# Interest calculation

## *(Interest paid after)*

Simple interest – $FV = PV*(1+r*t) \implies PV = FV/(1+r*t)$

Amount of interest - $I = PV*r*t$

Compound interest – $FV = PV*(1+r)^n$, certainly $PV = FV * (1+r)^{-n}$

Combined interest - $FV = PV*(1+r)^n*(1+r*t)$

, where n…. number of whole interest periods

Continuous interest - $FV = PV*e^{(f*t)}$

$(1+re) = (1+r(m)/m)^m \Rightarrow re=(1+r(m)/m)^m - 1$

$e^f \Rightarrow f=\ln(1+re)$

## *(Interest paid ahead)*

$PV = FV – D$, $D=FV*d*t \Rightarrow PV = FV*(1-d*t)$; $d=r/(1+r)$ and analogous $r = d/(1-d)$, always must be $d < r$

Compound discount … $PV = FV*(1 - d)^n$, $PV = FV*(1 - d/m)^{(m*n)}$

$(1 - de) = (1 – d(m)/m)^m = 1/e^f$ (calculation for PV…), obviously it must be true $de = 1 – e^{-f}$

*Note 1: Discounting = using the discount factor to calculate PV in contrary to use a COMERCIAL DISCOUNT – $D=FV.d.t \Rightarrow PV=FV*(1-d*t)$*

*Note 2: You pay tax just from the obtained interest!!! (do not tax the principal)*

PV…. Present Value

FV…. Future Value

D …. Commercial discount (similar to Interest amount I)

r…. Interest rate

e… 2,182818….

d…. discount rate

de…. Effective discount rate

d(m) … nominal discount rate with m-conversions

**TO UNDERSTAND THE INTEREST CALCULATION IT IS NOT IMPORTANT (and useful) DO A MECHANICAL MEMORIZATION, but understand the logic of each approach!!!**

1. Decide which of the four investments is the best…?
   (t1 = 88,000.00, t3 = 107,000.00, t5 = 129,300.00, t6 = 132,064.00)
   (risk = 2 % p. a., Opportunity costs 3 % p. a., Inflation = 1,5 %)
   r = 0,02 + 0,03 + 0,015 = 6,5 %
   PV(1) = 88000/1,065 = 82629,11 4.
   PV(2) = 107000/1,065^3 = 88579,85 3.
   PV(3) = 129300/1,065^5 = 94372,59 1.
   PV(4) = 132064/1,065^6 = 90507,97 2.

   With cost (- 800, - 3000, - 4000, - 7000)

   PV(1) = (88000-800)/1,065 = 81877,93 4.
   PV(2) = (107000-3000)/1,065^3 = 86096,31 2.
   PV(3) = (129300-4000)/1,065^5 = 91454,07 1.
   PV(4) = (132064-7000)/1,065^6 = 85710,63 3.

2.
What is better choice of this two: pay the car in 6 months 460,000.00 or pay it in 3 days with 5 % reduction of price? A bank offers you 6 % p. a., but the fee for opening the account is 1.300,--
PV (in 3 days) rebate = 460000-0,05*460000 = 437000
PV (t=0) = 437000/(1+0.06*3/360) = 436781.6
PV (of 460000) = 460000/(1+0.06/2) = 446601.9
You prefer the first offer! PV1<PV2

3.
FV = 758,000.00, PV = 550,000.00, T = 5 years. Use continuous interest… What will be the effective discount rate:

758000 = 550000*e^(f*5)
f = (ln(758/550))/5 = 0,064153
de = 1 – e^(-0,064153) = 0,062139
*Proof:*
*PV = 758000*(1 - 0,062139)^5 = 550000*

**2 optional points** – what is the nominal discount rate – **d(12)** if you have 12 conversion in one year? For the calculation use parameters from continuous interest – “f”.