In this chapter you work with production functions, relating output of a firm to the inputs it uses. This theory will look familiar to you, because it closely parallels the theory of utility functions. In utility theory, an *indifference curve* is a locus of commodity bundles, all of which give a consumer the same utility. In production theory, an *isoquant* is a locus of input combinations, all of which give the same output. In consumer theory, you found that the slope of an indifference curve at the bundle (x_1, x_2) is the ratio of marginal utilities, $MU_1(x_1, x_2)/MU_2(x_1, x_2)$. In production theory, the slope of an isoquant at the input combination (x_1, x_2) is the ratio of the marginal products, $MP_1(x_1, x_2)/MP_2(x_1, x_2)$. Most of the functions that we gave as examples of utility functions can also be used as examples of production functions.

There is one important difference between production functions and utility functions. Remember that utility functions were only "unique up to monotonic transformations." In contrast, two different production functions that are monotonic transformations of each other describe different technologies.

If the utility function $U(x_1, x_2) = x_1 + x_2$ represents a person's preferences, then so would the utility function $U^*(x_1, x_2) = (x_1 + x_2)^2$. A person who had the utility function $U^*(x_1, x_2)$ would have the same indifference curves as a person with the utility function $U(x_1, x_2)$ and would make the same choices from every budget. But suppose that one firm has the production function $f(x_1, x_2) = x_1 + x_2$, and another has the production function $f^*(x_1, x_2) = (x_1 + x_2)^2$. It is true that the two firms will have the same isoquants, but they certainly do not have the same technology. If both firms have the input combination $(x_1, x_2) = (1, 1)$, then the first firm will have an output of 2 and the second firm will have an output of 4.

Now we investigate "returns to scale." Here we are concerned with the change in output if the amount of every input is multiplied by a number $t > 1$. If multiplying inputs by t multiplies output by more than t, then there are increasing returns to scale. If output is multiplied by exactly t, there are constant returns to scale. If output is multiplied by less than t, then there are decreasing returns to scale.

Consider the production function $f(x_1, x_2) = x_1^{1/2} x_2^{3/4}$. If we multiply the amount of each input by t, then output will be $f(tx_1, tx_2) = (tx_1)^{1/2}(tx_2)^{3/4}$. To compare $f(tx_1, tx_2)$ to $f(x_1, x_2)$, factor out the expressions involving t from the last equation. You get $f(tx_1, tx_2) = t^{5/4} x_1^{1/2} x_2^{3/4} = t^{5/4} f(x_1, x_2)$. Therefore when you multiply the amounts of all inputs by t, you multiply the amount of output by $t^{5/4}$. This means there are *increasing* returns to scale.

Let the production function be $f(x_1, x_2) = \min\{x_1, x_2\}$. Then

$$f(tx_1, tx_2) = \min\{tx_1, tx_2\} = \min t\{x_1, x_2\} = t\min\{x_1, x_2\} = tf(x_1, x_2).$$

Therefore when all inputs are multiplied by t, output is also multiplied by t. It follows that this production function has *constant* returns to scale.

You will also be asked to determine whether the marginal product of each single factor of production increases or decreases as you increase the amount of that factor without changing the amount of other factors. Those of you who know calculus will recognize that the marginal product of a factor is the first derivative of output with respect to the amount of that factor. Therefore the marginal product of a factor will decrease, increase, or stay constant as the amount of the factor increases depending on whether the *second* derivative of the production function with respect to the amount of that factor is negative, positive, or zero.

Consider the production function $f(x_1, x_2) = x_1^{1/2} x_2^{3/4}$. The marginal product of factor 1 is $\frac{1}{2} x_1^{-1/2} x_2^{3/4}$. This is a decreasing function of x_1, as you can verify by taking the derivative of the marginal product with respect to x_1. Similarly, you can show that the marginal product of x_2 decreases as x_2 increases.

18.0 Warm Up Exercise. The first part of this exercise is to calculate marginal products and technical rates of substitution for several frequently encountered production functions. As an example, consider the production function $f(x_1, x_2) = 2x_1 + \sqrt{x_2}$. The marginal product of x_1 is the derivative of $f(x_1, x_2)$ with respect to x_1, holding x_2 fixed. This is just 2. The marginal product of x_2 is the derivative of $f(x_1, x_2)$ with respect to x_2, holding x_1 fixed, which in this case is $\frac{1}{2\sqrt{x_2}}$. The TRS is $-MP_1/MP_2 = -4\sqrt{x_2}$. Those of you who do not know calculus should fill in this table from the answers in the back. The table will be a useful reference for later problems.

Marginal Products and Technical Rates of Substitution

$f(x_1, x_2)$	$MP_1(x_1, x_2)$	$MP_2(x_1, x_2)$	$TRS(x_1, x_2)$
$x_1 + 2x_2$			
$ax_1 + bx_2$			
$50x_1x_2$			
$x_1^{1/4}x_2^{3/4}$	$\frac{1}{4}x_1^{-3/4}x_2^{3/4}$		
$Cx_1^a x_2^b$	$Cax_1^{a-1}x_2^b$		
$(x_1 + 2)(x_2 + 1)$	$x_2 + 1$		
$(x_1 + a)(x_2 + b)$			
$ax_1 + b\sqrt{x_2}$			
$x_1^a + x_2^a$			
$(x_1^a + x_2^a)^b$	$bax_1^{a-1}\left(x_1^a + x_2^a\right)^{b-1}$	$bax_2^{a-1}\left(x_1^a + x_2^a\right)^{b-1}$	

# Returns to Scale and Changes in Marginal Products

For each production function in the table below, put an I, C, or D in the first column if the production function has increasing, constant, or decreasing returns to scale. Put an I, C, or D in the second (third) column, depending on whether the marginal product of factor 1 (factor 2) is increasing, constant, or decreasing, as the amount of that factor alone is varied.

$f(x_1, x_2)$	Scale	MP_1	MP_2
$x_1 + 2x_2$			
$\sqrt{x_1 + 2x_2}$			
$.2x_1 x_2^2$			
$x_1^{1/4} x_2^{3/4}$			
$x_1 + \sqrt{x_2}$			
$(x_1 + 1)^{.5}(x_2)^{.5}$			
$\left(x_1^{1/3} + x_2^{1/3}\right)^3$			

18.1 (0) Prunella raises peaches. Where L is the number of units of labor she uses and T is the number of units of land she uses, her output is $f(L, T) = L^{\frac{1}{2}} T^{\frac{1}{2}}$ bushels of peaches.

(a) On the graph below, plot some input combinations that give her an output of 4 bushels. Sketch a production isoquant that runs through these points. The points on the isoquant that gives her an output of 4 bushels all satisfy the equation $T =$ _____.

T
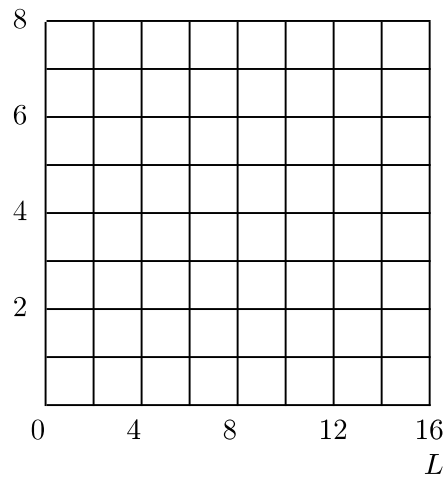
(b) This production function exhibits (constant, increasing, decreasing)

returns to scale._____.

(c) In the short run, Prunella cannot vary the amount of land she uses. On the graph below, use blue ink to draw a curve showing Prunella's output as a function of labor input if she has 1 unit of land. Locate the points on your graph at which the amount of labor is 0, 1, 4, 9, and 16 and

label them. The slope of this curve is known as the marginal _____

of _____ Is this curve getting steeper or flatter as the amount

of labor increase?_____.

Output
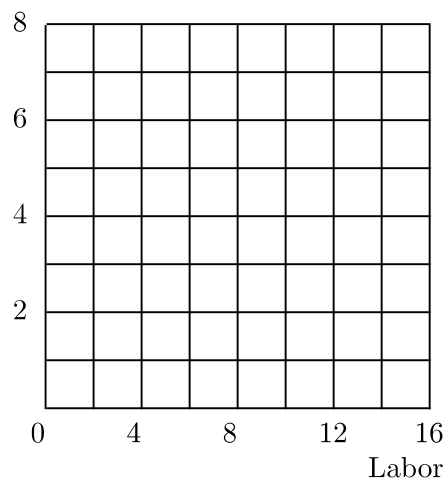
Labor

(d) Assuming she has 1 unit of land, how much extra output does she get from adding an extra unit of labor when she previously used 1 unit

of labor? _____ 4 units of labor? _____ If you know calculus, compute the marginal product of labor at the input combination $(1, 1)$ and compare it with the result from the unit increase

in labor output found above._____

_____ .

(e) In the long run, Prunella can change her input of land as well as of labor. Suppose that she increases the size of her orchard to 4 units of land. Use red ink to draw a new curve on the graph above showing output as a function of labor input. Also use red ink to draw a curve showing marginal product of labor as a function of labor input when the amount of land is fixed at 4.

18.2 (0) Suppose x_1 and x_2 are used in fixed proportions and $f(x_1, x_2) = \min\{x_1, x_2\}$.

(a) Suppose that $x_1 < x_2$. The marginal product for x_1 is _____ and

(increases, remains constant, decreases)_____ for small

increases in x_1. For x_2 the marginal product is _____ , and (increases,

remains constant, decreases)_____ for small increases in

x_2. The technical rate of substitution between x_2 and x_1 is _____

_____ This technology demonstrates (increasing, constant, decreasing)

_____ returns to scale.

(b) Suppose that $f(x_1, x_2) = \min\{x_1, x_2\}$ and $x_1 = x_2 = 20$. What is

the marginal product of a small increase in x_1? _____ What is the

marginal product of a small increase in x_2? _____ The marginal

product of x_1 will (increase, decrease, stay constant)_____ if the amount of x_2 is increased by a little bit.

18.3 (0) Suppose the production function is Cobb-Douglas and $f(x_1, x_2) = x_1^{1/2} x_2^{3/2}$.

(a) Write an expression for the marginal product of x_1 at the point

(x_1, x_2)._____ .

(b) The marginal product of x_1 (increases, decreases, remains constant)

_____ for small increases in x_1, holding x_2 fixed.

(c) The marginal product of factor 2 is _____ , and it (increases,

remains constant, decreases)_____ for small increases in x_2.

(d) An increase in the amount of x_2 (increases, leaves unchanged, de-

creases) _____ the marginal product of x_1.

(e) The technical rate of substitution between x_2 and x_1 is_____.

(f) Does this technology have diminishing technical rate of substitution?

_____.

(g) This technology demonstrates (increasing, constant, decreasing)

_____ returns to scale.

18.4 (0) The production function for fragles is $f(K, L) = L/2 + \sqrt{K}$,
where L is the amount of labor used and K the amount of capital used.

(a) There are (constant, increasing, decreasing)_____ returns

to scale. The marginal product of labor is _____ (constant,
increasing, decreasing).

(b) In the short run, capital is fixed at 4 units. Labor is variable. On the
graph below, use blue ink to draw output as a function of labor input in
the short run. Use red ink to draw the marginal product of labor as a
function of labor input in the short run. The average product of labor is
defined as total output divided by the amount of labor input. Use black
ink to draw the average product of labor as a function of labor input in
the short run.

Fragles
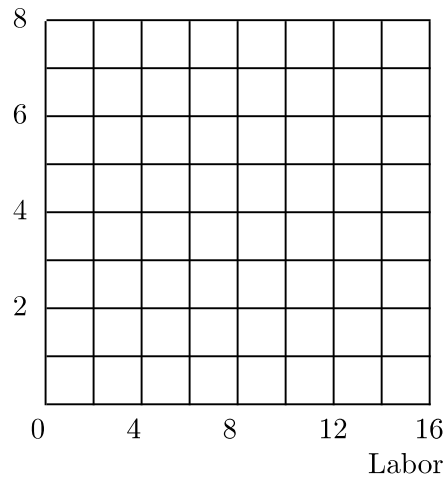
18.5 (0) General Monsters Corporation has two plants for producing juggernauts, one in Flint and one in Inkster. The Flint plant produces according to $f_F(x_1, x_2) = \min\{x_1, 2x_2\}$ and the Inkster plant produces according to $f_I(x_1, x_2) = \min\{2x_1, x_2\}$, where x_1 and x_2 are the inputs.

(a) On the graph below, use blue ink to draw the isoquant for 40 juggernauts at the Flint plant. Use red ink to draw the isoquant for producing 40 juggernauts at the Inkster plant.

x_2

x_1
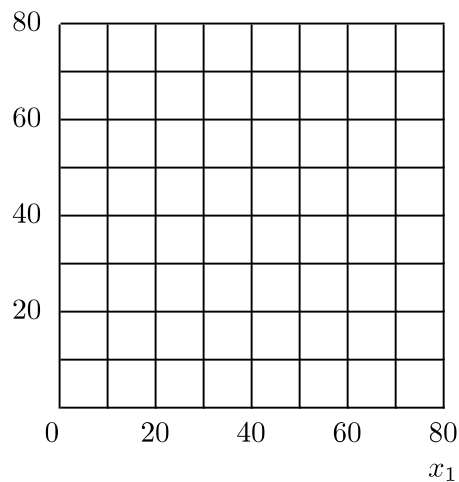
(b) Suppose that the firm wishes to produce 20 juggernauts at each plant. How much of each input will the firm need to produce 20 juggernauts at

the Flint plant? _____ How much of each input will the firm

need to produce 20 juggernauts at the Inkster plant? _____
Label with an *a* on the graph, the point representing the total amount of each of the two inputs that the firm needs to produce a total of 40 juggernauts, 20 at the Flint plant and 20 at the Inkster plant.

(c) Label with a *b* on your graph the point that shows how much of each of the two inputs is needed in toto if the firm is to produce 10 juggernauts in the Flint plant and 30 juggernauts in the Inkster plant. Label with a *c* the point that shows how much of each of the two inputs that the firm needs in toto if it is to produce 30 juggernauts in the Flint plant and 10 juggernauts in the Inkster plant. Use a black pen to draw the firm's isoquant for producing 40 units of output if it can split production in any manner between the two plants. Is the technology available to this firm

convex?_____.

18.6 (0) You manage a crew of 160 workers who could be assigned to make either of two products. Product A requires 2 workers per unit of output. Product B requires 4 workers per unit of output.

(a) Write an equation to express the combinations of products A and B

that could be produced using exactly 160 workers. _____ On the diagram below, use blue ink to shade in the area depicting the combinations of A and B that could be produced with 160 workers. (Assume that it is also possible for some workers to do nothing at all.)

B
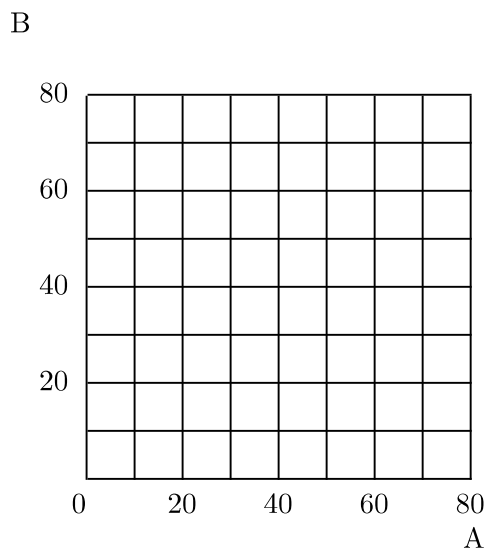
(b) Suppose now that every unit of product A that is produced requires the use of 4 shovels as well as 2 workers and that every unit of product B produced requires 2 shovels and 4 workers. On the graph you have just drawn, use red ink to shade in the area depicting combinations of A and B that could be produced with 180 shovels if there were no worries about the labor supply. Write down an equation for the set of combinations of

A and B that require exactly 180 shovels._____.

(c) On the same diagram, use black ink to shade the area that represents possible output combinations when one takes into account both the limited supply of labor and the limited supply of shovels.

(d) On your diagram locate the feasible combination of inputs that use up all of the labor and all of the shovels. If you didn't have the graph,

what equations would you solve to determine this point?_____

_____.

(e) If you have 160 workers and 180 shovels, what is the largest amount of

product *A* that you could produce? _____ If you produce this amount, you will not use your entire supply of one of the inputs. Which

one? _____ How many will be left unused?_____.

18.7 (0) A firm has the production function $f(x, y) = \min\{2x, x + y\}$. On the graph below, use red ink to sketch a couple of production isoquants for this firm. A second firm has the production function $f(x, y) = x + \min\{x, y\}$. Do either or both of these firms have constant returns to scale?

_____ On the same graph, use black ink to draw a couple of isoquants for the second firm.
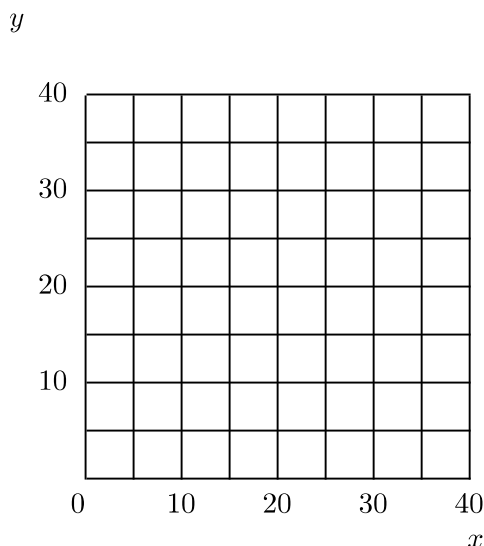
18.8 (0) Suppose the production function has the form

$$f(x_1, x_2, x_3) = Ax_1^a x_2^b x_3^c,$$

where $a + b + c > 1$. Prove that there are increasing returns to scale.

_____.

18.9 (0) Suppose that the production function is $f(x_1, x_2) = Cx_1^a x_2^b$, where a, b, and C are positive constants.

(a) For what positive values of a, b, and C are there decreasing returns to scale?_____ constant returns to scale? _____

_____ increasing returns to scale?_____.

(b) For what positive values of a, b, and C is there decreasing marginal product for factor 1?_____.

(c) For what positive values of a, b, and C is there diminishing technical rate of substitution?_____.

18.10 (0) Suppose that the production function is $f(x_1, x_2) = (x_1^a + x_2^a)^b$, where a and b are positive constants.

(a) For what positive values of a and b are there decreasing returns to scale?_____ Constant returns to scale? _____ Increasing returns to scale?_____.

18.11 (0) Suppose that a firm has the production function $f(x_1, x_2) = \sqrt{x_1} + x_2^2$.

(a) The marginal product of factor 1 (increases, decreases, stays constant) _____ as the amount of factor 1 increases. The marginal product of factor 2 (increases, decreases, stays constant) _____ as the amount of factor 2 increases.

(b) This production function does not satisfy the definition of increasing returns to scale, constant returns to scale, or decreasing returns to scale.

How can this be? _____

_____ Find a combination of inputs such that doubling the amount of both inputs will more than double the amount

of output. _____ Find a combination of inputs such that doubling the amount of both inputs will less than double output.

_____.

A firm in a competitive industry cannot charge more than the market price for its output. If it also must compete for its inputs, then it has to pay the market price for inputs as well. Suppose that a profit-maximizing competitive firm can vary the amount of only one factor and that the marginal product of this factor decreases as its quantity increases. Then the firm will maximize its profits by hiring enough of the variable factor so that the value of its marginal product is equal to the wage. Even if a firm uses several factors, only some of them may be variable in the short run.

A firm has the production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Suppose that this firm is using 16 units of factor 2 and is unable to vary this quantity in the short run. In the short run, the only thing that is left for the firm to choose is the amount of factor 1. Let the price of the firm's output be p, and let the price it pays per unit of factor 1 be w_1. We want to find the amount of x_1 that the firm will use and the amount of output it will produce. Since the amount of factor 2 used in the short run must be 16, we have output equal to $f(x_1, 16) = 4x_1^{1/2}$. The marginal product of x_1 is calculated by taking the derivative of output with respect to x_1. This marginal product is equal to $2x_1^{-1/2}$. Setting the value of the marginal product of factor 1 equal to its wage, we have $p2x_1^{-1/2} = w_1$. Now we can solve this for x_1. We find $x_1 = (2p/w_1)^2$. Plugging this into the production function, we see that the firm will choose to produce $4x_1^{1/2} = 8p/w_1$ units of output.

In the long run, a firm is able to vary all of its inputs. Consider the case of a competitive firm that uses two inputs. Then if the firm is maximizing its profits, it must be that the value of the marginal product of each of the two factors is equal to its wage. This gives two equations in the two unknown factor quantities. If there are decreasing returns to scale, these two equations are enough to determine the two factor quantities. If there are constant returns to scale, it turns out that these two equations are only sufficient to determine the *ratio* in which the factors are used.

In the problems on the weak axiom of profit maximization, you are asked to determine whether the observed behavior of firms is consistent with profit-maximizing behavior. To do this you will need to plot some of the firm's isoprofit lines. An isoprofit line relates all of the input-output combinations that yield the same amount of profit for some given input and output prices. To get the equation for an isoprofit line, just write down an equation for the firm's profits at the given input and output prices. Then solve it for the amount of output produced as a function of the amount of the input chosen. Graphically, you know that a firm's behavior is consistent with profit maximization if its input-output choice in each period lies below the isoprofit lines of the other periods.

19.1 (0) The short-run production function of a competitive firm is given by $f(L) = 6L^{2/3}$, where L is the amount of labor it uses. (For those who do not know calculus—if total output is aL^b, where a and b are constants, and where L is the amount of some factor of production, then the marginal product of L is given by the formula abL^{b-1}.) The cost per unit of labor is $w = 6$ and the price per unit of output is $p = 3$.

(a) Plot a few points on the graph of this firm's production function and sketch the graph of the production function, using blue ink. Use black ink to draw the isoprofit line that passes through the point $(0, 12)$, the isoprofit line that passes through $(0, 8)$, and the isoprofit line that passes through the point $(0, 4)$. What is the slope of each of the isoprofit lines?

_____ How many points on the isoprofit line through

$(0, 12)$ consist of input-output points that are actually possible?_____

_____ Make a squiggly line over the part of the isoprofit line through $(0, 4)$ that consists of outputs that are actually possible.

(b) How many units of labor will the firm hire? _____ How much

output will it produce? _____ If the firm has no other costs, how much

will its total profits be?_____.

      Output
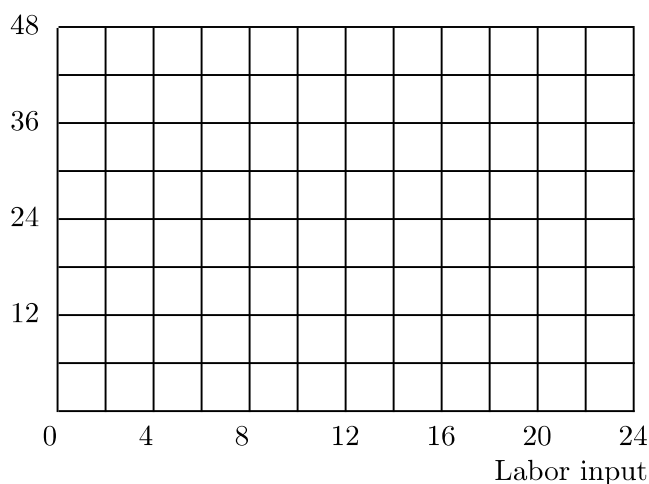
(c) Suppose that the wage of labor falls to 4, and the price of output remains at p. On the graph, use red ink to draw the new isoprofit line for the firm that passes through its old choice of input and output. Will

the firm increase its output at the new price?_____ Explain why,

referring to your diagram._____

_____.

19.2 (0) A Los Angeles firm uses a single input to produce a recreational commodity according to a production function $f(x) = 4\sqrt{x}$, where x is the number of units of input. The commodity sells for \$100 per unit. The input costs \$50 per unit.

(a) Write down a function that states the firm's profit as a function of

the amount of input._____.

(b) What is the profit-maximizing amount of input? _____ of output?

_____ How much profits does it make when it maximizes profits?

_____.

(c) Suppose that the firm is taxed \$20 per unit of its output and the price of its input is subsidized by \$10. What is its new input level?

_____ What is its new output level? _____ How much profit does

it make now?_____ (Hint: A good way to solve this is to write an expression for the firm's profit as a function of its input and solve for the profit-maximizing amount of input.)

(d) Suppose that instead of these taxes and subsidies, the firm is taxed at 50% of its profits. Write down its after-tax profits as a function

of the amount of input._____ What is the profit-

maximizing amount of output? _____ How much profit does it make

after taxes?_____.

19.3 (0) Brother Jed takes heathens and reforms them into righteous individuals. There are two inputs needed in this process: heathens (who are widely available) and preaching. The production function has the following form: $r_p = \min\{h, p\}$, where r_p is the number of righteous persons produced, h is the number of heathens who attend Jed's sermons, and p is the number of hours of preaching. For every person converted, Jed receives a payment of s from the grateful convert. Sad to say, heathens do not flock to Jed's sermons of their own accord. Jed must offer heathens a payment of w to attract them to his sermons. Suppose the amount of preaching is fixed at $\bar{p}$ and that Jed is a profit-maximizing prophet.

(a) If $h < \bar{p}$, what is the marginal product of heathens? _____ What is the value of the marginal product of an additional heathen? _____.

(b) If $h > \bar{p}$, what is the marginal product of heathens? _____ What is the value of the marginal product of an additional heathen in this case?

_____.

(c) Sketch the shape of this production function in the graph below. Label the axes, and indicate the amount of the input where $h = \bar{p}$.
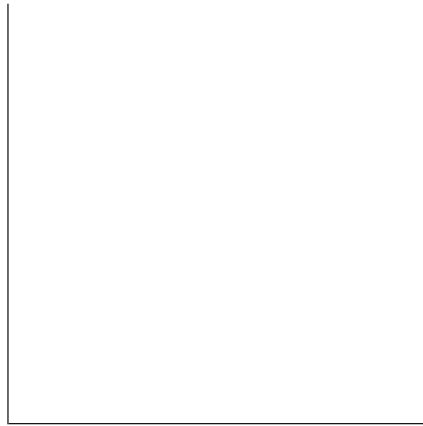
(d) If $w < s$, how many heathens will be converted? _____ If $w > s$, how many heathens will be converted? _____.

19.4 (0) Allie's Apples, Inc. purchases apples in bulk and sells two products, boxes of apples and jugs of cider. Allie's has capacity limitations of three kinds: warehouse space, crating facilities, and pressing facilities. A box of apples requires 6 units of warehouse space, 2 units of crating facilities, and no pressing facilities. A jug of cider requires 3 units of warehouse space, 2 units of crating facilities, and 1 unit of pressing facilities. The total amounts available each day are: 1,200 units of warehouse space, 600 units of crating facilities, and 250 units of pressing facilities.

(a) If the only capacity limitations were on warehouse facilities, and if all warehouse space were used for the production of apples, how many boxes of apples could be produced in one day? _____ How many jugs of cider could be produced each day if, instead, all warehouse space were used in the production of cider and there were no other capacity constraints? _____ Draw a blue line in the following graph to represent the warehouse space constraint on production combinations.

(b) Following the same reasoning, draw a red line to represent the constraints on output to limitations on crating capacity. How many boxes of apples could Allie produce if he only had to worry about crating capacity?

_____ How many jugs of cider?_____.

(c) Finally draw a black line to represent constraints on output combinations due to limitations on pressing facilities. How many boxes of apples could Allie produce if he only had to worry about the pressing capacity and no other constraints? _____ How many jugs of

cider?_____.

(d) Now shade the area that represents feasible combinations of daily production of apples and cider for Allie's Apples.

Cider
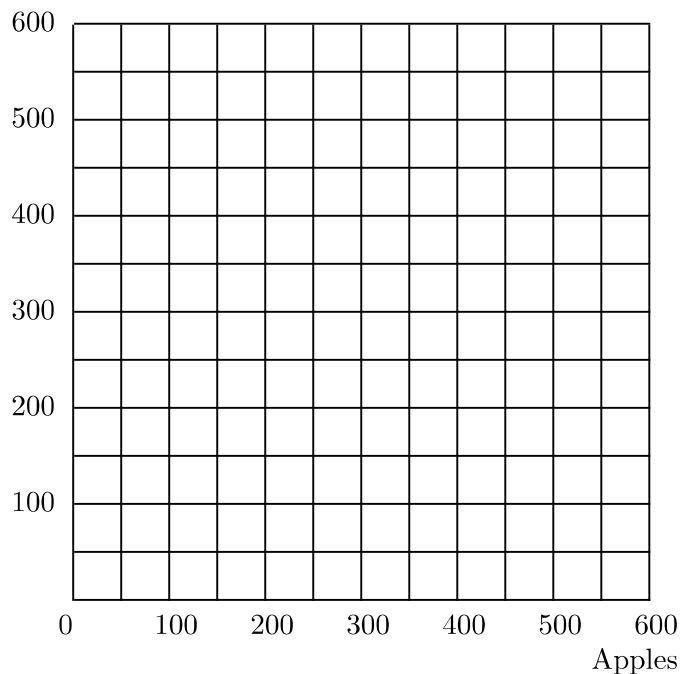
Apples

(e) Allie's can sell apples for \$5 per box of apples and cider for \$2 per jug. Draw a black line to show the combinations of sales of apples and cider that would generate a revenue of \$1,000 per day. At the profit-

maximizing production plan, Allie's is producing _____ boxes of apples

and _____ jugs of cider. Total revenues are_____.

19.5 (0) A profit-maximizing firm produces one output, y, and uses one input, x, to produce it. The price per unit of the factor is denoted by
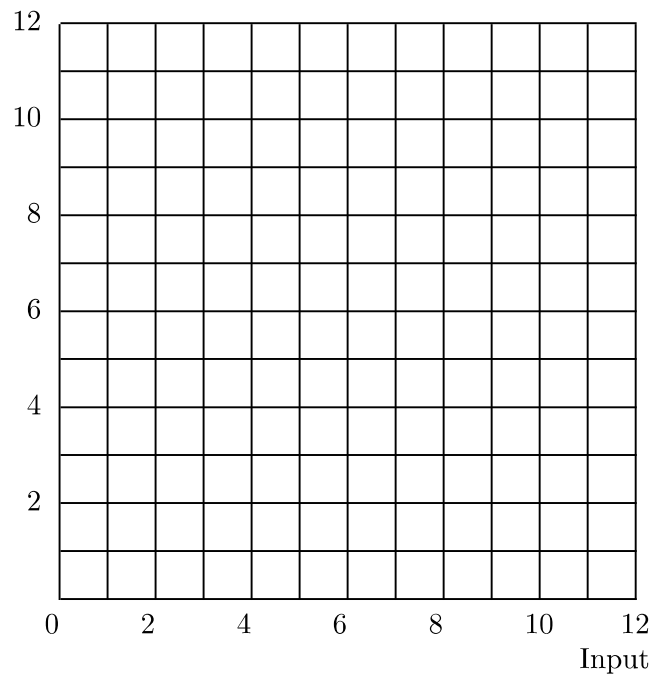
w and the price of the output is denoted by p. You observe the firm's behavior over three periods and find the following:

Period	y	x	w	p
1	1	1	1	1
2	2.5	3	.5	1
3	4	8	.25	1

(a) Write an equation that gives the firm's profits, π, as a function of the amount of input x it uses, the amount of output y it produces, the

per-unit cost of the input w, and the price of output p._____.

(b) In the diagram below, draw an isoprofit line for each of the three periods, showing combinations of input and output that would yield the same profits that period as the combination actually chosen. What are the

equations for these three lines?_____ Using the theory of revealed profitability, shade in the region on the graph that represents input-output combinations that could be feasible as far as one can tell from the evidence that is available. How would you describe this
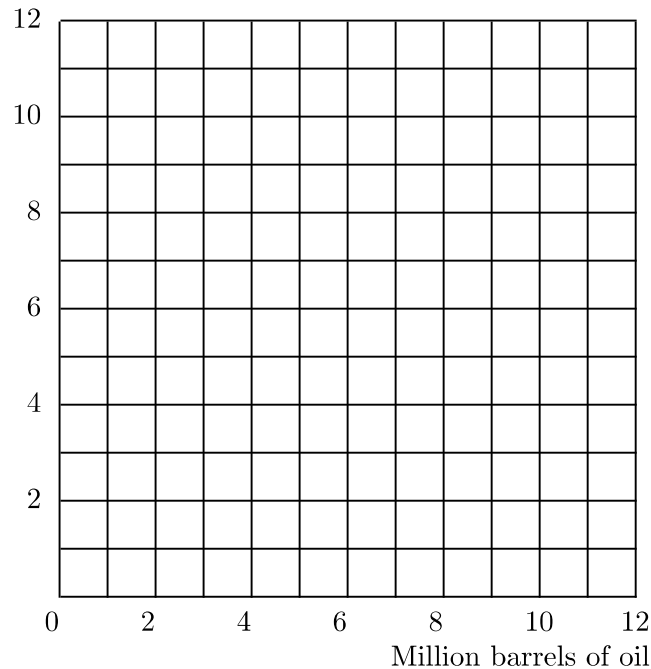
region in words?_____.

Output

19.6 (0) T-bone Pickens is a corporate raider. This means that he looks for companies that are not maximizing profits, buys them, and then tries to operate them at higher profits. T-bone is examining the financial records of two refineries that he might buy, the Shill Oil Company and the Golf Oil Company. Each of these companies buys oil and produces gasoline. During the time period covered by these records, the price of gasoline fluctuated significantly, while the cost of oil remained constant at $10 a barrel. For simplicity, we assume that oil is the only input to gasoline production.

Shill Oil produced 1 million barrels of gasoline using 1 million barrels of oil when the price of gasoline was $10 a barrel. When the price of gasoline was $20 a barrel, Shill produced 3 million barrels of gasoline using 4 million barrels of oil. Finally, when the price of gasoline was $40 a barrel, Shill used 10 million barrels of oil to produce 5 million barrels of gasoline.

Golf Oil (which is managed by Martin E. Lunch III) did exactly the same when the price of gasoline was $10 and $20, but when the price of gasoline hit $40, Golf produced 3.5 million barrels of gasoline using 8 million barrels of oil.

(a) Using black ink, plot Shill Oil's isoprofit lines and choices for the three different periods. Label them 10, 20, and 40. Using red ink draw Golf Oil's isoprofit line and production choice. Label it with a 40 in red ink.

Million barrels of gasoline

Million barrels of oil

(b) How much profits could Golf Oil have made when the price of gasoline was \$40 a barrel if it had chosen to produce the same amount that it did when the price was \$20 a barrel? _____ What profits did Golf actually make when the price of gasoline was \$40?_____.

(c) Is there any evidence that Shill Oil is not maximizing profits? Explain.

_____.

(d) Is there any evidence that Golf Oil is not maximizing profits? Explain.

_____

_____.

19.7 (0) After carefully studying Shill Oil, T-bone Pickens decides that it has probably been maximizing its profits. But he still is very interested in buying Shill Oil. He wants to use the gasoline they produce to fuel his delivery fleet for his chicken farms, Capon Truckin'. In order to do this Shill Oil would have to be able to produce 5 million barrels of gasoline from 8 million barrels of oil. Mark this point on your graph. Assuming that Shill always maximizes profits, would it be technologically feasible for it to produce this input-output combination? Why or why not?

_____

_____.

19.8 (0) Suppose that firms operate in a competitive market, attempt to maximize profits, and only use one factor of production. Then we know that for any changes in the input and output price, the input choice and the output choice must obey the Weak Axiom of Profit Maximization, $\Delta p \Delta y - \Delta w \Delta x \geq 0$.

Which of the following propositions can be proven by the Weak Axiom of Profit Maximizing Behavior (WAPM)? Respond yes or no, and give a short argument.

(a) If the price of the input does not change, then a decrease in the price of the output will imply that the firm will produce the same amount or less output._____

_____.

(b) If the price of the output remains constant, then a decrease in the input price will imply that the firm will use the same amount or more of the input._____

_____.

(c) If both the price of the output and the input increase and the firm produces less output, then the firm will use more of the input._____

_____.

19.9 (1) Farmer Hoglund has discovered that on his farm, he can get 30 bushels of corn per acre if he applies no fertilizer. When he applies N pounds of fertilizer to an acre of land, the *marginal product* of fertilizer is $1 - N/200$ bushels of corn per pound of fertilizer.

(a) If the price of corn is \$3 a bushel and the price of fertilizer is \$$p$ per pound (where $p < 3$), how many pounds of fertilizer should he use per acre in order to maximize profits?_____.

(b) (Only for those who remember a bit of easy integral calculus.) Write down a function that states Farmer Hoglund's yield per acre as a function of the amount of fertilizer he uses._____.

(c) Hoglund's neighbor, Skoglund, has better land than Hoglund. In fact, for any amount of fertilizer that he applies, he gets exactly twice as much corn per acre as Hoglund would get with the same amount of fertilizer. How much fertilizer will Skoglund use per acre when the price of corn is \$3 a bushel and the price of fertilizer is \$$p$ a pound? _____
(Hint: Start by writing down Skoglund's marginal product of fertilizer as a function of N.)

(d) When Hoglund and Skoglund are both maximizing profits, will Skoglund's output be more than twice as much, less than twice as much or exactly twice as much as Hoglund's? Explain._____

_____

_____.

(e) Explain how someone who looked at Hoglund's and Skoglund's corn yields and their fertilizer inputs but couldn't observe the quality of their land, would get a misleading idea of the productivity of fertilizer._____

_____.

19.10 (0) A firm has two variable factors and a production function, $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$. The price of its output is 4. Factor 1 receives a wage of w_1 and factor 2 receives a wage of w_2.

(a) Write an equation that says that the value of the marginal product of factor 1 is equal to the wage of factor 1 _____ and an equation that says that the value of the marginal product of factor 2 is equal to the wage of factor 2._____ Solve two equations in the two unknowns, x_1 and x_2, to give the amounts of factors 1 and 2 that maximize the firm's profits as a function of w_1 and w_2. This gives

$x_1 =$_____ and $x_2 =$_____ (Hint: You could use the first equation to solve for x_1 as a function of x_2 and of the factor wages. Then substitute the answer into the second equation and solve for x_2 as a function of the two wage rates. Finally use your solution for x_2 to find the solution for x_1.)

(b) If the wage of factor 1 is 2, and the wage of factor 2 is 1, how many units of factor 1 will the firm demand?_____ How many units of factor 2 will it demand?_____ How much output will it produce?

_____ How much profit will it make?_____.

19.11 (0) A firm has two variable factors and a production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. The price of its output is 4, the price of factor 1 is w_1, and the price of factor 2 is w_2.
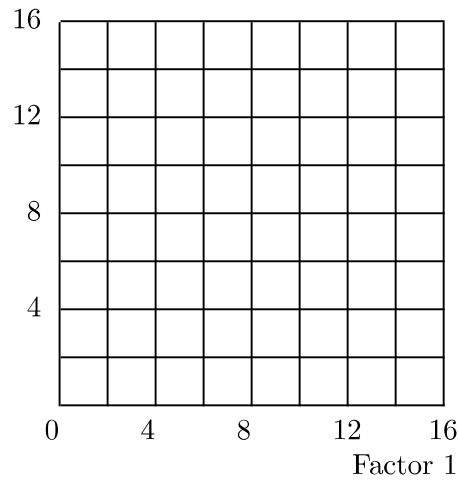
(a) Write the two equations that say that the value of the marginal product of each factor is equal to its wage. _____

_____ If $w_1 = 2w_2$, these two equations imply that $x_1/x_2 =$_____.

(b) For this production function, is it possible to solve the two marginal productivity equations uniquely for x_1 and x_2?_____.

19.12 (1) A firm has two variable factors and a production function $f(x_1, x_2) = \sqrt{2x_1 + 4x_2}$. On the graph below, draw production isoquants corresponding to an ouput of 3 and to an output of 4.

(a) If the price of the output good is 4, the price of factor 1 is 2, and the price of factor 2 is 3, find the profit-maximizing amount of factor 1

_____ , the profit-maximizing amount of factor 2_____ , and the
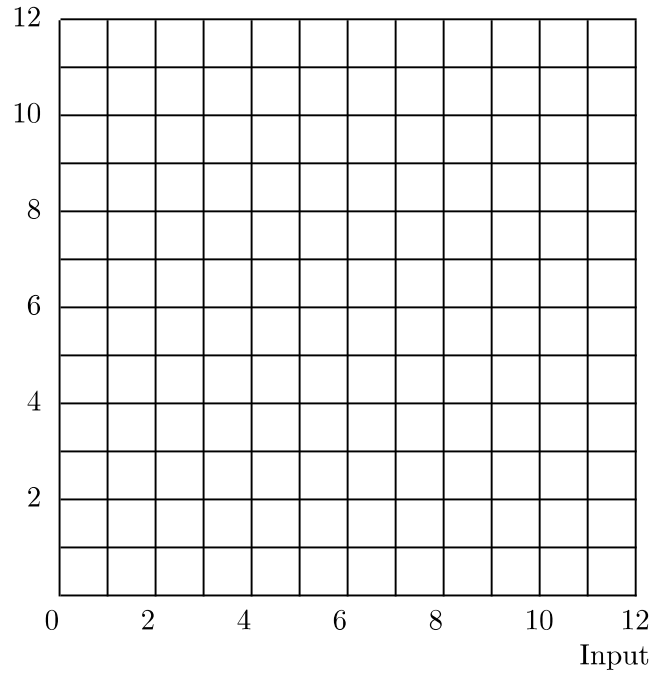
profit-maximizing output_____.

Factor 2

0   4   8   12   16
Factor 1

19.13 (0) A profit-maximizing firm produces one output, y, and uses one input, x, to produce it. The price per unit of the factor is denoted by w and the price of the output is denoted by p. You observe the firm's behavior over three periods and find the following:

Period	y	x	w	p
1	1	1	1	1
2	2.5	3	.5	1
3	4	8	.25	1

(a) Write an equation that gives the firm's profits, π, as a function of the amount of input x it uses, the amount of output y it produces, the

per-unit cost of the input w, and the price of output p.＿＿＿＿＿＿＿＿.

(b) In the diagram below, draw an isoprofit line for each of the three periods, showing combinations of input and output that would yield the same profits that period as the combination actually chosen. What are the

equations for these three lines?＿＿＿＿＿＿＿＿＿＿＿＿＿ Using the theory of revealed profitability, shade in the region on the graph that represents input-output combinations that could be feasible as far as one can tell from the evidence that is available. How would you describe this
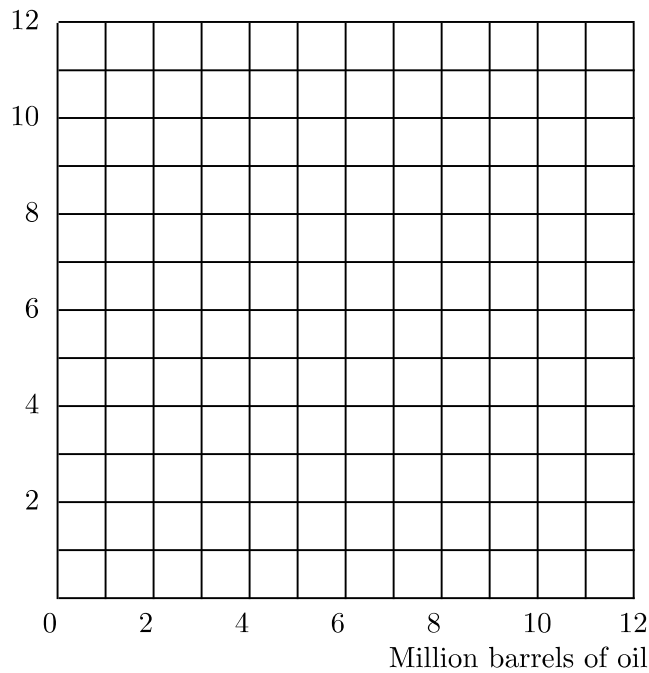
region in words?＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿.

Output

Input

19.14 (0) T-bone Pickens is a corporate raider. This means that he looks for companies that are not maximizing profits, buys them, and then tries to operate them at higher profits. T-bone is examining the financial records of two refineries that he might buy, the Shill Oil Company and the Golf Oil Company. Each of these companies buys oil and produces gasoline. During the time period covered by these records, the price of gasoline fluctuated significantly, while the cost of oil remained constant at $10 a barrel. For simplicity, we assume that oil is the only input to gasoline production.

Shill Oil produced 1 million barrels of gasoline using 1 million barrels of oil when the price of gasoline was $10 a barrel. When the price of gasoline was $20 a barrel, Shill produced 3 million barrels of gasoline using 4 million barrels of oil. Finally, when the price of gasoline was $40 a barrel, Shill used 10 million barrels of oil to produce 5 million barrels of gasoline.

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(a) Using black ink, plot Shill Oil's isoprofit lines and choices for the three different periods. Label them 10, 20, and 40. Using red ink draw Golf Oil's isoprofit line and production choice. Label it with a 40 in red ink.

Million barrels of gasoline

Million barrels of oil

(b) How much profits could Golf Oil have made when the price of gasoline was $40 a barrel if it had chosen to produce the same amount that it did when the price was $20 a barrel? _____ What profits did Golf actually make when the price of gasoline was $40?_____.

(c) Is there any evidence that Shill Oil is not maximizing profits? Explain.

_____.

(d) Is there any evidence that Golf Oil is not maximizing profits? Explain.

_____

_____.

19.15 (0) After carefully studying Shill Oil, T-bone Pickens decides that it has probably been maximizing its profits. But he still is very interested in buying Shill Oil. He wants to use the gasoline they produce to fuel his delivery fleet for his chicken farms, Capon Truckin'. In order to do this Shill Oil would have to be able to produce 5 million barrels of gasoline from 8 million barrels of oil. Mark this point on your graph. Assuming that Shill always maximizes profits, would it be technologically feasible for it to produce this input-output combination? Why or why not?

19.16 (0) Suppose that firms operate in a competitive market, attempt to maximize profits, and only use one factor of production. Then we know that for any changes in the input and output price, the input choice and the output choice must obey the Weak Axiom of Profit Maximization, $\Delta p \Delta y - \Delta w \Delta x \geq 0$.

Which of the following propositions can be proven by the Weak Axiom of Profit Maximizing Behavior (WAPM)? Respond yes or no, and give a short argument.

(a) If the price of the input does not change, then a decrease in the price of the output will imply that the firm will produce the same amount

or less output._____

_____.

(b) If the price of the output remains constant, then a decrease in the input price will imply that the firm will use the same amount or

more of the input._____

_____.

(c) If both the price of the output and the input increase and the firm

produces less output, then the firm will use more of the input._____

_____.

In the chapter on consumer choice, you studied a consumer who tries to maximize his utility subject to the constraint that he has a fixed amount of money to spend. In this chapter you study the behavior of a firm that is trying to produce a fixed amount of output in the cheapest possible way. In both theories, you look for a point of tangency between a curved line and a straight line. In consumer theory, there is an "indifference curve" and a "budget line." In producer theory, there is a "production isoquant" and an "isocost line." As you recall, in consumer theory, finding a tangency gives you only one of the two equations you need to locate the consumer's chosen point. The second equation you used was the budget equation. In cost-minimization theory, again the tangency condition gives you one equation. This time you don't know in advance how much the producer is spending; instead you are told how much output he wants to produce and must find the cheapest way to produce it. So your second equation is the equation that tells you that the desired amount is being produced.

Example. A firm has the production function $f(x_1, x_2) = (\sqrt{x_1} + 3\sqrt{x_2})^2$. The price of factor 1 is $w_1 = 1$ and the price of factor 2 is $w_2 = 1$. Let us find the cheapest way to produce 16 units of output. We will be looking for a point where the technical rate of substitution equals $-w_1/w_2$. If you calculate the technical rate of substitution (or look it up from the warm up exercise in Chapter 18), you find $TRS(x_1, x_2) = -(1/3)(x_2/x_1)^{1/2}$. Therefore we must have $-(1/3)(x_2/x_1)^{1/2} = -w_1/w_2 = -1$. This equation can be simplified to $x_2 = 9x_1$. So we know that the combination of inputs chosen has to lie somewhere on the line $x_2 = 9x_1$. We are looking for the cheapest way to produce 16 units of output. So the point we are looking for must satisfy the equation $(\sqrt{x_1} + 3\sqrt{x_2})^2 = 16$, or equivalently $\sqrt{x_1} + 3\sqrt{x_2} = 4$. Since $x_2 = 9x_1$, we can substitute for x_2 in the previous equation to get $\sqrt{x_1} + 3\sqrt{9x_1} = 4$. This equation simplifies further to $10\sqrt{x_1} = 4$. Solving this for x_1, we have $x_1 = 16/100$. Then $x_2 = 9x_1 = 144/100$.

The amounts x_1 and x_2 that we solved for in the previous paragraph are known as the *conditional factor demands for factors 1 and 2*, conditional on the wages $w_1 = 1$, $w_2 = 1$, and output $y = 16$. We express this by saying $x_1(1, 1, 16) = 16/100$ and $x_2(1, 1, 16) = 144/100$. Since we know the amount of each factor that will be used to produce 16 units of output and since we know the price of each factor, we can now calculate the cost of producing 16 units. This cost is $c(w_1, w_2, 16) = w_1 x_1(w_1, w_2, 16) + w_2 x_2(w_1, w_2, 16)$. In this instance since $w_1 = w_2 = 1$, we have $c(1, 1, 16) = x_1(1, 1, 16) + x_2(1, 1, 16) = 160/100$.

In consumer theory, you also dealt with cases where the consumer's indifference "curves" were straight lines and with cases where there were

kinks in the indifference curves. Then you found that the consumer's choice might occur at a boundary or at a kink. Usually a careful look at the diagram would tell you what is going on. The story with kinks and boundary solutions is almost exactly the same in the case of cost-minimizing firms. You will find some exercises that show how this works.

20.1 (0) Nadine sells user-friendly software. Her firm's production function is $f(x_1, x_2) = x_1 + 2x_2$, where x_1 is the amount of unskilled labor and x_2 is the amount of skilled labor that she employs.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 20 units of output. Draw another isoquant representing input combinations that will produce 40 units of output.
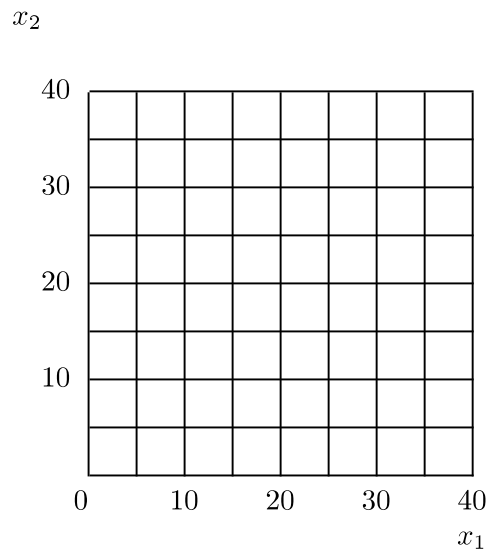
(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?_____.

(c) If Nadine uses only unskilled labor, how much unskilled labor would she need in order to produce y units of output?_____.

(d) If Nadine uses only skilled labor to produce output, how much skilled labor would she need in order to produce y units of output?_____.

(e) If Nadine faces factor prices $(1, 1)$, what is the cheapest way for her to produce 20 units of output? $x_1 =$ _____ , $x_2 =$_____.

(f) If Nadine faces factor prices $(1, 3)$, what is the cheapest way for her to produce 20 units of output? $x_1 = $ _____ , $x_2 =$ _____.

(g) If Nadine faces factor prices (w_1, w_2), what will be the minimal cost of producing 20 units of output? _____.

(h) If Nadine faces factor prices (w_1, w_2), what will be the minimal cost of producing y units of output? _____.

20.2 (0) The Ontario Brassworks produces brazen effronteries. As you know brass is an alloy of copper and zinc, used in fixed proportions. The production function is given by: $f(x_1, x_2) = \min\{x_1, 2x_2\}$, where x_1 is the amount of copper it uses and x_2 is the amount of zinc that it uses in production.

(a) Illustrate a typical isoquant for this production function in the graph below.
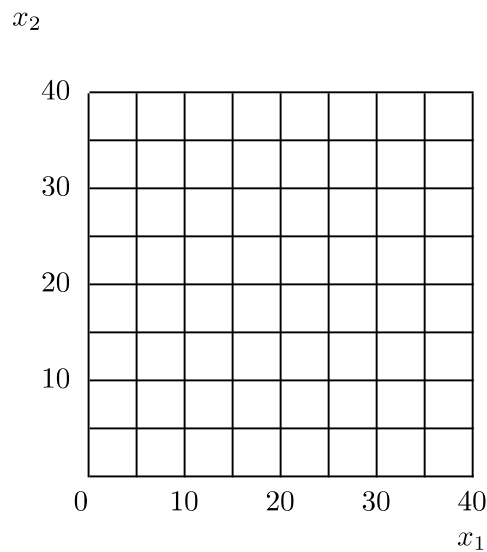
x_2

(b) Does this production function exhibit increasing, decreasing, or constant returns to scale? _____.

(c) If the firm wanted to produce 10 effronteries, how much copper would it need? _____ How much zinc would it need? _____.

(d) If the firm faces factor prices $(1, 1)$, what is the cheapest way for it to produce 10 effronteries? How much will this cost?_____

_____

_____.

(e) If the firm faces factor prices (w_1, w_2), what is the cheapest cost to produce 10 effronteries?_____.

(f) If the firm faces factor prices (w_1, w_2), what will be the minimal cost of producing y effronteries?_____.

20.3 (0) A firm uses labor and machines to produce output according to the production function $f(L, M) = 4L^{1/2}M^{1/2}$, where L is the number of units of labor used and M is the number of machines. The cost of labor is \$40 per unit and the cost of using a machine is \$10.
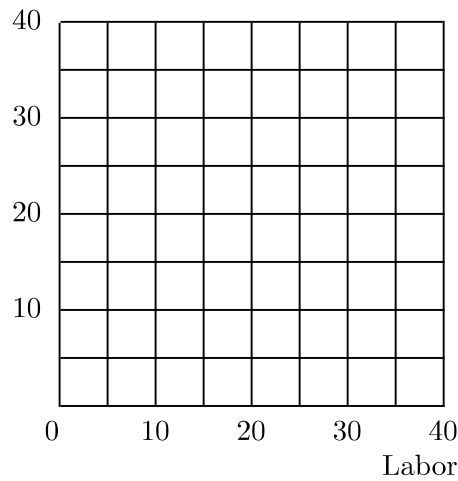
(a) On the graph below, draw an isocost line for this firm, showing combinations of machines and labor that cost \$400 and another isocost line showing combinations that cost \$200. What is the slope of these isocost lines?_____.

(b) Suppose that the firm wants to produce its output in the cheapest possible way. Find the number of machines it would use per worker. (Hint: The firm will produce at a point where the slope of the production isoquant equals the slope of the isocost line.)_____.

(c) On the graph, sketch the production isoquant corresponding to an output of 40. Calculate the amount of labor _____ and the number of machines _____ that are used to produce 40 units of output in the cheapest possible way, given the above factor prices. Calculate the cost of producing 40 units at these factor prices: $c(40, 10, 40) = $_____.

(d) How many units of labor _____ and how many machines _____ would the firm use to produce y units in the cheapest possible way? How much would this cost? _____ (Hint: Notice that there are constant returns to scale.)

Machines

40 ─
        ┌──┬──┬──┬──┬──┬──┬──┐
        │ │ │ │ │ │ │ │
30 ─   ├──┼──┼──┼──┼──┼──┼──┤
        │ │ │ │ │ │ │ │
        ├──┼──┼──┼──┼──┼──┼──┤
20 ─   │ │ │ │ │ │ │ │
        ├──┼──┼──┼──┼──┼──┼──┤
        │ │ │ │ │ │ │ │
10 ─   ├──┼──┼──┼──┼──┼──┼──┤
        │ │ │ │ │ │ │ │
        ├──┼──┼──┼──┼──┼──┼──┤
        │ │ │ │ │ │ │ │
        └──┴──┴──┴──┴──┴──┴──┘
      0   10   20   30   40
                                    Labor

20.4 (0) Earl sells lemonade in a competitive market on a busy street corner in Philadelphia. His production function is $f(x_1, x_2) = x_1^{1/3} x_2^{1/3}$, where output is measured in gallons, x_1 is the number of pounds of lemons he uses, and x_2 is the number of labor-hours spent squeezing them.

(a) Does Earl have constant returns to scale, decreasing returns to scale, or increasing returns to scale?——————————————————.

(b) Where w_1 is the cost of a pound of lemons and w_2 is the wage rate for lemon-squeezers, the cheapest way for Earl to produce lemonade is to use —————— hours of labor per pound of lemons. (Hint: Set the slope of his isoquant equal to the slope of his isocost line.)

(c) If he is going to produce y units in the cheapest way possible, then the number of pounds of lemons he will use is $x_1(w_1, w_2, y) =$——————

and the number of hours of labor that he will use is $x_2(w_1, w_2, y) =$————

—————— (Hint: Use the production function and the equation you found in the last part of the answer to solve for the input quantities.)

(d) The cost to Earl of producing y units at factor prices w_1 and w_2 is

$c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y) =$——————————.

20.5 (0) The prices of inputs (x_1, x_2, x_3, x_4) are $(4, 1, 3, 2)$.

(a) If the production function is given by $f(x_1, x_2) = \min\{x_1, x_2\}$, what is the minimum cost of producing one unit of output?_____.

(b) If the production function is given by $f(x_3, x_4) = x_3 + x_4$, what is the minimum cost of producing one unit of output?_____.

(c) If the production function is given by $f(x_1, x_2, x_3, x_4) = \min\{x_1 + x_2, x_3 + x_4\}$, what is the minimum cost of producing one unit of output?

_____.

(d) If the production function is given by $f(x_1, x_2) = \min\{x_1, x_2\} + \min\{x_3, x_4\}$, what is the minimum cost of producing one unit of output?

_____.

20.6 (0) Joe Grow, an avid indoor gardener, has found that the number of happy plants, h, depends on the amount of light, l, and water, w. In fact, Joe noticed that plants require twice as much light as water, and any more or less is wasted. Thus, Joe's production function is $h = \min\{l, 2w\}$.

(a) Suppose Joe is using 1 unit of light, what is the least amount of water he can use and still produce a happy plant?_____.

(b) If Suppose Joe wants to produce 4 happy plants, what are the minimum amounts of light and water required?_____.

(c) Joe's conditional factor demand function for light is $l(w_1, w_2, h) =$ _____ and his conditional factor demand function for water is $w(w_1, w_2, h) =$_____.

(d) If each unit of light costs w_1 and each unit of water costs w_2, Joe's cost function is $c(w_1, w_2, h) =$_____.

20.7 (1) Joe's sister, Flo Grow, is a university administrator. She uses an alternative method of gardening. Flo has found that happy plants only need fertilizer and talk. (*Warning:* Frivolous observations about university administrators' talk being a perfect substitute for fertilizer is in extremely poor taste.) Where f is the number of bags of fertilizer used and t is the number of hours she talks to her plants, the number of happy plants produced is exactly $h = t + 2f$. Suppose fertilizer costs w_f per bag and talk costs w_t per hour.

(a) If Flo uses no fertilizer, how many hours of talk must she devote if she wants one happy plant? _____ If she doesn't talk to her plants at all, how many bags of fertilizer will she need for one happy plant?

_____.

(b) If $w_t < w_f/2$, would it be cheaper for Flo to use fertilizer or talk to raise one happy plant?_____.

(c) Flo's cost function is $c(w_f, w_t, h) =$_____.

(d) Her conditional factor demand for talk is $t(w_f, w_t, h) =$_____ if $w_t < w_f/2$ and _____ if $w_t > w_f/2$.

20.8 (0) Remember T-bone Pickens, the corporate raider? Now he's concerned about his chicken farms, Pickens's Chickens. He feeds his chickens on a mixture of soybeans and corn, depending on the prices of each. According to the data submitted by his managers, when the price of soybeans was $10 a bushel and the price of corn was $10 a bushel, they used 50 bushels of corn and 150 bushels of soybeans for each coop of chickens. When the price of soybeans was $20 a bushel and the price of corn was $10 a bushel, they used 300 bushels of corn and no soybeans per coop of chickens. When the price of corn was $20 a bushel and the price of soybeans was $10 a bushel, they used 250 bushels of soybeans and no corn for each coop of chickens.

(a) Graph these three input combinations and isocost lines in the following diagram.

Corn

Soybeans
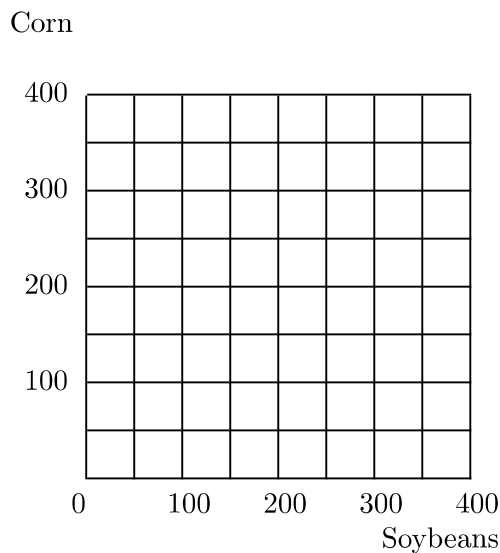
(b) How much money did Pickens' managers spend per coop of chickens when the prices were $(10, 10)$? _____ When the prices were $(10, 20)$? _____ When the prices were $(20, 10)$?_____.

(c) Is there any evidence that Pickens's managers were not minimizing costs? Why or why not?

_____.

(d) Pickens wonders whether there are any prices of corn and soybeans at which his managers will use 150 bushels of corn and 50 bushels of soybeans to produce a coop of chickens. How much would this production method cost per coop of chickens if the prices were $p_s = 10$ and $p_c = 10$? _____ _____ if the prices were $p_s = 10$, $p_c = 20$? _____ if the prices were $p_s = 20$, $p_c = 10$?_____.

(e) If Pickens's managers were always minimizing costs, can it be possible to produce a coop of chickens using 150 bushels and 50 bushels of soybeans?_____

_____

_____.

20.9 (0) A genealogical firm called Roots produces its output using only one input. Its production function is $f(x) = \sqrt{x}$.

(a) Does the firm have increasing, constant, or decreasing returns to scale?

_____.

(b) How many units of input does it take to produce 10 units of output? _____ If the input costs w per unit, what does it cost to produce 10 units of output?_____.

(c) How many units of input does it take to produce y units of output? _____ If the input costs w per unit, what does it cost to produce y units of output?_____.

(d) If the input costs w per unit, what is the average cost of producing y

units? $AC(w, y) = $_____.

20.10 (0) A university cafeteria produces square meals, using only one input and a rather remarkable production process. We are not allowed to say what that ingredient is, but an authoritative kitchen source says that "fungus is involved." The cafeteria's production function is $f(x) = x^2$, where x is the amount of input and $f(x)$ is the number of square meals produced.

(a) Does the cafeteria have increasing, constant, or decreasing returns to
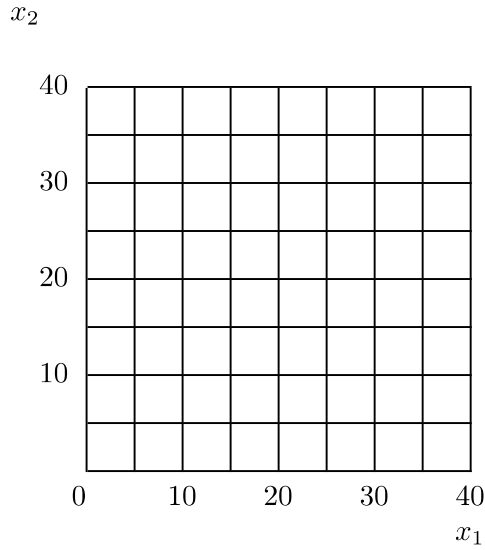
scale?_____.

(b) How many units of input does it take to produce 144 square meals?

_____ If the input costs w per unit, what does it cost to produce 144

square meals?_____.

(c) How many units of input does it take to produce y square meals?

_____ If the input costs w per unit, what does it cost to produce y

square meals?_____.

(d) If the input costs w per unit, what is the average cost of producing y

square meals? $AC(w, y) = $_____.

20.11 (0) Irma's Handicrafts produces plastic deer for lawn ornaments. "It's hard work," says Irma, "but anything to make a buck." Her production function is given by $f(x_1, x_2) = (\min\{x_1, 2x_2\})^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of labor used, and $f(x_1, x_2)$ is the number of deer produced.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 4 deer. Draw another production isoquant representing input combinations that will produce 5 deer.

(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?_____.

(c) If Irma faces factor prices $(1,1)$, what is the cheapest way for her to produce 4 deer? _____ How much does this cost?_____.
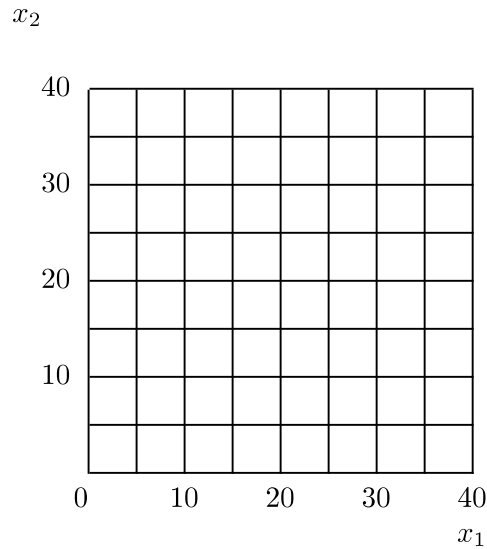
(d) At the factor prices $(1,1)$, what is the cheapest way to produce 5 deer? _____ How much does this cost?_____.

(e) At the factor prices $(1,1)$, the cost of producing y deer with this technology is $c(1,1,y) =$_____.

(f) At the factor prices (w_1, w_2), the cost of producing y deer with this technology is $c(w_1, w_2, y) =$_____.

20.12 (0) Al Deardwarf also makes plastic deer for lawn ornaments. Al has found a way to automate the production process completely. He doesn't use any labor–only wood and plastic. Al says he likes the business "because I need the doe." Al's production function is given by $f(x_1, x_2) = (2x_1 + x_2)^{1/2}$, where x_1 is the amount of plastic used, x_2 is the amount of wood used, and $f(x_1, x_2)$ is the number of deer produced.

(a) In the graph below, draw a production isoquant representing input combinations that will produce 4 deer. Draw another production isoquant representing input combinations that will produce 6 deer.

x_2

(b) Does this production function exhibit increasing, decreasing, or constant returns to scale?_____.

(c) If Al faces factor prices $(1,1)$, what is the cheapest way for him to produce 4 deer? _____ How much does this cost?_____.

(d) At the factor prices $(1,1)$, what is the cheapest way to produce 6 deer? _____ How much does this cost?_____.

(e) At the factor prices $(1,1)$, the cost of producing y deer with this technology is $c(1,1,y) =$_____.

(f) At the factor prices $(3,1)$, the cost of producing y deer with this technology is $c(3,1,y) =$_____.

20.13 (0) Suppose that Al Deardwarf from the last problem cannot vary the amount of wood that he uses in the short run and is stuck with using 20 units of wood. Suppose that he can change the amount of plastic that he uses, even in the short run.

(a) How much plastic would Al need in order to make 100 deer?_____

_____.

(b) If the cost of plastic is \$1 per unit and the cost of wood is \$1 per unit, how much would it cost Al to make 100 deer?———————————————.

(c) Write down Al's short-run cost function at these factor prices.————

————————————————————————————————.

Here you continue to work on cost functions. Total cost can be divided into fixed cost, the part that doesn't change as output changes, and variable cost. To get the average (total) cost, average fixed cost, and average variable cost, just divide the appropriate cost function by y, the level of output. The marginal cost function is the derivative of the total cost function with respect to output—or the rate of increase in cost as output increases, if you don't know calculus.

Remember that the marginal cost curve intersects both the average cost curve and the average variable cost curve at their minimum points. So to find the minimum point on the average cost curve, you simply set marginal cost equal to average cost and similarly for the minimum of average variable cost.

A firm has the total cost function $C(y) = 100 + 10y$. Let us find the equations for its various cost curves. Total fixed costs are 100, so the equation of the average fixed cost curve is $100/y$. Total variable costs are $10y$, so average variable costs are $10y/y = 10$ for all y. Marginal cost is 10 for all y. Average total costs are $(100 + 10y)/y = 10 + 10/y$. Notice that for this firm, average total cost decreases as y increases. Notice also that marginal cost is less than average total cost for all y.

21.1 (0) Mr. Otto Carr, owner of Otto's Autos, sells cars. Otto buys autos for $\$c$ each and has no other costs.

(a) What is his total cost if he sells 10 cars? _____ What if he sells 20 cars? _____ Write down the equation for Otto's total costs assuming he sells y cars: $TC(y) =$_____.

(b) What is Otto's average cost function? $AC(y) =$ _____ For every additional auto Otto sells, by how much do his costs increase? _____ Write down Otto's marginal cost function: $MC(y) =$_____.

(c) In the graph below draw Otto's average and marginal cost curves if $c = 20$.

$$AC, MC$$
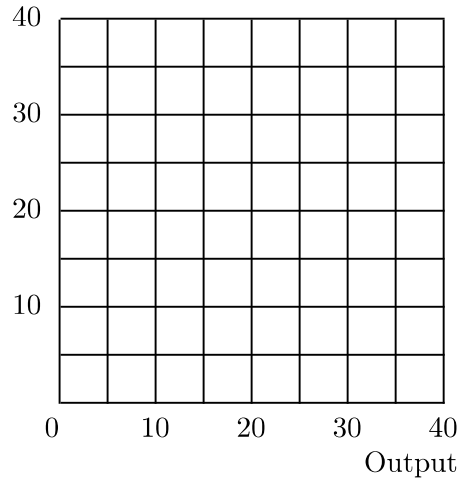
(d) Suppose Otto has to pay $\$b$ a year to produce obnoxious television commercials. Otto's total cost curve is now $TC(y) = $ _____ , his average cost curve is now $AC(y) = $ _____ , and his marginal cost curve is $MC(y) = $_____.

(e) If $b = \$100$, use red ink to draw Otto's average cost curve on the graph above.

21.2 (0) Otto's brother, Dent Carr, is in the auto repair business. Dent recently had little else to do and decided to calculate his cost conditions. He found that the total cost of repairing s cars is $TC(s) = 2s^2 + 10$. But Dent's attention was diverted to other things ... and that's where you come in. Please complete the following:

Dent's Total Variable Costs:_____.

Total Fixed Costs:_____.

Average Variable Costs:_____.

Average Fixed Costs:_____.

Average Total Costs:_____.

Marginal Costs:_____.

21.3 (0) A third brother, Rex Carr, owns a junk yard. Rex can use one of two methods to destroy cars. The first involves purchasing a hydraulic

car smasher that costs $200 a year to own and then spending $1 for every car smashed into oblivion; the second method involves purchasing a shovel that will last one year and costs $10 and paying the last Carr brother, Scoop, to bury the cars at a cost of $5 each.

(a) Write down the total cost functions for the two methods, where y is output per year: $TC_1(y) =$ ———— , $TC_2(y) =$ —————————.

(b) The first method has an average cost function ————————— and a marginal cost function ———————— For the second method these costs are

———————— and—————————————————————————————————.

(c) If Rex wrecks 40 cars per year, which method should he use? ————

———————— If Rex wrecks 50 cars per year, which method should he use?

————————— What is the smallest number of cars per year for which

it would pay him to buy the hydraulic smasher?—————————————.

21.4 (0) Mary Magnolia wants to open a flower shop, the Petal Pusher, in a new mall. She has her choice of three different floor sizes, 200 square feet, 500 square feet, or 1,000 square feet. The monthly rent will be $1 a square foot. Mary estimates that if she has F square feet of floor space and sells y bouquets a month, her variable costs will be $c_v(y) = y^2/F$ per month.

(a) If she has 200 square feet of floor space, write down her marginal cost function: ————————— and her average cost function: —————————

———— At what amount of output is average cost minimized?—————————

At this level of output, how much is average cost?—————————————.

(b) If she has 500 square feet, write down her marginal cost function:

————————— and her average cost function: ————————————— At

what amount of output is average cost minimized? ———————— At this

level of output, how much is average cost?—————————————————.

(c) If she has 1,000 square feet of floor space, write down her marginal cost function: _____ and her average cost function: _____ _____ At what amount of output is average cost minimized?

_____ At this level of output, how much is average cost?_____.

(d) Use red ink to show Mary's average cost curve and her marginal cost curves if she has 200 square feet. Use blue ink to show her average cost curve and her marginal cost curve if she has 500 square feet. Use black ink to show her average cost curve and her marginal cost curve if she has 1,000 square feet. Label the average cost curves AC and the marginal cost curves MC.

Dollars
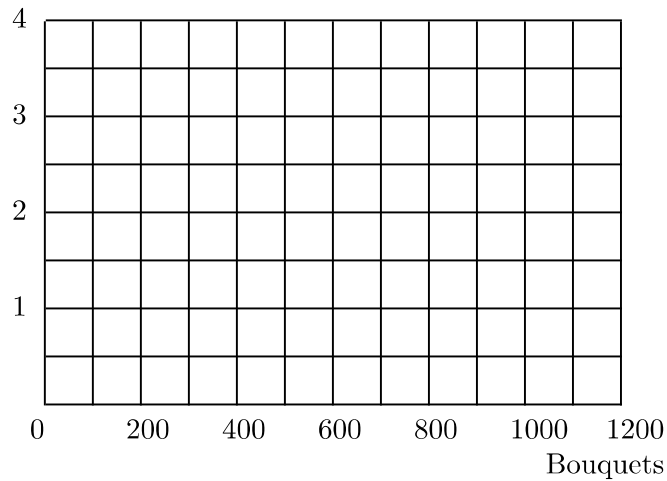
(e) Use yellow marker to show Mary's long-run average cost curve and her long-run marginal cost curve in your graph. Label them LRAC and LRMC.

21.5 (0) Touchie MacFeelie publishes comic books. The only inputs he needs are old jokes and cartoonists. His production function is

$$Q = .1J^{\frac{1}{2}}L^{3/4},$$

where J is the number of old jokes used, L the number of hours of cartoonists' labor used as inputs, and Q is the number of comic books produced.

(a) Does this production process exhibit increasing, decreasing, or constant returns to scale? Explain your answer._____

_____.

(b) If the number of old jokes used is 100, write an expression for the

marginal product of cartoonists' labor as a function of L. ——————
Is the marginal product of labor decreasing or increasing as the amount

of labor increases?—————————————————————————.

21.6 (0) Touchie MacFeelie's irascible business manager, Gander Mac-Grope, announces that old jokes can be purchased for \$1 each and that the wage rate of cartoonists' labor is \$2.

(a) Suppose that in the short run, Touchie is stuck with exactly 100 old jokes (for which he paid \$1 each) but is able to hire as much labor as he wishes. How much labor would he have to hire in order produce Q comic

books?—————————————————————————————.

(b) Write down Touchie's short-run total cost as a function of his output

——————————————————————————————————.

(c) His short-run marginal cost function is————————————————.

(d) His short-run average cost function is—————————————————.

21.7 (1) Touchie asks his brother, Sir Francis MacFeelie, to study the long-run picture. Sir Francis, who has carefully studied the appendix to Chapter 19 in your text, prepared the following report.

(a) If all inputs are variable, and if old jokes cost \$1 each and cartoonist labor costs \$2 per hour, the cheapest way to produce exactly one comic

book is to use ———————————— jokes and ———————————— hours of labor. (Fractional jokes are certainly allowable.)

(b) This would cost ——————————— dollars.

(c) Given our production function, the cheapest proportions in which to use jokes and labor are the same no matter how many comic books we print. But when we double the amount of both inputs, the number of

comic books produced is multiplied by——————————————————.

21.8 (0) Consider the cost function $c(y) = 4y^2 + 16$.

(a) The average cost function is—————————————————————.

(b) The marginal cost function is_____.

(c) The level of output that yields the minimum average cost of production

is_____.

(d) The average variable cost function is_____.

(e) At what level of output does average variable cost equal marginal

cost?_____.

21.9 (0) A competitive firm has a production function of the form $Y = 2L + 5K$. If $w = \$2$ and $r = \$3$, what will be the minimum cost of

producing 10 units of output?_____.