# LECTURE 10

## Introduction to Econometrics

## Multicollinearity & Heteroskedasticity

November 22, 2016

# On previous lectures

- We discussed the specification of a regression equation
- **Specification** consists of choosing:
  1. correct independent variables
  2. correct functional form
  3. correct form of the stochastic error term

# On today's lecture

- We will finish the discussion of the choice of independent variables by talking about **multicollinearity**
- We will start the discussion of the correct form of the error term by talking about **heteroskedasticity**
- For both of these issues, we will learn
  - what is the nature of the problem
  - what are its consequences
  - how it is diagnosed
  - what are the remedies available

# Multicollinearity

# Perfect multicollinearity

- Some explanatory variable is a perfect linear function of one or more other explanatory variables
- Violation of one of the classical assumptions
- OLS estimate cannot be found
  - Intuitively: the estimator cannot distinguish which of the explanatory variables causes the change of the dependent variable if they move together
  - Technically: the matrix $\mathbf{X}'\mathbf{X}$ is singular (not invertible)
- Rare and easy to detect

# Examples of perfect multicollinearity

**Dummy variable trap**

- Inclusion of dummy variable for each category in the model with intercept
- Example: wage equation for sample of individuals who have high-school education or higher:

$$wage_i = \beta_1 + \beta_2 high\_school_i + \beta_3 university_i + \beta_4 phd_i + e_i$$

- Automatically detected by most statistical softwares

# Imperfect multicollinearity

- Two or more explanatory variables are highly correlated in the particular data set
- OLS estimate can be found, but it may be very imprecise
  - Intuitively: the estimator can hardly distinguish the effects of the explanatory variables if they are highly correlated
  - Technically: the matrix $\mathbf{X}'\mathbf{X}$ is nearly singular and this causes the variance of the estimator $Var\left(\widehat{\beta}\right) = \sigma^2\left(\mathbf{X}'\mathbf{X}\right)^{-1}$ to be very large
- Usually referred to simply as “multicollinearity”

# Consequences of multicollinearity

1. Estimates remain unbiased and consistent (estimated coefficients are not affected)

2. Standard deviations of coefficients increase

    - Confidence intervals are very large - estimates are less reliable

    - $t$-statistics are smaller - variables may become insignificant

# Detection of multicollinearity

- Some multicollinearity exists in every equation - the aim is to recognize when it causes a severe problem
- Multicollinearity can be signaled by the underlying theory, but it is very sample depending
- We judge the severity of multicollinearity based on the properties of our sample and on the results we obtain
- One simple method: examine correlation coefficients between explanatory variables
    - if some of them is too high, we may suspect that the coefficients of these variables can be affected by multicollinearity

# Remedies for multicollinearity

- Drop a redundant variable
  - when the variable is not needed to represent the effect on the dependent variable
  - in case of severe multicollinearity, it makes no statistical difference which variable is dropped
  - theoretical underpinnings of the model should be the basis for such a decision
- Do nothing
  - when multicollinearity does not cause insignificant $t$-scores or unreliable estimated coefficients
  - deletion of collinear variable can cause specification bias
- Increase the size of the sample
  - the confidence intervals are narrower when we have more observations

# EXAMPLE

- Estimating the demand for gasoline in the U.S.:

$$\widehat{PCON}_i = 389.6 - \underset{(13.2)}{36.5}\ TAX_i + \underset{(10.3)}{60.8}\ UHM_i - \underset{(0.043)}{0.061}\ REG_i$$

$$t = 5.92 \qquad\qquad -2.77 \qquad\qquad -1.43$$

$$R^2 = 0.924 \quad , \quad n = 50 \quad , \quad Corr(UHM, REG) = 0.978$$

| | | |
|---|---|---|
| $PCON_i$ | ... | petroleum consumption in the *i*-th state |
| $TAX_i$ | ... | the gasoline tax rate in the *i*-th state |
| $UHM_i$ | ... | urban highway miles within the *i*-th state |
| $REG_i$ | ... | motor vehicle registrations in the *i*-the state |

# Example

- We suspect a multicollinearity between urban highway miles and motor vehicle registration across states, because those states that have a lot of highways might also have a lot of motor vehicles.
- Therefore, we might run into multicollinearity problems. How do we detect multicollinearity?
  - Look at correlation coefficient. It is indeed huge (0.978).
  - Look at the coefficients of the two variables. Are they both individually significant? *UHM* is significant, but *REG* is not. This further suggests a presence of multicollinearity.
- Remedy: try dropping one of the correlated variables.

# Example

$$\widehat{PCON}_i = 551.7 - \underset{(16.9)}{53.6}\ TAX_i + \underset{(0.012)}{0.186}\ REG_i$$

$$t = -3.18 \qquad\qquad 15.88$$

$$R^2 = 0.866 \quad , \quad n = 50$$

$$\widehat{PCON}_i = 410.0 - \underset{(13.1)}{39.6}\ TAX_i + \underset{(2.16)}{46.4}\ UHM_i$$

$$t = -3.02 \qquad\qquad 21.40$$

$$R^2 = 0.921 \quad , \quad n = 50$$

# Heteroskedasticity

# Heteroskedasticity

- Observations of the error term are drawn from a distribution that has no longer a constant variance

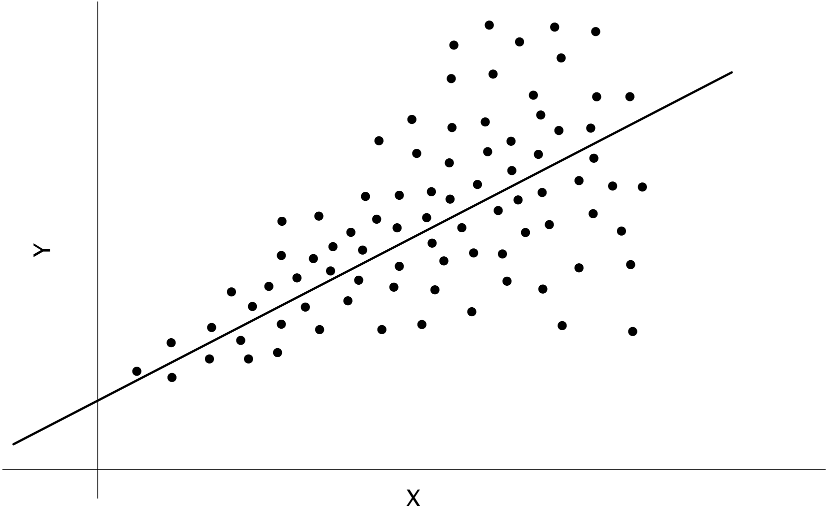
$$Var(\varepsilon_i) = \sigma_i^2 \qquad , \qquad i = 1, 2, \ldots, n$$

Note: constant variance means: $Var(\varepsilon_i) = \sigma^2 (i = 1, 2, \ldots, n)$

- Often occurs in data sets in which there is a wide disparity between the largest and smallest observed values
  - Smaller values often connected to smaller variance and larger values to larger variance (e.g. consumption of households based on their income level)
- One particular form of heteroskedasticity (variance of the error term is a function of some observable variable):

$$Var(\varepsilon_i) = h(x_i) \qquad , \qquad i = 1, 2, \ldots, n$$

# HETEROSKEDASTICITY



# Consequences of heteroskedasticity

- Violation of one of the classical assumptions

1. Estimates remain unbiased and consistent (estimated coefficients are not affected)
2. Estimated standard errors of the coefficients are biased
   - heteroskedastic error term causes the dependent variable to fluctuate in a way that the OLS estimation procedure attributes to the independent variable
   - heteroskedasticity biases $t$ statistics, which leads to unreliable hypothesis testing
   - typically, we encounter underestimation of the standard errors, so the $t$ scores are incorrectly too high

# Detection of heteroskedasticity

- There is a battery of tests for heteroskedasticity
  - Sometimes, simple visual analysis of residuals is sufficient to detect heteroskedasticity

- We will derive a test for the model

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \varepsilon_i$$

- The test is based on analysis of residuals

$$e_i = y_i - \widehat{y}_i = y_i - (\widehat{\beta}_0 + \widehat{\beta}_1 x_i + \widehat{\beta}_2 z_i)$$

- The null hypothesis for the test is no heteroskedasticity: $E(e^2) = \sigma^2$
  - Therefore, we will analyse the relationship between $e^2$ and explanatory variables

# White test for heteroskedasticity

1. Estimate the equation, get the residuals $e_i$
2. Regress the residuals squared on all explanatory variables and on squares and cross-products of all explanatory variables:

$$e_i^2 = \alpha_0 + \alpha_1 x_i + \alpha_2 z_i + \alpha_3 x_i^2 + \alpha_4 z_i^2 + \alpha_5 x_i z_i + \nu_i \tag{1}$$

3. Get the $R^2$ of this regression and the sample size $n$
4. Test the joint significance of (1): test statistic = $nR^2 \sim \chi^2_k$, where $k$ is the number of slope coefficients in (1)
5. If $nR^2$ is larger than the $\chi^2_k$ critical value, then we have to reject $H_0$ of no heteroskedasticity

# REMEDIES FOR HETEROSKEDASTICITY

1. Redefing the variables
   - in order to reduce the variance of observations with extreme values
   - e.g. by taking logarithms or by scaling some variables
2. Weighted Least Squares (WLS)
   - consider the model $y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \varepsilon_i$
   - suppose $Var(\varepsilon_i) = \sigma^2 z_i^2$
   - we prove on the lecture that if we redefine the model as

   $$\frac{y_i}{z_i} = \beta_0 \frac{1}{z_i} + \beta_1 \frac{x_i}{z_i} + \beta_2 + \frac{\varepsilon_i}{z_i} \quad ,$$

   it becomes homoskedastic
3. **Heteroskedasticity-corrected robust standard errors**

# Heteroskedasticity-corrected robust errors

- The logic behind:
  - Since heteroskedasticity causes problems with the standard errors of OLS but not with the coefficients, it makes sense to improve the estimation of the standard errors in a way that does not alter the estimate of the coefficients (White, 1980)
- Heteroskedasticity-corrected standard errors are typically larger than OLS s.e., thus producing lower $t$ scores
- In panel and cross-sectional data with group-level variables, the method of **clustering** standard errors is the answer to heteroskedasticity

# Summary

- Multicollinearity
  - does not lead to inconsistent estimates, but it makes them lose significance
  - if really necessary, can be remedied by dropping or transforming variables, or by getting more data
- Heteroskedasticity
  - does not lead to inconsistent estimates, but it makes the inference wrong
  - can be simply remedied by the use of robust standard errors
- Readings:
  - Studenmund Chapter 8 and 10
  - Wooldridge Chapter 8