

LR Cost

1. Assume that the production of less-than-truckload (LTL) motor carrier services depends upon three inputs: capital, labour, and fuel. The production function for LTL ton-miles is $TM = f(L, K, F, \gamma)$, where γ is the state of technology.
 - a. Holding fuel and capital constant, graph the total, marginal, and average productivity curves for labour. Graphically depict the expected effect on the total product of labour if the LTL firm invests in more capital.
 - b. In general, what is the cost constraint that an LTL firm faces? Graph the isocost equation under the assumption that capital is held fixed at \bar{K} .
 - c. Holding capital constant at \bar{K} , use isoquant and isocost curves to predict the impact that an increase in fuel price will have upon an LTL's optimal use of fuel and labour. Also depict the expected effect upon the consumption of fuel and labour if the per unit price of capital increases.
2. Suppose that the production function for owner-operators in the truckload sector of the motor carrier industry required that capital and labour be used in fixed proportions: one driver with each truck.
 - a. Identify the production function for owner-operator services and graphically depict the set of isoquants for this production function.
 - b. Define the elasticity of substitution and discuss the extent to which capital and labour are substitutable for this production technology.
 - c. Graphically identify the optimal use of capital and labour for a given set of labour and capital input prices, w and r , respectively. What effect on the optimal capital : labour ratio will occur if the relative labour wages increase by 10 %, all else held constant?
 - d. When truckload services are produced with this technology, what will be the impact on output if capital and labour increase by 15 %?
3. Suppose that a transport firm's cost of producing carrier services is solely a function of labour, which is given by the following production function:

$$T = 5L$$

- a. What is the labour input requirement per unit of output produced?
- b. What is the optimal amount of labour used, and what is the cost function for this transportation firm? What is the firm's marginal cost and average cost of production?
- c. Using the cost function obtained in (b), demonstrate that this firm produces output under constant returns to scale.

4. Suppose that an airline company's long-run production depends only upon labour according to the following function:

$$\text{Passenger-Miles } (PM) = AL^\alpha$$

- a. What is the air carrier's optimal use of labour in the short run?
- b. What is the long-run total cost function for this air carrier? What is the average cost of production?
- c. It can be shown that the marginal cost of production for this technology equals

$$MC = \frac{1}{\alpha} \kappa (PM)^{(1-\alpha)/\alpha}$$

Using this expression for marginal cost and the expression for average cost derived in (b), what are the returns to scale for this production technology? For what values of α will this firm operate under constant, increasing, and decreasing returns to scale?

5. Over the years, there have been several studies of input substitutability in the public transit industry. Among the studies' results, the elasticity of substitution between labour and fuel averaged around 0.43, while elasticity of substitution between capital and fuel has been estimated as 0.35.

- a. Define the elasticity of substitution between fuel and labour. Given the above information, what effect will a 10% increase in the relative price of labour have upon the fuel : labour ration?
- b. Are fuel and capital substitutes or complements in the production of public transit services?
- c. As manager of a local public transit district, you are concerned about rising energy prices, which raise the cost of fuel to the district. In order to conserve fuel for those bus routes with the highest passenger loads, you schedule your services so that your bus fleet, on average, stays idle for a longer period of time. Is this response consistent with an elasticity of substitution between fuel and capital that lies in the 0.3 range?

6. Consider the following production function for a railroad, which is hypothesized to depend upon four inputs – capital (K), labour (L), fuel (F), and network size (N):

$$\text{Ton-miles} = f(K, L, F, N; \gamma)$$

Based upon this production technology, distinguish between following returns to scale concepts:

- economies of capital utilization
- economies of traffic density
- economies of network size

- generalized economies of scale

In your answer, be sure to identify which inputs are held fixed and which inputs are not held fixed. Also identify whether the economies of scale measure is a short- or long-run concept.

7. Callan and Thomas (1992) estimated a long-run translog cost function for the household goods sector of the motor carrier industry. The dependent variable was total long-run costs and the independent variables included:

- the quantity of ton-miles produced
- the average length of haul, in miles
- the average load, in tons
- the percentage of household goods shipped, defined as a percentage of the total operating revenues generated by the shipment of personal effects and household goods
- the input prices for labour, fuel, capital, and materials

Table 5.13 First-order coefficients at the sample mean*

Regressor	Coefficient Estimate (t-statistic)
Constant term	15.78 (488.5)
Output	1.004 (39.1)
Price of Labour	0.310 (45.6)
Price of Capital	0.501 (54.5)
Price of Fuel	0.023 (19.2)
Average Load Per Vehicle	-0.228 (-3.4)
Average Length of Haul	-0.264 (-3.7)
Percentage of Household Goods Shipped	-1.547 (-7.0)
$R^2 = 0.90$	
Number of observations = 356	

* The estimated translog cost function has the following form:

$$\ln C(T; p, o) = \alpha_0 + \alpha_1 (\ln T - \ln \bar{T}) + \sum_{i=2}^5 \alpha_i (\ln p_i - \ln \bar{p}_i) + \sum_{i=6}^8 \alpha_i (\ln o_i - \ln \bar{o}_i) + \text{"second-order and interaction terms"} + \varepsilon$$

Source: Reprinted from Callan, S. and Thomas, J. 1992: Constant returns to scale in the post-deregulatory period: the case of specialized motor carriers. *Logistics and Transportation Review*, 25, 271-88, with permission from Elsevier Science

- For each of the explanatory variables, discuss the effect that you expect this variable to have upon long-run total costs.
- Data for the analysis was based upon Class I (revenues greater than \$5 million) and Class II (revenues between \$1 million and \$5 million) interstate household goods carriers operating in 1984. Table 5.13 gives the "first-order" coefficients and associated *t*-statistics for this model. Interpret the coefficient estimates. At the sample mean, are the results in the table consistent with the expected effects that you identified in (a)?

- c. Based upon the estimated value, at the sample mean, do household goods carriers operate under constant, increasing, or decreasing returns to scale? Test the null hypothesis that the coefficient is significantly different from one. What is the total operating cost of the average household goods carrier?
 - d. What impact on total cost would you expect if household goods carriers increased their percentage of nonhousehold carriage (for example, business shipments) by 15 %? What effect is this likely to have upon the costs of the “typical” carrier? Suppose that the typical household goods firm desired to increase its market area by lengthening its average length of haul by 10 %. In doing so, however, it experiences a 5% increase in its labour costs. What will be the net effect on total costs of the typical firm?
8. In 1968, Keeler (1971) identified the per seat-mile costs (shown in table 5.14) associated with four major intercity modes of travel: rail, air, automobile, and intercity bus.

Table 5.14 Intercity modal costs, 1968

Mode	Cost Per Seat-Mile (cents)
Intercity Bus (200-mile trip)	1.44
Air (Lockheed 1,011, 256-seat configuration, 250-mile trip)	3.00
Automobile (two occupants)	4.5
Rail (three-car train seating 240 passengers)	1.5

Source: Reprinted from Keeler (1971), table 7, p. 160, with the permission of The University of Chicago Press. Copyright © 1971 by The University of Chicago. All rights reserved

- a. What does this table tell us about the cost competitiveness of rail in comparison with the other three intercity modes?
- b. Consider the following sets of statistics for 1990:

Intercity modal costs

Mode	Per-Mile Cost	Average Length of Trip
Certificated Air Carrier	13.02	803
Rail	12.85	274
Intercity Bus	11.55	141
Automobile	13.33*	115*

* Per mile costs of operating vehicle occupant: assumes 1.62 occupants per vehicle in 1990. Average Length of Trip for automobile is based upon intercity vacation trips.

Based upon this information, can you conclude that rail trips are competitive with air trips? How about intercity bus and automobile trips? Use the concept of economies of distance to argue that rail trips *will be more competitive* with shorter-haul air trips, but *will be less competitive* with longer-haul intercity bus and auto trips.

9. Barbera et al. (1987) used a translog cost function to analyse the cost structure for all Class I railroads (revenues over \$253.7 million annually, 1993 dollars) for the period 1979–83. The dependent variable was long-run total costs and the explanatory variables included:

- the level of output, measured as net freight ton-miles
- the operating characteristic, measured as net freight tons
- the network size, measured as miles of track operated
- the prices of inputs for labour, capital, fuel and material

Table 5.15 below reports the first-order coefficients and associated *t*-statistics for this model. In this paper, the authors interpret net freight tons, net freight ton-miles, and miles of track as firm size measures.

Table 5.15 First-order condition at the sample mean*

Regressor	Coefficient Estimate (<i>t</i> -statistic)
Constant term	-0.364 (-6.4)
Net Freight Tons	0.224 (3.3)
Price of Fuel	0.072 (46.2)
Price of Materials	0.431 (76.6)
Price of Capital	0.177 (23.4)
Price of Labour	0.320 (46.1)
Net Freight Ton-Miles	0.416 (5.4)
Miles of Track	0.390 (5.2)

$R^2 = 0,96$

* The estimated translog cost function has the following form:

$$\ln C(T; p, o) = \alpha_0 + \alpha_1 (\ln T - \ln \bar{T}) + \sum_{i=2}^5 \alpha_i (\ln p_i - \ln \bar{p}_i) + \sum_{i=6}^7 \alpha_i (\ln o_i - \ln \bar{o}_i) + \text{"second-order and interaction terms"} + \varepsilon$$

Source: Barbara et al. (1987), table 2, p. 240

- What proportion of total costs is due to labour, capital, fuel, and equipment? What would be the expected effect of a 5% increase in fuel prices on the total costs of a typical Class I railroad?
- For this analysis, the authors define the following relationships:

$$\begin{aligned} \text{coefficient of Net Freight Tons} &= \alpha_{\text{NFT}} \\ \text{coefficient of Net Freight Ton-Miles} &= \alpha_{\text{NTM}} \\ \text{coefficient of Miles of Track} &= \alpha_{\text{MT}} \\ \text{economies of length of haul} &= \varepsilon_{\text{LH}} = \frac{1}{\alpha_{\text{NTM}}} \\ \text{economies of density} &= \varepsilon_{\text{d}} = \frac{1}{\alpha_{\text{NFT}} + \alpha_{\text{NTM}}} \\ \text{economies of scale} &= \varepsilon_0 = \frac{1}{\alpha_{\text{NFT}} + \alpha_{\text{NTM}} + \alpha_{\text{MT}}} \end{aligned}$$

Explain the intuition behind each of these concepts. From the results, what are the estimated economies of traffic density and economies of scale? The authors also report that the standard

error for the estimated economies of traffic density is 0.3348, and that the standard error associated with the estimated economies of scale is 0.936. Use these standard errors to test the null hypothesis of constant economies of traffic density and constant returns to scale. For the purposes of the test, assume 11 degrees of freedom.

- c. Given the model's specification, why is it appropriate to interpret the coefficient of Net Ton-Miles as reflecting length of haul economies? What do the empirical results tell us about length of haul scale economies? Suppose that you had two rail companies, one operating in the southern portion of the East Coast and a second company operating in the northern portion of the East Coast. Could the length of haul results support an "end-to-end" merger of the two rail lines?
- d. Based upon the returns to scale, traffic density, and length of haul results, what policies should rail firms follow in order to reduce their unit cost of production?

10. Pozdena and Merewitz (1978) analysed 11 rapid rail transit properties operating in North America between 1960 and 1970. From their analysis, they obtained the following long-run total cost function:

$$LRTC = 7.42w^{0.98}p_e^{0.48}Q^{0.76}$$

where w is the wage rate (\$ per hour), p_e is the price of energy (\$ per kilowatt-hour), and Q is output (million vehicle-miles).

- a. According to this study, do rapid rail transit systems operate under increasing, decreasing or constant returns to scale? (Hint: take the logarithm of the equation and interpret the coefficient estimates.)
- b. In the early 1970s, there was a significant increase in oil prices. What effect would a 20% increase in kilowatt-hour prices have upon long-run rapid rail transit costs?
- c. From the $LRTC$ equation, the authors also calculated the long-run marginal cost of rapid rail transit systems to be

$$LRMC = 5.66w^{0.98}p_e^{0.48}Q^{-0.24}$$

What effect on $LRMC$ will there be from a 10% increase in output? Is the impact on $LRMC$ consistent with your answer in (a)?

- d. At the time of this study, San Francisco's Bay Area Rapid Transit (BART) was not included. In 1975, BART had the following characteristics:

$w = \$7.48$ per hour (base wage of train attendants)
 $p_e = \$0.019$ per kilowatt-hour
 $Q = 22.7$ million vehicle-miles

Based upon *LRTC* and *LRMC* identified in (a) and (c), forecast BART's long-run total and marginal costs of operation. Also, calculate BART's long-run average cost per vehicle-mile. Is the average cost per vehicle-mile greater or less than the marginal cost per vehicle-mile, and is this consistent with the results previously obtained?

11. Case and Lave (1970) analysed inland waterway costs in the United States. In their paper, they identified two trends associated with inland waterway transport during the preceding three decades: a relatively constant cost per ton-mile and a trend for a small number of firms to garner a large share of the market, either through growth or merger. In order to examine these issues, the authors estimated the following Cobb-Douglas long-run average cost (*LRAC*) function:

$$LRAC = \alpha_0(EBM)^{\alpha_1}(SZ)^{\alpha_2}T^{\alpha_3}$$

Where *EBM* is “equivalent barge-miles”, a measure of output, *SZ* is size of firm, measured by the number of towboats, and *T* is a time trend. α_i ($i = 1, \dots, 4$) are parameters. Data for this study was based upon quarterly observations for five major inland water carriers between 1962 and 1966. The results of the analysis are shown in table 5.16.

Table 5.16 Inland waterway regression results*

Regressor	Coefficient Estimate (<i>t</i> -statistics)
Constant term	-0.200
Equivalent Barge Miles (EBM)	-0.615 (-11.0)
Number of Towboats	-0.074 (-1.3)
Time Trend	0.030 (0.88)
$R^2 = 0.865$	
Number of observations = 83	

* The estimated model also included three seasonal variables for the first, second, and third quarters, as well as four dummy variables for firms 1, 2, 3, and 4. The constant term reflects the fourth quarter and firm 5. A *t*-statistic was not reported for the constant term.

Source: Case and Lave (1970), table III, p. 188

- Based upon the reported results, do inland waterway companies operate under increasing, decreasing, or constant returns to scale?
- The authors argue that the measure of firm size, Number of Towboats, should have a negative effect upon long-run average costs. What's the economic intuition behind this hypothesis, and do the reported results support this?