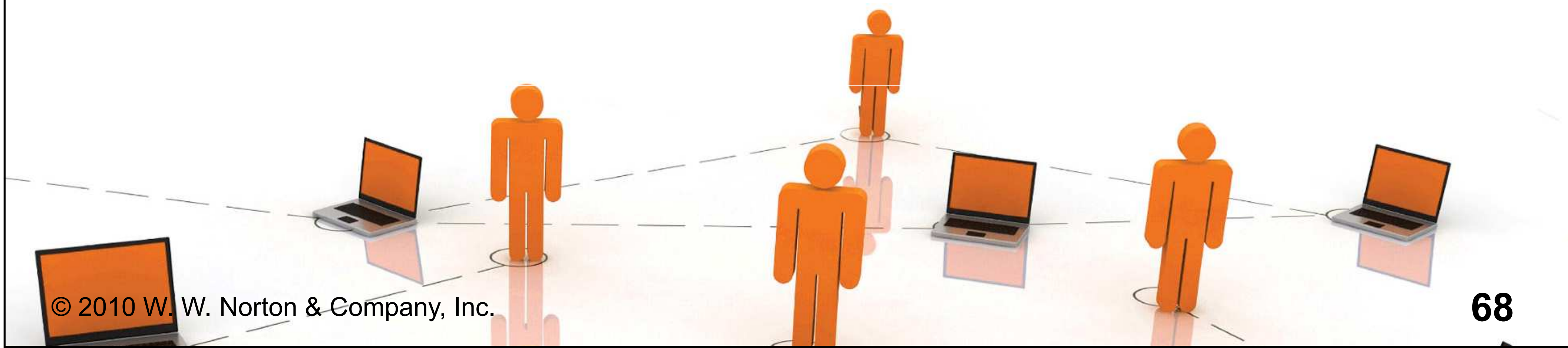
- Two firms, A and B. Shares cost $10.
- With prob. 1/2 A's profit is $100 and B's profit is $20.
- With prob. 1/2 A's profit is $20 and B's profit is $100.
- You have $100 to invest. How?

Diversification

- Buy only firm A's stock?
- $100/10 = 10 shares.
- You earn $1000 with prob. 1/2 and $200 with prob. 1/2.
- Expected earning: $500 + $100 = $600

Diversification

- Buy only firm B's stock?
- $100/10 = 10 shares.
- You earn $1000 with prob. 1/2 and $200 with prob. 1/2.
- Expected earning: $500 + $100 = $600



Diversification

- **Buy 5 shares in each firm?**
- **You earn $600 for sure.**
- **Diversification has maintained expected earning and lowered risk.**

Diversification

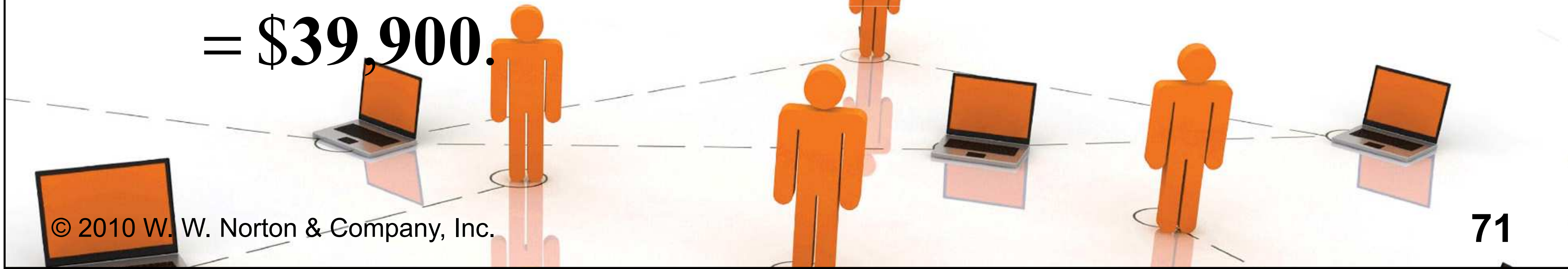
- Buy 5 shares in each firm?
- You earn $600 for sure.
- Diversification has maintained expected earning and lowered risk.
- Typically, diversification lowers expected earnings in exchange for lowered risk.

Risk Spreading/Mutual Insurance

- **100 risk-neutral persons each independently risk a $10,000 loss.**
- **Loss probability = 0.01.**
- **Initial wealth is $40,000.**
- **No insurance: expected wealth is**

$$0 \cdot 99 \times \$40{,}000 + 0 \cdot 01(\$40{,}000 - \$10{,}000)$$

$$= \$39{,}900.$$



Risk Spreading/Mutual Insurance

- **Mutual insurance: Expected loss is**

$$0 \cdot 01 \times \$10{,}000 = \$100.$$

- **Each of the 100 persons pays \$1 into a mutual insurance fund.**
- **Mutual insurance: expected wealth is**

$$\$40{,}000 - \$1 = \$39{,}999 > \$39{,}900.$$

- **Risk-spreading benefits everyone.**