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In this chapter you will study ways to measure a consumer's valuation of a good given the consumer's demand curve for it. The basic logic is as follows: The height of the demand curve measures how much the consumer is willing to pay for the last unit of the good purchased—the willingness to pay for the marginal unit. Therefore the sum of the willingnesses-to-pay for each unit gives us the total willingness to pay for the consumption of the good.

In geometric terms, the total willingness to pay to consume some amount of the good is just the area under the demand curve up to that amount. This area is called **gross consumer's surplus** or **total benefit** of the consumption of the good. If the consumer has to pay some amount in order to purchase the good, then we must subtract this expenditure in order to calculate the **(net) consumer's surplus**.

When the utility function takes the quasilinear form,  $u(x) + m$ , the area under the demand curve measures  $u(x)$ , and the area under the demand curve minus the expenditure on the other good measures  $u(x) + m$ . Thus in this case, consumer's surplus serves as an exact measure of utility, and the change in consumer's surplus is a monetary measure of a change in utility.

If the utility function has a different form, consumer's surplus will not be an exact measure of utility, but it will often be a good approximation. However, if we want more exact measures, we can use the ideas of the **compensating variation** and the **equivalent variation**.

Recall that the compensating variation is the amount of extra income that the consumer would need at the *new* prices to be as well off as she was facing the old prices; the equivalent variation is the amount of money that it would be necessary to take away from the consumer at the old prices to make her as well off as she would be, facing the new prices. Although different in general, the change in consumer's surplus and the compensating and equivalent variations will be the same if preferences are quasilinear.

In this chapter you will practice:

- Calculating consumer's surplus and the change in consumer's surplus
- Calculating compensating and equivalent variations

Suppose that the inverse demand curve is given by  $P(q) = 100 - 10q$  and that the consumer currently has 5 units of the good. How much money would you have to pay him to compensate him for reducing his consumption of the good to zero?

Answer: The inverse demand curve has a height of 100 when  $q = 0$  and a height of 50 when  $q = 5$ . The area under the demand curve is a trapezoid with a base of 5 and heights of 100 and 50. We can calculate

the area of this trapezoid by applying the formula

$$\text{Area of a trapezoid} = \text{base} \times \frac{1}{2}(\text{height}_1 + \text{height}_2).$$

In this case we have  $A = 5 \times \frac{1}{2}(100 + 50) = \$375$ .

Suppose now that the consumer is purchasing the 5 units at a price of \$50 per unit. If you require him to reduce his purchases to zero, how much money would be necessary to compensate him?

In this case, we saw above that his gross benefits decline by \$375. On the other hand, he has to spend  $5 \times 50 = \$250$  less. The decline in *net* surplus is therefore \$125.

Suppose that a consumer has a utility function  $u(x_1, x_2) = x_1 + x_2$ . Initially the consumer faces prices (1, 2) and has income 10. If the prices change to (4, 2), calculate the compensating and equivalent variations.

Answer: Since the two goods are perfect substitutes, the consumer will initially consume the bundle (10, 0) and get a utility of 10. After the prices change, she will consume the bundle (0, 5) and get a utility of 5. After the price change she would need \$20 to get a utility of 10; therefore the compensating variation is  $20 - 10 = 10$ . Before the price change, she would need an income of 5 to get a utility of 5. Therefore the equivalent variation is  $10 - 5 = 5$ .

**14.1 (0)** Sir Plus consumes mead, and his demand function for tankards of mead is given by  $D(p) = 100 - p$ , where  $p$  is the price of mead in shillings.

(a) If the price of mead is 50 shillings per tankard, how many tankards of mead will he consume?\_\_\_\_\_.

(b) How much gross consumer's surplus does he get from this consumption?\_\_\_\_\_.

(c) How much money does he spend on mead?\_\_\_\_\_.

(d) What is his net consumer's surplus from mead consumption?\_\_\_\_\_.

**14.2 (0)** Here is the table of reservation prices for apartments taken from Chapter 1:

Person	=	A	B	C	D	E	F	G	H
Price	=	40	25	30	35	10	18	15	5

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(a) If the equilibrium rent for an apartment turns out to be \$20, which consumers will get apartments?\_\_\_\_\_.

(b) If the equilibrium rent for an apartment turns out to be \$20, what is the consumer's (net) surplus generated in this market for person A?

\_\_\_\_\_ For person B?\_\_\_\_\_.

(c) If the equilibrium rent is \$20, what is the total net consumers' surplus generated in the market?\_\_\_\_\_.

(d) If the equilibrium rent is \$20, what is the total gross consumers' surplus in the market?\_\_\_\_\_.

(e) If the rent declines to \$19, how much does the gross surplus increase?

\_\_\_\_\_

(f) If the rent declines to \$19, how much does the net surplus increase?

\_\_\_\_\_

**14.3 (0)** Quasimodo consumes earplugs and other things. His utility function for earplugs  $x$  and money to spend on other goods  $y$  is given by

$$u(x, y) = 100x - \frac{x^2}{2} + y.$$

(a) What kind of utility function does Quasimodo have?\_\_\_\_\_

\_\_\_\_\_

(b) What is his inverse demand curve for earplugs?\_\_\_\_\_.

(c) If the price of earplugs is \$50, how many earplugs will he consume?

\_\_\_\_\_

(d) If the price of earplugs is \$80, how many earplugs will he consume?

\_\_\_\_\_

(e) Suppose that Quasimodo has \$4,000 in total to spend a month. What is his total utility for earplugs and money to spend on other things if the price of earplugs is \$50?\_\_\_\_\_.

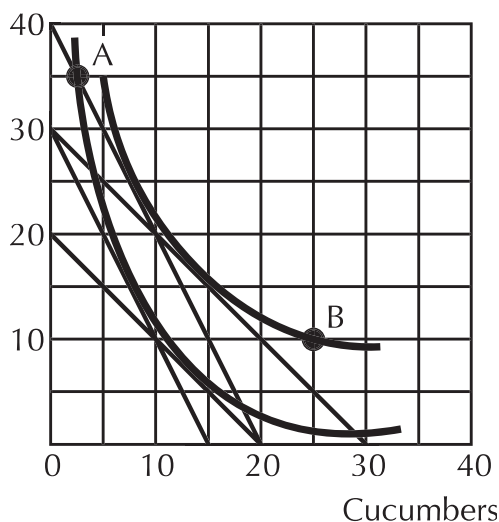
(f) What is his total utility for earplugs and other things if the price of earplugs is \$80?\_\_\_\_\_.

(g) Utility decreases by \_\_\_\_\_ when the price changes from \$50 to \$80.

(h) What is the change in (net) consumer's surplus when the price changes from \$50 to \$80?\_\_\_\_\_.

**14.4 (2)** In the graph below, you see a representation of Sarah Gamp's indifference curves between cucumbers and other goods. Suppose that the reference price of cucumbers and the reference price of "other goods" are both 1.

Other goods



(a) What is the minimum amount of money that Sarah would need in order to purchase a bundle that is indifferent to *A*?\_\_\_\_\_.

(b) What is the minimum amount of money that Sarah would need in order to purchase a bundle that is indifferent to *B*?\_\_\_\_\_.

(c) Suppose that the reference price for cucumbers is 2 and the reference price for other goods is 1. How much money does she need in order to purchase a bundle that is indifferent to bundle *A*?\_\_\_\_\_.

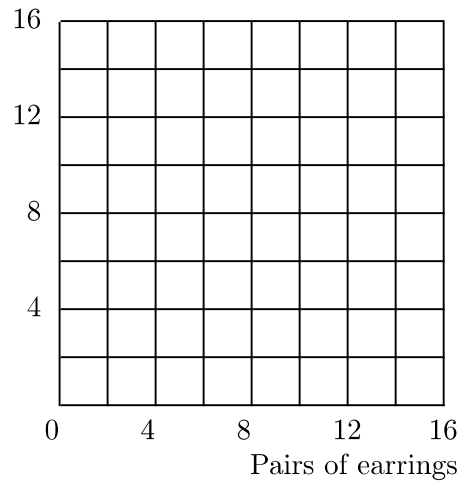
(d) What is the minimum amount of money that Sarah would need to purchase a bundle that is indifferent to *B* using these new prices?\_\_\_\_\_.

(e) No matter what prices Sarah faces, the amount of money she needs to purchase a bundle indifferent to  $A$  must be (higher, lower) than the amount she needs to purchase a bundle indifferent to  $B$ .\_\_\_\_\_.

**14.5 (2)** Bernice's preferences can be represented by  $u(x, y) = \min\{x, y\}$ , where  $x$  is pairs of earrings and  $y$  is dollars to spend on other things. She faces prices  $(p_x, p_y) = (2, 1)$  and her income is 12.

(a) Draw in pencil on the graph below some of Bernice's indifference curves and her budget constraint. Her optimal bundle is \_\_\_\_\_ pairs of earrings and \_\_\_\_\_ dollars to spend on other things.

Dollars for other things



(b) The price of a pair of earrings rises to \$3 and Bernice's income stays the same. Using blue ink, draw her new budget constraint on the graph above.

Her new optimal bundle is \_\_\_\_\_ pairs of earrings and \_\_\_\_\_ dollars to spend on other things.

(c) What bundle would Bernice choose if she faced the original prices and had just enough income to reach the new indifference curve? \_\_\_\_\_ Draw with red ink the budget line that passes through this bundle at the original prices. How much income would Bernice need at the original prices to have this (red) budget line?\_\_\_\_\_.

(d) The maximum amount that Bernice would pay to avoid the price increase is \_\_\_\_\_. This is the (compensating, equivalent) variation in income. \_\_\_\_\_.

(e) What bundle would Bernice choose if she faced the new prices and had just enough income to reach her original indifference curve? \_\_\_\_\_ Draw with black ink the budget line that passes through this bundle at the new prices. How much income would Bernice have with this budget? \_\_\_\_\_.

(f) In order to be as well-off as she was with her original bundle, Bernice's original income would have to rise by \_\_\_\_\_. This is the (compensating, equivalent) variation in income. \_\_\_\_\_.

**14.6 (0)** Ulrich likes video games and sausages. In fact, his preferences can be represented by  $u(x, y) = \ln(x + 1) + y$  where  $x$  is the number of video games he plays and  $y$  is the number of dollars that he spends on sausages. Let  $p_x$  be the price of a video game and  $m$  be his income.

(a) Write an expression that says that Ulrich's marginal rate of substitution equals the price ratio. (Hint: Remember Donald Fribble from Chapter 6?) \_\_\_\_\_.

(b) Since Ulrich has \_\_\_\_\_ preferences, you can solve this equation alone to get his demand function for video games, which is \_\_\_\_\_ His demand function for the dollars to spend on sausages is \_\_\_\_\_.

(c) Video games cost \$.25 and Ulrich's income is \$10. Then Ulrich demands \_\_\_\_\_ video games and \_\_\_\_\_ dollars' worth of sausages. His utility from this bundle is \_\_\_\_\_ (Round off to two decimal places.)

(d) If we took away all of Ulrich's video games, how much money would he need to have to spend on sausages to be just as well-off as before? \_\_\_\_\_.

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(e) Now an amusement tax of \$.25 is put on video games and is passed on in full to consumers. With the tax in place, Ulrich demands \_\_\_\_\_ video game and \_\_\_\_\_ dollars' worth of sausages. His utility from this bundle is \_\_\_\_\_ (Round off to two decimal places.)

(f) Now if we took away all of Ulrich's video games, how much money would he have to have to spend on sausages to be just as well-off as with the bundle he purchased after the tax was in place?\_\_\_\_\_.

(g) What is the change in Ulrich's consumer surplus due to the tax? \_\_\_\_\_ How much money did the government collect from Ulrich by means of the tax?\_\_\_\_\_.

**14.7 (1)** Lolita, an intelligent and charming Holstein cow, consumes only two goods, cow feed (made of ground corn and oats) and hay. Her preferences are represented by the utility function  $U(x, y) = x - x^2/2 + y$ , where  $x$  is her consumption of cow feed and  $y$  is her consumption of hay. Lolita has been instructed in the mysteries of budgets and optimization and always maximizes her utility subject to her budget constraint. Lolita has an income of  $\$m$  that she is allowed to spend as she wishes on cow feed and hay. The price of hay is always \$1, and the price of cow feed will be denoted by  $p$ , where  $0 < p \leq 1$ .

(a) Write Lolita's inverse demand function for cow feed. (Hint: Lolita's utility function is quasilinear. When  $y$  is the numeraire and the price of  $x$  is  $p$ , the inverse demand function for someone with quasilinear utility  $f(x) + y$  is found by simply setting  $p = f'(x)$ .)\_\_\_\_\_.

(b) If the price of cow feed is  $p$  and her income is  $m$ , how much hay does Lolita choose? (Hint: The money that she doesn't spend on feed is used to buy hay.)\_\_\_\_\_.

(c) Plug these numbers into her utility function to find out the utility level that she enjoys at this price and this income.\_\_\_\_\_.

(d) Suppose that Lolita's daily income is \$3 and that the price of feed is \$.50. What bundle does she buy?\_\_\_\_\_ What bundle would she buy if the price of cow feed rose to \$1?\_\_\_\_\_.

(e) How much money would Lolita be willing to pay to avoid having the price of cow feed rise to \$1? \_\_\_\_\_ This amount is known as the \_\_\_\_\_ variation.

(f) Suppose that the price of cow feed rose to \$1. How much extra money would you have to pay Lolita to make her as well-off as she was at the old prices? \_\_\_\_\_ This amount is known as the \_\_\_\_\_ variation. Which is bigger, the compensating or the equivalent variation, or are they the same?\_\_\_\_\_.

(g) At the price \$.50 and income \$3, how much (net) consumer's surplus is Lolita getting?\_\_\_\_\_.

**14.8 (2)** F. Flintstone has quasilinear preferences and his inverse demand function for Brontosaurus Burgers is  $P(b) = 30 - 2b$ . Mr. Flintstone is currently consuming 10 burgers at a price of 10 dollars.

(a) How much money would he be willing to pay to have this amount rather than no burgers at all? \_\_\_\_\_ What is his level of (net) consumer's surplus?\_\_\_\_\_.

(b) The town of Bedrock, the only supplier of Brontosaurus Burgers, decides to raise the price from \$10 a burger to \$14 a burger. What is Mr. Flintstone's change in consumer's surplus?\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_.

**14.9 (1)** Karl Kapitalist is willing to produce  $p/2 - 20$  chairs at every price,  $p > 40$ . At prices below 40, he will produce nothing. If the price of chairs is \$100, Karl will produce \_\_\_\_\_ chairs. At this price, how much is his producer's surplus?\_\_\_\_\_.

**14.10 (2)** Ms. Q. Moto loves to ring the church bells for up to 10 hours a day. Where  $m$  is expenditure on other goods, and  $x$  is hours of bell ringing, her utility is  $u(m, x) = m + 3x$  for  $x \leq 10$ . If  $x > 10$ , she develops painful blisters and is worse off than if she didn't ring the bells. Her income is equal to \$100 and the sexton allows her to ring the bell for 10 hours.



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(a) Due to complaints from the villagers, the sexton has decided to restrict Ms. Moto to 5 hours of bell ringing per day. This is bad news for Ms. Moto. In fact she regards it as just as bad as losing \_\_\_\_\_ dollars of income.

(b) The sexton relents and offers to let her ring the bells as much as she likes so long as she pays \$2 per hour for the privilege. How much ringing does she do now? \_\_\_\_\_ This tax on her activities is as bad as a loss of how much income?\_\_\_\_\_.

(c) The villagers continue to complain. The sexton raises the price of bell ringing to \$4 an hour. How much ringing does she do now? \_\_\_\_\_ This tax, as compared to the situation in which she could ring the bells for free, is as bad as a loss of how much income?\_\_\_\_\_.



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Some problems in this chapter will ask you to construct the market demand curve from individual demand curves. The market demand at any given price is simply the sum of the individual demands at that price. The key thing to remember in going from individual demands to the market demand is to *add quantities*. Graphically, you sum the individual demands horizontally to get the market demand. The market demand curve will have a kink in it whenever the market price is high enough that some individual demand becomes zero.

Sometimes you will need to find a consumer's reservation price for a good. Recall that the reservation price is the price that makes the consumer indifferent between having the good at that price and not having the good at all. Mathematically, the reservation price  $p^*$  satisfies  $u(0, m) = u(1, m - p^*)$ , where  $m$  is income and the quantity of the other good is measured in dollars.

Finally, some of the problems ask you to calculate price and/or income elasticities of demand. These problems are especially easy if you know a little calculus. If the demand function is  $D(p)$ , and you want to calculate the price elasticity of demand when the price is  $p$ , you only need to calculate  $dD(p)/dp$  and multiply it by  $p/q$ .

**15.0 Warm Up Exercise. (Calculating elasticities.)** Here are some drills on price elasticities. For each demand function, find an expression for the price elasticity of demand. The answer will typically be a function of the price,  $p$ . As an example, consider the linear demand curve,  $D(p) = 30 - 6p$ . Then  $dD(p)/dp = -6$  and  $p/q = p/(30 - 6p)$ , so the price elasticity of demand is  $-6p/(30 - 6p)$ .

(a)  $D(p) = 60 - p$ . \_\_\_\_\_.

(b)  $D(p) = a - bp$ . \_\_\_\_\_.

(c)  $D(p) = 40p^{-2}$ . \_\_\_\_\_.

(d)  $D(p) = Ap^{-b}$ . \_\_\_\_\_.

(e)  $D(p) = (p + 3)^{-2}$ . \_\_\_\_\_.

(f)  $D(p) = (p + a)^{-b}$ . \_\_\_\_\_.

**15.1 (0)** In Gas Pump, South Dakota, there are two kinds of consumers, Buick owners and Dodge owners. Every Buick owner has a demand function for gasoline  $D_B(p) = 20 - 5p$  for  $p \leq 4$  and  $D_B(p) = 0$  if  $p > 4$ . Every Dodge owner has a demand function  $D_D(p) = 15 - 3p$  for  $p \leq 5$  and  $D_D(p) = 0$  for  $p > 5$ . (Quantities are measured in gallons per week and price is measured in dollars.) Suppose that Gas Pump has 150 consumers, 100 Buick owners, and 50 Dodge owners.

(a) If the price is \$3, what is the total amount demanded by each individual Buick Owner? \_\_\_\_\_ And by each individual Dodge owner?

\_\_\_\_\_.

(b) What is the total amount demanded by all Buick owners? \_\_\_\_\_

What is the total amount demanded by all Dodge owners? \_\_\_\_\_.

(c) What is the total amount demanded by all consumers in Gas Pump at a price of 3? \_\_\_\_\_.

(d) On the graph below, use blue ink to draw the demand curve representing the total demand by Buick owners. Use black ink to draw the demand curve representing total demand by Dodge owners. Use red ink to draw the market demand curve for the whole town.

(e) At what prices does the market demand curve have kinks? \_\_\_\_\_

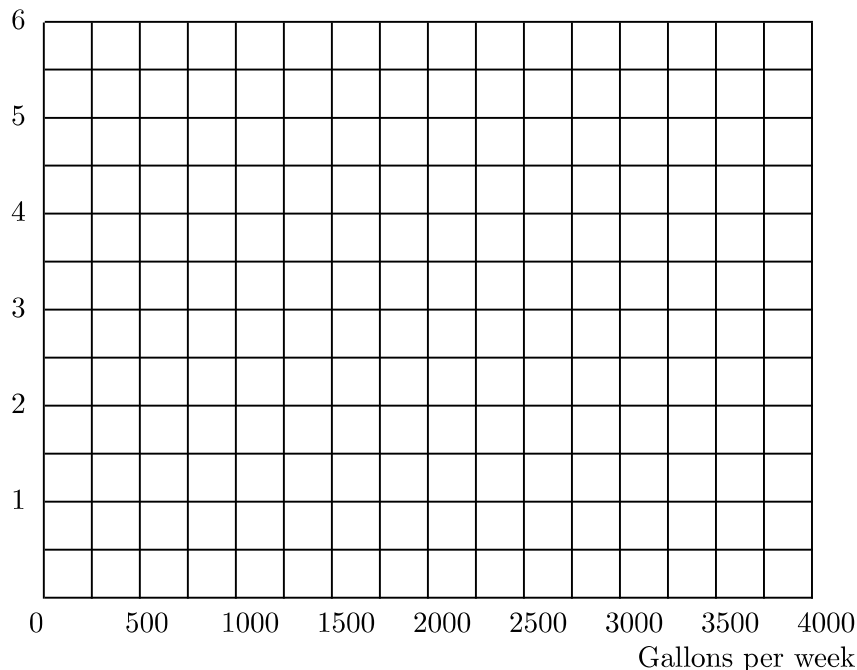
\_\_\_\_\_.

(f) When the price of gasoline is \$1 per gallon, how much does weekly demand fall when price rises by 10 cents? \_\_\_\_\_.

(g) When the price of gasoline is \$4.50 per gallon, how much does weekly demand fall when price rises by 10 cents? \_\_\_\_\_.

(h) When the price of gasoline is \$10 per gallon, how much does weekly demand fall when price rises by 10 cents? \_\_\_\_\_.

Dollars per gallon



**15.2 (0)** For each of the following demand curves, compute the inverse demand curve.

(a)  $D(p) = \max\{10 - 2p, 0\}$ . \_\_\_\_\_

(b)  $D(p) = 100/\sqrt{p}$ . \_\_\_\_\_

(c)  $\ln D(p) = 10 - 4p$ . \_\_\_\_\_

(d)  $\ln D(p) = \ln 20 - 2 \ln p$ . \_\_\_\_\_

**15.3 (0)** The demand function of dog breeders for electric dog polishers is  $q_b = \max\{200 - p, 0\}$ , and the demand function of pet owners for electric dog polishers is  $q_o = \max\{90 - 4p, 0\}$ .

(a) At price  $p$ , what is the price elasticity of dog breeders' demand for electric dog polishers? \_\_\_\_\_ What is the price elasticity of pet owners' demand? \_\_\_\_\_

(b) At what price is the dog breeders' elasticity equal to  $-1$ ? \_\_\_\_\_

At what price is the pet owners' elasticity equal to  $-1$ ? \_\_\_\_\_.

(c) On the graph below, draw the dog breeders' demand curve in blue ink, the pet owners' demand curve in red ink, and the market demand curve in pencil.

(d) Find a nonzero price at which there is positive total demand for dog polishers and at which there is a kink in the demand curve. \_\_\_\_\_

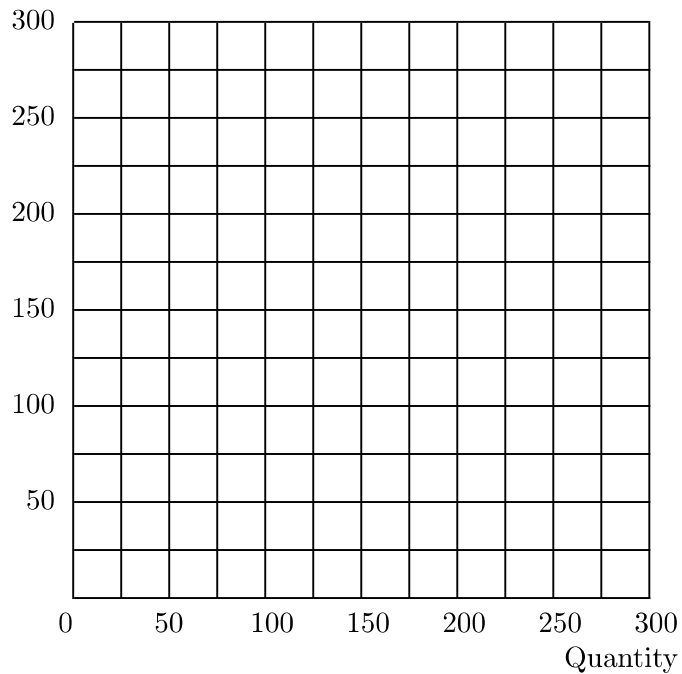
What is the market demand function for prices below the kink? \_\_\_\_\_

\_\_\_\_\_ What is the market demand function for prices above the kink? \_\_\_\_\_.

(e) Where on the market demand curve is the price elasticity equal to  $-1$ ? \_\_\_\_\_ At what price will the revenue from the sale of electric

dog polishers be maximized? \_\_\_\_\_ If the goal of the sellers is to maximize revenue, will electric dog polishers be sold to breeders only, to pet owners only, or to both? \_\_\_\_\_.

Price



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**15.4 (0)** The demand for kitty litter, in pounds, is  $\ln D(p) = 1,000 - p + \ln m$ , where  $p$  is the price of kitty litter and  $m$  is income.

(a) What is the price elasticity of demand for kitty litter when  $p = 2$  and  $m = 500$ ? \_\_\_\_\_ When  $p = 3$  and  $m = 500$ ? \_\_\_\_\_ When  $p = 4$  and  $m = 1,500$ ?\_\_\_\_\_.

(b) What is the income elasticity of demand for kitty litter when  $p = 2$  and  $m = 500$ ? \_\_\_\_\_ When  $p = 2$  and  $m = 1,000$ ? \_\_\_\_\_ When  $p = 3$  and  $m = 1,500$ ?\_\_\_\_\_.

(c) What is the price elasticity of demand when price is  $p$  and income is  $m$ ? \_\_\_\_\_ The income elasticity of demand?\_\_\_\_\_.

**15.5 (0)** The demand function for drangles is  $q(p) = (p + 1)^{-2}$ .

(a) What is the price elasticity of demand at price  $p$ ?\_\_\_\_\_.

(b) At what price is the price elasticity of demand for drangles equal to  $-1$ ?\_\_\_\_\_.

(c) Write an expression for total revenue from the sale of drangles as a function of their price. \_\_\_\_\_ Use calculus to find the revenue-maximizing price. Don't forget to check the second-order condition.\_\_\_\_\_.

(d) Suppose that the demand function for drangles takes the more general form  $q(p) = (p + a)^{-b}$  where  $a > 0$  and  $b > 1$ . Calculate an expression for the price elasticity of demand at price  $p$ . \_\_\_\_\_ At what price is the price elasticity of demand equal to  $-1$ ?\_\_\_\_\_.

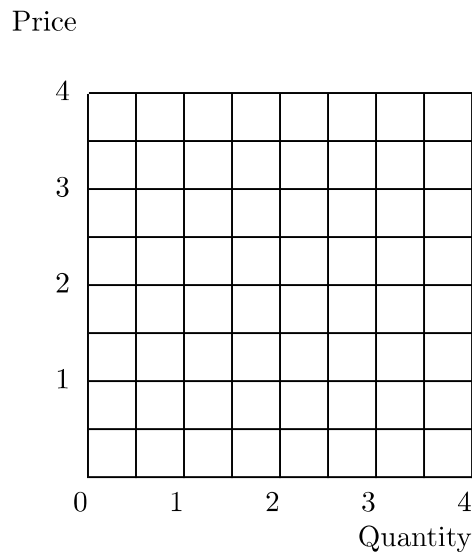
**15.6 (0)** Ken's utility function is  $u_K(x_1, x_2) = x_1 + x_2$  and Barbie's utility function is  $u_B(x_1, x_2) = (x_1 + 1)(x_2 + 1)$ . A person can buy 1 unit of good 1 or 0 units of good 1. It is impossible for anybody to buy fractional units or to buy more than 1 unit. Either person can buy any quantity of good 2 that he or she can afford at a price of \$1 per unit.

(a) Where  $m$  is Barbie's wealth and  $p_1$  is the price of good 1, write an equation that can be solved to find Barbie's reservation price for good 1.

\_\_\_\_\_ What is Barbie's reservation price for good 1?

\_\_\_\_\_ What is Ken's reservation price for good 1?\_\_\_\_\_.

(b) If Ken and Barbie each have a wealth of 3, plot the market demand curve for good 1.



**15.7 (0)** The demand function for yo-yos is  $D(p, M) = 4 - 2p + \frac{1}{100}M$ , where  $p$  is the price of yo-yos and  $M$  is income. If  $M$  is 100 and  $p$  is 1,

(a) What is the income elasticity of demand for yo-yos?\_\_\_\_\_.

(b) What is the price elasticity of demand for yo-yos?\_\_\_\_\_.

**15.8 (0)** If the demand function for zarfs is  $P = 10 - Q$ ,

(a) At what price will total revenue realized from their sale be at a maximum?\_\_\_\_\_.

(b) How many zarfs will be sold at that price?\_\_\_\_\_.

**15.9 (0)** The demand function for football tickets for a typical game at a large midwestern university is  $D(p) = 200,000 - 10,000p$ . The university has a clever and avaricious athletic director who sets his ticket prices so as to maximize revenue. The university's football stadium holds 100,000 spectators.

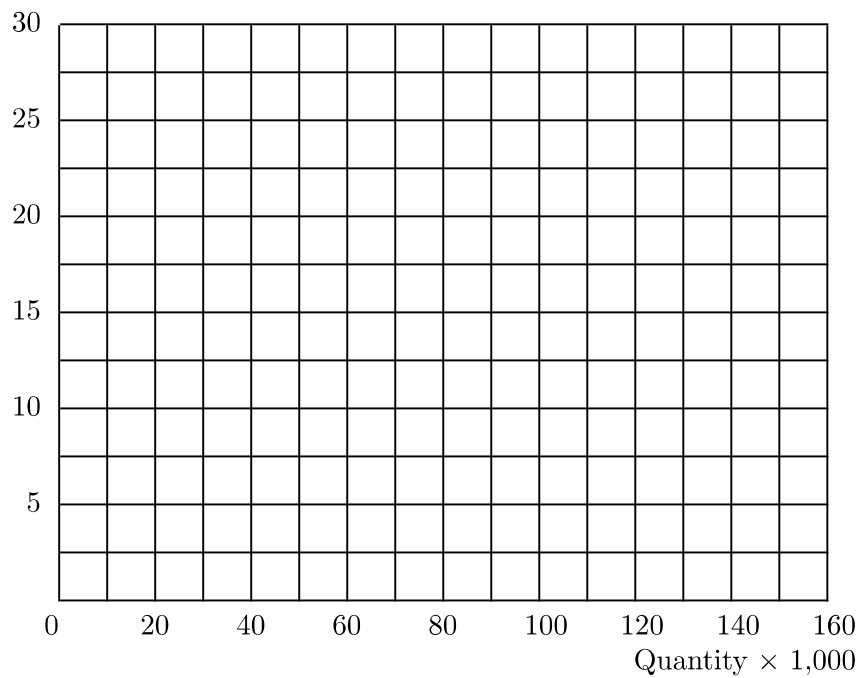


(a) Write down the inverse demand function. \_\_\_\_\_.

(b) Write expressions for total revenue \_\_\_\_\_ and marginal revenue \_\_\_\_\_ as a function of the number of tickets sold.

(c) On the graph below, use blue ink to draw the inverse demand function and use red ink to draw the marginal revenue function. On your graph, also draw a vertical blue line representing the capacity of the stadium.

Price



(d) What price will generate the maximum revenue? \_\_\_\_\_ What quantity will be sold at this price? \_\_\_\_\_.

(e) At this quantity, what is marginal revenue? \_\_\_\_\_ At this quantity, what is the price elasticity of demand? \_\_\_\_\_ Will the stadium be full? \_\_\_\_\_.

(f) A series of winning seasons caused the demand curve for football tickets to shift upward. The new demand function is  $q(p) = 300,000 - 10,000p$ . What is the new inverse demand function?\_\_\_\_\_

\_\_\_\_\_.

(g) Write an expression for marginal revenue as a function of output.  $MR(q) =$ \_\_\_\_\_ Use red ink to draw the new demand function and use black ink to draw the new marginal revenue function.

(h) Ignoring stadium capacity, what price would generate maximum revenue? \_\_\_\_\_ What quantity would be sold at this price?\_\_\_\_\_

\_\_\_\_\_.

(i) As you noticed above, the quantity that would maximize total revenue given the new higher demand curve is greater than the capacity of the stadium. Clever though the athletic director is, he cannot sell seats he hasn't got. He notices that his marginal revenue is positive for any number of seats that he sells up to the capacity of the stadium. Therefore, in order to maximize his revenue, he should sell \_\_\_\_\_ tickets at a price of

\_\_\_\_\_.

(j) When he does this, his marginal revenue from selling an extra seat is \_\_\_\_\_ The elasticity of demand for tickets at this price quantity combination is\_\_\_\_\_.

**15.10 (0)** The athletic director discussed in the last problem is considering the extra revenue he would gain from three proposals to expand the size of the football stadium. Recall that the demand function he is now facing is given by  $q(p) = 300,000 - 10,000p$ .

(a) How much could the athletic director increase the total revenue per game from ticket sales if he added 1,000 new seats to the stadium's capacity and adjusted the ticket price to maximize his revenue?\_\_\_\_\_.

(b) How much could he increase the revenue per game by adding 50,000 new seats? \_\_\_\_\_ 60,000 new seats? (Hint: The athletic director still wants to maximize revenue.)\_\_\_\_\_.

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(c) A zealous alumnus offers to build as large a stadium as the athletic director would like and donate it to the university. There is only one hitch. The athletic director must price his tickets so as to keep the stadium full. If the athletic director wants to maximize his revenue from ticket sales, how large a stadium should he choose?\_\_\_\_\_.



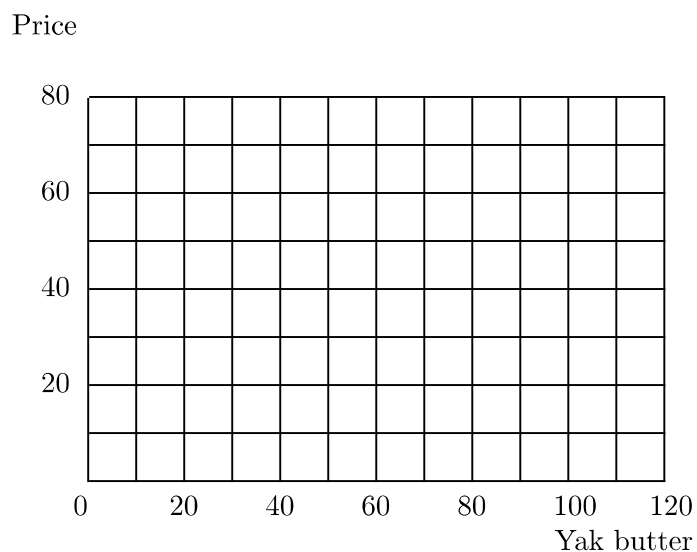
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Supply and demand problems are bread and butter for economists. In the problems below, you will typically want to solve for equilibrium prices and quantities by writing an equation that sets supply equal to demand. Where the price received by suppliers is the same as the price paid by demanders, one writes supply and demand as functions of the same price variable,  $p$ , and solves for the price that equalizes supply and demand. But if, as happens with taxes and subsidies, suppliers face different prices from demanders, it is a good idea to denote these two prices by separate variables,  $p_s$  and  $p_d$ . Then one can solve for equilibrium by solving a system of two equations in the two unknowns  $p_s$  and  $p_d$ . The two equations are the equation that sets supply equal to demand and the equation that relates the price paid by demanders to the net price received by suppliers.

The demand function for commodity  $x$  is  $q = 1,000 - 10p_d$ , where  $p_d$  is the price paid by consumers. The supply function for  $x$  is  $q = 100 + 20p_s$ , where  $p_s$  is the price received by suppliers. For each unit sold, the government collects a tax equal to half of the price paid by consumers. Let us find the equilibrium prices and quantities. In equilibrium, supply must equal demand, so that  $1,000 - 10p_d = 100 + 20p_s$ . Since the government collects a tax equal to half of the price paid by consumers, it must be that the sellers only get half of the price paid by consumers, so it must be that  $p_s = p_d/2$ . Now we have two equations in the two unknowns,  $p_s$  and  $p_d$ . Substitute the expression  $p_d/2$  for  $p_s$  in the first equation, and you have  $1,000 - 10p_d = 100 + 10p_d$ . Solve this equation to find  $p_d = 45$ . Then  $p_s = 22.5$  and  $q = 550$ .

**16.1 (0)** The demand for yak butter is given by  $120 - 4p_d$  and the supply is  $2p_s - 30$ , where  $p_d$  is the price paid by demanders and  $p_s$  is the price received by suppliers, measured in dollars per hundred pounds. Quantities demanded and supplied are measured in hundred-pound units.

(a) On the axes below, draw the demand curve (with blue ink) and the supply curve (with red ink) for yak butter.



(b) Write down the equation that you would solve to find the equilibrium price. \_\_\_\_\_.

(c) What is the equilibrium price of yak butter? \_\_\_\_\_ What is the equilibrium quantity? \_\_\_\_\_ Locate the equilibrium price and quantity on the graph, and label them  $p_1$  and  $q_1$ .

(d) A terrible drought strikes the central Ohio steppes, traditional homeland of the yaks. The supply schedule shifts to  $2p_s - 60$ . The demand schedule remains as before. Draw the new supply schedule. Write down the equation that you would solve to find the new equilibrium price of yak butter. \_\_\_\_\_.

(e) The new equilibrium price is \_\_\_\_\_ and the quantity is \_\_\_\_\_ Locate the new equilibrium price and quantity on the graph and label them  $p_2$  and  $q_2$ .

(f) The government decides to relieve stricken yak butter consumers and producers by paying a subsidy of \$5 per hundred pounds of yak butter to producers. If  $p_d$  is the price paid by demanders for yak butter, what is the total amount received by producers for each unit they produce? \_\_\_\_\_  
 \_\_\_\_\_ When the price paid by consumers is  $p_d$ , how much yak butter is produced? \_\_\_\_\_.

(g) Write down an equation that can be solved for the equilibrium price paid by consumers, given the subsidy program. \_\_\_\_\_  
 What are the equilibrium price paid by consumers and the equilibrium quantity of yak butter now?\_\_\_\_\_.

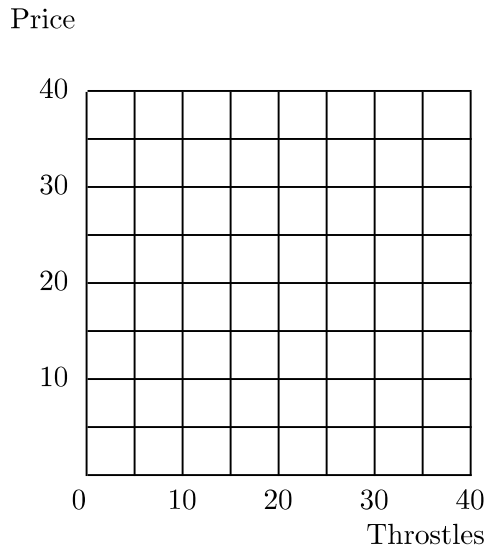
(h) Suppose the government had paid the subsidy to consumers rather than producers. What would be the equilibrium net price paid by consumers? \_\_\_\_\_ The equilibrium quantity would be\_\_\_\_\_.

**16.2 (0)** Here are the supply and demand equations for throstles, where  $p$  is the price in dollars:

$$D(p) = 40 - p$$

$$S(p) = 10 + p.$$

On the axes below, draw the demand and supply curves for throstles, using blue ink.



(a) The equilibrium price of throstles is \_\_\_\_\_ and the equilibrium quantity is\_\_\_\_\_.

(b) Suppose that the government decides to restrict the industry to selling only 20 throstles. At what price would 20 throstles be demanded? \_\_\_\_\_  
 How many throstles would suppliers supply at that price? \_\_\_\_\_ At what price would the suppliers supply only 20 units?\_\_\_\_\_.

(c) The government wants to make sure that only 20 throstles are bought, but it doesn't want the firms in the industry to receive more than the minimum price that it would take to have them supply 20 throstles. One way to do this is for the government to issue 20 ration coupons. Then in order to buy a throstle, a consumer would need to present a ration coupon along with the necessary amount of money to pay for the good. If the ration coupons were freely bought and sold on the open market, what would be the equilibrium price of these coupons?\_\_\_\_\_.

(d) On the graph above, shade in the area that represents the deadweight loss from restricting the supply of throstles to 20. How much is this expressed in dollars? (Hint: What is the formula for the area of a triangle?)

\_\_\_\_\_.

**16.3 (0)** The demand curve for ski lessons is given by  $D(p_D) = 100 - 2p_D$  and the supply curve is given by  $S(p_S) = 3p_S$ .

(a) What is the equilibrium price? \_\_\_\_\_ What is the equilibrium quantity?\_\_\_\_\_.

(b) A tax of \$10 per ski lesson is imposed on consumers. Write an equation that relates the price paid by demanders to the price received by suppliers.

\_\_\_\_\_ Write an equation that states that supply equals demand.\_\_\_\_\_.

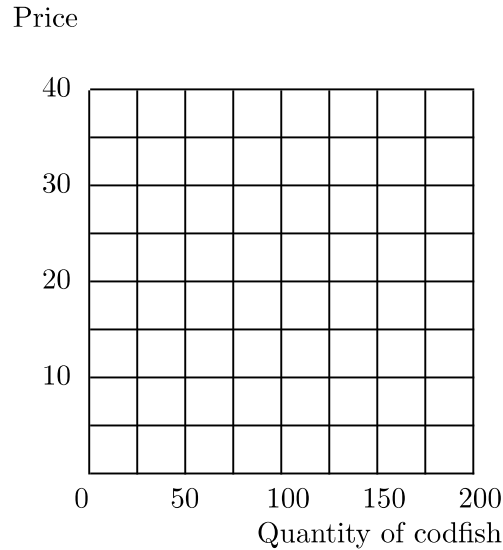
(c) Solve these two equations for the two unknowns  $p_S$  and  $p_D$ . With the \$10 tax, the equilibrium price  $p_D$  paid by consumers would be \_\_\_\_\_ per lesson. The total number of lessons given would be\_\_\_\_\_.

(d) A senator from a mountainous state suggests that although ski lesson consumers are rich and deserve to be taxed, ski instructors are poor and deserve a subsidy. He proposes a \$6 subsidy on production while maintaining the \$10 tax on consumption of ski lessons. Would this policy have any different effects for suppliers or for demanders than a tax of \$4 per lesson?\_\_\_\_\_.

**16.4 (0)** The demand curve for salted codfish is  $D(P) = 200 - 5P$  and the supply curve  $S(P) = 5P$ .



(a) On the graph below, use blue ink to draw the demand curve and the supply curve. The equilibrium market price is \_\_\_\_\_ and the equilibrium quantity sold is \_\_\_\_\_.



(b) A quantity tax of \$2 per unit sold is placed on salted codfish. Use red ink to draw the new supply curve, where the price on the vertical axis remains the price per unit paid by demanders. The new equilibrium price paid by the demanders will be \_\_\_\_\_ and the new price received by the suppliers will be \_\_\_\_\_. The equilibrium quantity sold will be \_\_\_\_\_.

(c) The deadweight loss due to this tax will be \_\_\_\_\_. On your graph, shade in the area that represents the deadweight loss.

**16.5 (0)** The demand function for merino ewes is  $D(P) = 100/P$ , and the supply function is  $S(P) = P$ .

(a) What is the equilibrium price? \_\_\_\_\_.

(b) What is the equilibrium quantity? \_\_\_\_\_.

(c) An ad valorem tax of 300% is imposed on merino ewes so that the price paid by demanders is four times the price received by suppliers. What is the equilibrium price paid by the demanders for merino ewes now?

\_\_\_\_\_ What is the equilibrium price received by the suppliers for merino ewes? \_\_\_\_\_ What is the equilibrium quantity? \_\_\_\_\_.

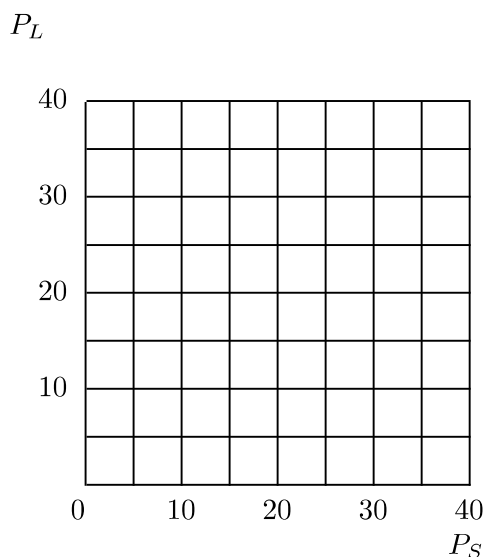
**16.6 (0)** Schrecklich and LaMerde are two justifiably obscure nineteenth-century impressionist painters. The world's total stock of paintings by Schrecklich is 100, and the world's stock of paintings by LaMerde is 150. The two painters are regarded by connoisseurs as being very similar in style. Therefore the demand for either painter's work depends both on its own price and the price of the other painter's work. The demand function for Schrecklich's is  $D_S(P) = 200 - 4P_S - 2P_L$ , and the demand function for LaMerde's is  $D_L(P) = 200 - 3P_L - P_S$ , where  $P_S$  and  $P_L$  are respectively the price in dollars of a Schrecklich painting and a LaMerde painting.

(a) Write down two simultaneous equations that state the equilibrium condition that the demand for each painter's work equals supply.

\_\_\_\_\_.

(b) Solving these two equations, one finds that the equilibrium price of Schrecklich's is \_\_\_\_\_ and the equilibrium price of LaMerde's is \_\_\_\_\_.

(c) On the diagram below, draw a line that represents all combinations of prices for Schrecklich's and LaMerde's such that the supply of Schrecklich's equals the demand for Schrecklich's. Draw a second line that represents those price combinations at which the demand for LaMerde's equals the supply of LaMerde's. Label the unique price combination at which both markets clear with the letter  $E$ .

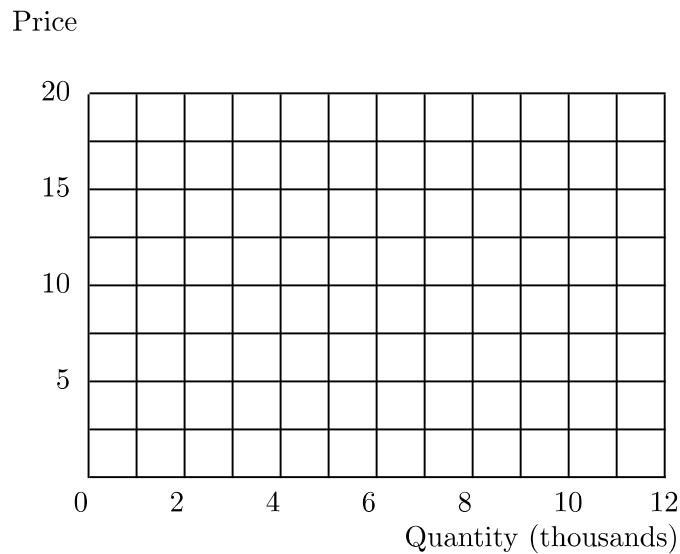


(d) A fire in a bowling alley in Hamtramck, Michigan, destroyed one of the world's largest collections of works by Schrecklich. The fire destroyed a total of 10 Schrecklichs. After the fire, the equilibrium price of Schrecklichs was \_\_\_\_\_ and the equilibrium price of LaMerdes was\_\_\_\_\_.

(e) On the diagram you drew above, use red ink to draw a line that shows the locus of price combinations at which the demand for Schrecklichs equals the supply of Schrecklichs after the fire. On your diagram, label the new equilibrium combination of prices  $E'$ .

**16.7 (0)** The price elasticity of demand for oatmeal is constant and equal to  $-1$ . When the price of oatmeal is \$10 per unit, the total amount demanded is 6,000 units.

(a) Write an equation for the demand function. \_\_\_\_\_  
 Graph this demand function below with blue ink. (Hint: If the demand curve has a constant price elasticity equal to  $\epsilon$ , then  $D(p) = ap^\epsilon$  for some constant  $a$ . You have to use the data of the problem to solve for the constants  $a$  and  $\epsilon$  that apply in this particular case.)



(b) If the supply is perfectly inelastic at 5,000 units, what is the equilibrium price? \_\_\_\_\_ Show the supply curve on your graph and label the equilibrium with an  $E$ .

(c) Suppose that the demand curve shifts outward by 10%. Write down the new equation for the demand function. \_\_\_\_\_ Suppose that the supply curve remains vertical but shifts to the right by 5%. Solve for the new equilibrium price \_\_\_\_\_ and quantity\_\_\_\_\_.

(d) By what percentage approximately did the equilibrium price rise?

\_\_\_\_\_ Use red ink to draw the new demand curve and the new supply curve on your graph.

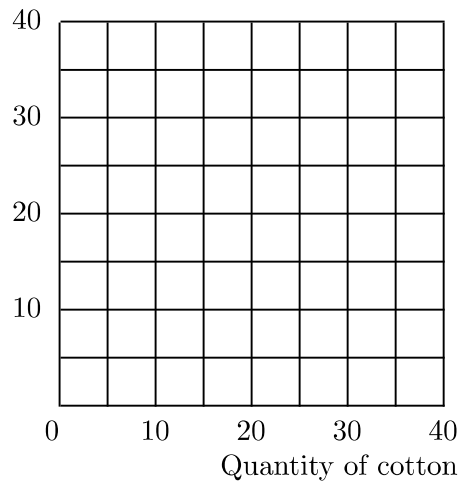
(e) Suppose that in the above problem the demand curve shifts outward by  $x\%$  and the supply curve shifts right by  $y\%$ . By approximately what percentage will the equilibrium price rise?\_\_\_\_\_.

**16.8 (0)** An economic historian\* reports that econometric studies indicate for the pre-Civil War period, 1820–1860, the price elasticity of demand for cotton from the American South was approximately  $-1$ . Due to the rapid expansion of the British textile industry, the demand curve for American cotton is estimated to have shifted outward by about 5% per year during this entire period.

(a) If during this period, cotton production in the United States grew by 3% per year, what (approximately) must be the rate of change of the price of cotton during this period?\_\_\_\_\_.

(b) Assuming a constant price elasticity of  $-1$ , and assuming that when the price is \$20, the quantity is also 20, graph the demand curve for cotton. What is the total revenue when the price is \$20? \_\_\_\_\_  
What is the total revenue when the price is \$10?\_\_\_\_\_.

Price of cotton



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\* Gavin Wright, *The Political Economy of the Cotton South*, W. W. Norton, 1978.

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(c) If the change in the quantity of cotton supplied by the United States is to be interpreted as a movement along an upward-sloping long-run supply curve, what would the elasticity of supply have to be? (Hint: From 1820 to 1860 quantity rose by about 3% per year and price rose by \_\_\_\_\_ % per year. [See your earlier answer.] If the quantity change is a movement along the long-run supply curve, then the long-run price elasticity must be what?)\_\_\_\_\_.

(d) The American Civil War, beginning in 1861, had a devastating effect on cotton production in the South. Production fell by about 50% and remained at that level throughout the war. What would you predict would be the effect on the price of cotton?\_\_\_\_\_.

(e) What would be the effect on total revenue of cotton farmers in the South?\_\_\_\_\_.

(f) The expansion of the British textile industry ended in the 1860s, and for the remainder of the nineteenth century, the demand curve for American cotton remained approximately unchanged. By about 1900, the South approximately regained its prewar output level. What do you think happened to cotton prices then?\_\_\_\_\_.

**16.9 (0)** The number of bottles of chardonnay demanded per year is  $\$1,000,000 - 60,000P$ , where  $P$  is the price per bottle (in U.S. dollars). The number of bottles supplied is  $40,000P$ .

(a) What is the equilibrium price? \_\_\_\_\_ What is the equilibrium quantity?\_\_\_\_\_.

(b) Suppose that the government introduces a new tax such that the wine maker must pay a tax of \$5 per bottle for every bottle that he produces. What is the new equilibrium price paid by consumers? \_\_\_\_\_  
\_\_\_\_\_ What is the new price received by suppliers? \_\_\_\_\_ What is the new equilibrium quantity?\_\_\_\_\_.

**16.10 (0)** The inverse demand function for bananas is  $P_d = 18 - 3Q_d$  and the inverse supply function is  $P_s = 6 + Q_s$ , where prices are measured in cents.

(a) If there are no taxes or subsidies, what is the equilibrium quantity?  
\_\_\_\_\_ What is the equilibrium market price?\_\_\_\_\_.

(b) If a subsidy of 2 cents per pound is paid to banana growers, then in equilibrium it still must be that the quantity demanded equals the quantity supplied, but now the price received by sellers is 2 cents higher than the price paid by consumers. What is the new equilibrium quantity?  
\_\_\_\_\_ What is the new equilibrium price received by suppliers?  
\_\_\_\_\_ What is the new equilibrium price paid by demanders?  
\_\_\_\_\_.

(c) Express the change in price as a percentage of the original price.  
\_\_\_\_\_ If the cross-elasticity of demand between bananas and apples is +.5, what will happen to the quantity of apples demanded as a consequence of the banana subsidy, if the price of apples stays constant?  
(State your answer in terms of percentage change.)\_\_\_\_\_.

**16.11 (1)** King Kanuta rules a small tropical island, Nutting Atoll, whose primary crop is coconuts. If the price of coconuts is  $P$ , then King Kanuta's subjects will demand  $D(P) = 1,200 - 100P$  coconuts per week for their own use. The number of coconuts that will be supplied per week by the island's coconut growers is  $S(p) = 100P$ .

(a) The equilibrium price of coconuts will be \_\_\_\_\_ and the equilibrium quantity supplied will be\_\_\_\_\_.

(b) One day, King Kanuta decided to tax his subjects in order to collect coconuts for the Royal Larder. The king required that every subject who consumed a coconut would have to pay a coconut to the king as a tax. Thus, if a subject wanted 5 coconuts for himself, he would have to purchase 10 coconuts and give 5 to the king. When the price that is received by the sellers is  $p_S$ , how much does it cost one of the king's subjects to get an extra coconut for himself?\_\_\_\_\_.

(c) When the price paid to suppliers is  $p_S$ , how many coconuts will the king's subjects demand for their own consumption? (Hint: Express  $p_D$  in terms of  $p_S$  and substitute into the demand function.)\_\_\_\_\_.

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(d) Since the king consumes a coconut for every coconut consumed by the subjects, the total amount demanded by the king and his subjects is twice the amount demanded by the subjects. Therefore, when the price received by suppliers is  $p_S$ , the total number of coconuts demanded per week by Kanuta and his subjects is\_\_\_\_\_.

(e) Solve for the equilibrium value of  $p_S$ \_\_\_\_\_, the equilibrium total number of coconuts produced\_\_\_\_\_, and the equilibrium total number of coconuts consumed by Kanuta's subjects.\_\_\_\_\_.

(f) King Kanuta's subjects resented paying the extra coconuts to the king, and whispers of revolution spread through the palace. Worried by the hostile atmosphere, the king changed the coconut tax. Now, the shopkeepers who sold the coconuts would be responsible for paying the tax. For every coconut sold to a consumer, the shopkeeper would have to pay one coconut to the king. This plan resulted in \_\_\_\_\_ coconuts being sold to the consumers. The shopkeepers got \_\_\_\_\_ per coconut after paying their tax to the king, and the consumers paid a price of \_\_\_\_\_ per coconut.

**16.12 (1)** On August 29, 2005, Hurricane Katrina caused severe damage to oil installations in the Gulf of Mexico. Although this damage could eventually be repaired, it resulted in a substantial reduction in the short run supply of gasoline in the United States. In many areas, retail gasoline prices quickly rose by about 30% to an average of \$3.06 per gallon.

Georgia governor Sonny Perdue suspended his state's 7.5 cents-a-gallon gas tax and 4% sales tax on gasoline purchases until Oct. 1. Governor Perdue explained that, "I believe it is absolutely wrong for the state to reap a tax windfall in this time of urgency and tragedy." Lawmakers in several other states were considering similar actions.

Let us apply supply and demand analysis to this problem. Before the hurricane, the United States consumed about 180 million gallons of gasoline per day, of which about 30 million gallons came from the Gulf of Mexico. In the short run, the supply of gasoline is extremely inelastic and is limited by refinery and transport capacity. Let us assume that the daily short run supply of gasoline was perfectly inelastic at 180 million gallons before the storm and perfectly inelastic at 150 million gallons after the storm. Suppose that the demand function, measured in millions of gallons per day, is given by  $Q = 240 - 30P$  where  $P$  is the dollar price, including tax, that consumers pay for gasoline.

(a) What was the market equilibrium price for gasoline before the hurricane? \_\_\_\_\_ After the hurricane?\_\_\_\_\_.

(b) Suppose that both before and after the hurricane, a government tax of 10 cents is charged for every gallon of gasoline sold in the United States. How much money would *suppliers* receive *per gallon* of gasoline before the hurricane? \_\_\_\_\_ After the hurricane?\_\_\_\_\_.

(c) Suppose that after the hurricane, the federal government removed the gas tax. What would then be the equilibrium price paid by consumers? \_\_\_\_\_ How much money would suppliers receive per gallon of gasoline? \_\_\_\_\_ How much revenue would the government lose per day by removing the tax? \_\_\_\_\_ What is the net effect of removing the tax on gasoline prices?\_\_\_\_\_ Who are the gainers and who are the losers from removing the tax?\_\_\_\_\_

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(d) Suppose that after the hurricane, the ten-cent tax is removed in some states but not in others. The states where the tax is removed constitute just half of the demand in the United States. Thus the demand schedule in each half of the country is  $Q = 120 - 15P$  where  $P$  is the price paid by consumers in that part of the country. Let  $P^*$  be the equilibrium price for consumers in the part of the country where the tax is removed. In equilibrium, suppliers must receive the same price per gallon in all parts of the country. Therefore the equilibrium price for consumers in states that keep the tax must be  $\$P^* + \$0.10$ . In equilibrium it must be that the total amount of gasoline demanded in the two parts of the country equals the total supply. Write an equation for total demand as a function of  $P^*$ . \_\_\_\_\_ Set demand equal to supply and solve for the price paid by consumers in the states that remove the tax \_\_\_\_\_ and for the price paid by consumers in states that do not remove the tax. \_\_\_\_\_ How much money do suppliers receive per gallon of gasoline sold in every state? \_\_\_\_\_ How does the tax removal affect daily gasoline consumption in each group of states?\_\_\_\_\_

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(e) If half of the states remove the gasoline tax, as described above, some groups will be better off and some worse off than they would be if the tax were left in place. Describe the gains or losses for each of the following groups.

Consumers in the states that remove the tax\_\_\_\_\_

\_\_\_\_\_.

Consumers in other states\_\_\_\_\_.

Gasoline suppliers\_\_\_\_\_.

Governments of the states that remove the tax\_\_\_\_\_

\_\_\_\_\_.

Governments of states that do not remove the tax\_\_\_\_\_

\_\_\_\_\_.