LECTURE 9

Introduction to Econometrics

Multicollinearity & Heteroskedasticity

November 3, 2017

ON PREVIOUS LECTURES

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  - Does including an irrelevant variable cause a bias in the other coefficients?
- We defined the four specification criteria that determine if a variable belongs to the equation:
  - Can you list some of these specification criteria?

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  - what are the remedies available

Multicollinearity

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- Rare and easy to detect

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- Automatically detected by most statistical softwares

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- Usually referred to simply as "multicollinearity"

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   - t-statistics are smaller - variables may become insignificant

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- One simple method: examine correlation coefficients between explanatory variables
  - if some of them is too high, we may suspect that the coefficients of these variables can be affected by multicollinearity

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- Increase the size of the sample
  - the confidence intervals are narrower when we have more observations

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- Estimating the demand for gasoline in the U.S.:

$$\widehat{PCON}_i = 389.6 - \underset{(13.2)}{36.5}\ TAX_i + \underset{(10.3)}{60.8}\ UHM_i - \underset{(0.043)}{0.061}\ REG_i$$

$$t = 5.92 \qquad\qquad -2.77 \qquad\qquad -1.43$$

$$R^2 = 0.924 \quad , \quad n = 50 \quad , \quad Corr(UHM, REG) = 0.978$$

$PCON_i$	...	petroleum consumption in the *i*-th state
TAX_i	...	the gasoline tax rate in the *i*-th state
UHM_i	...	urban highway miles within the *i*-th state
REG_i	...	motor vehicle registrations in the *i*-the state

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- Remedy: try dropping one of the correlated variables.

Example

$$\widehat{PCON}_i = 551.7 - \underset{(16.9)}{53.6}\ TAX_i + \underset{(0.012)}{0.186}\ REG_i$$

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$$\widehat{PCON}_i = 410.0 - \underset{(13.1)}{39.6}\ TAX_i + \underset{(2.16)}{46.4}\ UHM_i$$

$$t = -3.02 \qquad\qquad 21.40$$

$$R^2 = 0.921 \quad , \quad n = 50$$

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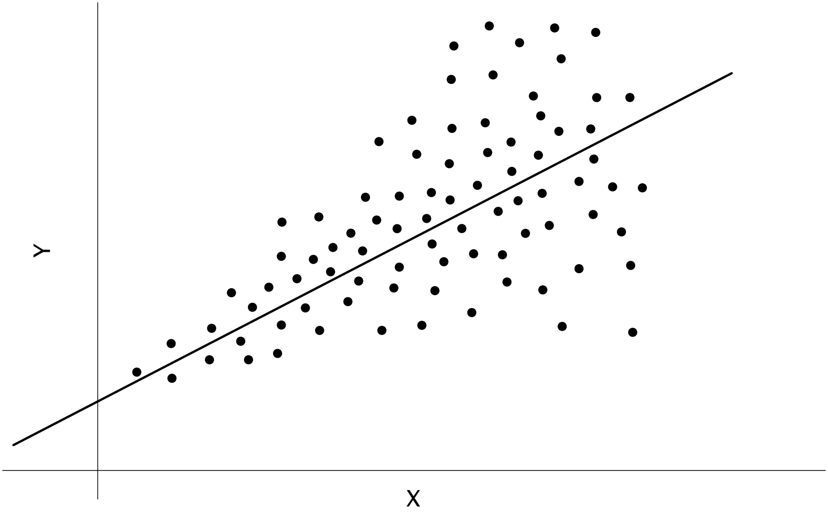
- One particular form of heteroskedasticity (variance of the error term is a function of some observable variable):

$$Var(\varepsilon_i) = h(x_i) \qquad , \qquad i = 1, 2, \ldots, n$$

HETEROSKEDASTICITY



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   - typically, we encounter underestimation of the standard errors, so the t scores are incorrectly too high

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  - Therefore, we will analyse the relationship between e^2 and explanatory variables

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2. Regress the squared residuals on all explanatory variables and on squares and cross-products of all explanatory variables:

$$e_i^2 = \alpha_0 + \alpha_1 x_i + \alpha_2 z_i + \alpha_3 x_i^2 + \alpha_4 z_i^2 + \alpha_5 x_i z_i + \nu_i \quad (1)$$

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3. Get the R^2 of this regression and the sample size n
4. Test the joint significance of (1): test statistic = $nR^2 \sim \chi^2_k$, where k is the number of slope coefficients in (1)
5. If nR^2 is larger than the χ^2_k critical value, then we have to reject H_0 of no heteroskedasticity

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3. **Heteroskedasticity-corrected robust standard errors**

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  - Since heteroskedasticity causes problems with the standard errors of OLS but not with the coefficients, it makes sense to improve the estimation of the standard errors in a way that does not alter the estimate of the coefficients (White, 1980)

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- In panel and cross-sectional data with group-level variables, the method of **clustering** the standard errors is the desired answer to heteroskedasticity

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- Readings:
  - Studenmund Chapter 8 and 10
  - Wooldridge Chapter 8