

Formulas II

The problem of interest period:

$$FV = PV\left(1 + \frac{r_n}{m}\right)^{mt},$$

where r_n ...nominal interest rate,

m ...# of conversions (how many time interest is calculated to given nominal interest rate),

t ...gives time over interest periods, mt ...#number of interest periods

if $m \rightarrow \infty \rightarrow$ concept of continues interest :

$$FV = PVe^{ft},$$

where e ...Euler's number (2, 182781...), f ...interest intensity, t ...time (given to period of f).

Effective interest rate:

An effective interest rate (r_e) is an interest rate that satisfy:

$$\left(1 + \frac{r_n}{m}\right)^m = e^f = (1 + r_e)$$

Value of capital and taxes:

There are three different situations:

1. $IP = TP$,
2. $IP < TP$,
3. Tax is paid only once when the money is withdraw.

1. $IP = TP$ How much is the FV if the tax rate is 15 % and tax is paid when interest is accrued. You save 7.000,00. The bank promised you to pay monthly interest in the amount of 0.3 %. The maturity of your account (when you withdraw your money) is 6 years.

Solution:

$$FV_{tax} = 7000(1 + 0.003 * (1 - 0.15))^{12*6},$$

where 0.003...monthly interest rate, $(1-0.15)$...how much money left me after taxpayment, $12 * 6$...# of interest periods (12m in one year/6 years).

Note: We multiply exactly the interest rate.

2. $IP < TP$ How much is the FV if the tax rate is 15 % and tax is paid once a year (after one year). You save 7.000,00. The bank promised you to pay monthly interest in the amount of 0.3 %. The maturity of your account (when you withdraw your money) is 6 years.

Solution:

$$FV_{tax} = 7000(((1 + 0.003)^{12} - 1) * (1 - 0.15) + 1)^6,$$

where we have to calculate interest first and after we have the amount of interest in one year we can pay the tax.

Here 12...# of month (as well IP's in one tax period - year), 6...# of tax periods.

Note that the expression $((1 + 0.003)^{12} - 1)$

is nothing, but the effective interest rate r_e , so

again you follow the logic

from 1 (interest rate multiply by $(1 - tax)$).

3. *Tax paid only once in T* How much is the FV if the tax rate is 15 % and tax is paid when you withdraw your money. You save 7.000,00. The bank promised you to pay monthly interest in the amount of 0.3 %. The maturity of your account (when you withdraw your money) is 6 years.

Solution:

$$FV_{tax} = 7000(((1 + 0.003)^{12*6} - 1) * (1 - 0.15) + 1),$$

the magic here is that you calculate FV using compound interest and then pay 15 % tax.

**Crucial moment you pay tax ALWAYS
only from INTEREST!!!**

Tax & inflation

!!!Always calculate the tax liability from nominal value. After you separate the tax from earned interest first then you can depreciate the money with inflation:

$$FV_{tax,\pi} = \frac{FV_{tax}}{(1 + \pi)^n}$$