

LECTURE 7

Introduction to Econometrics

Nonlinear specifications and dummy variables

October 27, 2017

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- ▶ We introduced the measure or the goodness of fit - R^2
- ▶ We showed how the F -test and the R^2 are related

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- ▶ We will define the notion of a dummy variable and we will show its different uses in linear regression models

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 - ▶ The choice of a functional form should be based on the underlying economic theory and/or intuition
 - ▶ Do we expect a curve instead of a straight line? Does the effect of a variable peak at some point and then start to decline?

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- ▶ Linear form is used as default functional form until strong evidence that it is inappropriate is found

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- ▶ Before using a double-log model, make sure that there are no negative or zero observations in the data set

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- ▶ *Ceteris paribus* is a Latin phrase meaning 'other things being equal'.

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EXAMPLES OF SEMILOG FORMS

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- ▶ Estimating demand for chicken meat:

$$\hat{Y} = -6.94 - \frac{0.57}{(0.19)} PC + \frac{0.25}{(0.11)} PB + \frac{12.2}{(2.81)} \ln YD$$

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- ▶ Interpretation: An increase in the annual disposable income by 1% increases chicken consumption by 0.12 kg per year, ceteris paribus.

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- ▶ Estimating the influence of education and experience on wages:

$$\widehat{\ln wage} = 0.217 + \frac{0.098}{(0.008)} educ + \frac{0.010}{(0.002)} exper$$

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- ▶ Interpretation: An increase in education by one year increases annual wage by 9.8%, *ceteris paribus*. An increase in experience by one year increases annual wage by 1%, *ceteris paribus*.

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- ▶ We might also have higher order polynomials, e.g.:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_1^3 + \beta_4 x_1^4 + \varepsilon$$

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- ▶ Decreasing returns to hours of studying: more hours implies higher grade, but the positive effect of additional hour of studying decreases with more hours

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- ▶ Ideally: the specification is given by underlying theory of the equation
- ▶ In reality: underlying theory does not give precise functional form
- ▶ In most cases, either linear form is adequate, or common sense will point out an easy choice from among the alternatives

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 - ▶ dependent variables are often transformed to log-form in order to make their distribution closer to the normal distribution

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- ▶ Examples of dummy variables:

$$Male = \begin{cases} 1 & \text{if the person is male} \\ 0 & \text{if the person is female} \end{cases}$$

$$Weekend = \begin{cases} 1 & \text{if the day is on weekend} \\ 0 & \text{if the day is a work day} \end{cases}$$

$$NewStadium = \begin{cases} 1 & \text{if the team plays on new stadium} \\ 0 & \text{if the team plays on old stadium} \end{cases}$$

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- ▶ It changes the intercept for the subset of data defined by a dummy variable condition:

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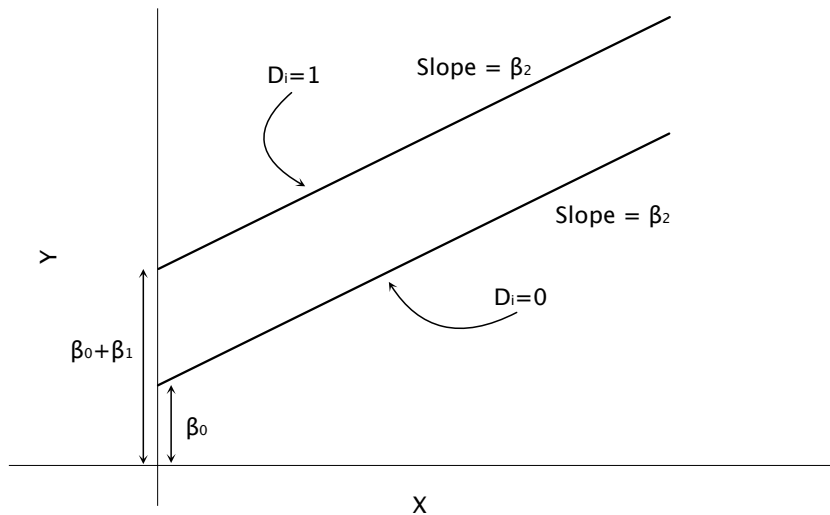
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- ▶ We have

$$\begin{aligned} y_i &= (\beta_0 + \beta_1) + \beta_2 x_i + \varepsilon_i & \text{if } D_i = 1 \\ y_i &= \beta_0 + \beta_2 x_i + \varepsilon_i & \text{if } D_i = 0 \end{aligned}$$

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where $M_i = \begin{cases} 1 & \text{if the } i\text{-th person is male} \\ 0 & \text{if the } i\text{-th person is female} \end{cases}$

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- ▶ Interpretation of the dummy variable M : men earn on average \$2.156 per hour more than women, *ceteris paribus*

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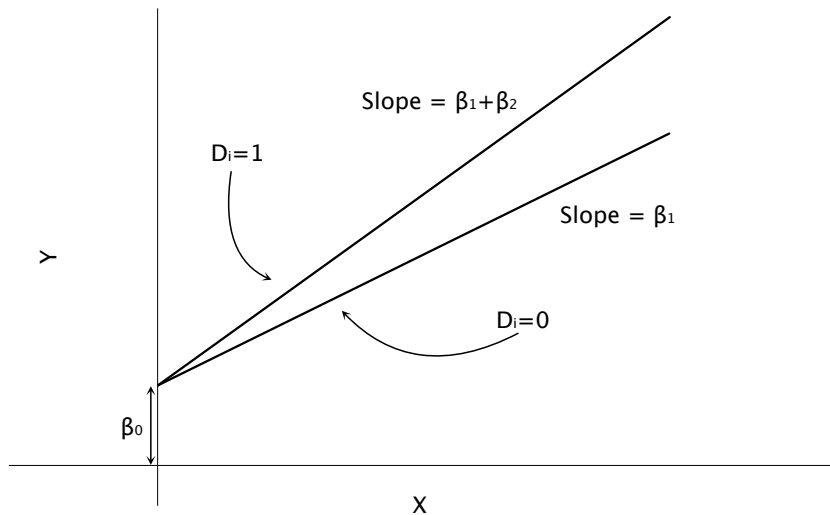
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- ▶ Interpretation: men gain on average 17 cents per hour more than women for each additional year of education, *ceteris paribus*

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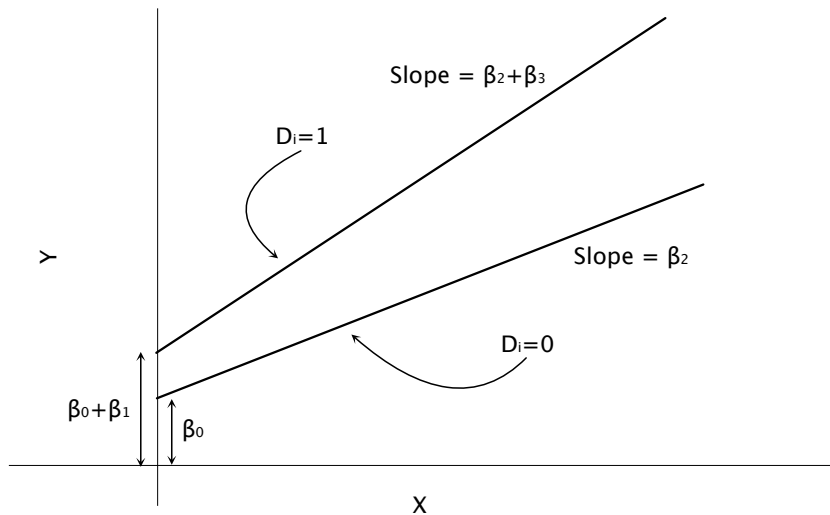
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$$H = \begin{cases} 1 & \text{if high school} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad C = \begin{cases} 1 & \text{if college} \\ 0 & \text{otherwise} \end{cases}$$

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- ▶ Should we include also a third dummy in the regression, which is equal to 1 for people with elementary education?
 - ▶ No, unless we exclude the intercept!
 - ▶ Using full set of dummies leads to perfect multicollinearity (dummy variable trap, see next lectures)

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- ▶ We defined the concept of a dummy variable and we showed its use

SUMMARY

- ▶ We discussed different nonlinear specifications of a regression equation and their interpretation
- ▶ We defined the concept of a dummy variable and we showed its use
- ▶ Further readings:
 - ▶ Studenmund, Chapter 7
 - ▶ Wooldridge, Chapters 6 & 7