Problem sets

Microeconomics

Autumn 2017

Consumer theory. Deadline: 17.11.2017

1. Suppose that the set of consumption bundles X is finite and the preference relation $\succeq$ satisfies completness and transitivity. Show that there is a utility function $u : X \to \Re$ representing $\succeq$.

2. Consider two utility functions $u(x_1, x_2)$ and $v(x_1, x_2)$, where $v = f(u)$ and f is strictly increasing function. Derive the first-order conditions for the utility maximization problem and show that Marshallian demands will be the same for u and v

3. Find the Marshallian and Hicksian demands for the following utility functions. You can suppose that there are only two goods when deriving Hicksian demands.

   - Cobb-Douglas $u(x_1, \ldots, x_n) = x_1^{a_1} \ldots x_n^{a_n}$
   - Leontieff $u(x_1, \ldots, x_n) = \min\{a_1 x_1, \ldots, a_n x_n\}$
   - Perfect substitutes $u(x_1, \ldots, x_n) = a_1 x_1 + \ldots + a_n x_n$

4. Let the utility function of a consumer be in quasilinear form $u(x_1, x_2) = x_1 + f(x_2)$. Normalize $p_1 = 1$. Derive the Marshallian demand and interpret it in terms of wealth effects.

5. Suppose that an indirect utility function has the Gorman form $u(p, w) = a(p) + b(p)w$. Show that the wealth expansion curves are linear. Show that homothetic and quasilinear preferences are special case of Gorman form indirect utility function.

6. Show that Hicksian demand function is homogenous of degree zero in prices. Use this fact and Euler's theorem to prove that

$$\sum_{j=1}^{n} \frac{\partial H_i}{\partial p_j} p_j = 0$$

   Interpret this in terms of Slutsky matrix.

Production and general equilibrium. Deadline: 8.12.2017

1. Consider a production function that exhibits decreasing returns to scale. Show that cost function is strictly convex in output, i.e. $c(p, \alpha y) > \alpha c(p, y)$ for any $\alpha > 1$

2. Consider a two-input production function that employs capital (K) and labor (L) as the two inputs. Find the supply function, input demand functions, conditional input demand function and cost function for the following production functions:

   - Cobb-Douglas $f(K, L) = K^\alpha L^\beta$ where $\alpha, \beta > 0$ and $\alpha + \beta < 1$
   - Leontieff $f(K, L) = (\min\{\alpha K, \beta L\})^a$ where $\alpha, \beta > 0$ and $a \in (0, 1)$
   - $f(K, L) = (\alpha K + \beta L)^a$ where $\alpha, \beta > 0$ and $a \in (0, 1)$

3. Do all homogeneous production functions of whatever degree have a) marginal products and b) marginal technical rates of substitution which are independent of the level of output?

4. Consider a two-consumer (a and b) and two-good exchange economy. Let $u^a(x_1^a, x_2^a) = (x_1^a)^{1/3}(x_2^a)^{2/3}$, $\omega^a = (3, 1)$, $u^b(x_1^b, x_2^b) = (x_1^b)^{1/3}(x_2^b)^{2/3}$ and $\omega^b = (1, 1)$. Take good 1 as numeraire. Find competitive equilibria nad graph it in the Edgeworth box. Find the set of interior pareto efficient allocation and graph it in the Edgeworth box.

5. Consider an economy with two consumers, two firms, and two commodities: time and consumption good. The consumers have to sleep 12 hours a day and hence each consumer is endowed with 12 hours of waking time and has preferences over consumption (x_1) and leisure (x_2) given by the utility function $u_1(x_1^1, x_2^1) = (x_1^1)^{3/4}(x_2^1)^{1/4}$ for consumer 1 and $u_1(x_1^2, x_2^2) = (x_2^1)^{1/4}(x_2^2)^{3/4}$ for consumer 2. Each firm produces the consumption good (y_1) out of labor ($-y_2$) using the production possibility set characterized by $y_1 - 2\sqrt{-y_2} \leq 0$. Each consumer owns one of the two firms. Solve for the competitive equilibrium in this economy when the wage rate is normalized to 1 (including the price of consumption, consumption bundles and production plans).

Decision under uncertainty. Deadline: 6.1.2018

1. Person A is a expected utility maximizer with Bernoulli uitlity function $v(y) = \sqrt{y}$ where y is income. She is asked to enter a business, which involves 50-50 chance of an income 900 or 400 and so the expected value of the income is 650.

   - If asked to pay a fair price of 650 in order to take a part in the business, would she accept?

- What is the largest sum of money she would be prepared to pay to take part in the venture?

2. Consider the following utility functions:

   - Show that $v(y) = -e^{-ay}$ exhibits constant absolute risk aversion.
   - Show that $v(y) = a + by^{1-\rho}$ exhibits constant relative risk aversion.

3. Consider a strictly risk-averse decision maker who has an initial wealth of w but who runs a risk of loss of D crowns. The probability of loss is π. The decison maker can buy an insurance. One unit of insurance costs q crowns and pays 1 crown if the loss occurs. The decision maker's problem is to choose the optimal level if α.

   - Suppose that the price of an insurance is fair, ie. $q = \pi$. Show that the decision maker insures completely, i.e. $\alpha = D$
   - Show that if insurance is not fair, then the individual will not insure comletely.

4. Suppose that two individuals have utility functions $u(y)$ and $v(y)$ where $v(y) = f(u(y))$ is an increasing concave transformation of u. Show that he individual with the utility function $v(y)$ has greater absolute and relative risk aversion. Use Arrow-Pratt approximation to show that she will also have larger risk premium.

5. Suppose that utility function $u(y)$ represents preferences satisfying the axioms of expected utility. Show that $v(y) = a+bu(y)$ represents the same preferences. Give an example, which shows that non-linear transformation of $u(y)$ does not represent the same preferences.