In Chapter 11, you learned some tricks that allow you to use techniques you already know for studying intertemporal choice. Here you will learn some similar tricks, so that you can use the same methods to study risk taking, insurance, and gambling.

One of these new tricks is similar to the trick of treating commodities at different dates as different commodities. This time, we invent new commodities, which we call *contingent commodities*. If either of two events A or B could happen, then we define one contingent commodity as *consumption if A happens* and another contingent commodity as *consumption if B happens*. The second trick is to find a budget constraint that correctly specifies the set of contingent commodity bundles that a consumer can afford.

This chapter presents one other new idea, and that is the notion of von Neumann-Morgenstern utility. A consumer's willingness to take various gambles and his willingness to buy insurance will be determined by how he feels about various combinations of contingent commodities. Often it is reasonable to assume that these preferences can be expressed by a utility function that takes the special form known as *von Neumann-Morgenstern utility*. The assumption that utility takes this form is called the *expected utility hypothesis*. If there are two events, 1 and 2 with probabilities π_1 and π_2, and if the contingent consumptions are c_1 and c_2, then the von Neumann-Morgenstern utility function has the special functional form, $U(c_1, c_2) = \pi_1 u(c_1) + \pi_2 u(c_2)$. The consumer's behavior is determined by maximizing this utility function subject to his budget constraint.

You are thinking of betting on whether the Cincinnati Reds will make it to the World Series this year. A local gambler will bet with you at odds of 10 to 1 against the Reds. You think the probability that the Reds will make it to the World Series is $\pi = .2$. If you don't bet, you are certain to have \$1,000 to spend on consumption goods. Your behavior satisfies the expected utility hypothesis and your von Neumann-Morgenstern utility function is $\pi_1\sqrt{c_1} + \pi_2\sqrt{c_2}$.

The contingent commodities are *dollars if the Reds make the World Series* and *dollars if the Reds don't make the World Series*. Let c_W be your consumption contingent on the Reds making the World Series and c_{NW} be your consumption contingent on their not making the Series. Betting on the Reds at odds of 10 to 1 means that if you bet \$$x$ on the Reds, then if the Reds make it to the Series, you make a net gain of \$$10x$, but if they don't, you have a net loss of \$$x$. Since you had \$1,000 before betting, if you bet \$$x$ on the Reds and they made it to the Series, you would have $c_W = 1,000 + 10x$ to spend on consumption. If you bet \$$x$ on the Reds and they didn't make it to the Series, you would lose \$$x$, and you would have $c_{NW} = 1,000 - x$. By increasing the amount \$$x$ that you bet, you can make c_W larger and c_{NW} smaller. (You could also bet against the Reds at the same odds. If you bet \$$x$ against the Reds and they fail to make it to the Series, you make a net gain of $.1x$ and if they make it to the Series, you lose \$$x$. If you work through the rest of this discussion for the case where you bet against the Reds, you will see that the same equations apply, with x being a negative number.) We can use the above two equations to solve for a budget equation. From the second equation, we have $x = 1,000 - c_{NW}$. Substitute this expression for x into the first equation and rearrange terms to find $c_W + 10c_{NW} = 11,000$, or equivalently, $.1c_W + c_{NW} = 1,100$. (The same budget equation can be written in many equivalent ways by multiplying both sides by a positive constant.)

Then you will choose your contingent consumption bundle (c_W, c_{NW}) to maximize $U(c_W, c_{NW}) = .2\sqrt{c_W} + .8\sqrt{c_{NW}}$ subject to the budget constraint, $.1c_W + c_{NW} = 1,100$. Using techniques that are now familiar, you can solve this consumer problem. From the budget constraint, you see that consumption contingent on the Reds making the World Series costs 1/10 as much as consumption contingent on their not making it. If you set the marginal rate of substitution between c_W and c_{NW} equal to the price ratio and simplify the resulting expression, you will find that $c_{NW} = .16c_W$. This equation, together with the budget equation implies that $c_W = \$4,230.77$ and $c_{NW} = \$676.92$. You achieve this bundle by betting \$323.08 on the Reds. If the Reds make it to the Series, you will have $\$1,000 + 10 \times 323.08 = \$4,230.80$. If not, you will have \$676.92. (We rounded the solutions to the nearest penny.)

12.1 (0) In the next few weeks, Congress is going to decide whether or not to develop an expensive new weapons system. If the system is approved, it will be very profitable for the defense contractor, *General Statics*. Indeed, if the new system is approved, the value of stock in *General Statics* will rise from \$10 per share to \$15 a share, and if the project is not approved, the value of the stock will fall to \$5 a share. In his capacity as a messenger for Congressman Kickback, Buzz Condor has discovered that the weapons system is much more likely to be approved than is generally thought. On the basis of what he knows, Condor has decided that the probability that the system will be approved is 3/4 and the probability that it will not be approved is 1/4. Let c_A be Condor's consumption if the system is approved and c_{NA} be his consumption if the system is not approved. Condor's von Neumann-Morgenstern utility function is $U(c_A, c_{NA}) = .75 \ln c_A + .25 \ln c_{NA}$. Condor's total wealth is \$50,000, all of which is invested in perfectly safe assets. Condor is about to buy stock in *General Statics*.

(a) If Condor buys x shares of stock, and if the weapons system is approved, he will make a profit of \$5 per share. Thus the amount he can consume, contingent on the system being approved, is $c_A = \$50,000 + 5x$. If Condor buys x shares of stock, and if the weapons system is not approved, then he will make a loss of \$________ per share. Thus the amount

____

he can consume, contingent on the system not being approved, is $c_{NA} =$

______________.

(b) You can solve for Condor's budget constraint on contingent commodity bundles (c_A, c_{NA}) by eliminating x from these two equations. His budget constraint can be written as ______ c_A+______ $c_{NA} = 50,000$.

(c) Buzz Condor has no moral qualms about trading on inside information, nor does he have any concern that he will be caught and punished. To decide how much stock to buy, he simply maximizes his von Neumann-Morgenstern utility function subject to his budget. If he sets his marginal rate of substitution between the two contingent commodities equal to their relative prices and simplifies the equation, he finds that $c_A/c_{NA} =$

______ (Reminder: Where a is any constant, the derivative of $a \ln x$ with respect to x is a/x.)

(d) Condor finds that his optimal contingent commodity bundle is $(c_A, c_{NA}) =$______________ To acquire this contingent commodity bundle, he must buy ________ shares of stock in *General Statics.*

12.2 (0) Willy owns a small chocolate factory, located close to a river that occasionally floods in the spring, with disastrous consequences. Next summer, Willy plans to sell the factory and retire. The only income he will have is the proceeds of the sale of his factory. If there is no flood, the factory will be worth \$500,000. If there is a flood, then what is left of the factory will be worth only \$50,000. Willy can buy flood insurance at a cost of \$.10 for each \$1 worth of coverage. Willy thinks that the probability that there will be a flood this spring is 1/10. Let c_F denote the contingent commodity *dollars if there is a flood* and c_{NF} denote *dollars if there is no flood.* Willy's von Neumann-Morgenstern utility function is $U(c_F, c_{NF}) = .1\sqrt{c_F} + .9\sqrt{c_{NF}}$.

(a) If he buys no insurance, then in each contingency, Willy's consumption will equal the value of his factory, so Willy's contingent commodity bundle will be $(c_F, c_{NF}) =$______________.

(b) To buy insurance that pays him \$$x$ in case of a flood, Willy must pay an insurance premium of $.1x$. (The insurance premium must be paid whether or not there is a flood.) If Willy insures for \$$x$, then if there is a flood, he gets \$$x$ in insurance benefits. Suppose that Willy has contracted for insurance that pays him \$$x$ in the event of a flood. Then after paying his insurance premium, he will be able to consume $c_F =$__________
If Willy has this amount of insurance and there is no flood, then he will be able to consume $c_{NF} =$______________.

(c) You can eliminate x from the two equations for c_F and c_{NF} that you found above. This gives you a budget equation for Willy. Of course there are many equivalent ways of writing the same budget equation, since multiplying both sides of a budget equation by a positive constant yields an equivalent budget equation. The form of the budget equation in which the "price" of c_{NF} is 1 can be written as $.9c_{NF}+$________ $c_F =$

____.

(d) Willy's marginal rate of substitution between the two contingent commodities, *dollars if there is no flood* and *dollars if there is a flood*, is $MRS(c_F, c_NF) = -\frac{.1\sqrt{c_{NF}}}{.9\sqrt{c_F}}$. To find his optimal bundle of contingent commodities, you must set this marginal rate of substitution equal to the number __________ Solving this equation, you find that Willy will choose to consume the two contingent commodities in the ratio______________.

(e) Since you know the ratio in which he will consume c_F and c_{NF}, and you know his budget equation, you can solve for his optimal consumption bundle, which is (c_F, c_{NF})=__________________ Willy will buy an insurance policy that will pay him ____________ if there is a flood. The amount of insurance premium that he will have to pay is_____________.

12.3 (0) Clarence Bunsen is an expected utility maximizer. His preferences among contingent commodity bundles are represented by the expected utility function

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1\sqrt{c_1} + \pi_2\sqrt{c_2}.$$

Clarence's friend, Hjalmer Ingqvist, has offered to bet him \$1,000 on the outcome of the toss of a coin. That is, if the coin comes up heads, Clarence must pay Hjalmer \$1,000 and if the coin comes up tails, Hjalmer must pay Clarence \$1,000. The coin is a fair coin, so that the probability of heads and the probability of tails are both 1/2. If he doesn't accept the bet, Clarence will have \$10,000 with certainty. In the privacy of his car dealership office over at Bunsen Motors, Clarence is making his decision. (Clarence uses the pocket calculator that his son, Elmer, gave him last Christmas. You will find that it will be helpful for you to use a calculator too.) Let Event 1 be "coin comes up heads" and let Event 2 be "coin comes up tails."

(a) If Clarence accepts the bet, then in Event 1, he will have __________ dollars and in Event 2, he will have __________ dollars.

____

(b) Since the probability of each event is 1/2, Clarence's expected utility for a gamble in which he gets c_1 in Event 1 and c_2 in Event 2 can be described by the formula __________ Therefore Clarence's expected utility if he accepts the bet with Hjalmer will be __________ (Use that calculator.)

(c) If Clarence decides not to bet, then in Event 1, he will have ________ dollars and in Event 2, he will have ________ dollars. Therefore if he doesn't bet, his expected utility will be____________________.

(d) Having calculated his expected utility if he bets and if he does not bet, Clarence determines which is higher and makes his decision accordingly. Does Clarence take the bet?________________________.

12.4 (0) It is a slow day at Bunsen Motors, so since he has his calculator warmed up, Clarence Bunsen (whose preferences toward risk were described in the last problem) decides to study his expected utility function more closely.

(a) Clarence first thinks about really *big* gambles. What if he bet his entire $10,000 on the toss of a coin, where he loses if heads and wins if tails? Then if the coin came up heads, he would have 0 dollars and if it came up tails, he would have $20,000. His expected utility if he took the bet would be ________ , while his expected utility if he didn't take the bet would be ________ Therefore he concludes that he would not take such a bet.

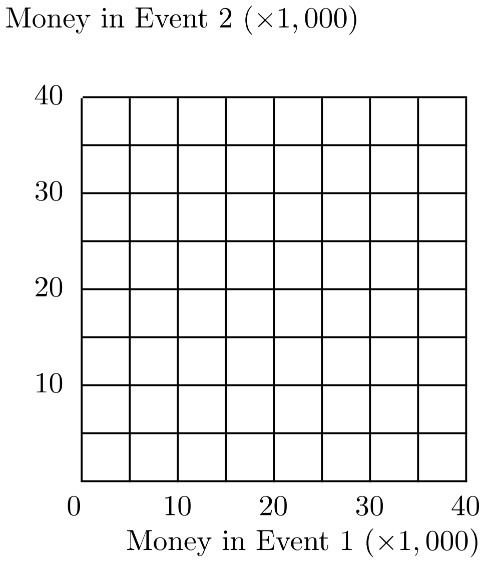
(b) Clarence then thinks, "Well, of course, I wouldn't want to take a chance on losing all of my money on just an ordinary bet. But, what if somebody offered me a really good deal. Suppose I had a chance to bet where if a fair coin came up heads, I lost my $10,000, but if it came up tails, I would win $50,000. Would I take the bet? If I took the bet, my expected utility would be ________ If I didn't take the bet, my expected utility would be ________ Therefore I should ________ the bet."

(c) Clarence later asks himself, "If I make a bet where I lose my $10,000 if the coin comes up heads, what is the smallest amount that I would have to win in the event of tails in order to make the bet a good one for me to take?" After some trial and error, Clarence found the answer. You, too, might want to find the answer by trial and error, but it is easier to

find the answer by solving an equation. On the left side of your equation, you would write down Clarence's utility if he doesn't bet. On the right side of the equation, you write down an expression for Clarence's utility if he makes a bet such that he is left with zero consumption in Event 1 and x in Event 2. Solve this equation for x. The answer to Clarence's question is where $x = 10,000$. The equation that you should write is

____________ The solution is $x =$______________________.

(d) Your answer to the last part gives you two points on Clarence's indifference curve between the contingent commodities, money in Event 1 and money in Event 2. (Poor Clarence has never heard of indifference curves or contingent commodities, so you will have to work this part for him, while he heads over to the Chatterbox Cafe for morning coffee.) One of these points is where money in both events is \$10,000. On the graph below, label this point A. The other is where money in Event 1 is zero and money in Event 2 is ________ On the graph below, label this point B.



(e) You can quickly find a third point on this indifference curve. The coin is a fair coin, and Clarence cares whether heads or tails turn up only because that determines his prize. Therefore Clarence will be indifferent between two gambles that are the same except that the assignment of prizes to outcomes are reversed. In this example, Clarence will be indifferent between point B on the graph and a point in which he gets zero if Event 2 happens and ________ if Event 1 happens. Find this point on the Figure above and label it C.

____

(f) Another gamble that is on the same indifference curve for Clarence as not gambling at all is the gamble where he loses $5,000 if heads turn up and where he wins __________ dollars if tails turn up. (Hint: To solve this problem, put the utility of not betting on the left side of an equation and on the right side of the equation, put the utility of having $\$10,000 - \$5,000$ in Event 1 and $\$10,000 + x$ in Event 2. Then solve the resulting equation for x.) On the axes above, plot this point and label it D. Now sketch in the entire indifference curve through the points that you have labeled.

12.5 (0) Hjalmer Ingqvist's son-in-law, Earl, has not worked out very well. It turns out that Earl likes to gamble. His preferences over contingent commodity bundles are represented by the expected utility function

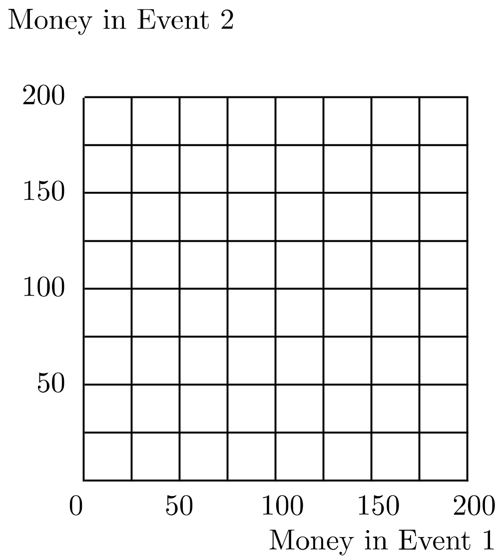
$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1^2 + \pi_2 c_2^2.$$

(a) Just the other day, some of the boys were down at Skoog's tavern when Earl stopped in. They got to talking about just how bad a bet they could get him to take. At the time, Earl had $100. Kenny Olson shuffled a deck of cards and offered to bet Earl $20 that Earl would not cut a spade from the deck. Assuming that Earl believed that Kenny wouldn't cheat, the probability that Earl would win the bet was 1/4 and the probability that Earl would lose the bet was 3/4. If he won the bet, Earl would have ______ dollars and if he lost the bet, he would have ______ dollars. Earl's expected utility if he took the bet would be ______ , and his expected utility if he did not take the bet would be ______ Therefore he refused the bet.

(b) Just when they started to think Earl might have changed his ways, Kenny offered to make the same bet with Earl except that they would bet $100 instead of $20. What is Earl's expected utility if he takes that bet? ______ Would Earl be willing to take this bet?______

____.

(c) Let Event 1 be the event that a card drawn from a fair deck of cards is a spade. Let Event 2 be the event that the card is not a spade. Earl's preferences between income contingent on Event 1, c_1, and income contingent on Event 2, c_2, can be represented by the equation __________ Use blue ink on the graph below to sketch Earl's indifference curve passing through the point $(100, 100)$.

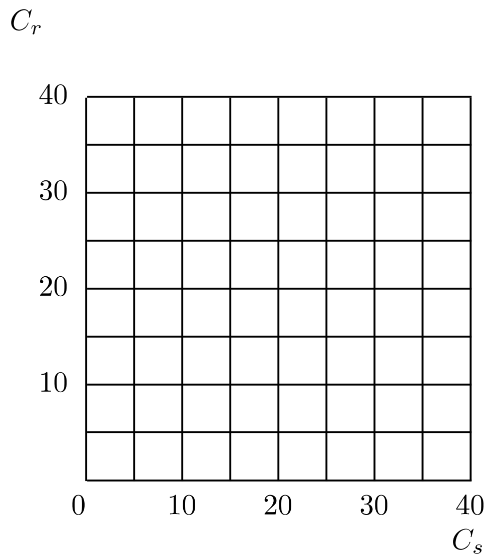


(d) On the same graph, let us draw Hjalmer's son-in-law Earl's indifference curves between contingent commodities where the probabilities are different. Suppose that a card is drawn from a fair deck of cards. Let Event 1 be the event that the card is black. Let event 2 be the event that the card drawn is red. Suppose each event has probability 1/2. Then Earl's preferences between income contingent on Event 1 and income contingent on Event 2 are represented by the formula ________________ On the graph, use red ink to show two of Earl's indifference curves, including the one that passes through $(100, 100)$.

12.6 (1) Sidewalk Sam makes his living selling sunglasses at the boardwalk in Atlantic City. If the sun shines Sam makes \$30, and if it rains Sam only makes \$10. For simplicity, we will suppose that there are only two kinds of days, sunny ones and rainy ones.

(a) One of the casinos in Atlantic City has a new gimmick. It is accepting bets on whether it will be sunny or rainy the next day. The casino sells dated "rain coupons" for \$1 each. If it rains the next day, the casino will give you \$2 for every rain coupon you bought on the previous day. If it doesn't rain, your rain coupon is worthless. In the graph below, mark Sam's "endowment" of contingent consumption if he makes no bets with the casino, and label it E.

____



(b) On the same graph, mark the combination of consumption contingent on rain and consumption contingent on sun that he could achieve by buying 10 rain coupons from the casino. Label it A.

(c) On the same graph, use blue ink to draw the budget line representing all of the other patterns of consumption that Sam can achieve by buying rain coupons. (Assume that he can buy fractional coupons, but not negative amounts of them.) What is the slope of Sam's budget line at points above and to the left of his initial endowment?______________________.

(d) Suppose that the casino also sells sunshine coupons. These tickets also cost \$1. With these tickets, the casino gives you \$2 if it doesn't rain and nothing if it does. On the graph above, use red ink to sketch in the budget line of contingent consumption bundles that Sam can achieve by buying sunshine tickets.

(e) If the price of a dollar's worth of consumption when it rains is set equal to 1, what is the price of a dollar's worth of consumption if it shines?____.

12.7 (0) Sidewalk Sam, from the previous problem, has the utility function for consumption in the two states of nature

$$u(c_s, c_r, \pi) = c_s^{1-\pi} c_r^{\pi},$$

where c_s is the dollar value of his consumption if it shines, c_r is the dollar value of his consumption if it rains, and π is the probability that it will rain. The probability that it will rain is $\pi = .5$.

(a) How many units of consumption is it optimal for Sam to consume conditional on rain?______________________.

(b) How many rain coupons is it optimal for Sam to buy?__________.

12.8 (0) Sidewalk Sam's brother Morgan von Neumanstern is an expected utility maximizer. His von Neumann-Morgenstern utility function for wealth is $u(c) = \ln c$. Sam's brother also sells sunglasses on another beach in Atlantic City and makes exactly the same income as Sam does. He can make exactly the same deal with the casino as Sam can.

(a) If Morgan believes that there is a 50% chance of rain and a 50% chance of sun every day, what would his expected utility of consuming (c_s, c_r) be?______________________.

(b) How does Morgan's utility function compare to Sam's? Is one a monotonic transformation of the other?______________________

______________________.

(c) What will Morgan's optimal pattern of consumption be? Answer: Morgan will consume ________ on the sunny days and ________ on the rainy days. How does this compare to Sam's consumption?__________

______________________.

12.9 (0) Billy John Pigskin of Mule Shoe, Texas, has a von Neumann-Morgenstern utility function of the form $u(c) = \sqrt{c}$. Billy John also weighs about 300 pounds and can outrun jackrabbits and pizza delivery trucks. Billy John is beginning his senior year of college football. If he is not seriously injured, he will receive a \$1,000,000 contract for playing professional football. If an injury ends his football career, he will receive a \$10,000 contract as a refuse removal facilitator in his home town. There is a 10% chance that Billy John will be injured badly enough to end his career.

(a) What is Billy John's expected utility?______________________

______________________.

____

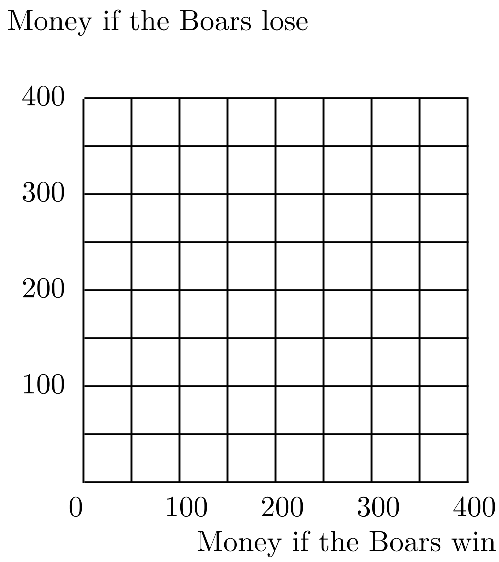
(b) If Billy John pays $p for an insurance policy that would give him \$1,000,000 if he suffered a career-ending injury while in college, then he would be sure to have an income of $\$1,000,000 - p$ no matter what happened to him. Write an equation that can be solved to find the largest price that Billy John would be willing to pay for such an insurance policy.

____.

(c) Solve this equation for p.______________________________.

12.10 (1) You have \$200 and are thinking about betting on the Big Game next Saturday. Your team, the Golden Boars, are scheduled to play their traditional rivals the Robber Barons. It appears that the going odds are 2 to 1 against the Golden Boars. That is to say if you want to bet \$10 on the Boars, you can find someone who will agree to pay you \$20 if the Boars win in return for your promise to pay him \$10 if the Robber Barons win. Similarly if you want to bet \$10 on the Robber Barons, you can find someone who will pay you \$10 if the Robber Barons win, in return for your promise to pay him \$20 if the Robber Barons lose. Suppose that you are able to make as large a bet as you like, either on the Boars or on the Robber Barons so long as your gambling losses do not exceed \$200. (To avoid tedium, let us ignore the possibility of ties.)

(a) If you do not bet at all, you will have \$200 whether or not the Boars win. If you bet \$50 on the Boars, then after all gambling obligations are settled, you will have a total of ______ dollars if the Boars win and ______ dollars if they lose. On the graph below, use blue ink to draw a line that represents all of the combinations of "money if the Boars win" and "money if the Robber Barons win" that you could have by betting from your initial \$200 at these odds.



(b) Label the point on this graph where you would be if you did not bet at all with an E.

(c) After careful thought you decide to bet \$50 on the Boars. Label the point you have chosen on the graph with a C. Suppose that after you have made this bet, it is announced that the star Robber Baron quarterback suffered a sprained thumb during a tough economics midterm examination and will miss the game. The market odds shift from 2 to 1 against the Boars to "even money" or 1 to 1. That is, you can now bet on either team and the amount you would win if you bet on the winning team is the same as the amount that you would lose if you bet on the losing team. You cannot cancel your original bet, but you can make new bets at the new odds. Suppose that you keep your first bet, but you now also bet \$50 on the Robber Barons at the new odds. If the Boars win, then after you collect your winnings from one bet and your losses from the other, how much money will you have left? ________ If the Robber Barons win, how much money will you have left after collecting your winnings and paying off your losses?____________________.

(d) Use red ink to draw a line on the diagram you made above, showing the combinations of "money if the Boars win" and "money if the Robber Barons win" that you could arrange for yourself by adding possible bets at the new odds to the bet you made before the news of the quarterback's misfortune. On this graph, label the point D that you reached by making the two bets discussed above.

12.11 (2) The *certainty equivalent* of a lottery is the amount of money you would have to be given with certainty to be just as well-off with that lottery. Suppose that your von Neumann-Morgenstern utility function over lotteries that give you an amount x if Event 1 happens and y if Event 1 does not happen is $U(x, y, \pi) = \pi\sqrt{x} + (1-\pi)\sqrt{y}$, where π is the probability that Event 1 happens and $1-\pi$ is the probability that Event 1 does not happen.

(a) If $\pi = .5$, calculate the utility of a lottery that gives you \$10,000 if Event 1 happens and \$100 if Event 1 does not happen.____________

____________________________________.

(b) If you were sure to receive \$4,900, what would your utility be? ______ (Hint: If you receive \$4,900 with certainty, then you receive \$4,900 in both events.)

(c) Given this utility function and $\pi = .5$, write a general formula for the certainty equivalent of a lottery that gives you \$$x$ if Event 1 happens and \$$y$ if Event 1 does not happen.____________________.

(d) Calculate the certainty equivalent of receiving \$10,000 if Event 1 happens and \$100 if Event 1 does not happen.____________________.

12.12 (0) Dan Partridge is a risk averter who tries to maximize the expected value of $\sqrt{c}$, where c is his wealth. Dan has \$50,000 in safe assets and he also owns a house that is located in an area where there are lots of forest fires. If his house burns down, the remains of his house and the lot it is built on would be worth only \$40,000, giving him a total wealth of \$90,000. If his home doesn't burn, it will be worth \$200,000 and his total wealth will be \$250,000. The probability that his home will burn down is .01.

(a) Calculate his expected utility if he doesn't buy fire insurance.____________________.

(b) Calculate the certainty equivalent of the lottery he faces if he doesn't buy fire insurance.____________________.

(c) Suppose that he can buy insurance at a price of \$1 per \$100 of insurance. For example if he buys \$100,000 worth of insurance, he will pay \$1,000 to the company no matter what happens, but if his house burns, he will also receive \$100,000 from the company. If Dan buys \$160,000 worth of insurance, he will be fully insured in the sense that no matter what happens his after-tax wealth will be____________________.

(d) Therefore if he buys full insurance, the certainty equivalent of his wealth is ____________ , and his expected utility is____________________.

12.13 (1) Portia has been waiting a long time for her ship to come in and has concluded that there is a 25% chance that it will arrive today. If it does come in today, she will receive \$1,600. If it does not come in today, it will never come and her wealth will be zero. Portia has a von Neumann-Morgenstern utility such that she wants to maximize the expected value of $\sqrt{c}$, where c is total wealth. What is the minimum price at which she will sell the rights to her ship?____________________.