LECTURE 5

Introduction to Econometrics

Hypothesis testing

October 20, 2017

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- We will explain what significance of coefficients means
- We will learn how to read regression output
- Readings for this week:
  - Studenmund, Chapter 5.1 - 5.4
  - Wooldridge, Chapter 4

Questions we ask

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- What can we learn about the real world from a sample?
- Is it likely that our results could have been obtained by chance?
- If our theory is correct, what are the odds that this particular outcome would have been observed?

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- We can often reject a given hypothesis with a certain degree of confidence
- In such a case, we conclude that it is very unlikely the sample result would have been observed if the hypothesized theory were correct

Null and Alternative Hypotheses

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- *Alternative hypothesis*: specification of the range of values of the coefficient that would be expected to occur if the researcher's theory were correct
- In other words: we define the null hypothesis as the result we do not expect

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$$\begin{aligned} H_0 : \quad \beta &= 0 \\ H_A : \quad \beta &\neq 0 \end{aligned}$$

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- In hypothesis testing, we focus on Type I error and we ensure that its probability is not unreasonably large

Decision rule

DECISION RULE

1. Calculate sample statistic
2. Compare sample statistic with the *critical value* (from the statistical tables)

- The critical value divides the range of possible values of the statistic into two regions: *acceptance region* and *rejection region*
  - If the sample statistic falls into the rejection region, we reject H_0
  - If the sample statistic falls into the acceptance region, we do not reject H_0
- The idea is that if the value of the coefficient does not support H_0, the sample statistic should fall into the rejection region

One-sided rejection region

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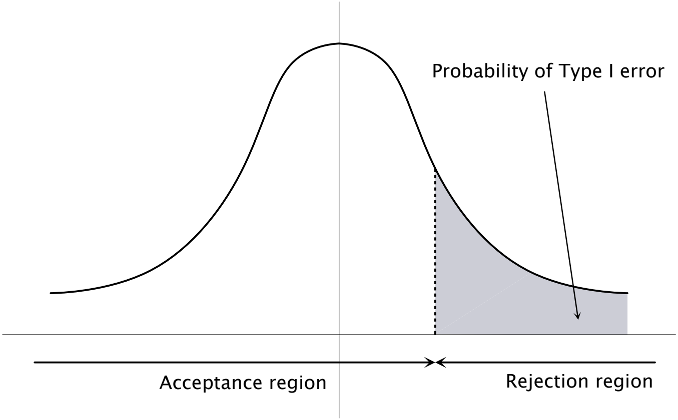
- $H_0: \beta \leq 0 \quad \text{vs} \quad H_A: \beta > 0$

One-sided rejection region

- $H_0: \;\; \beta \leq 0 \quad \text{vs} \quad H_A: \;\; \beta > 0$
- Distribution of $\widehat{\beta}$:



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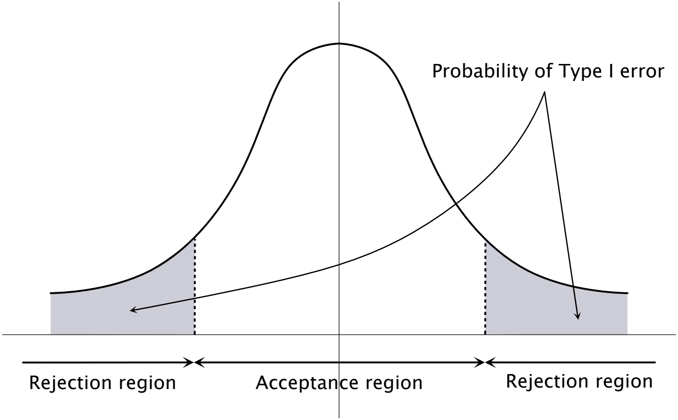

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- The t-test is appropriate to use when the stochastic error term is normally distributed and when the variance of that distribution is unknown
  - These are the usual assumptions in regression analyses
- The t-test accounts for differences in the units of measurement of the variables

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- We know that

$$\widehat{\beta}_1 \sim N\left(\beta_1, Var(\widehat{\beta}_1)\right) \quad \Rightarrow \quad \frac{\widehat{\beta}_1 - \beta_1}{\sqrt{Var(\widehat{\beta}_1)}} \sim N(0, 1)$$

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- We denote *standard error* of $\widehat{\beta}_1$ (sample counterpart of standard deviation $\sigma_{\widehat{\beta}_1}$) as $s.e.\left(\widehat{\beta}_1\right)$

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- We define the t-statistic

$$t \quad := \quad \frac{\widehat{\beta}_1 - \beta_1}{s.e.\left(\widehat{\beta}_1\right)} \quad \sim t_{n-k}$$

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- This statistic depends only on the estimate $\widehat{\beta}_1$, our hypothesis about β_1, and it has a known distribution

TWO-SIDED t-TEST

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  - where $\widehat{\beta}_1$ is the estimated regression coefficient of β_1
  - b is the constant from our null hypothesis
  - $s.e.\left(\widehat{\beta}_1\right)$ is the estimated standard error of $\widehat{\beta}_1$

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2. We find the critical values in the statistical tables: $t_{n-k,0.975}$ and $t_{n-k,0.025}$
   - The critical value depends on the chosen level of Type I error and $n - k$
   - Note that $t_{n-k,0.975} = -t_{n-k,0.025}$

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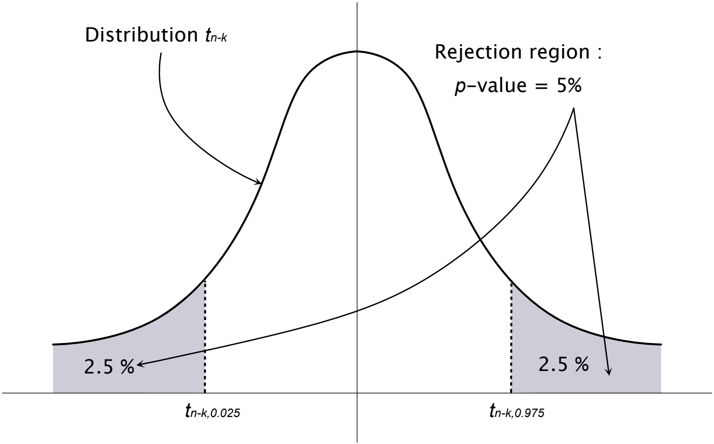

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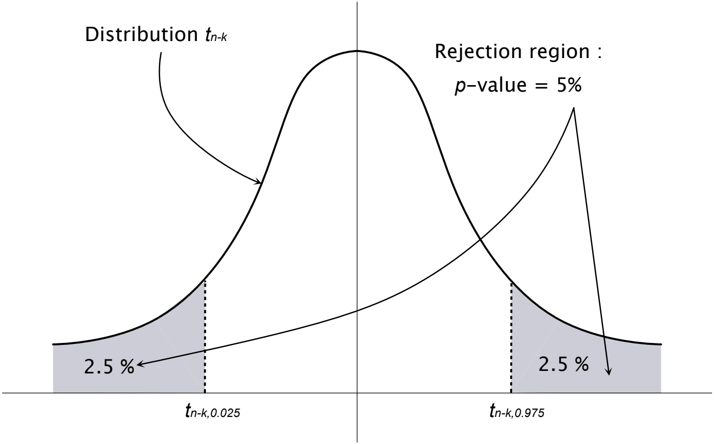

- We reject H_0 if $|t| > t_{n-k,0.975}$

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- We compare our statistic to the critical value $t_{n-k,0.95}$

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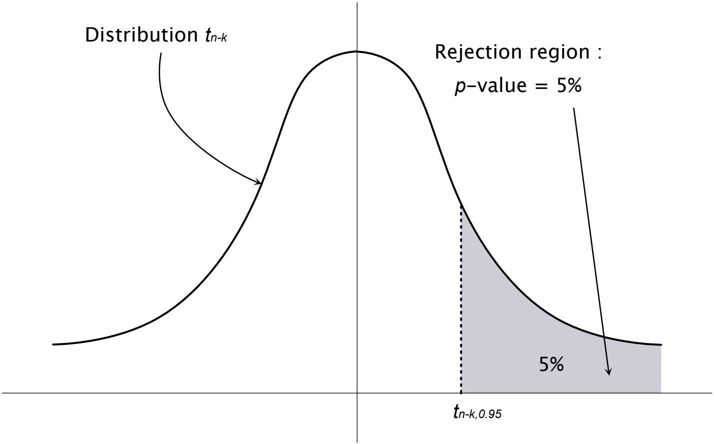

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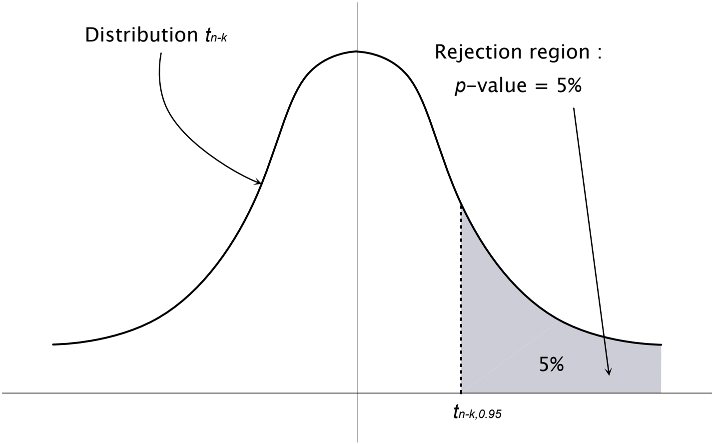

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- This t-statistic is displayed in most regression outputs

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  - However, there is no ”correct” significance level.
- Or we can ask a more informative question:
  - **What is the smallest significance level at which the null hypothesis would still be rejected?**
  - This level of significance is known as the p-value.
  - Remember that the significance level describes the probability of type I. error. The smaller the p-value, the smaller the probability of rejecting the true null hypothesis (the bigger the confidence the null hypothesis is indeed correctly rejected).
  - The p-value for $H_0 : \beta = 0$ is displayed in most regression outputs

Example

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Model 3: OLS, using observations 1-526
Dependent variable: wage

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const      −3.39054      0.766566     −4.423    1.18e-05 ***
  educ        0.644272     0.0538061    11.97     2.28e-29 ***
  exper       0.0700954    0.0109776     6.385    3.78e-10 ***

Mean dependent var   5.896103   S.D. dependent var   3.693086
Sum squared resid    5548.160   S.E. of regression   3.257044
R-squared            0.225162   Adjusted R-squared   0.222199
F(2, 523)            75.98998   P-value(F)           1.07e-29
Log-likelihood      −1365.969   Akaike criterion     2737.937
Schwarz criterion    2750.733   Hannan-Quinn         2742.948
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$$= P\left(-\frac{c}{s.e.\left(\widehat{\beta}\right)} < \frac{\widehat{\beta} - \beta}{s.e.\left(\widehat{\beta}\right)} < \frac{c}{s.e.\left(\widehat{\beta}\right)}\right) = 0.95$$

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- Since $\frac{\widehat{\beta}-\beta}{s.e.\left(\widehat{\beta}\right)} \sim t_{n-k}$, we derive the confidence interval:

$$\widehat{\beta} \pm t_{n-k,0.975} \cdot s.e.\left(\widehat{\beta}\right)$$

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- Confidence interval for coefficient on education: $\widehat{\beta} \pm t_{n-k,0.975} \cdot s.e.\left(\widehat{\beta}\right) = 0.644 \pm 1.960 \cdot 0.054$
- $\widehat{\beta} \in [0.538; 0.750]$ with 95% probability

SUMMARY

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- We derived the t-statistic
- We defined the concept of the p-value
- We explained what significance of a coefficient means
- We observed a regression output on an example