Home assignment # 1
(To be submitted on the lecture, Tuesday October 18)

1. Imagine you work in a bulb factory as a supervisory technician, who is asked to check the lifespan of light bulbs. You know that the lifespan of light bulbs produced in your factory is distributed $N(200, 400)$, that is, normally distributed with a mean of 200 hours and a variance of 400 hours. What is the probability that you find a light bulb with a life span longer than 245 hours? (*Hint*: convert the distribution to a standard normal one and then refer to statistical tables.)

2. The basic ingredients of beer are water; a starch source, such as malted barley, able to be fermented (converted into alcohol); a brewer's yeast to produce the fermentation; and a flavoring, such as hops, to offset the sweetness of the malt. You receive a unique dataset that includes information about barley production in South Bohemia as well as information about agricultural production techniques and local weather. Obviously, you are very curious about what is the effect of soil fertilization and weather on barley yields. You run an OLS regression of annual yields on fertilizer intensity and rainfall and obtain the following results:

$$\widehat{Y}_t = -120 + 0.10 \cdot F_t + 5.33 \cdot R_t$$

where Y_t = the barley yield (bushels/acre) in year t
F_t = fertilizer intensity (pounds/ acre) in year t
R_t = rainfall (inches) in year t

   (a) Carefully interpret the meaning of the coefficients of 0.10 and 5.33 in terms of impact of F and R on Y.

   (b) Use the estimates to determine the yield in years with 100, 500 and 1500 inches of rainfall, given that the fertilizer intensity was always 100 pounds per acre.

   (c) Does the constant term of -120 mean that *negative* amounts of barley are possible? If not, what is the meaning of that estimate?

   (d) Suppose that you were told that the true value of β_F is *known* to be 0.20. Does this show that the estimate is biased? Why or why not?

3. Let us investigate the results of an experiment in broiler (poultry meat) production. The average weight of an experimental lot of broilers and their corresponding level of average feed consumption was tabulated over the time period in which they changed from baby chickens to mature broilers ready for market. At time t, we denote the average weight of the experimental group of broilers as the output y_t, the total feed consumed by the input x_t. The observations are summarized in Table 1.

End of Time Period	Average Weight of Broiler in Pounds	Average Cumulative Feed Inputs in Pounds
t	y_t	x_t
1	0.57	1.00
2	1.01	2.00
3	1.20	3.00
4	1.27	4.00
5	1.91	5.00
6	2.52	6.00
7	2.55	7.00
8	2.92	8.00
9	3.38	9.00
10	3.43	10.00
11	3.53	11.00
12	4.11	12.00
13	4.26	13.00
14	4.33	14.00
15	4.41	15.00

Table 1: Data on poultry production

Let us suppose that the underlying model is

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \varepsilon_t \quad .$$

Answer the following questions. Do not use statistical software (Stata etc.) for the solution!!! You can use Excel for multiplication and inversion of the matrices in question[1], but otherwise you are asked to do all estimation and computation "by hand" - meaning according to the formulae we introduced on the lecture. Make sure you show in your solution all the matrices you construct and compute.

(a) Using the Least Squares formula, find the coefficients β_0, β_1 and β_2.

(b) Find and list the residuals of the model.

(c) What is the sign of $\widehat{\beta}_2$? How do you interpret this coefficient?

(d) Based on your estimation result, find $\frac{\partial y_t}{\partial x_t}$, the marginal productivity of the feed input. Interpret your your result.

(e) We denote the price of broilers by p_b and the price of feed by p_f. Explain why $\frac{\partial y_t}{\partial x_t} = \frac{p_f}{p_b}$. [Hint: Solve for the firm's profit maximization problem.]

[1]For Excel, you may want to check out the commands =MMULT() and =MINVERSE()