Demand Discrete

1. In 1990, there were 1.77 vehicles per household in the United States. In addition, households drove each vehicle an average of 8,531 miles annually.

    a. Is vehicle ownership a discrete good or a divisible good? Suppose that the average purchase price of vehicles increases. Briefly explain the expected effect of the price rise on vehicle ownership, and comment on whether this occurs at the intensive or extensive margin.
    b. Are annual vehicle-miles travelled a discrete good or a divisible good? What effect will an increase in household annual income have upon annual miles travelled? Does this reflect consumption at the intensive margin or the extensive margin?

2. Helen is faced with the choice of either taking a train or an airplane to visit her mother in Seattle. Assume that Helen's utility function is $U(T,A,x;\phi)$, where T represents the trip by train, A represents the trip by air, x is all other consumption and ϕ reflects Helen's consumption preferences.

    a. Assuming that the choice of mode for the Seattle trip is a discrete good, what are the possible values that T and A can take? Briefly comment on whether it is possible for Helen to simultaneously consume T and A can take? Briefly comment on whether it is possible for Helen to simultaneously consume T and A.
    b. What is Helen's budget constraint if she takes the train to Seattle? What is her budget constraint if she goes to Seattle by air?
    c. Given that she makes the trip by air, identify the determinants of Helen's conditional indirect utility function. If Helen made the trip by train, would her conditional indirect utility function depend on the same determinants?
    d. Suppose that Helen decides to take the train to Seattle. Assuming that Helen is a utility-maximizing consumer, what can you conclude regarding the relationship between the conditional indirect utility of travelling by train and the conditional indirect utility of travelling by air?

3. Suppose that Tom has three possible modes for his daily journey to work: automobile (A), bus (B), and rapid transit (RT). Tom's utility function for work-trip modal choice and all other consumption x is $U(A,B,RT,x;\phi)$, where the modal choices are mutually exclusive and ϕ gives Tom's consumption preferences. Tom faces the following budget constraint:

$$Y = p_a A + p_b B + p_m M + p_x x,$$

where Y is Tom's income per period, p_a is the price of A, p_b is the price of B, p_{rt} is the price of RT, and p_x is the price of x.

a. Briefly explain why Tom's utility function depends upon the consumption levels of each commodity, whereas his indirect utility function depends upon the economic environment; that is, the prices of each commodity and his income.
b. Suppose that you know that Tom takes rapid transit to work. You also know that if rapid transit were not available, Tom would drive to work. Use this information and the conditional indirect utility function for each mode to characterize Tom's modal choice decisions.

4. In 1983, 87.4 % of household trips to work were by private motor vehicle, 4.6 % by public transit, and 8.0 % by other modes of travel (for example, bicycle or walk). For private transportation, the average length of work trip (one way) was 8.5 miles, with an average commute time equal to 20 minutes. The operating cost per mile for private transportation was 8.36 cents. For public transit, the average commute time was 46.1 minutes per one-way trip, with an average fare equal to 60 cents. For other work-trip modes, the average one-way trip length was 5.6 miles, with an average trip time equal to 30 minutes.

   a. For each of the three modes, what is the monetary cost per trip?
   b. Assuming an average hourly wage rate equal to $10.00, what is the total cost pet work trip on each mode?
   c. Given the work-trip prices in (b) and the modal percentages, graph representative demand curves for each of the three modes.

5.
   a. In specifying a typical consumer's conditional indirect utility for a given transportation alternative, explain the difference between *the observed and common* portion of utility and that which is *unobserved and individual specific*.
   b. Given that individual travellers know with certainty whether the conditional indirect utility of one mode is greater than the conditional indirect utility of all other available modes, why is it necessary to develop a *random utility model* (RUM) of transportation choice? What role does unobservable taste variations play in the development of the RUM?
   c. What is the relationship between random utility model of transportation choice and a cumulative probability distribution function?

6. Wanting to vacation in Los Angeles, Mike, who lives in Kansas City, can take direct flights on either of two airlines. Let the conditional indirect utility of airlines A and B, respectively, be

$$\hat{U}_A = Y - p_x + \alpha p_A + \gamma t_A + \beta_1 Y + \varepsilon_A,$$
$$\hat{U}_B = Y - p_x + \alpha p_B + \gamma t_B + \beta_2 Y + \varepsilon_B,$$

where Y is income, p_x is the price of all goods, p_A is the price of airline A, p_B is the price on airline B, t_A and t_B are the travel times on airlines A and B respectively, ε_A and ε_B represent unobserved taste variations, α, γ, β_1 and β_2 are parameters.

a. Mike's conditional indirect utility for airline A does not depend upon either the price or the travel time on airline B. Similarly, his conditional indirect utility for airline B does not depend upon the price or travel time on airline A. If Mike decides to fly on airline A, does this decision imply that the price and travel time on airline B had no effect on Mike's choice of airline A?
b. If $\beta_1 = \beta_2$, does Mike's conditional indirect utility depend upon his income? Would his choice of airline depend upon income? If so, why; and if not, why not?
c. Consider the following three sets of conditional indirect utilities for the two airlines:

1) $\hat{U}_A = Y - p_x + \alpha p_A + \gamma t_A + \varepsilon_A$
   $\hat{U}_B = Y - p_x + \alpha p_B + \gamma t_B + \beta_B Y + \varepsilon_B$

2) $\hat{U}_A = Y - p_x + \alpha p_A + \gamma t_A + \beta_A Y + \varepsilon_A$
   $\hat{U}_B = Y - p_x + \alpha p_B + \gamma t_B + \varepsilon_B$

3) $\hat{U}_A = Y - p_x + \alpha p_A + \gamma t_A + \beta_1 Y + \varepsilon_A$
   $\hat{U}_B = Y - p_x + \alpha p_B + \gamma t_B + \beta_2 Y + \varepsilon_B$

By subtracting the conditional indirect utility for airline B from the conditional indirect utility for airline A, demonstrate that the indirect utility difference is the same for each. What is the relationship between β_1, β_2, β_A, and β_B?

7. Norman has to decide between purchasing a pickup truck (PU) or a sport utility (SU) vehicle. His conditional indirect utility for each type of vehicle is

$$\hat{U}_{PU} = Y - p_x + \alpha p_{PU} + \beta_1 Y + \varepsilon_{A,}$$
$$\hat{U}_{SUV} = Y - p_x + \alpha p_{SUV} + \beta_2 Y + \varepsilon_B,$$

where Y is income, p_x is the price of all other goods, p_{PU} is the price of the pickup truck, and p_{SU} is the price of the sport utility, ε_A and ε_B represent unobserved taste variations for each of the vehicle types, α, β_1 and β_2 are parameters.

a. Based upon the above conditional indirect utilities, make the appropriate assumptions necessary to derive a binary logit model of vehicle type choice. What is the probability that Norman will purchase a pickup truck? What is the probability that he will purchase a sport utility vehicle?

b. Evaluate the following statement: "Suppose Norman is considering the purchase of a pickup truck with an *equally priced* sport utility vehicle. If income has the same marginal effect upon Norman's conditional indirect utility of pickup trucks as it has on sport utility vehicles, then Norman has an equal probability choosing either vehicle."
c. What is the difference between a *generic* variable and an *alternative specific* variable? In the above model, is vehicle price a generic or alternative specific variable? How about income?

8. In 1988, there were 91.3 million households in the United States, 31.1 million households owned one vehicle, 33.3 million households owned two vehicles, and 14.0 million households owned three or more vehicles.

   a. In 1988, what was the probability that a household, selected at random, would own two vehicles?
   b. What was the probability that a randomly selected household in 1988 would own no vehicles?
   c. In what way can automobile ownership be described as a discrete variable? Based upon this, set up a multinomial logit empirical model of automobile ownership. What is the interpretation of the dependent variable? What do you think the primary determinants of automobile ownership would be?
   d. What effect would you expect an increase in income to have upon owning no vehicles? Would you expect an increase in income to increase the likelihood that a household owns three or more vehicles relative to owning no vehicles? And would you expect an increase in income to increase the likelihood that a household owns three or more vehicles relative to owning two vehicles?

9. Suppose, in a binary logit model, that the "own-" and "cross-" price elasticities of demand for mode "a" are –0.34 and +0.15 respectively. Interpret these numbers.

   a. What is the effect on the demand for mode "a" of a 15% increase in its price? Does this represent a change in demand or a change in quantity demanded?
   b. What is the effect on the demand for a mode "a" when the price of mode "b" increases by 15%? Does this represent a change in demand or a change in quantity demanded?

10. Assume that three transportation alternatives are available: airline, rail, and automobile. Holding all else constant, and assuming a multinomial logit model of transportation mode choice, what will be the effect upon, airline, rail and automobile demands from a small increase in air fares? Demonstrate that the sum of these effects equals zero. Does this make sense? If so, why; and if not, why not?

11. Similar to the study reported in table 4.4, most studies of modal choice find that the value of in-vehicle travel time is less than the value that travellers place upon waiting time.

    a. What do these results tell us about the marginal disutility of in-vehicle travel time in comparison with the marginal disutility of out-of-vehicle travel time?
    b. Discuss why the value-of-time estimates derived from discrete choice models are oftentimes interpreted as marginal rates of substitution.
    c. Suppose that you're an economist for a commuter railroad system. The manager of the agency is considering either of two policies: adding additional stops, with the expected result of reducing on-line speeds but also reducing the headway (that is, the average time between trains); or removing some stops, which would increase on-line speeds but also entail longer headways. Overall, both policies are predicted to have the same effect on total travel time for the average consumer. Discuss how you would use information on riders' values of time in your police recommendation.

12. Train (1980) developed and estimated an empirical demand model, based upon a sample of 635 individuals living in the San Francisco Bay Area, for automobile ownership. His model included four alternatives:

    1 Own no automobiles (1).
    2 Own one automobile (2).
    3 Own two automobiles (3).
    4 Own three or more automobiles (4).

    and a portion of his multinomial logit model estimation results is presented in table 4.18. The numbers in parentheses reflect the alternative(s) associated with the variable. In answering the following questions, be sure to pay attention to coefficient estimates and the *t*-statistics.

    a. From reported results, is an increase in the cost of automobile ownership as a proportion of income consistent with the law of demand? What does this imply about the expected level of ownership among lower-income households? What about higher-income households?
    b. In this part, we want to analyse the effect of an increase in household income on automobile ownership.
        i. Relative to owning no automobiles, what is the estimated marginal indirect utility effect of an increase in household income for households that own three automobiles?
        ii. Relative to owning no automobiles, what is the effect on indirect utility of an increase in household income for households that own four automobiles?

Table 4.18 The multinomial logit model of automobile ownership

Variable	Coefficient Estimate	*t*-statistic
Constant for one automobile (2)	−2.10	−1.36
Constant for two automobiles (3)	−12.1	−6.05
Constant for three or more automobiles (4)	−23.6	−8.42
Annual auto cost divided by household income (1–4)	−2.26	−2.23
Number of persons in household (2)	0.573	2.90
Number of persons in household (3)	1.76	7.21
Number of persons in household (4)	2.89	10.1
Percentage of household members with driver's licenses (2)	4.24	4.68
Percentage of household members with driver's licenses (3)	9.77	8.45
Percentage of household members with driver's licenses (4)	16.9	10.8
Accessibility to nonwork destinations by transit (1)	0.270	0.884
Accessibility to nonwork destinations by auto or transit (2)	0.664	1.59
Accessibility to nonwork destinations by auto or transit (3)	0.745	1.73
Accessibility to nonwork destinations by auto or transit (4)	1.04	2.16
Home location in or near CBD (2)	−0.829	−1.76
Home location in or near CBD (3)	−1.30	−3.52
Home location in or near CBD (4)	−1.34	−2.12
Household income (2)	0.0000905	1.84
Household income (3)	0.000197	3.76
Household income (4)	0.000183	2.92

Source: Train (1980), table II, p. 364

iii. Given your responses for (i) and (ii), what does this imply regarding the indirect marginal utility effect of an increase in household income for three-automobile households relative to four-automobile households? Or, alternatively, what is the effect on the demand for owning three cars relative to the demand for owning four cars when the household experiences and increase in income?

c. What do the reported estimation results say about the relationship between household size and the demand for automobile ownership?

d. Use the reported estimation results to evaluate whether households living in the suburbs are more likely to own automobiles. If so, are they also more likely to own a larger number of automobiles?

e. A primary substitute for the automobile is public transportation. Do the results support the conclusion that public transit availability will significantly affect the demand for automobile ownership?

f. Do the results identify a relationship between automobile ownership and the number of drivers in a household?

g. Compare and contrast these results on the demand for automobile ownership with that of Train (1986) that is summarized in the text.

13. McFadden (1976) estimated a multinomial logit model of work-trip mode choice for the San Francisco Bay Area. A portion of his results is given in table 4.19. The work-trip modes available to the 771 commuters sampled were:

1 Automobile alone.
2 Bus with walk access.
3 Bus with automobile access.
4 Carpool.

The numbers in parentheses are the mode(s) associated with each variable.

Table 4.19 Work-trip mode choice in San Francisco Bay Area

Explanatory Variable	Coefficient Estimate	*t*-statistic
Mode cost (cents)/Post-tax wage (cents per minute) (1–4)	−0.0284	−4.31
Auto in–vehicle time (minutes) (1, 3, 4)	−0.0644	−5.65
Walk time (minutes) (2, 3)	−0.0259	−2.94
Transfer walk time (minutes) (2, 3)	−0.0689	−5.28
Number of transfers (2, 3)	−0.105	−0.77
Headway of first bus (minutes) (2, 3)	−0.0318	−3.18
Number of drivers in household (1)	1.02	4.81
Number of drivers in household (3)	0.990	3.29
Number of drivers in household (4)	0.872	4.25
Head of household dummy variable (1)	0.627	3.37
Employment density at work location (1)	−0.00160	−2.27
Home near or in CBD (1)	−0.502	−4.18
Autos per driver (1)	5.00	9.65
Autos per driver (3)	2.33	2.74
Autos per driver (4)	2.38	5.28
Auto alone constant (1)	−5.26	−5.93
Bus with auto access constant (3)	−5.49	−5.33
Carpool constant (4)	−3.84	−6.36

Source: McFadden (1976), table 3, p. 38

a. Evaluate the effects of increases in modal cost and travel times on the demand for modes. Be sure to pay attention to the mode or modes associated with each variable when interpreting the results. Are these effects consistent with your expectations?
b. Relative to taking the bus with walk access, what effect on modal demand occurs when there is an increase in the number of drivers in the household? Whereas the number of licensed drivers reflects the demand for automobile use, the number of available automobiles reflects the supply. Do the results indicate that an increasing supply increases the use of automobiles in the work trip? Suppose that the number of autos in the household increases.

What effect will this have upon the demand for bus with auto access relative to the demand for carpool as a work-trip mode?

c. Based upon the reported results, who is more likely to use the automobile in the work trip, a female head of household or a male nonhead of household?
d. What will be the effect on a worker's work-trip mode choice who lives near downtown and works in a high-density employment area? Which of the two effects is stronger?

14. Consider the following indirect utility function

$$U = a\text{cost} + b\text{time}$$

where cost is dollars and time is hours.

a. What is the interpretation of b/a? Let w be the wage rate in dollars per hour. What is the interpretation of $(b/a)/w$? Now consider the following indirect utility function:

$$U = \alpha(\text{cost/w}) + \beta\text{time}$$

What is the interpretation of β/α? What is the relationship between β/α and $(b/a)/w$?

b. Consider the work-trip modal choice model in question 13. As a proportion of the wage rate, calculate the value of time associated with automobile in-vehicle time, transit in-vehicle time, walk time, and transfer wait time. Is out-of-vehicle travel time more onerous to commuters than in-vehicle travel time? Does this seem reasonable?
c. What is the total value of out-of-vehicle time associated with bus travel?

15. Harriet is a lumber analyst for a consulting firm. As part of her responsibilities she must keep apprized of those factors that influence shipper demands. Recently, she developed and estimated a binomial logit model of shipper carrier demands.

a. Theoretically, Harriet reasoned that a shipper's carrier demand would depend upon shipment cost, service quality, as measured by average transit time and average variability of transit time, and mode accessibility.
   i. Briefly identify the expected effect that each of these factors would have upon mode's demand, all else constant.
   ii. Harriet reasoned that a shipper's carrier demand would depend upon mean transit time and mean variability in transit time, because an increase in each of these would lead to higher interest and storage costs for the shipper. Do you agree with Harriet's reasoning? If so, why; and if not, why not?

b. For the lumber industry, the two primary modes that Harriet considered were (1) private carriage (that is, the company shipped its own goods) and (2) rail. The explanatory variables in Harriet's model were:

Shipment Size, in units of 10,000 pounds
Commodity Value, in units of $/pound
Freight Charges, in units of $10,000 per unit shipped
Mean Transit Time, in days
Standard Deviation of Transit Time, in days
Distance from a Rail Siding, in miles

Harriet's results are given in the table 4.20.

Table 4.20 The shipping mode choice for Harriet

Explanatory Variable	Coefficient Estimate	*t*-statistic
Shipment Size (2)	2.5	2.8
Commodity Value (2)	7.51	4.32
Freight Charges (1, 2)	−14.14	−8.40
Mean Transit Time (1, 2)	−6.43	−4.12
Standard Deviation of Transit Time (1, 2)	−3.48	−1.73
Distance from a Rail Siding (2)	−11.21	−4.83
Number of observations = 120		

i. Are the results consistent with Harriet's theoretical reasoning? Does service quality affect shipper modal demands? Suppose that the variability of transit time for private carriage increased. Qualitatively, how would this effect the demand for private carriage, and the demand for rail?
ii. From reported results, would you expect larger shipments to go by rail or private carriage? How about higher-valued shipments (for example, furniture versus bulk lumber)?
iii. From the reported results, does distance to a rail siding matter to a shipper?
iv. How much would a typical shipper be willing to pay to save one day of transit time? How much would a shipper be willing to pay to reduce transit time variability by one standard deviation?