Bond Duration

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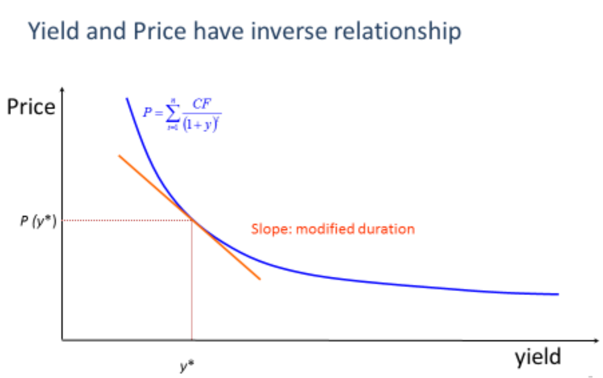
Introduction

Risk Management is one of the most important aspect of the investment process. A fundamental idea in finance is the relationship between risk and return. Low risk does not necessary imply low profitability, but low risk in combination with reasonable profitability is an aspect of high degree risk management. Even though the risk by its definition is an uncertain event in the future that can occur with certain probability, still it is measurable and manageable. In this paper, we will try to look at one of the most basic ideas of risk measurement, and mainly the concept of bond duration.

Price Sensitivity

The idea of duration derives from the relationship between the yield to maturity and the price of the bond itself. As the price of the bond is nothing else than the sum of its future cash flow discounted at the current yield, the relationship between these two variables is becoming intuitive. In the following graph, it is shown the relationship between the price and yield curve.



In order to be able to track such aspect of price sensitivity, the concept of duration was introduced. In the financial world there are 2 common ways to estimate this price sensitivity of a given bond. These are:

1. The Parametric Approach- which is nothing else than the estimation of the change in price by using the derivatives of the price function in respect to yield.
2. The full Valuation Approach- by calculating the price of the bond for every given change in the yield.

We will discuss these two approaches in the next sections.

Macau Duration

The formula that is usually used to calculate a bond's basic duration is the Macaulay duration, which was created by Frederick Macaulay in 1938, although it was not commonly used until the 1970s. Macaulay duration is calculated by adding the results of multiplying the present value of each cash flow by the time it is received and dividing by the total price of the security.

$$D_{Mac} = \frac{1}{P}\sum_{t=1}^{n}\frac{tCF_t}{(1+y)^t}$$

Frederick Macaulay's original 1938 measure of the "length" of life insurance liabilities. While Macaulay hinted at the relationship between duration and the price-yield relationship, no one seemed to notice. This formula is just a weighted average of time where the cash flows serve as weights. Macau duration tells us how long it will take a security to repay itself completely.

Modified duration

In the 1960s, Larry Fisher presented a proof of the relationship between price change and yield change and duration; it took another 15 or 20 years for duration to become common knowledge in the industry. In order to define the relationship between price and yield, it was used a very common approach, and mainly Taylor Series. We know that the relationship between price and yield is nothing else than a function. Taylor Series states that any function can be approximated through a polynomial function where each term in polynomial function is the derivative of the original function itself. It estimates a function f(x)'s value at f(x+Δx) as:

$$f(x+\Delta x) \cong f(x) + \frac{f^{'}(x)}{1!} * (\Delta x)^1 + \frac{f^{''}(x)}{2!} * (\Delta x)^2 + \ldots + \frac{f^{n}(x)}{n!} * (\Delta x)^n$$

Duration and convexity are linked to this idea. They can be used to approximate the behavior of a pricing function. Hence, the modified duration is just the first two polynoms of the polynomial function, which is essentially the derivative of the price function. These results will end up giving us a duration, which express the change in monetary units by a given change in yield. In order to express it in percentage change, this result is divided by the respective security price.

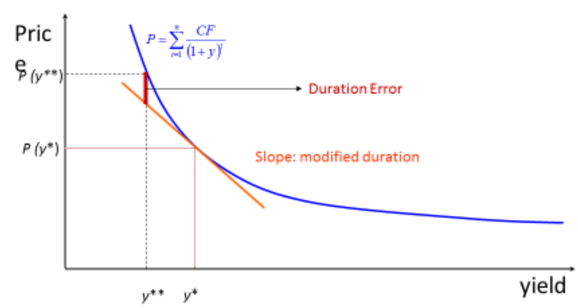
$$Duration_{modified} = -\frac{1}{P}\frac{dP}{dy}$$

If we do the calculus and re-arrange the expression, we obtain the formula.

$$D_{Mod} = \frac{D_{Mac}}{(1+y)}$$

Using the parametric approach we can derive the modified duration using the Macau Duration and this expression is showing the relationship between these two measures.

Even though modified duration is giving us a very good insight on our yield risk, there is still a major dropdown. Depending on the shape of the price-yield function and the interval of change in yield (Δy) the modified duration can give us a slight error, which can be observed in the graph below.



This error is easily corrected by adding the next polynom from the Taylor series, often called convexity.

Effective duration

Effective duration is probably the most accurate approximation of a bonds duration. Even though Taylor Series can predict with a certain accuracy the movement of the price given the yield curve change, it still leave some space for errors. Another reason to use the effective duration is that the use of modified duration implies that the cash flows of the security will not change, which is not true for securities with embedded options. These two arguments are enough to move to the full valuation approach. We have already stated that full valuation approach states that one should calculate the price for any given yield change. Given this information, we can easily derive the formula for this approach.

$$D_{Eff} = -\frac{1}{P}\frac{P_{(+)} - P_{(-)}}{2\Delta y}$$

This approach gives us a far more accurate view of the change in price when the yield curve move, but we have to admit that this approach implies a higher volume of calculus.

Characteristics of duration.

It is important to note that duration, as an indicator is still sensible to several factors.

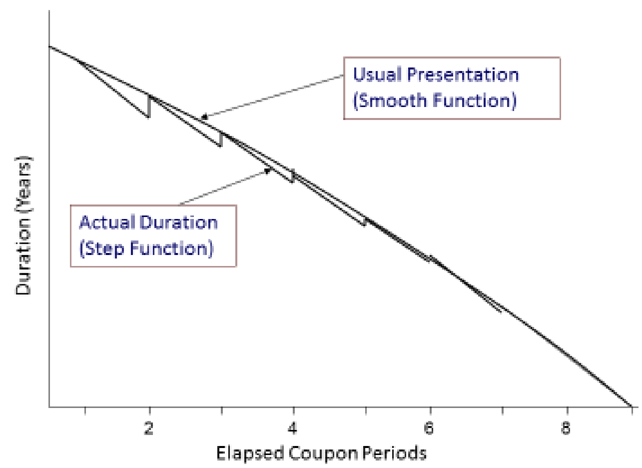
One of these factors is the coupon rate. The coupon rate determines the amount of the future cash-flow that will be received by the bondholder, hence it has a direct impact on the duration itself. A bond with higher coupon rate will have higher coupon payments and lower duration, and vice versa when referring to a bond with lower coupon yield.

Another factor is the yield curve. Since the yield is one of the components of the formula for both Macau and modified duration, it is obvious to consider it as an important factor.

The third factor is the time. We have mentioned that Macau duration is a formula of weighted average of time. The duration has a tendency to reduce its value over time as we approach the bond maturity.

The fourth and the last factor to be considered is the bond coupon payment. Coupon represent the future payments for the bondholder. When the coupon is paid there is no sense for the coupon holder to consider it a future cash flow, and it is eliminated from the formula of duration calculation. Each time we have a coupon payment the bond duration increases.

Even though it is very common to depict duration graphically as a smooth line, we should know that in essence the function of duration is not linear. In the graph below we can find a representation of duration, where the smooth line is an approximation and the second function represents the actual duration function. Each jump in duration is due to the coupon payment that is set to increase duration.



Even though duration is, a rather simple and very handy instrument to use in risk measurement it feels vital to point out some of its aspects. The risk measured so far rely on unrealistic assumptions that security prices are a function of a single systematic risk factor – Yield to maturity. If you have a portfolio of bonds, we are implicitly assuming each bond's yield moves by the same amount, equivalent to saying that the yield curve moves in a parallel manner. This is not a bad assumption, but the yield curve can (and does) move in different ways. Therefore, these measures fail to predict accurately price fluctuations from changes in shape or slope of yield curve (yield curve risk).

Conclusion

Duration is a handy, quite simple instrument that is able to give some insight about the price change risk depending on the yield change. This is a very widely used instrument in risk management and in financial modeling. However, its biggest disadvantage is that it assumes parallel shifts in the yield curve, which explains only 85-90% of yield changes, thus leaving some space for errors.

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