1 Interest and Tax

- Consider that you deposit on your saving account \$1,000.00. The interest rate is 4 % p. a. The interest period is

  one year. Every year you have to pay the tax from earned interest (at the end of the year). The tax rate is 15 %.

  What is the value on your bank account after 5 years?

  Note! Tax is paid only from received interest, NO FROM THE PRINCIPAL!

  $FV = 1000 * (1 + 0,04 * 0,85)^5$

  $FV = 1181.95977$

$$FV = PV\left(1 + r\left(1 - T\right)\right)^n$$

- Take the same example if the interest periodis quarterly.

  $FV_1 = 1000 * ((1 + \frac{0.04}{4})^4 - 1000) * 0.85 + 1000 \ldots$ after one year the FV will be 1034.51341

  $FV_2 = 1034.51341 * ((1 + \frac{0.04}{4})^4 - 1000) * 0.85 + 1000 \ldots$ after second year the FV will be 1070.21799

  $FV_3 = 1034.51341 * ((1 + \frac{0.04}{4})^4 - 1000) * 0.85 + 1000 \ldots$ after third year the FV will be 1107.15486

  $FV_4 = 1034.51341 * ((1 + \frac{0.04}{4})^4 - 1000) * 0.85 + 1000 \ldots$ after fourth year the FV will be 1145.36665

  $FV_5 = 1034.51341 * ((1 + \frac{0.04}{4})^4 - 1000) * 0.85 + 1000 \ldots$ after fifth year the FV will be 1184.89705

  or an easier way:

  $FV_{0,5} = 1000 * (((1 + \frac{0.04}{4})^4 - 1) * 0.85 + 1)^5 \ldots$ is ;)

$$FV = PV(((1 + \frac{r_{annual}}{m})^m - 1)(1 - T) + 1)^n$$

$T \ldots Tax$ rate
$m \ldots$ number of interest periods in one year (in one Tax pariod)
$n \ldots$ number of years (number of Tax periods)
Note! The effect of the number of interest periods (more IPs generate greater FV).

2 Effective Interest Rate

In order to have the same impact on the future value of capital at varying interest periods, we use the EFFECTIVE INTEREST RATE.

- The annual interest rate is 5 %. Your bank calculates the interest on the monthly basis. What is the future value of

  your 10,000.00 after one year?

  $FV = 10000 * (1 + \frac{0.05}{12})^{12} = 10511.619$

  The same effect on the future value must be reached if we use the effective interest rate.

  The calculation of effective interest rate:

$$r_{ef} = (1 + \frac{r_{annual}}{m})^m - 1)$$

Proof:
$r_{ef} = (1 + \frac{0.05}{12})^{12} - 1) = 0.0511619$
$FV_{r_{ef}} = 10000 * (1 + 0.0511619) = 10511.619$

3 Continuous Interest

If the number of interest periods goes to infinity then we speak about continuous interest calculation.

The expression is:

Let PV=1, than the FV in one year will be if $Lim_{m\to\infty}(1 + \frac{r}{m})^m \longrightarrow Lim_{m\to\infty}\left\{(1 + \frac{1}{\frac{m}{r}})^{\frac{r}{m}}\right\}^r = e^r$

Through the usage of the effective interest rate we can achieve the same effect on the FV, as in the discrete

calculation (simple, compound).

$FV = PVe^{ft}$

Note. Instead of interest rate, we use the term interest intensity (f).

The relation between the effective interest rate and the interest intensity is following:

$$r_{ef} = e^f - 1 \dashrightarrow then,$$

$$f = ln(r_{ef} + 1)$$

In our example:
$f = ln(1.0511619) = 0.0489612$
$\text{FV}_{contInt} = 10000 * e^{0.0489612} = 10511.619$

4 Tax payment by the continuous interest

$$FV_{contInt} = PV * ((e^f - 1) * (1 - T) + 1)^n$$

5 Effect of inflation

The Purchasing Power Parity of our money will be lower. To take the effect of inflation in account we use it as a discounting factor.

1. Situation when inflation is constant in average

$$df = \frac{1}{(1 + \pi_{averagen})^n}$$

2. inflation is different every year

$$df = \frac{1}{(1 + \pi_{0,1})(1 + \pi_{0,2})(1 + \pi_{0,3}) * ... * (1 + \pi_{0,n})}$$

3. Inflation is constant and we use the concept of continuous calculation.

$Inflationintensity \ldots\ f_{\Pi} = ln(AnnualAverInflation + 1)$,then

$$df = \frac{1}{e^{f_{\pi}}}$$

Our pevious example with 4 Interest Periods in one year, interest rate 4 % p. a., 5 years to maturity and continuous interest + continuous inflation (if the approximation of an annual average inflation is 1.7 %) will be:

$f_{\Pi} = ln(1,017) = 0.01685712$
$r_{ef} = (1 + \frac{0.04}{4})^4 - 1 = 0.04060401$
$f = ln(1,04060401) = 0.03980132$
$FV_{\Pi} = 1000 * e^{(0.03980132 - 0.01685712)*5} = 1121.56051$

6 The effect of Tax and Inflation

Note! Tax is paid from the nominal interes, first then the effect of inflation can be considered.

$FV_{conTaxInfla} = 1000 * ((e^{0.03980132} - 1) * 0.85 + 1)^5 * e^{-0.01685712*5} = 1089.12031$