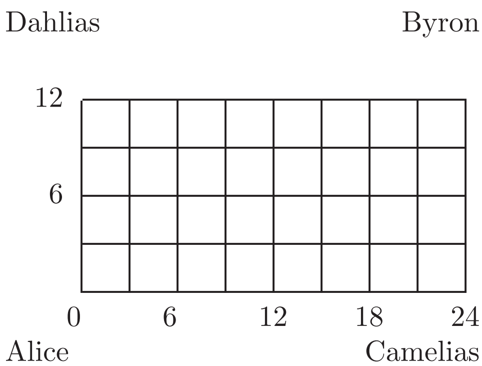
The *Edgeworth box* is a thing of beauty. An amazing amount of information is displayed with a few lines, points and curves. In fact one can use an Edgeworth box to show just about everything there is to say about the case of two traders dealing in two commodities. Economists know that the real world has more than two people and more than two commodities. But it turns out that the insights gained from this model extend nicely to the case of many traders and many commodities. So for the purpose of introducing the subject of exchange equilibrium, the Edgeworth box is exactly the right tool. We will start you out with an example of two gardeners engaged in trade. You will get most out of this example if you fill in the box as you read along.

Alice and Byron consume two goods, camelias and dahlias. Alice has 16 camelias and 4 dahlias. Byron has 8 camelias and 8 dahlias. They consume no other goods, and they trade only with each other. To describe the possible allocations of flowers, we first draw a box whose width is the total number of camelias and whose height is the total number of dahlias that Alice and Byron have between them. The width of the box is therefore $16 + 8 = 24$ and the height of the box is $4 + 8 = 12$.



Any feasible allocation of flowers between Alice and Byron is fully described by a single point in the box. Consider, for example, the allocation where Alice gets the bundle $(15, 9)$ and Byron gets the bundle $(9, 3)$. This allocation is represented by the point $A = (15, 9)$ in the Edgeworth box, which you should draw in. The distance 15 from A to the left side of the box is the number of camelias for Alice and the distance 9 from A to the bottom of the box is the number of dahlias for Alice. This point also determines Byron's consumption of camelias and dahlias. The distance 9 from A to the right side of the box is the total number of camelias consumed by Byron, and the distance from A to the top of the box is the number of dahlias consumed by Byron. Since the width of the box is the total supply of camelias and the height of the box is the total supply of dahlias, these conventions ensure that any point in the box represents a feasible allocation of the total supply of camelias and dahlias.

It is also useful to mark the initial allocation in the Edgeworth box, which, in this case, is the point $E = (16, 4)$. Now suppose that Alice's utility function is $U(c, d) = c + 2d$ and Byron's utility funtion is $U(c, d) = cd$. Alice's indifference curves will be straight lines with slope $-1/2$. The indifference curve that passes through her initial endowment, for example, will be a line that runs from the point $(24, 0)$ to the point $(0, 12)$. Since Byron has Cobb-Douglas utility, his indifference curves will be rectangular hyperbolas, but since quantities for Byron are measured from the upper right corner of the box, these indifference curves will be flipped over as in the Edgeworth box diagrams in your textbook.

The *Pareto set* or *contract curve* is the set of points where Alice's indifference curves are tangent to Byron's. There will be tangency if the slopes are the same. The slope of Alice's indifference curve at any point is $-1/2$. The slope of Byron's indifference curve depends on his consumption of the two goods. When Byron is consuming the bundle (c_B, d_B), the slope of his indifference curve is equal to his marginal rate of substitution, which is $-d_B/c_B$. Therefore Alice's and Byron's indifference curves will nuzzle up in a nice tangency whenever $-d_B/c_B = -1/2$. So the Pareto set in this example is just the diagonal of the Edgeworth box.

Some problems ask you to find a competitive equilibrium. For an economy with two goods, the following procedure is often a good way to calculate equilibrium prices and quantities.

- Since demand for either good depends only on the ratio of prices of good 1 to good 2, it is convenient to set the price of good 1 equal to 1 and let p_2 be the price of good 2.
- With the price of good 1 held at 1, calculate each consumer's demand for good 2 as a function of p_2.
- Write an equation that sets the total amount of good 2 demanded by all consumers equal to the total of all participants' initial endowments of good 2.
- Solve this equation for the value of p_2 that makes the demand for good 2 equal to the supply of good 2. (When the supply of good 2 equals the demand of good 2, it must also be true that the supply of good 1 equals the demand for good 1.)
- Plug this price into the demand functions to determine quantities.

Frank's utility function is $U(x_1, x_2) = x_1x_2$ and Maggie's is $U(x_1, x_2) = \min\{x_1, x_2\}$. Frank's initial endowment is 0 units of good 1 and 10 units of good 2. Maggie's initial endowment is 20 units of good 1 and 5 units of good 2. Let us find a competitive equilibrium for Maggie and Frank.

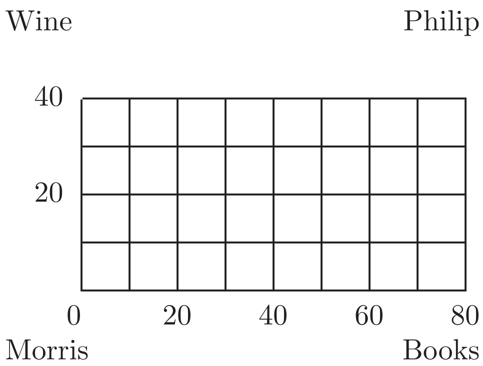
Set $p_1 = 1$ and find Frank's and Maggie's demand functions for good 2 as a function of p_2. Using the techniques learned in Chapter 6, we find that Frank's demand function for good 2 is $m/2p_2$, where m is his income. Since Frank's initial endowment is 0 units of good 1 and 10 units of good 2, his income is $10p_2$. Therefore Frank's demand for good 2 is $10p_2/2p_2 = 5$. Since goods 1 and 2 are perfect complements for Maggie, she will choose to consume where $x_1 = x_2$. This fact, together with her budget constraint implies that Maggie's demand function for good 2 is

$m/(1+p_2)$. Since her endowment is 20 units of good 1 and 5 units of good 2, her income is $20 + 5p_2$. Therefore at price p_2, Maggie's demand is $(20+5p_2)/(1+p_2)$. Frank's demand plus Maggie's demand for good 2 adds up to $5+(20+5p_2)/(1+p_2)$. The total supply of good 2 is Frank's 10 unit endowment plus Maggie's 5 unit endowment, which adds to 15 units. Therefore demand equals supply when

$$5 + \frac{(20+5p_2)}{(1+p_2)} = 15.$$

Solving this equation, one finds that the equilibrium price is $p_2 = 2$. At the equilibrium price, Frank will demand 5 units of good 2 and Maggie will demand 10 units of good 2.

31.1 (0) Morris Zapp and Philip Swallow consume wine and books. Morris has an initial endowment of 60 books and 10 bottles of wine. Philip has an initial endowment of 20 books and 30 bottles of wine. They have no other assets and make no trades with anyone other than each other. For Morris, a book and a bottle of wine are perfect substitutes. His utility function is $U(b,w) = b + w$, where b is the number of books he consumes and w is the number of bottles of wine he consumes. Philip's preferences are more subtle and convex. He has a Cobb-Douglas utility function, $U(b,w) = bw$. In the Edgeworth box below, Morris's consumption is measured from the lower left, and Philip's is measured from the upper right corner of the box.



(a) On this diagram, mark the initial endowment and label it E. Use red ink to draw Morris Zapp's indifference curve that passes through his initial endowment. Use blue ink to draw in Philip Swallow's indifference curve that passes through his initial endowment. (Remember that quantities for Philip are measured from the upper right corner, so his indifference curves are "Phlipped over.")

(b) At any Pareto optimum, where both people consume some of each good, it must be that their marginal rates of substitution are equal. No matter what he consumes, Morris's marginal rate of substitution is equal to ______ When Philip consumes the bundle, (b_P, w_P), his MRS is ________ Therefore every Pareto optimal allocation where both consume positive amounts of both goods satisfies the equation ______ ______ Use black ink on the diagram above to draw the locus of Pareto optimal allocations.

(c) At a competitive equilibrium, it will have to be that Morris consumes some books and some wine. But in order for him to do so, it must be that the ratio of the price of wine to the price of books is ______ Therefore we know that if we make books the *numeraire*, then the price of wine in competitive equilibrium must be______________________.

(d) At the equilibrium prices you found in the last part of the question, what is the value of Philip Swallow's initial endowment?______ At these prices, Philip will choose to consume ______ books and ______ bottles of wine. If Morris Zapp consumes all of the books and all of the wine that Philip doesn't consume, he will consume ______ books and ______ bottles of wine.

(e) At the competitive equilibrium prices that you found above, Morris's income is ______ Therefore at these prices, the cost to Morris of consuming all of the books and all of the wine that Philip doesn't consume is (the same as, more than, less than) ________ his income. At these prices, can Morris afford a bundle that he likes better than the bundle $(55, 15)$?______

(f) Suppose that an economy consisted of 1,000 people just like Morris and 1,000 people just like Philip. Each of the Morris types had the same endowment and the same tastes as Morris. Each of the Philip types had the same endowment and tastes as Philip. Would the prices that you found to be equilibrium prices for Morris and Philip still be competitive equilibrium prices?______ If each of the Morris types and each of the Philip types behaved in the same way as Morris and Philip did above, would supply equal demand for both wine and books?________.

31.2 (0) Consider a small exchange economy with two consumers, Astrid and Birger, and two commodities, herring and cheese. Astrid's initial endowment is 4 units of herring and 1 unit of cheese. Birger's initial endowment has no herring and 7 units of cheese. Astrid's utility function is $U(H_A, C_A) = H_A C_A$. Birger is a more inflexible person. His utility function is $U(H_B, C_B) = \min\{H_B, C_B\}$. (Here H_A and C_A are the amounts of herring and cheese for Astrid, and H_B and C_B are amounts of herring and cheese for Birger.)

(a) Draw an Edgeworth box, showing the initial allocation and sketching in a few indifference curves. Measure Astrid's consumption from the lower left and Birger's from the upper right. In your Edgeworth box, draw two different indifference curves for each person, using blue ink for Astrid's and red ink for Birger's.

(b) Use black ink to show the locus of Pareto optimal allocations. (Hint: Since Birger is kinky, calculus won't help much here. But notice that because of the rigidity of the proportions in which he demands the two goods, it would be inefficient to give Birger a positive amount of either good if he had less than that amount of the other good. What does that tell you about where the Pareto efficient locus has to be?)____

____.

(c) Let cheese be the numeraire (with price 1) and let p denote the price of herring. Write an expression for the amount of herring that Birger will demand at these prices.____________ (Hint: Since Birger initially owns 7 units of cheese and no herring and since cheese is the numeraire, the value of his initial endowment is 7. If the price of herring is p, how many units of herring will he choose to maximize his utility subject to his budget constraint?)

(d) Where the price of cheese is 1 and p is the price of herring, what is the value of Astrid's initial endoment?__________ . How much herring will Astrid demand at price p?__________

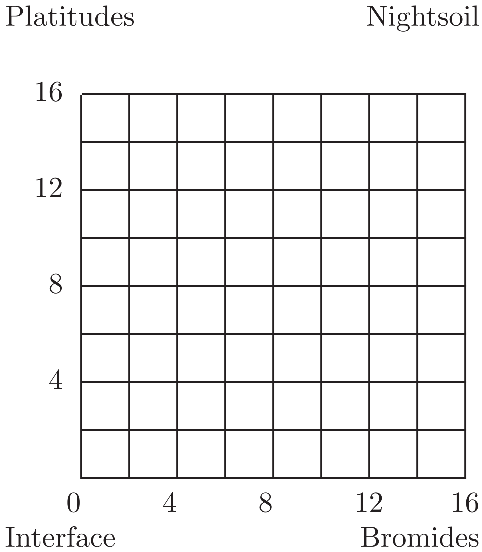
31.3 (0) Dean Foster Z. Interface and Professor J. Fetid Nightsoil exchange platitudes and bromides. When Dean Interface consumes T_I platitudes and B_I bromides, his utility is given by

$$U_I(B_I, T_I) = B_I + 2\sqrt{T_I}.$$

When Professor Nightsoil consumes T_N platitudes and B_N bromide, his utility is given by

$$U_N(B_N, T_N) = B_N + 4\sqrt{T_N}.$$

Dean Interface's initial endowment is 12 platitudes and 8 bromides. Professor Nightsoil's initial endowment is 4 platitudes and 8 bromides.



(a) If Dean Interface consumes T_I platitudes and B_I bromides, his marginal rate of substitution will be __________ If Professor Nightsoil consumes T_N platitudes and B_N bromides, his marginal rate of substitution will be__________.

(b) On the contract curve, Dean Interface's marginal rate of substitution equals Professor Nightsoil's. Write an equation that states this condition. __________ This equation is especially simple because each person's marginal rate of substitution depends only on his consumption of platitudes and not on his consumption of bromides.

____

(c) From this equation we see that $T_I/T_N =$ ______ at all points on the contract curve. This gives us one equation in the two unknowns T_I and T_N.

(d) But we also know that along the contract curve it must be that $T_I + T_N =$________ , since the total consumption of platitudes must equal the total endowment of platitudes.

(e) Solving these two equations in two unknowns, we find that everywhere on the contract curve, T_I and T_N are constant and equal to ________ and____.

(f) In the Edgeworth box, label the initial endowment with the letter E. Dean Interface has thick gray penciled indifference curves. Professor Nightsoil has red indifference curves. Draw a few of these in the Edgeworth box you made. Use blue ink to show the locus of Pareto optimal points. The contract curve is a (vertical, horizontal, diagonal) ________ ______ line in the Edgeworth box.

(g) Find the competitive equilibrium prices and quantities. You know what the prices have to be at competitive equilibrium because you know what the marginal rates of substitution have to be at every Pareto optimum.____.

31.4 (0) A little exchange economy has just two consumers, named Ken and Barbie, and two commodities, quiche and wine. Ken's initial endowment is 3 units of quiche and 2 units of wine. Barbie's initial endowment is 1 unit of quiche and 6 units of wine. Ken and Barbie have identical utility functions. We write Ken's utility function as, $U(Q_K, W_K) = Q_K W_K$ and Barbie's utility function as $U(Q_B, W_B) = Q_B W_B$, where Q_K and W_K are the amounts of quiche and wine for Ken and Q_B and W_B are amounts of quiche and wine for Barbie.

(a) Draw an Edgeworth box below, to illustrate this situation. Put quiche on the horizontal axis and wine on the vertical axis. Measure goods for Ken from the lower left corner of the box and goods for Barbie from the upper right corner of the box. (Be sure that you make the length of the box equal to the total supply of quiche and the height equal to the total supply of wine.) Locate the initial allocation in your box, and label it W. On the sides of the box, label the quantities of quiche and wine for each of the two consumers in the initial endowment.

(b) Use blue ink to draw an indifference curve for Ken that shows allocations in which his utility is 6. Use red ink to draw an indifference curve for Barbie that shows allocations in which her utility is 6.

(c) At any Pareto optimal allocation where both consume some of each good, Ken's marginal rate of substitution between quiche and wine must equal Barbie's. Write an equation that states this condition in terms of the consumptions of each good by each person.____________________.

(d) On your graph, show the locus of points that are Pareto efficient. (Hint: If two people must each consume two goods in the same proportions as each other, and if together they must consume twice as much wine as quiche, what must those proportions be?)

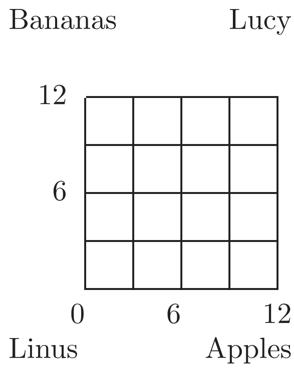
(e) In this example, at any Pareto efficient allocation, where both persons consume both goods, the slope of Ken's indifference curve will be ________ ________ Therefore, since we know that competitive equilibrium must be Pareto efficient, we know that at a competitive equilibrium, p_Q/p_W = ____.

(f) In competitive equilibrium, Ken's consumption bundle must be ____________________ How about Barbie's consumption bundle? ________ ________________ (Hint: You found competitive equilibrium prices above. You know Ken's initial endowment and you know the equilibrium prices. In equilibrium Ken's income will be the value of his endowment at competitive prices. Knowing his income and the prices, you can compute his demand in competitive equilibrium. Having solved for Ken's consumption and knowing that total consumption by Ken and Barbie equals the sum of their endowments, it should be easy to find Barbie's consumption.)

____

(g) On the Edgeworth box for Ken and Barbie, draw in the competitive equilibrium allocation and draw Ken's competitive budget line (with black ink).

31.5 (0) Linus Straight's utility function is $U(a, b) = a + 2b$, where a is his consumption of apples and b is his consumption of bananas. Lucy Kink's utility function is $U(a, b) = \min\{a, 2b\}$. Lucy initially has 12 apples and no bananas. Linus initially has 12 bananas and no apples. In the Edgeworth box below, goods for Lucy are measured from the upper right corner of the box and goods for Linus are measured from the lower left corner. Label the initial endowment point on the graph with the letter E. Draw two of Lucy's indifference curves in red ink and two of Linus's indifference curves in blue ink. Use black ink to draw a line through all of the Pareto optimal allocations.



(a) In this economy, in competitive equilibrium, the ratio of the price of apples to the price of bananas must be________________________.

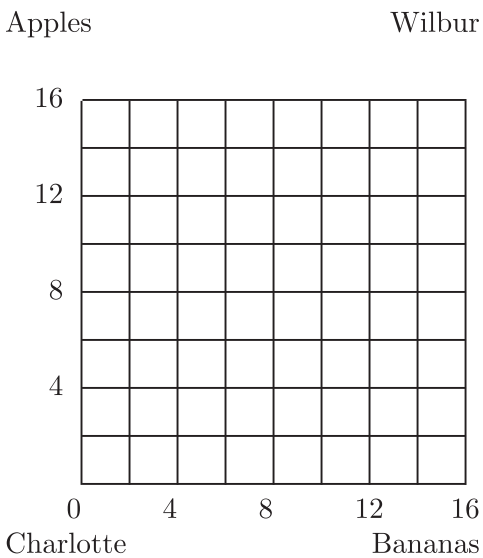
(b) Let a_S be Linus's consumption of apples and let b_S be his consumption of bananas. At competititive equilibrium, Linus's consumption will have to satisfy the budget constraint, $a_s +$________ $b_S =$________ This gives us one equation in two unknowns. To find a second equation, consider Lucy's consumption. In competitive equilibrium, total consumption of apples equals the total supply of apples and total consumption of bananas equals the total supply of bananas. Therefore Lucy will consume $12 - a_s$ apples and ________ $-b_s$ bananas. At a competitive equilibrium, Lucy will be consuming at one of her kink points. The kinks occur at bundles where Lucy consumes ________ apples for every banana that she consumes. Therefore we know that $\frac{12-a_s}{12-b_s} =$________________________.

(c) You can solve the two equations that you found above to find the quantities of apples and bananas consumed in competitive equilibrium by Linus and Lucy. Linus will consume ________ units of apples and ________ units of bananas. Lucy will consume ________ units of apples and 3 units of bananas.

31.6 (0) Consider a pure exchange economy with two consumers and two goods. At some given Pareto efficient allocation it is known that both consumers are consuming both goods and that consumer A has a marginal rate of substitution between the two goods of -2. What is consumer B's marginal rate of substitution between these two goods?________________.

31.7 (0) Charlotte loves apples and hates bananas. Her utility function is $U(a,b) = a - \frac{1}{4}b^2$, where a is the number of apples she consumes and b is the number of bananas she consumes. Wilbur likes both apples and bananas. His utility function is $U(a,b) = a + 2\sqrt{b}$. Charlotte has an initial endowment of no apples and 8 bananas. Wilbur has an initial endowment of 16 apples and 8 bananas.

(a) On the graph below, mark the initial endowment and label it E. Use red ink to draw the indifference curve for Charlotte that passes through this point. Use blue ink to draw the indifference curve for Wilbur that passes through this point.

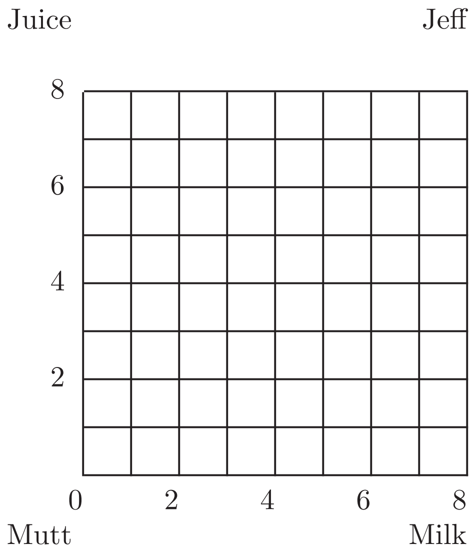


(b) If Charlotte hates bananas and Wilbur likes them, how many bananas can Charlotte be consuming at a Pareto optimal allocation?________ On the graph above, use black ink to mark the locus of Pareto optimal allocations of apples and bananas between Charlotte and Wilbur.

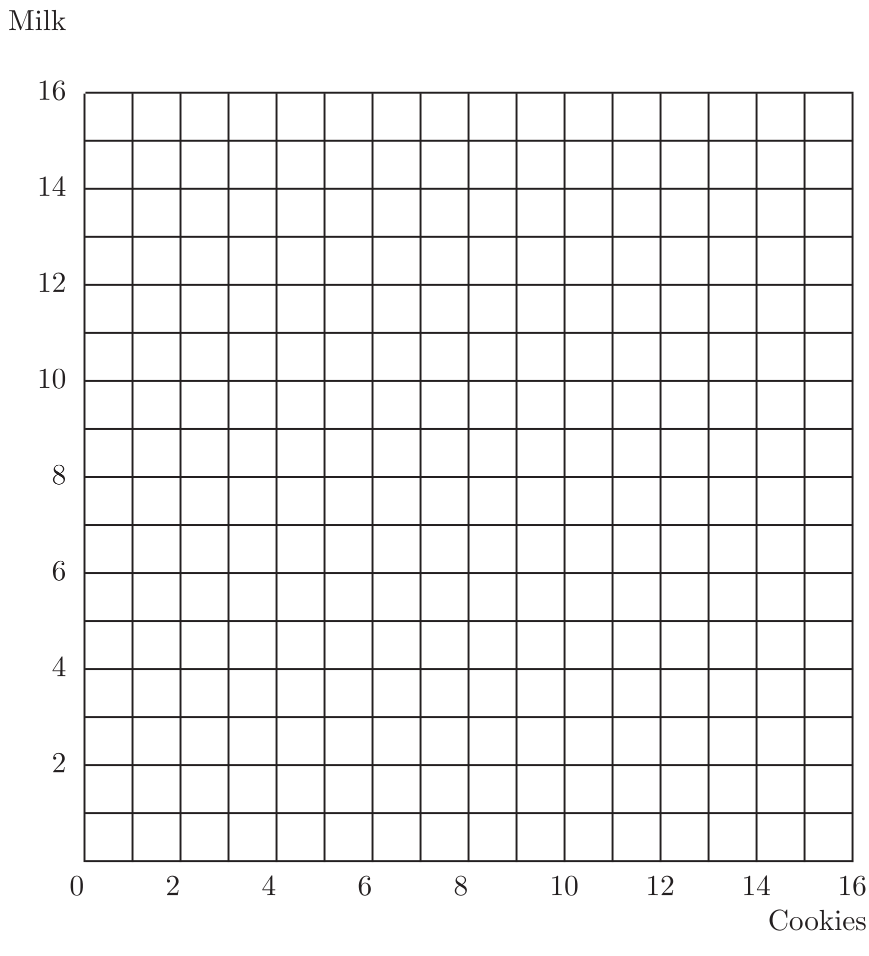
(c) We know that a competitive equilibrium allocation must be Pareto optimal and the total consumption of each good must equal the total supply, so we know that at a competitive equilibrium, Wilbur must be consuming ________ bananas. If Wilbur is consuming this number of bananas, his marginal utility for bananas will be ______ and his marginal utility of apples will be ________ If apples are the *numeraire*, then the only price of bananas at which he will want to consume exactly 16 bananas is _________ In competitive equilibrium, for the Charlotte-Wilbur economy, Wilbur will consume ________ bananas and ________ apples and Charlotte will consume ________ bananas and ________ apples.

31.8 (0) Mutt and Jeff have 8 cups of milk and 8 cups of juice to divide between themselves. Each has the same utility function given by $u(m, j) = \max\{m, j\}$, where m is the amount of milk and j is the amount of juice that each has. That is, each of them cares only about the larger of the two amounts of liquid that he has and is indifferent to the liquid of which he has the smaller amount.

(a) Sketch an Edgeworth box for Mutt and Jeff. Use blue ink to show a couple of indifference curves for each. Use red ink to show the locus of Pareto optimal allocations. (Hint: Look for boundary solutions.)



31.9 (1) Remember Tommy Twit from Chapter 3. Tommy is happiest when he has 8 cookies and 4 glasses of milk per day and his indifference curves are concentric circles centered around (8,4). Tommy's mother, Mrs. Twit, has strong views on nutrition. She believes that too much of anything is as bad as too little. She believes that the perfect diet for Tommy would be 7 glasses of milk and 2 cookies per day. In her view, a diet is healthier the smaller is the sum of the absolute values of the differences between the amounts of each food consumed and the ideal amounts. For example, if Tommy eats 6 cookies and drinks 6 glasses of milk, Mrs. Twit believes that he has 4 too many cookies and 1 too few glasses of milk, so the sum of the absolute values of the differences from her ideal amounts is 5. On the axes below, use blue ink to draw the locus of combinations that Mrs. Twit thinks are exactly as good for Tommy as $(6, 6)$. Also, use red ink to draw the locus of combinations that she thinks is just as good as $(8, 4)$. On the same graph, use red ink to draw an indifference "curve" representing the locus of combinations that Tommy likes just as well as 7 cookies and 8 glasses of milk.



(a) On the graph, shade in the area consisting of combinations of cookies and milk that both Tommy and his mother agree are better than 7 cookies and 8 glasses of milk, where "better" for Mrs. Twit means she thinks it is healthier, and where "better" for Tommy means he likes it better.

(b) Use black ink to sketch the locus of "Pareto optimal" bundles of cookies and milk for Tommy. In this situation, a bundle is Pareto optimal if any bundle that Tommy prefers to this bundle is a bundle that Mrs. Twit thinks is worse for him. The locus of Pareto optimal points that you just drew should consist of two line segments. These run from the point (8,4) to the point ______ and from that point to the point__________.

31.10 (2) This problem combines equilibrium analysis with some of the things you learned in the chapter on intertemporal choice. It concerns the economics of saving and the life cycle on an imaginary planet where life is short and simple. In advanced courses in macroeconomics, you would study more-complicated versions of this model that build in more earthly realism. For the present, this simple model gives you a good idea of how the analysis must go.

On the planet Drongo there is just one commodity, cake, and two time periods. There are two kinds of creatures, "old" and "young." Old creatures have an income of I units of cake in period 1 and no income in period 2. Young creatures have no income in period 1 and an income of I^* units of cake in period 2. There are N_1 old creatures and N_2 young creatures. The consumption bundles of interest to creatures are pairs (c_1, c_2), where c_1 is cake in period 1 and c_2 is cake in period 2. All creatures, old and young, have identical utility functions, representing preferences over cake in the two periods. This utility function is $U(c_1, c_2) = c_1^a c_2^{1-a}$, where a is a number such that $0 \leq a \leq 1$.

(a) If current cake is taken to be the *numeraire*, (that is, its price is set at 1), write an expression for the present value of a consumption bundle (c_1, c_2). ____________ Write down the present value of income for old creatures ________ and for young creatures__________

The budget line for any creature is determined by the condition that the present value of its consumption bundle equals the present value of its income. Write down this budget equation for old creatures: __________

__________ and for young creatures:______________________.

(b) If the interest rate is r, write down an expression for an old creature's demand for cake in period 1 __________ and in period 2 __________

__________ Write an expression for a young creature's demand for cake in period 1 ____________ and in period 2____________ (Hint: If its budget line is $p_1 c_1 + p_2 c_2 = W$ and its utility function is of the form proposed above, then a creature's demand function for good 1 is $c_1 = aW/p$ and demand for good 2 is $c_2 = (1 - a)W/p$.) If the interest rate is zero, how much cake would a young creature choose in period 1?

__________ For what value of a would it choose the same amount in each period if the interest rate is zero?__________ If $a = .55$, what would r have to be in order that young creatures would want to consume the same amount in each period?__________.

(c) The total supply of cake in period 1 equals the total cake earnings of all old creatures, since young creatures earn no cake in this period. There are N_1 old creatures and each earns I units of cake, so this total is $N_1 I$. Similarly, the total supply of cake in period 2 equals the total amount earned by young creatures. This amount is__________.

(d) At the equilibrium interest rate, the total demand of creatures for period-1 cake must equal total supply of period-1 cake, and similarly the demand for period-2 cake must equal supply. If the interest rate is r, then the demand for period-1 cake by each old creature is __________ and the demand for period-1 cake by each young creature is __________ Since there are N_1 old creatures and N_2 young creatures, the total demand for period-1 cake at interest rate r is__________.

(e) Using the results of the last section, write an equation that sets the demand for period-1 cake equal to the supply. __________ Write a general expression for the equilibrium value of r, given N_1, N_2, I, and I^*. __________ Solve this equation for the special case when $N_1 = N_2$ and $I = I^*$ and $a = 11/21$.__________.

(f) In the special case at the end of the last section, show that the interest rate that equalizes supply and demand for period-1 cake will also equalize supply and demand for period-2 cake. (This illustrates Walras's law.)__________

____

__________.