Review of Mathematics

Function

1. What is a function? What is a dependent and independent variable?

2. Draw graphs of the following functions (correspondences):

    (a) $p = 2x - 5$

    (b) $x_2 = 5 - \sqrt{x_1}$

    (c) $x_2 = 5 - x_1^2$

    (d) $x_2 = 4/x_1$

    (e) $10 = \min\{x_1, x_2\}$

    (f) $10 = \min\{x_1 + 2x_2, 2x_1 + x_2\}$

    (g) $S(p) = s_1(p) + s_2(p)$,
    where $s_1(p) = p$ a $s_2(p) = 2p$.

    (h) $S(p) = s_1(p) + s_2(p)$,
    where $s_1(p) = 2p$ a $s_2(p) = p - 1$ for $p \geq 1$
    and $s_2(p) = 0$ for $p < 1$.

3. What is a continuous function? What is the opposite to the continuous function?

4. What is a monotonic function? What is the shape of a positive and negative monotonic function?

5. What is the shape of a convex and concave function?

6. What is the inverse function? Formulate the inverse function of:

    (a) $y = ax + b$

    (b) $y = 5/x$

    (c) $y = e^x$

Logarithm

7. What is the logarithm? Draw the function $y = \ln x$.

8. Calculate the logarithm of $x_1^a x_2^b$.

Derivatives

9. What is the derivative? What is the relationship between the derivative and the slope of a tangent line to a function? What is the relationship between derivatives and convexity (concavity) of a function?

10. What is the product rule? What is the chain rule?

11. Take the derivative of the following functions with respect to p:

    (a) $D(p) = 50 - 2p$

    (b) $D(p) = 30p^{-2}$

    (c) $D(p) = (2p + a)^{(-b)}$

    (d) $R(p) = pq(p)$

12. What is a partial derivative? Take a partial derivative of the following functions with respect to x_1 a x_2:

    (a) $U(x_1, x_2) = ax_1 + bx_2$

    (b) $f(x_1, x_2) = x_1^a x_2^b$

    (c) $U(x_1, x_2) = a \ln x_1 + bx_2$

    (d) $U(x_1, x_2) = a\sqrt{x_1} + bx_2$

    (e) $U(x_1, x_2) = (x_1^2 + x_2^2)^a$

Optimalization

13. Solve the following problem:

$$\max_{x_1, x_2} c \ln x_1 + d \ln x_2$$

$$\text{subject to } p_1 x_1 + p_2 x_2 = m,$$

where $0 < c < 1$, $0 < d < 1$, $p_1 > 0$, $p_2 > 0$ a $m > 0$ are constants.