In the chapter on consumer choice, you studied a consumer who tries to maximize his utility subject to the constraint that he has a fixed amount of money to spend. In this chapter you study the behavior of a firm that is trying to produce a fixed amount of output in the cheapest possible way. In both theories, you look for a point of tangency between a curved line and a straight line. In consumer theory, there is an "indifference curve" and a "budget line." In producer theory, there is a "production isoquant" and an "isocost line." As you recall, in consumer theory, finding a tangency gives you only one of the two equations you need to locate the consumer's chosen point. The second equation you used was the budget equation. In cost-minimization theory, again the tangency condition gives you one equation. This time you don't know in advance how much the producer is spending; instead you are told how much output he wants to produce and must find the cheapest way to produce it. So your second equation is the equation that tells you that the desired amount is being produced.

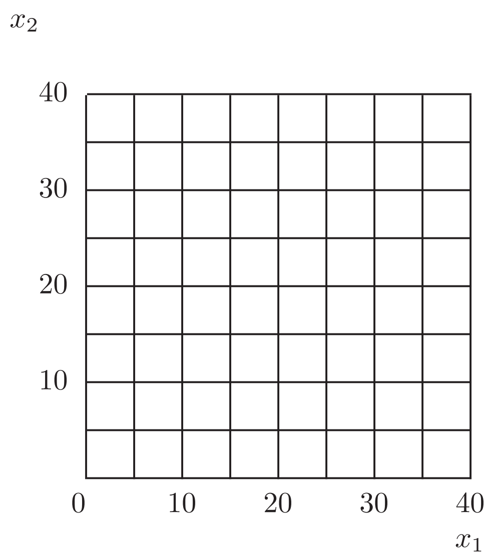
Example. A firm has the production function $f(x_1, x_2) = (\sqrt{x_1} + 3\sqrt{x_2})^2$. The price of factor 1 is $w_1 = 1$ and the price of factor 2 is $w_2 = 1$. Let us find the cheapest way to produce 16 units of output. We will be looking for a point where the technical rate of substitution equals $-w_1/w_2$. If you calculate the technical rate of substitution (or look it up from the warm up exercise in Chapter 18), you find $TRS(x_1, x_2) = -(1/3)(x_2/x_1)^{1/2}$. Therefore we must have $-(1/3)(x_2/x_1)^{1/2} = -w_1/w_2 = -1$. This equation can be simplified to $x_2 = 9x_1$. So we know that the combination of inputs chosen has to lie somewhere on the line $x_2 = 9x_1$. We are looking for the cheapest way to produce 16 units of output. So the point we are looking for must satisfy the equation $(\sqrt{x_1} + 3\sqrt{x_2})^2 = 16$, or equivalently $\sqrt{x_1} + 3\sqrt{x_2} = 4$. Since $x_2 = 9x_1$, we can substitute for x_2 in the previous equation to get $\sqrt{x_1} + 3\sqrt{9x_1} = 4$. This equation simplifies further to $10\sqrt{x_1} = 4$. Solving this for x_1, we have $x_1 = 16/100$. Then $x_2 = 9x_1 = 144/100$.

The amounts x_1 and x_2 that we solved for in the previous paragraph are known as the *conditional factor demands for factors 1 and 2*, conditional on the wages $w_1 = 1$, $w_2 = 1$, and output $y = 16$. We express this by saying $x_1(1, 1, 16) = 16/100$ and $x_2(1, 1, 16) = 144/100$. Since we know the amount of each factor that will be used to produce 16 units of output and since we know the price of each factor, we can now calculate the cost of producing 16 units. This cost is $c(w_1, w_2, 16) = w_1x_1(w_1, w_2, 16) + w_2x_2(w_1, w_2, 16)$. In this instance since $w_1 = w_2 = 1$, we have $c(1, 1, 16) = x_1(1, 1, 16) + x_2(1, 1, 16) = 160/100$.

In consumer theory, you also dealt with cases where the consumer's indifference "curves" were straight lines and with cases where there were kinks in the indifference curves. Then you found that the consumer's choice might occur at a boundary or at a kink. Usually a careful look at the diagram would tell you what is going on. The story with kinks and boundary solutions is almost exactly the same in the case of cost-minimizing firms. You will find some exercises that show how this works.

20.1 (0) Nadine sells user-friendly software. Her firm's production function is $f(x_1, x_2) = x_1 + 2x_2$, where x_1 is the amount of unskilled labor and x_2 is the amount of skilled labor that she employs.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 20 units of output. Draw another isoquant representing input combinations that will produce 40 units of output.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?____________________.

(c) If Nadine uses only unskilled labor, how much unskilled labor would she need in order to produce y units of output?____________________.

(d) If Nadine uses only skilled labor to produce output, how much skilled labor would she need in order to produce y units of output?____________________.

(e) If Nadine faces factor prices $(1, 1)$, what is the cheapest way for her to produce 20 units of output? $x_1 =$ __________, $x_2 =$____________________.

____

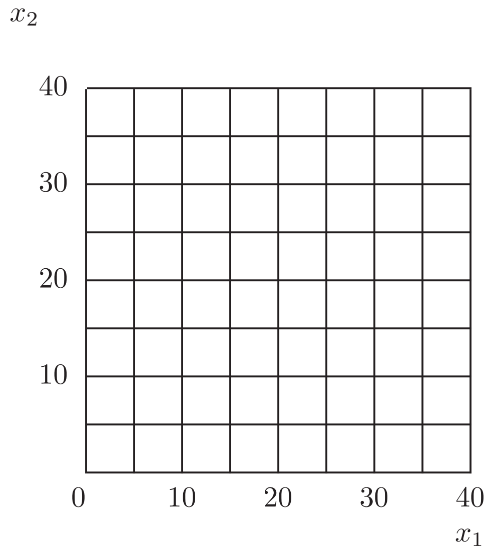
(f) If Nadine faces factor prices $(1, 3)$, what is the cheapest way for her to produce 20 units of output? $x_1 =$ ________ , $x_2 =$ ____________.

(g) If Nadine faces factor prices (w_1, w_2), what will be the minimal cost of producing 20 units of output? ________________________.

(h) If Nadine faces factor prices (w_1, w_2), what will be the minimal cost of producing y units of output? ________________________.

20.2 (0) The Ontario Brassworks produces brazen effronteries. As you know brass is an alloy of copper and zinc, used in fixed proportions. The production function is given by: $f(x_1, x_2) = \min\{x_1, 2x_2\}$, where x_1 is the amount of copper it uses and x_2 is the amount of zinc that it uses in production.

(a) Illustrate a typical isoquant for this production function in the graph below.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale? ________________________.

(c) If the firm wanted to produce 10 effronteries, how much copper would it need? ____________ How much zinc would it need? ____________.

(d) If the firm faces factor prices $(1, 1)$, what is the cheapest way for it to produce 10 effronteries? How much will this cost?__________________

____

____.

(e) If the firm faces factor prices (w_1, w_2), what is the cheapest cost to produce 10 effronteries?________________________________.

(f) If the firm faces factor prices (w_1, w_2), what will be the minimal cost of producing y effronteries?________________________________.

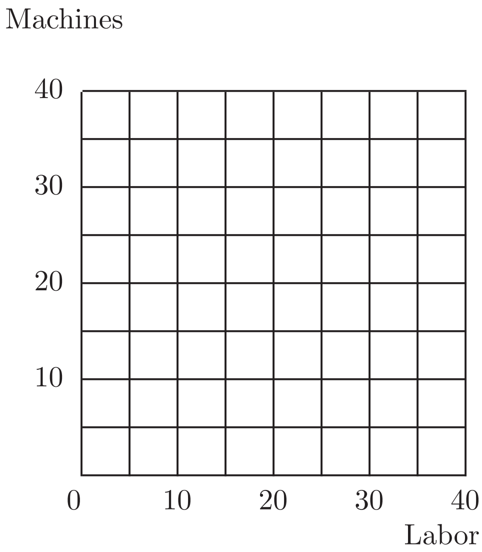
20.3 (0) A firm uses labor and machines to produce output according to the production function $f(L, M) = 4L^{1/2}M^{1/2}$, where L is the number of units of labor used and M is the number of machines. The cost of labor is \$40 per unit and the cost of using a machine is \$10.

(a) On the graph below, draw an isocost line for this firm, showing combinations of machines and labor that cost \$400 and another isocost line showing combinations that cost \$200. What is the slope of these isocost lines?____.

(b) Suppose that the firm wants to produce its output in the cheapest possible way. Find the number of machines it would use per worker. (Hint: The firm will produce at a point where the slope of the production isoquant equals the slope of the isocost line.)____________________.

(c) On the graph, sketch the production isoquant corresponding to an output of 40. Calculate the amount of labor __________ and the number of machines ________ that are used to produce 40 units of output in the cheapest possible way, given the above factor prices. Calculate the cost of producing 40 units at these factor prices: $c(40, 10, 40) =$____________.

(d) How many units of labor ______ and how many machines ______ would the firm use to produce y units in the cheapest possible way? How much would this cost? __________ (Hint: Notice that there are constant returns to scale.)



20.4 (0) Earl sells lemonade in a competitive market on a busy street corner in Philadelphia. His production function is $f(x_1, x_2) = x_1^{1/3} x_2^{1/3}$, where output is measured in gallons, x_1 is the number of pounds of lemons he uses, and x_2 is the number of labor-hours spent squeezing them.

(a) Does Earl have constant returns to scale, decreasing returns to scale, or increasing returns to scale?____________________.

(b) Where w_1 is the cost of a pound of lemons and w_2 is the wage rate for lemon-squeezers, the cheapest way for Earl to produce lemonade is to use __________ hours of labor per pound of lemons. (Hint: Set the slope of his isoquant equal to the slope of his isocost line.)

(c) If he is going to produce y units in the cheapest way possible, then the number of pounds of lemons he will use is $x_1(w_1, w_2, y) =$__________ and the number of hours of labor that he will use is $x_2(w_1, w_2, y) =$__________ (Hint: Use the production function and the equation you found in the last part of the answer to solve for the input quantities.)

(d) The cost to Earl of producing y units at factor prices w_1 and w_2 is $c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y) =$____________________.

20.5 (0) The prices of inputs (x_1, x_2, x_3, x_4) are $(4, 1, 3, 2)$.

(a) If the production function is given by $f(x_1, x_2) = \min\{x_1, x_2\}$, what is the minimum cost of producing one unit of output?________________.

(b) If the production function is given by $f(x_3, x_4) = x_3 + x_4$, what is the minimum cost of producing one unit of output?________________.

(c) If the production function is given by $f(x_1, x_2, x_3, x_4) = \min\{x_1 + x_2, x_3 + x_4\}$, what is the minimum cost of producing one unit of output?

________________________________.

(d) If the production function is given by $f(x_1, x_2) = \min\{x_1, x_2\} + \min\{x_3, x_4\}$, what is the minimum cost of producing one unit of output?

________________________________.

20.6 (0) Joe Grow, an avid indoor gardener, has found that the number of happy plants, h, depends on the amount of light, l, and water, w. In fact, Joe noticed that plants require twice as much light as water, and any more or less is wasted. Thus, Joe's production function is $h = \min\{l, 2w\}$.

(a) Suppose Joe is using 1 unit of light, what is the least amount of water he can use and still produce a happy plant?________________.

(b) If Suppose Joe wants to produce 4 happy plants, what are the minimum amounts of light and water required?________________.

(c) Joe's conditional factor demand function for light is $l(w_1, w_2, h) =$ ________ and his conditional factor demand function for water is $w(w_1, w_2, h) =$____________________________.

(d) If each unit of light costs w_1 and each unit of water costs w_2, Joe's cost function is $c(w_1, w_2, h) =$____________________________.

20.7 (1) Joe's sister, Flo Grow, is a university administrator. She uses an alternative method of gardening. Flo has found that happy plants only need fertilizer and talk. (*Warning:* Frivolous observations about university administrators' talk being a perfect substitute for fertilizer is in extremely poor taste.) Where f is the number of bags of fertilizer used and t is the number of hours she talks to her plants, the number of happy plants produced is exactly $h = t + 2f$. Suppose fertilizer costs w_f per bag and talk costs w_t per hour.

(a) If Flo uses no fertilizer, how many hours of talk must she devote if she wants one happy plant? __________ If she doesn't talk to her plants at all, how many bags of fertilizer will she need for one happy plant? ____________________.

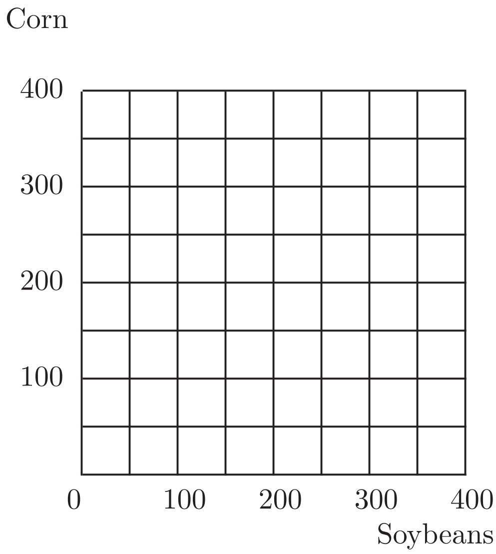
(b) If $w_t < w_f/2$, would it be cheaper for Flo to use fertilizer or talk to raise one happy plant? ____________________.

(c) Flo's cost function is $c(w_f, w_t, h) =$ ____________________.

(d) Her conditional factor demand for talk is $t(w_f, w_t, h) =$ __________ if $w_t < w_f/2$ and __________ if $w_t > w_f/2$.

20.8 (0) Remember T-bone Pickens, the corporate raider? Now he's concerned about his chicken farms, Pickens's Chickens. He feeds his chickens on a mixture of soybeans and corn, depending on the prices of each. According to the data submitted by his managers, when the price of soybeans was \$10 a bushel and the price of corn was \$10 a bushel, they used 50 bushels of corn and 150 bushels of soybeans for each coop of chickens. When the price of soybeans was \$20 a bushel and the price of corn was \$10 a bushel, they used 300 bushels of corn and no soybeans per coop of chickens. When the price of corn was \$20 a bushel and the price of soybeans was \$10 a bushel, they used 250 bushels of soybeans and no corn for each coop of chickens.

(a) Graph these three input combinations and isocost lines in the following diagram.



(b) How much money did Pickens' managers spend per coop of chickens when the prices were $(10, 10)$? ________ When the prices were $(10, 20)$? ________ When the prices were $(20, 10)$?________.

(c) Is there any evidence that Pickens's managers were not minimizing costs? Why or why not?

________.

(d) Pickens wonders whether there are any prices of corn and soybeans at which his managers will use 150 bushels of corn and 50 bushels of soybeans to produce a coop of chickens. How much would this production method cost per coop of chickens if the prices were $p_s = 10$ and $p_c = 10$? ________ if the prices were $p_s = 10$, $p_c = 20$? ________ if the prices were $p_s = 20$, $p_c = 10$?________.

(e) If Pickens's managers were always minimizing costs, can it be possible to produce a coop of chickens using 150 bushels and 50 bushels of soybeans?________

____

________.

20.9 (0) A genealogical firm called Roots produces its output using only one input. Its production function is $f(x) = \sqrt{x}$.

(a) Does the firm have increasing, constant, or decreasing returns to scale?

________.

(b) How many units of input does it take to produce 10 units of output? ________ If the input costs w per unit, what does it cost to produce 10 units of output?________.

(c) How many units of input does it take to produce y units of output? ________ If the input costs w per unit, what does it cost to produce y units of output?________.

____

(d) If the input costs w per unit, what is the average cost of producing y units? $AC(w, y) =$______________________.

20.10 (0) A university cafeteria produces square meals, using only one input and a rather remarkable production process. We are not allowed to say what that ingredient is, but an authoritative kitchen source says that "fungus is involved." The cafeteria's production function is $f(x) = x^2$, where x is the amount of input and $f(x)$ is the number of square meals produced.

(a) Does the cafeteria have increasing, constant, or decreasing returns to scale?______________________.

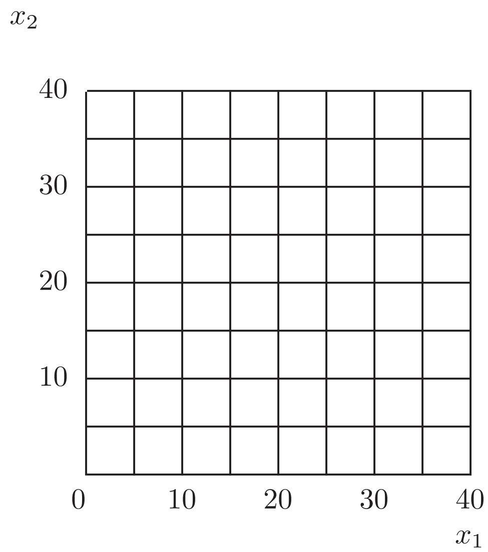
(b) How many units of input does it take to produce 144 square meals? ________ If the input costs w per unit, what does it cost to produce 144 square meals?______________________.

(c) How many units of input does it take to produce y square meals? ________ If the input costs w per unit, what does it cost to produce y square meals?______________________.

(d) If the input costs w per unit, what is the average cost of producing y square meals? $AC(w, y) =$______________________.

20.11 (0) Irma's Handicrafts produces plastic deer for lawn ornaments. "It's hard work," says Irma, "but anything to make a buck." Her production function is given by $f(x_1, x_2) = (\min\{x_1, 2x_2\})^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of labor used, and $f(x_1, x_2)$ is the number of deer produced.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 4 deer. Draw another production isoquant representing input combinations that will produce 5 deer.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?____________________.

(c) If Irma faces factor prices $(1, 1)$, what is the cheapest way for her to produce 4 deer? __________ How much does this cost?________.

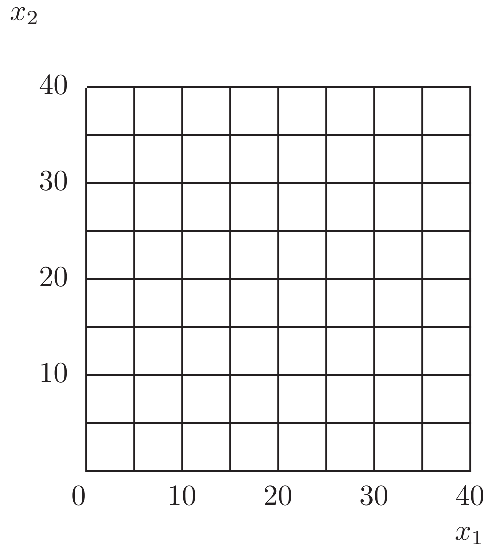
(d) At the factor prices $(1, 1)$, what is the cheapest way to produce 5 deer? __________ How much does this cost?________.

(e) At the factor prices $(1, 1)$, the cost of producing y deer with this technology is $c(1, 1, y) =$____________________.

(f) At the factor prices (w_1, w_2), the cost of producing y deer with this technology is $c(w_1, w_2, y) =$____________________.

20.12 (0) Al Deardwarf also makes plastic deer for lawn ornaments. Al has found a way to automate the production process completely. He doesn't use any labor–only wood and plastic. Al says he likes the business "because I need the doe." Al's production function is given by $f(x_1, x_2) = (2x_1 + x_2)^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of wood used, and $f(x_1, x_2)$ is the number of deer produced.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 4 deer. Draw another production isoquant representing input combinations that will produce 6 deer.



(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?__________.

(c) If Al faces factor prices $(1, 1)$, what is the cheapest way for him to produce 4 deer? ______ How much does this cost?______.

(d) At the factor prices $(1, 1)$, what is the cheapest way to produce 6 deer? ______ How much does this cost?______.

(e) At the factor prices $(1, 1)$, the cost of producing y deer with this technology is $c(1, 1, y) =$__________.

(f) At the factor prices $(3, 1)$, the cost of producing y deer with this technology is $c(3, 1, y) =$__________.

20.13 (0) Suppose that Al Deardwarf from the last problem cannot vary the amount of wood that he uses in the short run and is stuck with using 20 units of wood. Suppose that he can change the amount of plastic that he uses, even in the short run.

(a) How much plastic would Al need in order to make 100 deer?______

____________________.

(b) If the cost of plastic is \$1 per unit and the cost of wood is \$1 per unit, how much would it cost Al to make 100 deer?________________________.

(c) Write down Al's short-run cost function at these factor prices.________

____.

Here you continue to work on cost functions. Total cost can be divided into fixed cost, the part that doesn't change as output changes, and variable cost. To get the average (total) cost, average fixed cost, and average variable cost, just divide the appropriate cost function by y, the level of output. The marginal cost function is the derivative of the total cost function with respect to output—or the rate of increase in cost as output increases, if you don't know calculus.

Remember that the marginal cost curve intersects both the average cost curve and the average variable cost curve at their minimum points. So to find the minimum point on the average cost curve, you simply set marginal cost equal to average cost and similarly for the minimum of average variable cost.

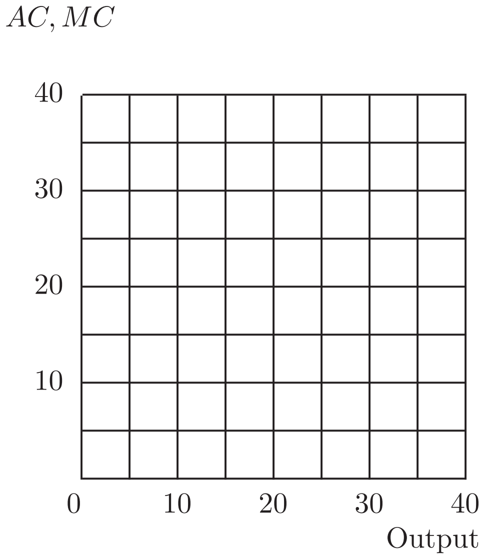
A firm has the total cost function $C(y) = 100 + 10y$. Let us find the equations for its various cost curves. Total fixed costs are 100, so the equation of the average fixed cost curve is $100/y$. Total variable costs are $10y$, so average variable costs are $10y/y = 10$ for all y. Marginal cost is 10 for all y. Average total costs are $(100 + 10y)/y = 10 + 10/y$. Notice that for this firm, average total cost decreases as y increases. Notice also that marginal cost is less than average total cost for all y.

21.1 (0) Mr. Otto Carr, owner of Otto's Autos, sells cars. Otto buys autos for $\$c$ each and has no other costs.

(a) What is his total cost if he sells 10 cars? __________ What if he sells 20 cars? __________ Write down the equation for Otto's total costs assuming he sells y cars: $TC(y) =$__________________________________.

(b) What is Otto's average cost function? $AC(y) =$ ________ For every additional auto Otto sells, by how much do his costs increase? ________ Write down Otto's marginal cost function: $MC(y) =$________________.

(c) In the graph below draw Otto's average and marginal cost curves if $c = 20$.



(d) Suppose Otto has to pay \$$b$ a year to produce obnoxious television commercials. Otto's total cost curve is now $TC(y) =$ __________ , his average cost curve is now $AC(y) =$ __________ , and his marginal cost curve is $MC(y) =$____________________________________.

(e) If $b = \$100$, use red ink to draw Otto's average cost curve on the graph above.

21.2 (0) Otto's brother, Dent Carr, is in the auto repair business. Dent recently had little else to do and decided to calculate his cost conditions. He found that the total cost of repairing s cars is $TC(s) = 2s^2 + 10$. But Dent's attention was diverted to other things ... and that's where you come in. Please complete the following:

Dent's Total Variable Costs:______________________________.

Total Fixed Costs:____________________________________.

Average Variable Costs:________________________________.

Average Fixed Costs:__________________________________.

Average Total Costs:__________________________________.

Marginal Costs:______________________________________.

21.3 (0) A third brother, Rex Carr, owns a junk yard. Rex can use one of two methods to destroy cars. The first involves purchasing a hydraulic car smasher that costs \$200 a year to own and then spending \$1 for every car smashed into oblivion; the second method involves purchasing a shovel that will last one year and costs \$10 and paying the last Carr brother, Scoop, to bury the cars at a cost of \$5 each.

(a) Write down the total cost functions for the two methods, where y is output per year: $TC_1(y) =$ __________ , $TC_2(y) =$____________________.

(b) The first method has an average cost function ____________ and a marginal cost function ________ For the second method these costs are __________ and____.

(c) If Rex wrecks 40 cars per year, which method should he use? _______ __________ If Rex wrecks 50 cars per year, which method should he use? _____________ What is the smallest number of cars per year for which it would pay him to buy the hydraulic smasher?______________________.

21.4 (0) Mary Magnolia wants to open a flower shop, the Petal Pusher, in a new mall. She has her choice of three different floor sizes, 200 square feet, 500 square feet, or 1,000 square feet. The monthly rent will be \$1 a square foot. Mary estimates that if she has F square feet of floor space and sells y bouquets a month, her variable costs will be $c_v(y) = y^2/F$ per month.

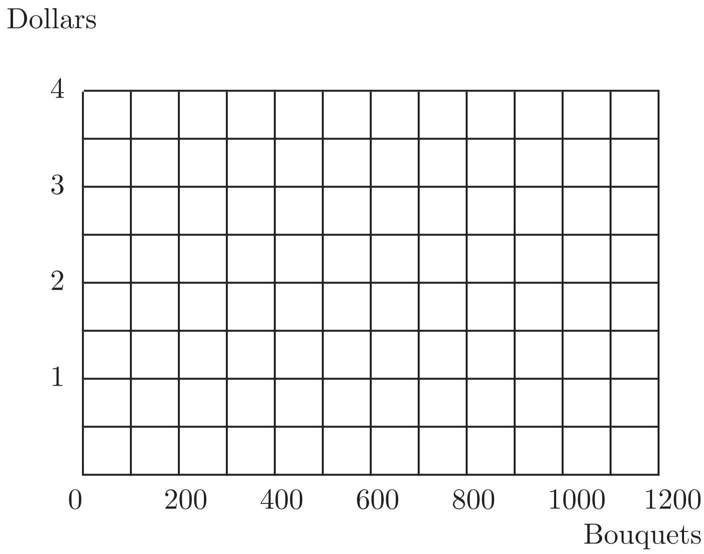
(a) If she has 200 square feet of floor space, write down her marginal cost function: _____________ and her average cost function: _____________ ______ At what amount of output is average cost minimized?__________ At this level of output, how much is average cost?____________________.

(b) If she has 500 square feet, write down her marginal cost function: _____________ and her average cost function: ____________________ At what amount of output is average cost minimized? __________ At this level of output, how much is average cost?___________________________.

(c) If she has 1,000 square feet of floor space, write down her marginal cost function: ____________ and her average cost function: __________

______________ At what amount of output is average cost minimized? __________ At this level of output, how much is average cost?________.

(d) Use red ink to show Mary's average cost curve and her marginal cost curves if she has 200 square feet. Use blue ink to show her average cost curve and her marginal cost curve if she has 500 square feet. Use black ink to show her average cost curve and her marginal cost curve if she has 1,000 square feet. Label the average cost curves AC and the marginal cost curves MC.



(e) Use yellow marker to show Mary's long-run average cost curve and her long-run marginal cost curve in your graph. Label them LRAC and LRMC.

21.5 (0) Touchie MacFeelie publishes comic books. The only inputs he needs are old jokes and cartoonists. His production function is

$$Q = .1J^{\frac{1}{2}}L^{3/4},$$

where J is the number of old jokes used, L the number of hours of cartoonists' labor used as inputs, and Q is the number of comic books produced.

(a) Does this production process exhibit increasing, decreasing, or constant returns to scale? Explain your answer.______________________

____.

____

(b) If the number of old jokes used is 100, write an expression for the marginal product of cartoonists' labor as a function of L. ________
Is the marginal product of labor decreasing or increasing as the amount of labor increases?________________________.

21.6 (0) Touchie MacFeelie's irascible business manager, Gander MacGrope, announces that old jokes can be purchased for \$1 each and that the wage rate of cartoonists' labor is \$2.

(a) Suppose that in the short run, Touchie is stuck with exactly 100 old jokes (for which he paid \$1 each) but is able to hire as much labor as he wishes. How much labor would he have to hire in order produce Q comic books?________________________.

(b) Write down Touchie's short-run total cost as a function of his output

________________________.

(c) His short-run marginal cost function is________________________.

(d) His short-run average cost function is________________________.

21.7 (1) Touchie asks his brother, Sir Francis MacFeelie, to study the long-run picture. Sir Francis, who has carefully studied the appendix to Chapter 19 in your text, prepared the following report.

(a) If all inputs are variable, and if old jokes cost \$1 each and cartoonist labor costs \$2 per hour, the cheapest way to produce exactly one comic book is to use ____________ jokes and ____________ hours of labor. (Fractional jokes are certainly allowable.)

(b) This would cost ________ dollars.

(c) Given our production function, the cheapest proportions in which to use jokes and labor are the same no matter how many comic books we print. But when we double the amount of both inputs, the number of comic books produced is multiplied by________________________.

21.8 (0) Consider the cost function $c(y) = 4y^2 + 16$.

(a) The average cost function is________________________.

(b) The marginal cost function is________________________________.

(c) The level of output that yields the minimum average cost of production is________________________________.

(d) The average variable cost function is________________________________.

(e) At what level of output does average variable cost equal marginal cost?________________________________.

21.9 (0) A competitive firm has a production function of the form $Y = 2L + 5K$. If $w = \$2$ and $r = \$3$, what will be the minimum cost of producing 10 units of output?________________________________.