
These workouts are designed to build your skills in describing economic situations with graphs and algebra. Budget sets are a good place to start, because both the algebra and the graphing are very easy. Where there are just two goods, a consumer who consumes x_1 units of good 1 and x_2 units of good 2 is said to consume the *consumption bundle*, (x_1, x_2) . Any consumption bundle can be represented by a point on a two-dimensional graph with quantities of good 1 on the horizontal axis and quantities of good 2 on the vertical axis. If the prices are p_1 for good 1 and p_2 for good 2, and if the consumer has income m , then she can afford any consumption bundle, (x_1, x_2) , such that $p_1x_1 + p_2x_2 \leq m$. On a graph, the *budget line* is just the line segment with equation $p_1x_1 + p_2x_2 = m$ and with x_1 and x_2 both nonnegative. The budget line is the boundary of the *budget set*. All of the points that the consumer can afford lie on one side of the line and all of the points that the consumer cannot afford lie on the other.

If you know prices and income, you can construct a consumer's budget line by finding two commodity bundles that she can "just afford" and drawing the straight line that runs through both points.

Myrtle has 50 dollars to spend. She consumes only apples and bananas. Apples cost 2 dollars each and bananas cost 1 dollar each. You are to graph her budget line, where apples are measured on the horizontal axis and bananas on the vertical axis. Notice that if she spends all of her income on apples, she can afford 25 apples and no bananas. Therefore her budget line goes through the point $(25, 0)$ on the horizontal axis. If she spends all of her income on bananas, she can afford 50 bananas and no apples. Therefore her budget line also passes through the point $(0, 50)$ on the vertical axis. Mark these two points on your graph. Then draw a straight line between them. This is Myrtle's budget line.

What if you are not told prices or income, but you know two commodity bundles that the consumer can just afford? Then, if there are just two commodities, you know that a unique line can be drawn through two points, so you have enough information to draw the budget line.

Laurel consumes only ale and bread. If she spends all of her income, she can just afford 20 bottles of ale and 5 loaves of bread. Another commodity bundle that she can afford if she spends her entire income is 10 bottles of ale and 10 loaves of bread. If the price of ale is 1 dollar per bottle, how much money does she have to spend? You could solve this problem graphically. Measure ale on the horizontal axis and bread on the vertical axis. Plot the two points, $(20, 5)$ and $(10, 10)$, that you know to be on the budget line. Draw the straight line between these points and extend the line to the horizontal axis. This point denotes the amount of ale Laurel can afford if she spends all of her money on ale. Since ale costs 1 dollar a bottle, her income in dollars is equal to the largest number of bottles she

can afford. Alternatively, you can reason as follows. Since the bundles $(20, 5)$ and $(10, 10)$ cost the same, it must be that giving up 10 bottles of ale makes her able to afford an extra 5 loaves of bread. So bread costs twice as much as ale. The price of ale is 1 dollar, so the price of bread is 2 dollars. The bundle $(20, 5)$ costs as much as her income. Therefore her income must be $20 \times 1 + 5 \times 2 = 30$.

When you have completed this workout, we hope that you will be able to do the following:

- Write an equation for the budget line and draw the budget set on a graph when you are given prices and income or when you are given two points on the budget line.
- Graph the effects of changes in prices and income on budget sets.
- Understand the concept of *numeraire* and know what happens to the budget set when income and all prices are multiplied by the same positive amount.
- Know what the budget set looks like if one or more of the prices is negative.
- See that the idea of a “budget set” can be applied to constrained choices where there are other constraints on what you can have, in addition to a constraint on money expenditure.

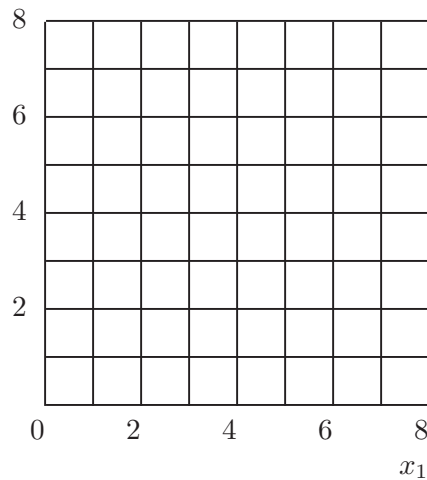
2.1 (0) You have an income of \$40 to spend on two commodities. Commodity 1 costs \$10 per unit, and commodity 2 costs \$5 per unit.

(a) Write down your budget equation. _____.

(b) If you spent all your income on commodity 1, how much could you buy? _____.

(c) If you spent all of your income on commodity 2, how much could you buy? _____ Use blue ink to draw your budget line in the graph below.

x_2



(d) Suppose that the price of commodity 1 falls to \$5 while everything else stays the same. Write down your new budget equation. _____

_____ On the graph above, use red ink to draw your new budget line.

(e) Suppose that the amount you are allowed to spend falls to \$30, while the prices of both commodities remain at \$5. Write down your budget equation. _____ Use black ink to draw this budget line.

(f) On your diagram, use blue ink to shade in the area representing commodity bundles that you can afford with the budget in Part (e) but could not afford to buy with the budget in Part (a). Use black ink or pencil to shade in the area representing commodity bundles that you could afford with the budget in Part (a) but cannot afford with the budget in Part (e).

2.2 (0) On the graph below, draw a budget line for each case.

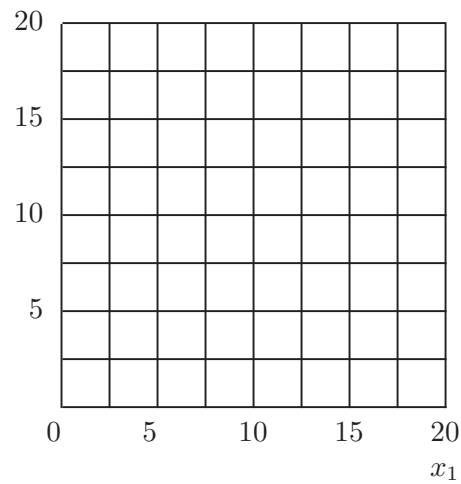
(a) $p_1 = 1, p_2 = 1, m = 15$. (Use blue ink.)

(b) $p_1 = 1, p_2 = 2, m = 20$. (Use red ink.)

(c) $p_1 = 0, p_2 = 1, m = 10$. (Use black ink.)

(d) $p_1 = p_2, m = 15p_1$. (Use pencil or black ink. Hint: How much of good 1 could you afford if you spend your entire budget on good 1?)

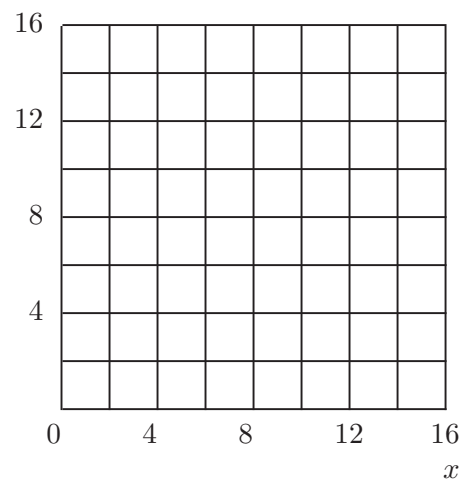
x_2



2.3 (0) Your budget is such that if you spend your entire income, you can afford either 4 units of good x and 6 units of good y or 12 units of x and 2 units of y .

(a) Mark these two consumption bundles and draw the budget line in the graph below.

y



(b) What is the ratio of the price of x to the price of y ?_____.

(c) If you spent all of your income on x , how much x could you buy?

_____.

(d) If you spent all of your income on y , how much y could you buy?

_____.

(e) Write a budget equation that gives you this budget line, where the price of x is 1._____.

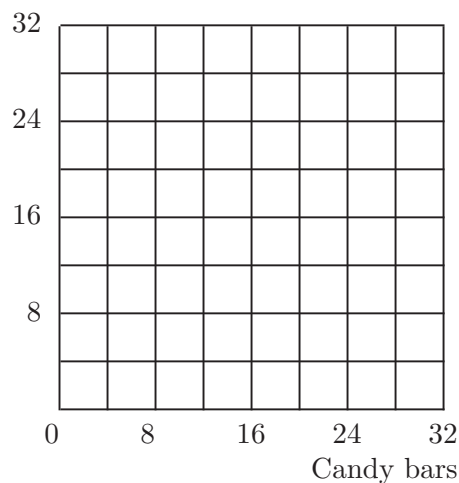
(f) Write another budget equation that gives you the same budget line, but where the price of x is 3._____.

2.4 (1) Murphy was consuming 100 units of X and 50 units of Y . The price of X rose from 2 to 3. The price of Y remained at 4.

(a) How much would Murphy's income have to rise so that he can still exactly afford 100 units of X and 50 units of Y ?_____.

2.5 (1) If Amy spent her entire allowance, she could afford 8 candy bars and 8 comic books a week. She could also just afford 10 candy bars and 4 comic books a week. The price of a candy bar is 50 cents. Draw her budget line in the box below. What is Amy's weekly allowance?_____.

Comic books



2.6 (0) In a small country near the Baltic Sea, there are only three commodities: potatoes, meatballs, and jam. Prices have been remarkably stable for the last 50 years or so. Potatoes cost 2 crowns per sack, meatballs cost 4 crowns per crock, and jam costs 6 crowns per jar.

(a) Write down a budget equation for a citizen named Gunnar who has an income of 360 crowns per year. Let P stand for the number of sacks of potatoes, M for the number of crocks of meatballs, and J for the number of jars of jam consumed by Gunnar in a year._____.

(b) The citizens of this country are in general very clever people, but they are not good at multiplying by 2. This made shopping for potatoes excruciatingly difficult for many citizens. Therefore it was decided to introduce a new unit of currency, such that potatoes would be the *numeraire*. A sack of potatoes costs one unit of the new currency while the same relative prices apply as in the past. In terms of the new currency, what is the price of meatballs?_____.

(c) In terms of the new currency, what is the price of jam?_____.

(d) What would Gunnar's income in the new currency have to be for him to be exactly able to afford the same commodity bundles that he could afford before the change?_____.

(e) Write down Gunnar's new budget equation._____ Is Gunnar's budget set any different than it was before the change?_____.

2.7 (0) Edmund Stench consumes two commodities, namely garbage and punk rock video cassettes. He doesn't actually eat the former but keeps it in his backyard where it is eaten by billy goats and assorted vermin. The reason that he accepts the garbage is that people pay him \$2 per sack for taking it. Edmund can accept as much garbage as he wishes at that price. He has no other source of income. Video cassettes cost him \$6 each.

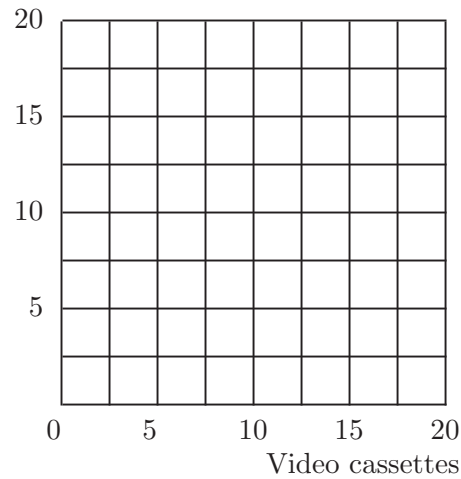
(a) If Edmund accepts zero sacks of garbage, how many video cassettes can he buy?_____.

(b) If he accepts 15 sacks of garbage, how many video cassettes can he buy?_____.

(c) Write down an equation for his budget line. _____.

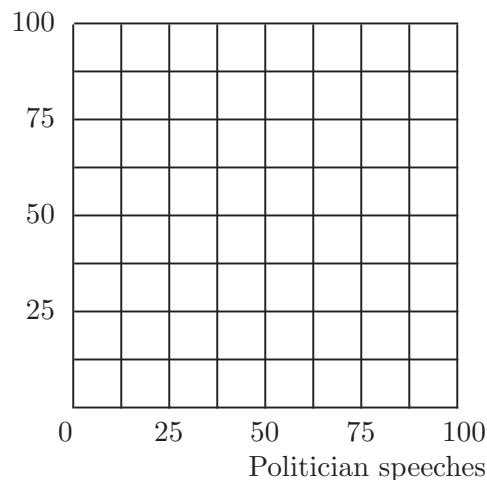
(d) Draw Edmund's budget line and shade in his budget set.

Garbage



2.8 (0) If you think Edmund is odd, consider his brother Emmett. Emmett consumes speeches by politicians and university administrators. He is paid \$1 per hour for listening to politicians and \$2 per hour for listening to university administrators. (Emmett is in great demand to help fill empty chairs at public lectures because of his distinguished appearance and his ability to refrain from making rude noises.) Emmett consumes one good for which he must pay. We have agreed not to disclose what that good is, but we can tell you that it costs \$15 per unit and we shall call it Good X . In addition to what he is paid for consuming speeches, Emmett receives a pension of \$50 per week.

Administrator speeches



(a) Write down a budget equation stating those combinations of the three commodities, Good X , hours of speeches by politicians (P), and hours of speeches by university administrators (A) that Emmett could afford to consume per week._____.

(b) On the graph above, draw a two-dimensional diagram showing the locus of consumptions of the two kinds of speeches that would be possible for Emmett if he consumed 10 units of Good X per week.

2.9 (0) Jonathan Livingstone Yuppie is a prosperous lawyer. He has, in his own words, “outgrown those confining two-commodity limits.” Jonathan consumes three goods, unblended Scotch whiskey, designer tennis shoes, and meals in French gourmet restaurants. The price of Jonathan’s brand of whiskey is \$20 per bottle, the price of designer tennis shoes is \$80 per pair, and the price of gourmet restaurant meals is \$50 per meal. After he has paid his taxes and alimony, Jonathan has \$400 a week to spend.

(a) Write down a budget equation for Jonathan, where W stands for the number of bottles of whiskey, T stands for the number of pairs of tennis shoes, and M for the number of gourmet restaurant meals that he consumes._____.

(b) Draw a three-dimensional diagram to show his budget set. Label the intersections of the budget set with each axis.

(c) Suppose that he determines that he will buy one pair of designer tennis shoes per week. What equation must be satisfied by the combinations of restaurant meals and whiskey that he could afford?_____.

2.10 (0) Martha is preparing for exams in economics and sociology. She has time to read 40 pages of economics and 30 pages of sociology. In the same amount of time she could also read 30 pages of economics and 60 pages of sociology.

(a) Assuming that the number of pages per hour that she can read of either subject does not depend on how she allocates her time, how many pages of sociology could she read if she decided to spend all of her time on sociology and none on economics? _____ (Hint: You have two points on her budget line, so you should be able to determine the entire line.)

(b) How many pages of economics could she read if she decided to spend all of her time reading economics? _____.

2.11 (1) Harry Hype has \$5,000 to spend on advertising a new kind of dehydrated sushi. Market research shows that the people most likely to buy this new product are recent recipients of M.B.A. degrees and lawyers who own hot tubs. Harry is considering advertising in two publications, a boring business magazine and a trendy consumer publication for people who wish they lived in California.

Fact 1: Ads in the boring business magazine cost \$500 each and ads in the consumer magazine cost \$250 each.

Fact 2: Each ad in the business magazine will be read by 1,000 recent M.B.A.'s and 300 lawyers with hot tubs.

Fact 3: Each ad in the consumer publication will be read by 300 recent M.B.A.'s and 250 lawyers who own hot tubs.

Fact 4: Nobody reads more than one ad, and nobody who reads one magazine reads the other.

(a) If Harry spends his entire advertising budget on the business publication, his ad will be read by _____ recent M.B.A.'s and by _____ lawyers with hot tubs.

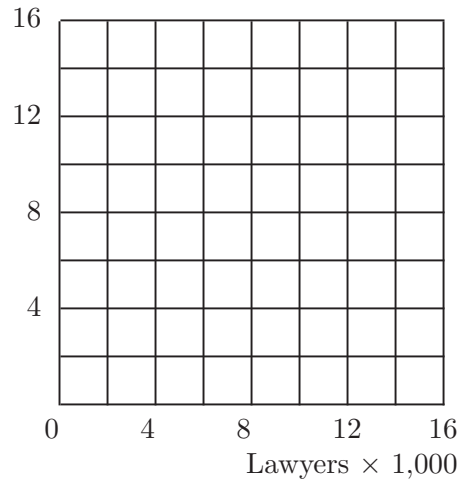
(b) If he spends his entire advertising budget on the consumer publication, his ad will be read by _____ recent M.B.A.'s and by _____ lawyers with hot tubs.

(c) Suppose he spent half of his advertising budget on each publication. His ad would be read by _____ recent M.B.A.'s and by _____ lawyers with hot tubs.

(d) Draw a "budget line" showing the combinations of number of readings by recent M.B.A.'s and by lawyers with hot tubs that he can obtain if he spends his entire advertising budget. Does this line extend all the way to the axes? _____ Sketch, shade in, and label the budget set, which includes all the combinations of MBA's and lawyers he can reach if he spends *no more than* his budget.

(e) Let M stand for the number of instances of an ad being read by an M.B.A. and L stand for the number of instances of an ad being read by a lawyer. This budget line is a line segment that lies on the line with equation _____ With a fixed advertising budget, how many readings by M.B.A.'s must he sacrifice to get an additional reading by a lawyer with a hot tub?_____.

M.B.A.'s \times 1,000



2.12 (0) On the planet Mungo, they have two kinds of money, blue money and red money. Every commodity has two prices—a red-money price and a blue-money price. Every Mungoan has two incomes—a red income and a blue income.

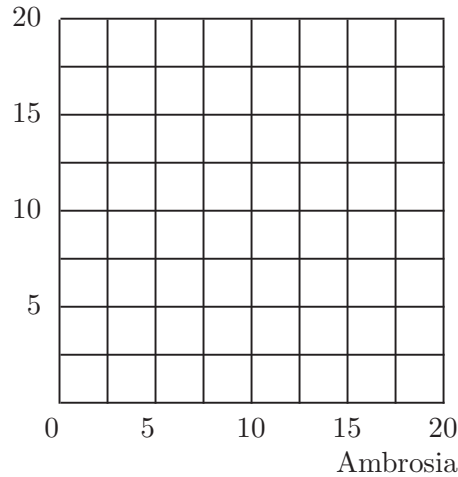
In order to buy an object, a Mungoan has to pay that object's red-money price in red money and its blue-money price in blue money. (The shops simply have two cash registers, and you have to pay at both registers to buy an object.) It is forbidden to trade one kind of money for the other, and this prohibition is strictly enforced by Mungo's ruthless and efficient monetary police.

- There are just two consumer goods on Mungo, ambrosia and bubble gum. All Mungoans prefer more of each good to less.
- The blue prices are 1 bcu (bcu stands for blue currency unit) per unit of ambrosia and 1 bcu per unit of bubble gum.
- The red prices are 2 rcus (red currency units) per unit of ambrosia and 6 rcus per unit of bubble gum.

(a) On the graph below, draw the red budget (with red ink) and the blue budget (with blue ink) for a Mungoan named Harold whose blue income is 10 and whose red income is 30. Shade in the "budget set" containing all of the commodity bundles that Harold can afford, given

its* two budget constraints. Remember, Harold has to have enough blue money *and* enough red money to pay both the blue-money cost and the red-money cost of a bundle of goods.

Bubble gum



(b) Another Mungoan, Gladys, faces the same prices that Harold faces and has the same red income as Harold, but Gladys has a blue income of 20. Explain how it is that Gladys will not spend its entire blue income no matter what its tastes may be. (Hint: Draw Gladys’s budget lines.)

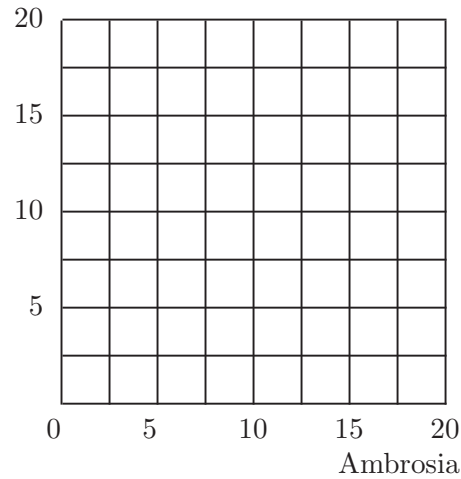
(c) A group of radical economic reformers on Mungo believe that the currency rules are unfair. “Why should everyone have to pay two prices for everything?” they ask. They propose the following scheme. Mungo will continue to have two currencies, every good will have a blue price and a red price, and every Mungoan will have a blue income and a red income. But nobody has to pay both prices. Instead, everyone on Mungo must declare itself to be either a Blue-Money Purchaser (a “Blue”) or a Red-Money Purchaser (a “Red”) before it buys anything at all. Blues must make all of their purchases in blue money at the blue prices, spending only their blue incomes. Reds must make all of their purchases in red money, spending only their red incomes.

Suppose that Harold has the same income after this reform, and that prices do not change. Before declaring which kind of purchaser it will be, Harold contemplates the set of commodity bundles that it could afford by making one declaration or the other. Let us call a commodity bundle

* We refer to all Mungoans by the gender-neutral pronoun, “it.” Although Mungo has two sexes, neither of them is remotely like either of ours.

“attainable” if Harold can afford it by declaring itself to be a “Blue” and buying the bundle with blue money or if Harold can afford the bundle by declaring itself to be a “Red” and buying it with red money. On the diagram below, shade in all of the attainable bundles.

Bubble gum



2.13 (0) Are Mungoan budgets really so fanciful? Can you think of situations on earth where people must simultaneously satisfy more than one budget constraint? Is money the only scarce resource that people use up when consuming?_____

In the previous section you learned how to use graphs to show the set of commodity bundles that a consumer can afford. In this section, you learn to put information about the consumer's preferences on the same kind of graph. Most of the problems ask you to draw indifference curves.

Sometimes we give you a formula for the indifference curve. Then all you have to do is graph a known equation. But in some problems, we give you only "qualitative" information about the consumer's preferences and ask you to sketch indifference curves that are consistent with this information. This requires a little more thought. Don't be surprised or disappointed if you cannot immediately see the answer when you look at a problem, and don't expect that you will find the answers hiding somewhere in your textbook. The best way we know to find answers is to "think and doodle." Draw some axes on scratch paper and label them, then mark a point on your graph and ask yourself, "What other points on the graph would the consumer find indifferent to this point?" If possible, draw a curve connecting such points, making sure that the shape of the line you draw reflects the features required by the problem. This gives you one indifference curve. Now pick another point that is preferred to the first one you drew and draw an indifference curve through it.

Jocasta loves to dance and hates housecleaning. She has strictly convex preferences. She prefers dancing to any other activity and never gets tired of dancing, but the more time she spends cleaning house, the less happy she is. Let us try to draw an indifference curve that is consistent with her preferences. There is not enough information here to tell us exactly where her indifference curves go, but there is enough information to determine some things about their shape. Take a piece of scratch paper and draw a pair of axes. Label the horizontal axis "Hours per day of housecleaning." Label the vertical axis "Hours per day of dancing." Mark a point a little ways up the vertical axis and write a 4 next to it. At this point, she spends 4 hours a day dancing and no time housecleaning. Other points that would be indifferent to this point would have to be points where she did more dancing *and* more housecleaning. The pain of the extra housekeeping should just compensate for the pleasure of the extra dancing. So an indifference curve for Jocasta must be upward sloping. Because she loves dancing and hates housecleaning, it must be that she prefers all the points above this indifference curve to all of the points on or below it. If Jocasta has strictly convex preferences, then it must be that if you draw a line between any two points on the same indifference curve, all the points on the line (except the endpoints) are preferred to the endpoints. For this to be the case, it must be that the indifference curve slopes upward ever more steeply as you move to the right along it. You should convince yourself of this by making some drawings on scratch paper. Draw an upward-sloping curve passing through the point $(0, 4)$

and getting steeper as one moves to the right.

When you have completed this workout, we hope that you will be able to do the following:

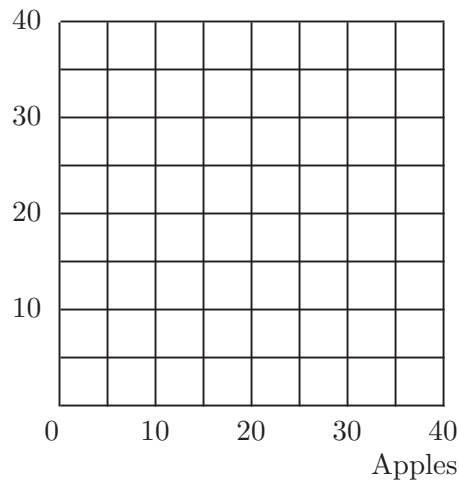
- Given the formula for an indifference curve, draw this curve, and find its slope at any point on the curve.
- Determine whether a consumer prefers one bundle to another or is indifferent between them, given specific indifference curves.
- Draw indifference curves for the special cases of perfect substitutes and perfect complements.
- Draw indifference curves for someone who dislikes one or both commodities.
- Draw indifference curves for someone who likes goods up to a point but who can get “too much” of one or more goods.
- Identify weakly preferred sets and determine whether these are convex sets and whether preferences are convex.
- Know what the marginal rate of substitution is and be able to determine whether an indifference curve exhibits “diminishing marginal rate of substitution.”
- Determine whether a preference relation or any other relation between pairs of things is transitive, whether it is reflexive, and whether it is complete.

3.1 (0) Charlie likes both apples and bananas. He consumes nothing else. The consumption bundle where Charlie consumes x_A bushels of apples per year and x_B bushels of bananas per year is written as (x_A, x_B) . Last year, Charlie consumed 20 bushels of apples and 5 bushels of bananas. It happens that the set of consumption bundles (x_A, x_B) such that Charlie is indifferent between (x_A, x_B) and $(20, 5)$ is the set of all bundles such that $x_B = 100/x_A$. The set of bundles (x_A, x_B) such that Charlie is just indifferent between (x_A, x_B) and the bundle $(10, 15)$ is the set of bundles such that $x_B = 150/x_A$.

(a) On the graph below, plot several points that lie on the indifference curve that passes through the point $(20, 5)$, and sketch this curve, using blue ink. Do the same, using red ink, for the indifference curve passing through the point $(10, 15)$.

(b) Use pencil to shade in the set of commodity bundles that Charlie weakly prefers to the bundle $(10, 15)$. Use blue ink to shade in the set of commodity bundles such that Charlie weakly prefers $(20, 5)$ to these bundles.

Bananas



For each of the following statements about Charlie's preferences, write "true" or "false."

(c) $(30, 5) \sim (10, 15)$. _____.

(d) $(10, 15) \succ (20, 5)$. _____.

(e) $(20, 5) \succeq (10, 10)$. _____.

(f) $(24, 4) \succeq (11, 9.1)$. _____.

(g) $(11, 14) \succ (2, 49)$. _____.

(h) A set is convex if for any two points in the set, the line segment between them is also in the set. Is the set of bundles that Charlie weakly prefers to $(20, 5)$ a convex set? _____.

(i) Is the set of bundles that Charlie considers inferior to $(20, 5)$ a convex set? _____.

(j) The slope of Charlie's indifference curve through a point, (x_A, x_B) , is known as his marginal _____ of _____ at that point.

(k) Remember that Charlie's indifference curve through the point $(10, 10)$ has the equation $x_B = 100/x_A$. Those of you who know calculus will remember that the slope of a curve is just its derivative, which in this case is $-100/x_A^2$. (If you don't know calculus, you will have to take our word for this.) Find Charlie's marginal rate of substitution at the point, $(10, 10)$._____.

(l) What is his marginal rate of substitution at the point $(5, 20)$?_____.

(m) What is his marginal rate of substitution at the point $(20, 5)$?_____.

(n) Do the indifference curves you have drawn for Charlie exhibit diminishing marginal rate of substitution?_____.

3.2 (0) Ambrose consumes only nuts and berries. Fortunately, he likes both goods. The consumption bundle where Ambrose consumes x_1 units of nuts per week and x_2 units of berries per week is written as (x_1, x_2) . The set of consumption bundles (x_1, x_2) such that Ambrose is indifferent between (x_1, x_2) and $(1, 16)$ is the set of bundles such that $x_1 \geq 0$, $x_2 \geq 0$, and $x_2 = 20 - 4\sqrt{x_1}$. The set of bundles (x_1, x_2) such that $(x_1, x_2) \sim (36, 0)$ is the set of bundles such that $x_1 \geq 0$, $x_2 \geq 0$ and $x_2 = 24 - 4\sqrt{x_1}$.

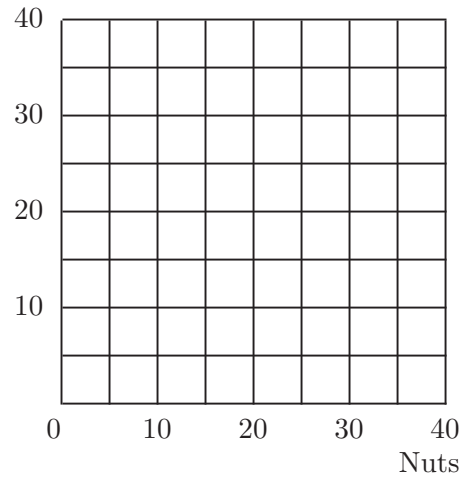
(a) On the graph below, plot several points that lie on the indifference curve that passes through the point $(1, 16)$, and sketch this curve, using blue ink. Do the same, using red ink, for the indifference curve passing through the point $(36, 0)$.

(b) Use pencil to shade in the set of commodity bundles that Ambrose weakly prefers to the bundle $(1, 16)$. Use red ink to shade in the set of all commodity bundles (x_1, x_2) such that Ambrose weakly prefers $(36, 0)$ to these bundles. Is the set of bundles that Ambrose prefers to $(1, 16)$ a convex set?_____.

(c) What is the slope of Ambrose's indifference curve at the point $(9, 8)$? (Hint: Recall from calculus the way to calculate the slope of a curve. If you don't know calculus, you will have to draw your diagram carefully and estimate the slope.)_____.

(d) What is the slope of his indifference curve at the point (4, 12)? _____

Berries



(e) What is the slope of his indifference curve at the point (9, 12)? _____

_____ at the point (4, 16)? _____

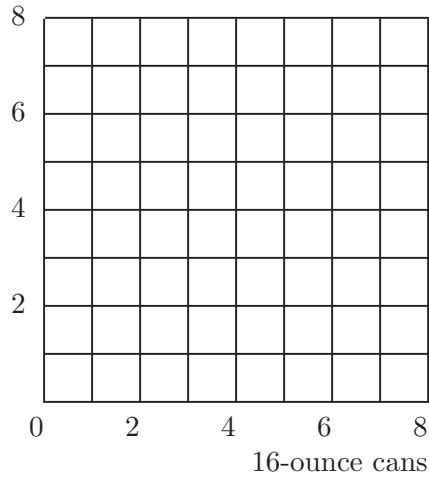
(f) Do the indifference curves you have drawn for Ambrose exhibit diminishing marginal rate of substitution? _____

(g) Does Ambrose have convex preferences? _____

3.3 (0) Shirley Sixpack is in the habit of drinking beer each evening while watching “The Best of Bowlerama” on TV. She has a strong thumb and a big refrigerator, so she doesn’t care about the size of the cans that beer comes in, she only cares about how much beer she has.

(a) On the graph below, draw some of Shirley’s indifference curves between 16-ounce cans and 8-ounce cans of beer. Use blue ink to draw these indifference curves.

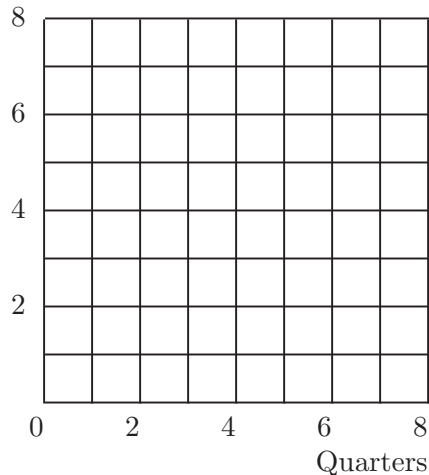
8-ounce cans



(b) Lorraine Quiche likes to have a beer while she watches “Masterpiece Theatre.” She only allows herself an 8-ounce glass of beer at any one time. Since her cat doesn’t like beer and she hates stale beer, if there is more than 8 ounces in the can she pours the excess into the sink. (She has no moral scruples about wasting beer.) On the graph above, use red ink to draw some of Lorraine’s indifference curves.

3.4 (0) Elmo finds himself at a Coke machine on a hot and dusty Sunday. The Coke machine requires exact change—two quarters and a dime. No other combination of coins will make anything come out of the machine. No stores are open; no one is in sight. Elmo is so thirsty that the only thing he cares about is how many soft drinks he will be able to buy with the change in his pocket; the more he can buy, the better. While Elmo searches his pockets, your task is to draw some indifference curves that describe Elmo’s preferences about what he finds.

Dimes



(a) If Elmo has 2 quarters and a dime in his pockets, he can buy 1 soft drink. How many soft drinks can he buy if he has 4 quarters and 2 dimes?

(b) Use red ink to shade in the area on the graph consisting of all combinations of quarters and dimes that Elmo thinks are just indifferent to having 2 quarters and 1 dime. (Imagine that it is possible for Elmo to have fractions of quarters or of dimes, but, of course, they would be useless in the machine.) Now use blue ink to shade in the area consisting of all combinations that Elmo thinks are just indifferent to having 4 quarters and 2 dimes. Notice that Elmo has indifference “bands,” not indifference curves.

(c) Does Elmo have convex preferences between dimes and quarters?

(d) Does Elmo always prefer more of both kinds of money to less?_____

(e) Does Elmo have a bliss point?_____

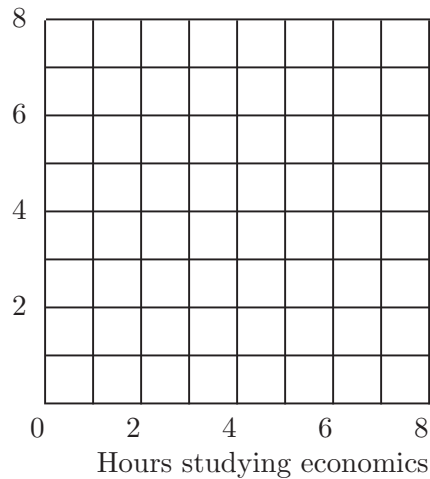
(f) If Elmo had arrived at the Coke machine on a Saturday, the drugstore across the street would have been open. This drugstore has a soda fountain that will sell you as much Coke as you want at a price of 4 cents an ounce. The salesperson will take any combination of dimes and quarters in payment. Suppose that Elmo plans to spend all of the money in his pocket on Coke at the drugstore on Saturday. On the graph above, use pencil or black ink to draw one or two of Elmo’s indifference curves between quarters and dimes in his pocket. (For simplicity, draw your graph as if Elmo’s fractional quarters and fractional dimes are accepted at the corresponding fraction of their value.) Describe these new indifference curves in words._____

3.5 (0) Randy Ratpack hates studying both economics and history. The more time he spends studying either subject, the less happy he is. But Randy has strictly convex preferences.

(a) Sketch an indifference curve for Randy where the two commodities are hours per week spent studying economics and hours per week spent studying history. Will the slope of an indifference curve be positive or negative?_____

(b) Do Randy's indifference curves get steeper or flatter as you move from left to right along one of them?_____.

Hours studying history

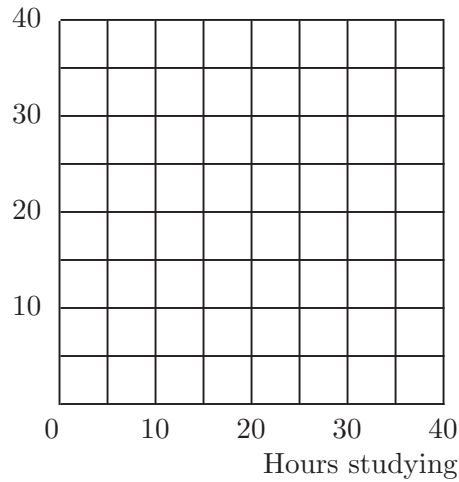


3.6 (0) Flossy Toothsome likes to spend some time studying and some time dating. In fact her indifference curves between hours per week spent studying and hours per week spent dating are concentric circles around her favorite combination, which is 20 hours of studying and 15 hours of dating per week. The closer she is to her favorite combination, the happier she is.

(a) Suppose that Flossy is currently studying 25 hours a week and dating 3 hours a week. Would she prefer to be studying 30 hours a week and dating 8 hours a week? _____ (Hint: Remember the formula for the distance between two points in the plane?)

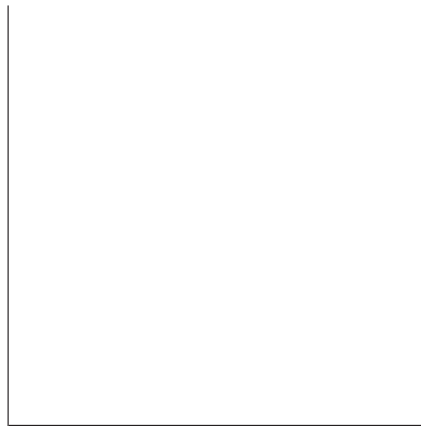
(b) On the axes below, draw a few of Flossy's indifference curves and use your diagram to illustrate which of the two time allocations discussed above Flossy would prefer.

Hours dating



3.7 (0) Joan likes chocolate cake and ice cream, but after 10 slices of cake, she gets tired of cake, and eating more cake makes her less happy. Joan always prefers more ice cream to less. Joan's parents require her to eat everything put on her plate. In the axes below, use blue ink to draw a set of indifference curves that depict her preferences between plates with different amounts of cake and ice cream. Be sure to label the axes.

(a) Suppose that Joan's preferences are as before, but that her parents allow her to leave anything on her plate that she doesn't want. On the graph below, use red ink to draw some indifference curves depicting her preferences between plates with different amounts of cake and ice cream.



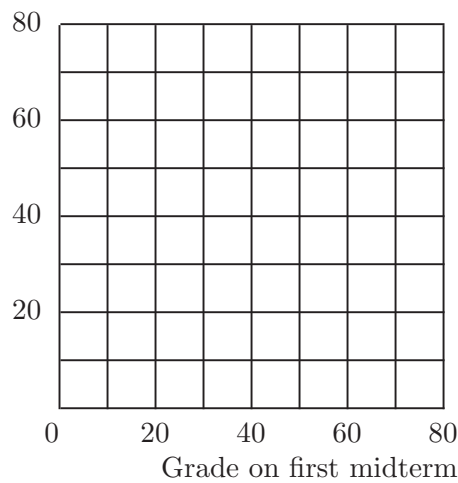
3.8 (0) Professor Goodheart always gives two midterms in his communications class. He only uses the higher of the two scores that a student gets on the midterms when he calculates the course grade.

(a) Nancy Lerner wants to maximize her grade in this course. Let x_1 be her score on the first midterm and x_2 be her score on the second midterm. Which combination of scores would Nancy prefer, $x_1 = 20$ and $x_2 = 70$ or $x_1 = 60$ and $x_2 = 60$?_____.

(b) On the graph below, use red ink to draw an indifference curve showing all of the combinations of scores that Nancy likes exactly as much as $x_1 = 20$ and $x_2 = 70$. Also use red ink to draw an indifference curve showing the combinations that Nancy likes exactly as much as $x_1 = 60$ and $x_2 = 60$.

(c) Does Nancy have convex preferences over these combinations?_____.

Grade on second midterm



(d) Nancy is also taking a course in economics from Professor Stern. Professor Stern gives two midterms. Instead of discarding the lower grade, Professor Stern discards the higher one. Let x_1 be her score on the first midterm and x_2 be her score on the second midterm. Which combination of scores would Nancy prefer, $x_1 = 20$ and $x_2 = 70$ or $x_1 = 60$ and $x_2 = 50$?_____.

(e) On the graph above, use blue ink to draw an indifference curve showing all of the combinations of scores on her econ exams that Nancy likes exactly as well as $x_1 = 20$ and $x_2 = 70$. Also use blue ink to draw an indifference curve showing the combinations that Nancy likes exactly as well as $x_1 = 60$ and $x_2 = 50$. Does Nancy have convex preferences over these combinations?_____.

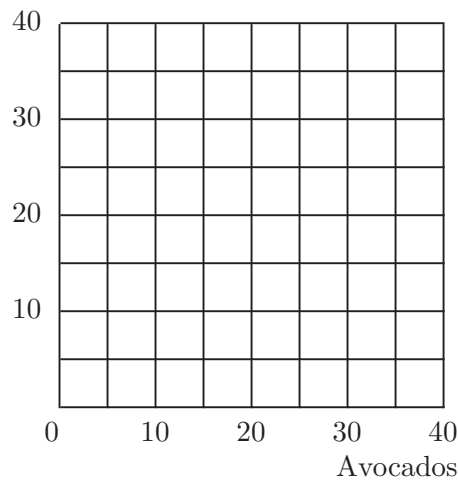
3.9 (0) Mary Granola loves to consume two goods, grapefruits and avocados.

(a) On the graph below, the slope of an indifference curve through any point where she has more grapefruits than avocados is -2 . This means that when she has more grapefruits than avocados, she is willing to give up _____ grapefruit(s) to get one avocado.

(b) On the same graph, the slope of an indifference curve at points where she has fewer grapefruits than avocados is $-1/2$. This means that when she has fewer grapefruits than avocados, she is just willing to give up _____ grapefruit(s) to get one avocado.

(c) On this graph, draw an indifference curve for Mary through bundle $(10A, 10G)$. Draw another indifference curve through $(20A, 20G)$.

Grapefruits



(d) Does Mary have convex preferences?_____.

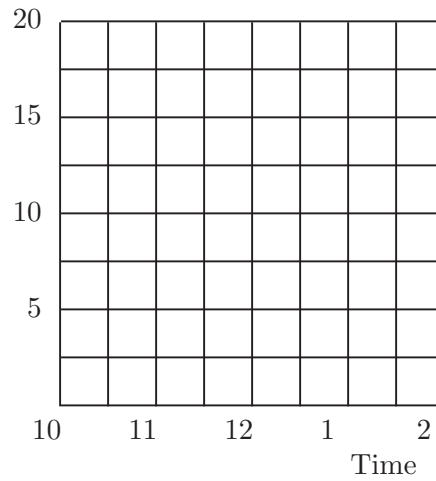
3.10 (2) Ralph Rigid likes to eat lunch at 12 noon. However, he also likes to save money so he can buy other consumption goods by attending the “early bird specials” and “late lunchers” promoted by his local diner. Ralph has 15 dollars a day to spend on lunch and other stuff. Lunch at noon costs \$5. If he delays his lunch until t hours after noon, he is able to buy his lunch for a price of $\$5 - t$. Similarly if he eats his lunch t hours before noon, he can buy it for a price of $\$5 - t$. (This is true for fractions of hours as well as integer numbers of hours.)

(a) If Ralph eats lunch at noon, how much money does he have per day to spend on other stuff?_____.

(b) How much money per day would he have left for other stuff if he ate at 2 P.M.?_____.

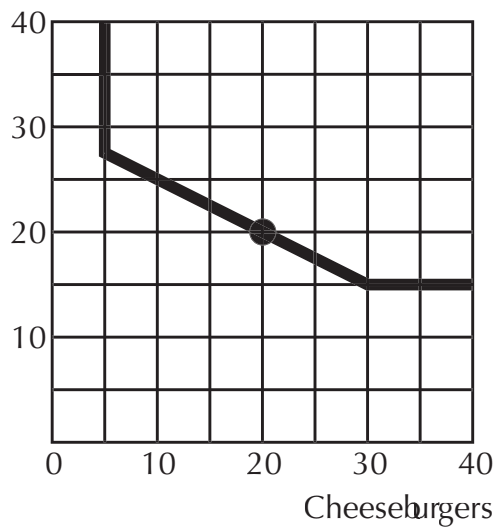
(c) On the graph below, use blue ink to draw the broken line that shows combinations of meal time and money for other stuff that Ralph can just afford. On this same graph, draw some indifference curves that would be consistent with Ralph choosing to eat his lunch at 11 A.M.

Money



3.11 (0) Henry Hanover is currently consuming 20 cheeseburgers and 20 Cherry Cokes a week. A typical indifference curve for Henry is depicted below.

Cherry Cole



(a) If someone offered to trade Henry one extra cheeseburger for every Coke he gave up, would Henry want to do this?_____.

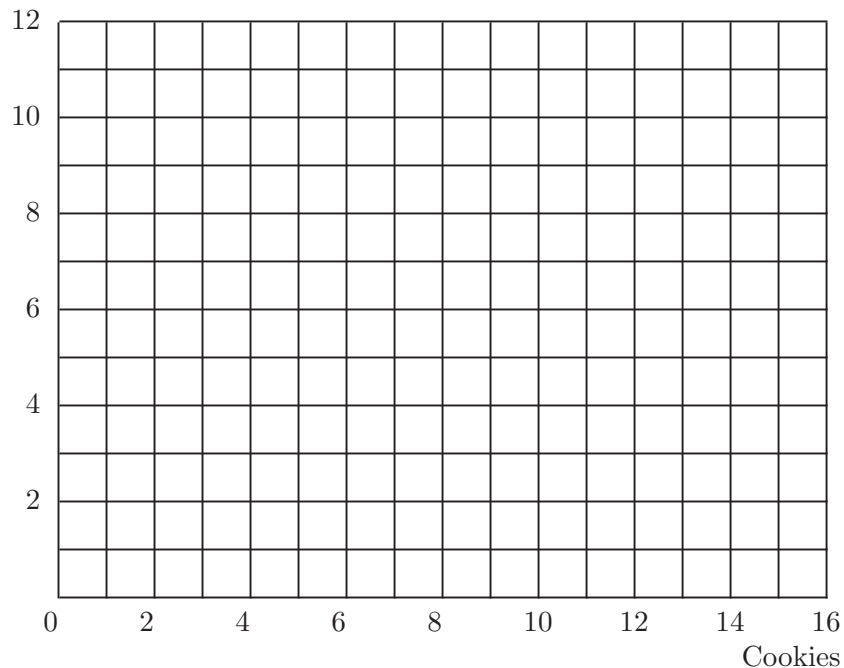
(b) What if it were the other way around: for every cheeseburger Henry gave up, he would get an extra Coke. Would he accept this offer?_____

(c) At what rate of exchange would Henry be willing to stay put at his current consumption level?_____.

3.12 (1) Tommy Twit is happiest when he has 8 cookies and 4 glasses of milk per day. Whenever he has more than his favorite amount of either food, giving him still more makes him worse off. Whenever he has less than his favorite amount of either food, giving him more makes him better off. His mother makes him drink 7 glasses of milk and only allows him 2 cookies per day. One day when his mother was gone, Tommy's sadistic sister made him eat 13 cookies and only gave him 1 glass of milk, despite the fact that Tommy complained bitterly about the last 5 cookies that she made him eat and begged for more milk. Although Tommy complained later to his mother, he had to admit that he liked the diet that his sister forced on him better than what his mother demanded.

(a) Use black ink to draw some indifference curves for Tommy that are consistent with this story.

Milk



(b) Tommy's mother believes that the optimal amount for him to consume is 7 glasses of milk and 2 cookies. She measures deviations by absolute values. If Tommy consumes some other bundle, say, (c, m) , she measures his departure from the optimal bundle by $D = |7 - m| + |2 - c|$. The larger D is, the worse off she thinks Tommy is. Use blue ink in the graph above to sketch a few of Mrs. Twit's indifference curves for Tommy's consumption. (Hint: Before you try to draw Mrs. Twit's indifference curves, we suggest that you take a piece of scrap paper and draw a graph of the locus of points (x_1, x_2) such that $|x_1| + |x_2| = 1$.)

3.13 (0) Coach Steroid likes his players to be big, fast, and obedient. If player A is better than player B in two of these three characteristics, then Coach Steroid prefers A to B , but if B is better than A in two of these three characteristics, then Steroid prefers B to A . Otherwise, Steroid is indifferent between them. Wilbur Westinghouse weighs 340 pounds, runs very slowly, and is fairly obedient. Harold Hotpoint weighs 240 pounds, runs very fast, and is very disobedient. Jerry Jacuzzi weighs 150 pounds, runs at average speed, and is extremely obedient.

(a) Does Steroid prefer Westinghouse to Hotpoint or vice versa?_____

_____.

(b) Does Steroid prefer Hotpoint to Jacuzzi or vice versa?_____

_____.

(c) Does Steroid prefer Westinghouse to Jacuzzi or vice versa?_____

_____.

(d) Does Coach Steroid have transitive preferences?_____.

(e) After several losing seasons, Coach Steroid decides to change his way of judging players. According to his new preferences, Steroid prefers player A to player B if player A is better in all three of the characteristics that Steroid values, and he prefers B to A if player B is better at all three things. He is indifferent between A and B if they weigh the same, are equally fast, and are equally obedient. In all other cases, Coach Steroid simply says "A and B are not comparable."

(f) Are Coach Steroid's new preferences complete?_____.

(g) Are Coach Steroid's new preferences transitive?_____.

(h) Are Coach Steroid's new preferences reflexive?_____.

3.14 (0) The Bear family is trying to decide what to have for dinner. Baby Bear says that his ranking of the possibilities is (honey, grubs, Goldilocks). Mama Bear ranks the choices (grubs, Goldilocks, honey), while Papa Bear's ranking is (Goldilocks, honey, grubs). They decide to take each pair of alternatives and let a majority vote determine the family rankings.

(a) Papa suggests that they first consider honey vs. grubs, and then the winner of that contest vs. Goldilocks. Which alternative will be chosen?

_____.

(b) Mama suggests instead that they consider honey vs. Goldilocks and then the winner vs. grubs. Which gets chosen?_____.

(c) What order should Baby Bear suggest if he wants to get his favorite food for dinner?_____.

(d) Are the Bear family's "collective preferences," as determined by voting, transitive?_____.

3.15 (0) Olson likes strong coffee, the stronger the better. But he can't distinguish small differences. Over the years, Mrs. Olson has discovered that if she changes the amount of coffee by more than one teaspoon in her six-cup pot, Olson can tell that she did it. But he cannot distinguish differences smaller than one teaspoon per pot. Where A and B are two different cups of coffee, let us write $A \succ B$ if Olson prefers cup A to cup B . Let us write $A \succeq B$ if Olson either prefers A to B , or can't tell the difference between them. Let us write $A \sim B$ if Olson can't tell the difference between cups A and B . Suppose that Olson is offered cups A , B , and C all brewed in the Olsons' six-cup pot. Cup A was brewed using 14 teaspoons of coffee in the pot. Cup B was brewed using 14.75 teaspoons of coffee in the pot and cup C was brewed using 15.5 teaspoons of coffee in the pot. For each of the following expressions determine whether it is true or false.

(a) $A \sim B$._____.

(b) $B \sim A$._____.

(c) $B \sim C$._____.

(d) $A \sim C$._____.

(e) $C \sim A$._____.

(f) $A \succeq B$._____.

(g) $B \succeq A$._____.

(h) $B \succeq C$._____.

(i) $A \succeq C$._____.

(j) $C \succeq A$._____.

(k) $A \succ B$._____.

(l) $B \succ A$._____.

(m) $B \succ C$._____.

(n) $A \succ C$._____.

(o) $C \succ A$._____.

(p) Is Olson's "at-least-as-good-as" relation, \succeq , transitive?_____.

(q) Is Olson's "can't-tell-the-difference" relation, \sim , transitive?_____.

(r) Is Olson's "better-than" relation, \succ , transitive?_____.

In the previous chapter, you learned about preferences and indifference curves. Here we study another way of describing preferences, the *utility function*. A utility function that represents a person's preferences is a function that assigns a utility number to each commodity bundle. The numbers are assigned in such a way that commodity bundle (x, y) gets a higher utility number than bundle (x', y') if and only if the consumer prefers (x, y) to (x', y') . If a consumer has the utility function $U(x_1, x_2)$, then she will be indifferent between two bundles if they are assigned the same utility.

If you know a consumer's utility function, then you can find the indifference curve passing through any commodity bundle. Recall from the previous chapter that when good 1 is graphed on the horizontal axis and good 2 on the vertical axis, the slope of the indifference curve passing through a point (x_1, x_2) is known as the *marginal rate of substitution*. An important and convenient fact is that *the slope of an indifference curve is minus the ratio of the marginal utility of good 1 to the marginal utility of good 2*. For those of you who know even a tiny bit of calculus, calculating marginal utilities is easy. To find the marginal utility of either good, you just take the derivative of utility with respect to the amount of that good, treating the amount of the other good as a constant. (If you don't know any calculus at all, you can calculate an approximation to marginal utility by the method described in your textbook. Also, at the beginning of this section of the workbook, we list the marginal utility functions for commonly encountered utility functions. Even if you can't compute these yourself, you can refer to this list when later problems require you to use marginal utilities.)

Arthur's utility function is $U(x_1, x_2) = x_1x_2$. Let us find the indifference curve for Arthur that passes through the point $(3, 4)$. First, calculate $U(3, 4) = 3 \times 4 = 12$. The indifference curve through this point consists of all (x_1, x_2) such that $x_1x_2 = 12$. This last equation is equivalent to $x_2 = 12/x_1$. Therefore to draw Arthur's indifference curve through $(3, 4)$, just draw the curve with equation $x_2 = 12/x_1$. At the point (x_1, x_2) , the marginal utility of good 1 is x_2 and the marginal utility of good 2 is x_1 . Therefore Arthur's marginal rate of substitution at the point $(3, 4)$ is $-x_2/x_1 = -4/3$.

Arthur's uncle, Basil, has the utility function $U^*(x_1, x_2) = 3x_1x_2 - 10$. Notice that $U^*(x_1, x_2) = 3U(x_1, x_2) - 10$, where $U(x_1, x_2)$ is Arthur's utility function. Since U^* is a positive multiple of U minus a constant, it must be that any change in consumption that increases U will also increase U^* (and vice versa). Therefore we say that Basil's utility function is a *monotonic increasing transformation* of Arthur's utility function. Let us find Basil's indifference curve through the point $(3, 4)$. First we find that

$U^*(3, 4) = 3 \times 3 \times 4 - 10 = 26$. The indifference curve passing through this point consists of all (x_1, x_2) such that $3x_1x_2 - 10 = 26$. Simplify this last expression by adding 10 to both sides of the equation and dividing both sides by 3. You find $x_1x_2 = 12$, or equivalently, $x_2 = 12/x_1$. This is exactly the same curve as Arthur's indifference curve through $(3, 4)$. We could have known in advance that this would happen, because if two consumers' utility functions are monotonic increasing transformations of each other, then these consumers must have the same preference relation between any pair of commodity bundles.

When you have finished this workout, we hope that you will be able to do the following:

- Draw an indifference curve through a specified commodity bundle when you know the utility function.
- Calculate marginal utilities and marginal rates of substitution when you know the utility function.
- Determine whether one utility function is just a “monotonic transformation” of another and know what that implies about preferences.
- Find utility functions that represent preferences when goods are perfect substitutes and when goods are perfect complements.
- Recognize utility functions for commonly studied preferences such as perfect substitutes, perfect complements, and other kinked indifference curves, quasilinear utility, and Cobb-Douglas utility.

4.0 Warm Up Exercise. This is the first of several “warm up exercises” that you will find in *Workouts*. These are here to help you see how to do calculations that are needed in later problems. The answers to *all* warm up exercises are in your answer pages. If you find the warm up exercises easy and boring, go ahead—skip them and get on to the main problems. You can come back and look at them if you get stuck later.

This exercise asks you to calculate marginal utilities and marginal rates of substitution for some common utility functions. These utility functions will reappear in several chapters, so it is a good idea to get to know them now. If you know calculus, you will find this to be a breeze. Even if your calculus is shaky or nonexistent, you can handle the first three utility functions just by using the definitions in the textbook. These three are easy because the utility functions are linear. If you do not know any calculus, fill in the rest of the answers from the back of the workbook and keep a copy of this exercise for reference when you encounter these utility functions in later problems.

$u(x_1, x_2)$	$MU_1(x_1, x_2)$	$MU_2(x_1, x_2)$	$MRS(x_1, x_2)$
$2x_1 + 3x_2$			
$4x_1 + 6x_2$			
$ax_1 + bx_2$			
$2\sqrt{x_1} + x_2$			
$\ln x_1 + x_2$			
$v(x_1) + x_2$			
x_1x_2			
$x_1^a x_2^b$			
$(x_1 + 2)(x_2 + 1)$			
$(x_1 + a)(x_2 + b)$			
$x_1^a + x_2^a$			

4.1 (0) Remember Charlie from Chapter 3? Charlie consumes apples and bananas. We had a look at two of his indifference curves. In this problem we give you enough information so you can find *all* of Charlie's indifference curves. We do this by telling you that Charlie's utility function happens to be $U(x_A, x_B) = x_A x_B$.

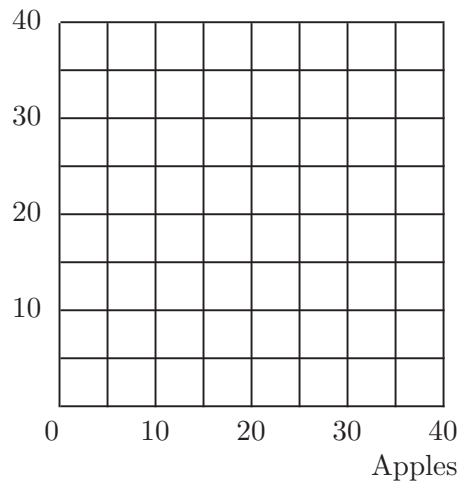
(a) Charlie has 40 apples and 5 bananas. Charlie's utility for the bundle (40, 5) is $U(40, 5) =$ _____ The indifference curve through (40, 5)

includes all commodity bundles (x_A, x_B) such that $x_A x_B =$ _____

So the indifference curve through (40, 5) has the equation $x_B =$ _____

_____ On the graph below, draw the indifference curve showing all of the bundles that Charlie likes exactly as well as the bundle (40, 5).

Bananas



(b) Donna offers to give Charlie 15 bananas if he will give her 25 apples. Would Charlie have a bundle that he likes better than (40, 5) if he makes

this trade?_____ What is the largest number of apples that Donna could demand from Charlie in return for 15 bananas if she expects him to

be willing to trade or at least indifferent about trading?_____ (Hint: If Donna gives Charlie 15 bananas, he will have a total of 20 bananas. If he has 20 bananas, how many apples does he need in order to be as well-off as he would be without trade?)

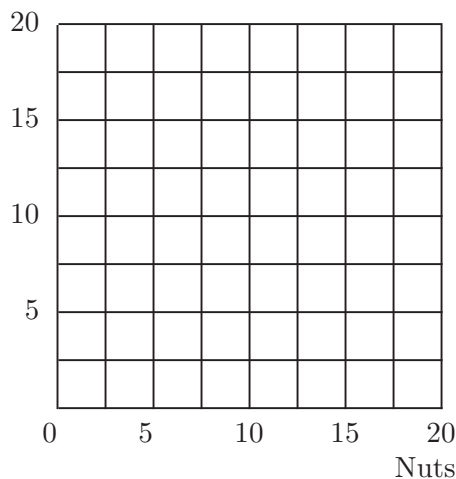
4.2 (0) Ambrose, whom you met in the last chapter, continues to thrive on nuts and berries. You saw two of his indifference curves. One indifference curve had the equation $x_2 = 20 - 4\sqrt{x_1}$, and another indifference curve had the equation $x_2 = 24 - 4\sqrt{x_1}$, where x_1 is his consumption of

nuts and x_2 is his consumption of berries. Now it can be told that Ambrose has quasilinear utility. In fact, his preferences can be represented by the utility function $U(x_1, x_2) = 4\sqrt{x_1} + x_2$.

(a) Ambrose originally consumed 9 units of nuts and 10 units of berries. His consumption of nuts is reduced to 4 units, but he is given enough berries so that he is just as well-off as he was before. After the change, how many units of berries does Ambrose consume?_____.

(b) On the graph below, indicate Ambrose's original consumption and sketch an indifference curve passing through this point. As you can verify, Ambrose is indifferent between the bundle (9,10) and the bundle (25,2). If you doubled the amount of each good in each bundle, you would have bundles (18,20) and (50,4). Are these two bundles on the same indifference curve? _____ (Hint: How do you check whether two bundles are indifferent when you know the utility function?)

Berries



(c) What is Ambrose's marginal rate of substitution, $MRS(x_1, x_2)$, when he is consuming the bundle (9, 10)? (Give a numerical answer.)_____ What is Ambrose's marginal rate of substitution when he is consuming the bundle (9, 20)?_____.

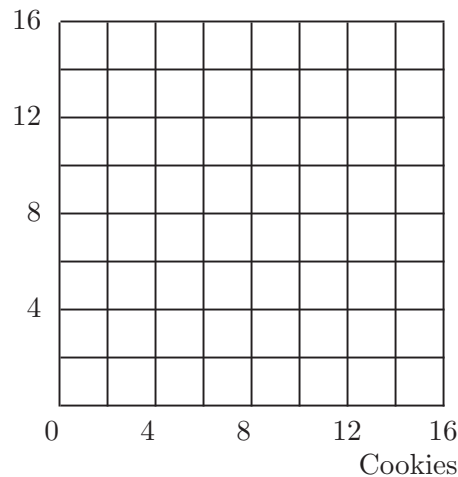
(d) We can write a general expression for Ambrose's marginal rate of substitution when he is consuming commodity bundle (x_1, x_2) . This is $MRS(x_1, x_2) =$ _____ Although we always write $MRS(x_1, x_2)$ as a function of the two variables, x_1 and x_2 , we see that Ambrose's utility function has the special property that his marginal rate of substitution does not change when the variable _____ changes.

4.3 (0) Burt's utility function is $U(x_1, x_2) = (x_1 + 2)(x_2 + 6)$, where x_1 is the number of cookies and x_2 is the number of glasses of milk that he consumes.

(a) What is the slope of Burt's indifference curve at the point where he is consuming the bundle (4, 6)?_____ Use pencil or black ink to draw a line with this slope through the point (4, 6). (Try to make this graph fairly neat and precise, since details will matter.) The line you just drew is the *tangent line* to the consumer's indifference curve at the point (4, 6).

(b) The indifference curve through the point (4, 6) passes through the points (_____, 0), (7, _____), and (2, _____). Use blue ink to sketch in this indifference curve. Incidentally, the equation for Burt's indifference curve through the point (4, 6) is $x_2 =$ _____.

Glasses of milk



(c) Burt currently has the bundle (4, 6). Ernie offers to give Burt 9 glasses of milk if Burt will give Ernie 3 cookies. If Burt makes this trade, he would have the bundle _____ Burt refuses to trade. Was this a wise decision?_____ Mark the bundle (1, 15) on your graph.

(d) Ernie says to Burt, "Burt, your marginal rate of substitution is -2 . That means that an extra cookie is worth only twice as much to you as an extra glass of milk. I offered to give you 3 glasses of milk for every cookie you give me. If I offer to give you more than your marginal rate of substitution, then you should want to trade with me." Burt replies, "Ernie, you are right that my marginal rate of substitution is -2 . That means that I am willing to make *small* trades where I get more than 2

glasses of milk for every cookie I give you, but 9 glasses of milk for 3 cookies is too big a trade. My indifference curves are not straight lines, you see.” Would Burt be willing to give up 1 cookie for 3 glasses of milk?_____ Would Burt object to giving up 2 cookies for 6 glasses of milk?_____.

(e) On your graph, use red ink to draw a line with slope -3 through the point $(4, 6)$. This line shows all of the bundles that Burt can achieve by trading cookies for milk (or milk for cookies) at the rate of 1 cookie for every 3 glasses of milk. Only a segment of this line represents trades that make Burt better off than he was without trade. Label this line segment on your graph AB .

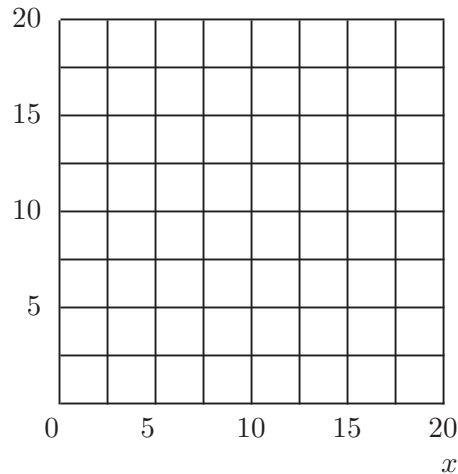
4.4 (0) Phil Rupp’s utility function is $U(x, y) = \max\{x, 2y\}$.

(a) On the graph below, use blue ink to draw and label the line whose equation is $x = 10$. Also use blue ink to draw and label the line whose equation is $2y = 10$.

(b) If $x = 10$ and $2y < 10$, then $U(x, y) =$ _____. If $x < 10$ and $2y = 10$, then $U(x, y) =$ _____.

(c) Now use red ink to sketch in the indifference curve along which $U(x, y) = 10$. Does Phil have convex preferences?_____.

y



4.5 (0) As you may recall, Nancy Lerner is taking Professor Stern’s economics course. She will take two examinations in the course, and her score for the course is the minimum of the scores that she gets on the two exams. Nancy wants to get the highest possible score for the course.

(a) Write a utility function that represents Nancy's preferences over alternative combinations of test scores x_1 and x_2 on tests 1 and 2 respectively.

$U(x_1, x_2) =$ _____.

4.6 (0) Remember Shirley Sixpack and Lorraine Quiche from the last chapter? Shirley thinks a 16-ounce can of beer is just as good as two 8-ounce cans. Lorraine only drinks 8 ounces at a time and hates stale beer, so she thinks a 16-ounce can is no better or worse than an 8-ounce can.

(a) Write a utility function that represents Shirley's preferences between commodity bundles comprised of 8-ounce cans and 16-ounce cans of beer. Let X stand for the number of 8-ounce cans and Y stand for the number of 16-ounce cans._____.

(b) Now write a utility function that represents Lorraine's preferences.
_____.

(c) Would the function utility $U(X, Y) = 100X + 200Y$ represent Shirley's preferences?_____ Would the utility function $U(x, y) = (5X + 10Y)^2$ represent her preferences?_____ Would the utility function $U(x, y) = X + 3Y$ represent her preferences?_____.

(d) Give an example of two commodity bundles such that Shirley likes the first bundle better than the second bundle, while Lorraine likes the second bundle better than the first bundle.
_____.

4.7 (0) Harry Mazzola has the utility function $u(x_1, x_2) = \min\{x_1 + 2x_2, 2x_1 + x_2\}$, where x_1 is his consumption of corn chips and x_2 is his consumption of french fries.

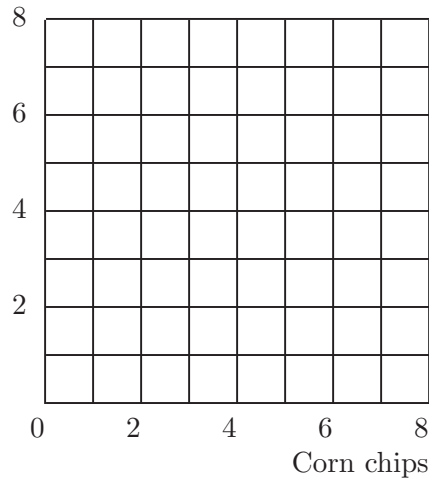
(a) On the graph below, use a pencil to draw the locus of points along which $x_1 + 2x_2 = 2x_1 + x_2$. Use blue ink to show the locus of points for which $x_1 + 2x_2 = 12$, and also use blue ink to draw the locus of points for which $2x_1 + x_2 = 12$.

(b) On the graph you have drawn, shade in the region where *both* of the following inequalities are satisfied: $x_1 + 2x_2 \geq 12$ and $2x_1 + x_2 \geq 12$.

At the bundle $(x_1, x_2) = (8, 2)$, one sees that $2x_1 + x_2 =$ _____ and $x_1 + 2x_2 =$ _____. Therefore $u(8, 2) =$ _____.

(c) Use black ink to sketch in the indifference curve along which Harry's utility is 12. Use red ink to sketch in the indifference curve along which Harry's utility is 6. (Hint: Is there anything about Harry Mazzola that reminds you of Mary Granola?)

French fries



(d) At the point where Harry is consuming 5 units of corn chips and 2 units of french fries, how many units of corn chips would he be willing to trade for one unit of french fries?_____.

4.8 (1) Vanna Boogie likes to have large parties. She also has a strong preference for having exactly as many men as women at her parties. In fact, Vanna's preferences among parties can be represented by the utility function $U(x, y) = \min\{2x - y, 2y - x\}$ where x is the number of women and y is the number of men at the party. On the graph below, let us try to draw the indifference curve along which Vanna's utility is 10.

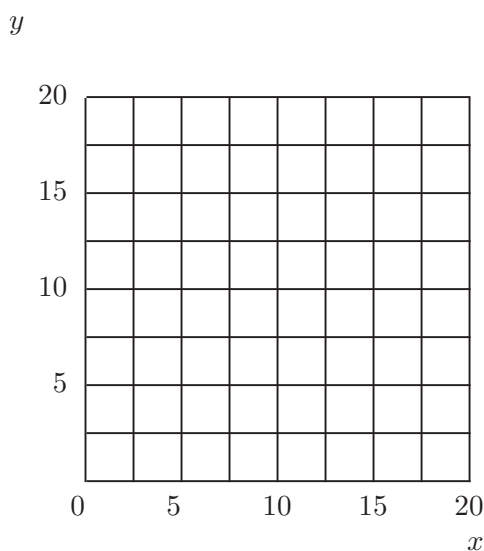
(a) Use pencil to draw the locus of points at which $x = y$. What point on this gives Vanna a utility of 10?_____ Use blue ink to draw the line along which $2y - x = 10$. When $\min\{2x - y, 2y - x\} = 2y - x$, there are (more men than women, more women than men)?_____

_____ Draw a squiggly red line over the part of the blue line for which $U(x, y) = \min\{2x - y, 2y - x\} = 2y - x$. This shows all the combinations that Vanna thinks are just as good as (10, 10) but where there are (more men than women, more women than men)?_____ Now draw a blue line along which $2x - y = 10$. Draw a squiggly red line over the part of this new blue line for which $\min\{2x - y, 2y - x\} = 2x - y$. Use pencil to shade in the area on the graph that represents all combinations that Vanna likes at least as well as (10, 10).

(b) Suppose that there are 9 men and 10 women at Vanna's party. Would Vanna think it was a better party or a worse party if 5 more men came to her party?_____.

(c) If Vanna has 16 women at her party and more men than women, and if she thinks the party is exactly as good as having 10 men and 10 women, how many men does she have at the party?_____ If Vanna has 16 women at her party and more women than men, and if she thinks the party is exactly as good as having 10 men and 10 women, how many men does she have at her party?_____.

(d) Vanna's indifference curves are shaped like what letter of the alphabet?_____.



4.9 (0) Suppose that the utility functions $u(x, y)$ and $v(x, y)$ are related by $v(x, y) = f(u(x, y))$. In each case below, write “Yes” if the function f is a positive monotonic transformation and “No” if it is not. (Hint for calculus users: A differentiable function $f(u)$ is an increasing function of u if its derivative is positive.)

(a) $f(u) = 3.141592u$ _____.

(b) $f(u) = 5,000 - 23u$ _____.

(c) $f(u) = u - 100,000$ _____.

(d) $f(u) = \log_{10} u$.

(e) $f(u) = -e^{-u}$.

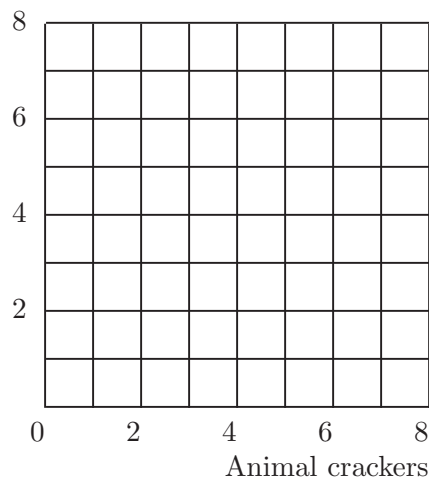
(f) $f(u) = 1/u$.

(g) $f(u) = -1/u$.

4.10 (0) Martha Modest has preferences represented by the utility function $U(a, b) = ab/100$, where a is the number of ounces of animal crackers that she consumes and b is the number of ounces of beans that she consumes.

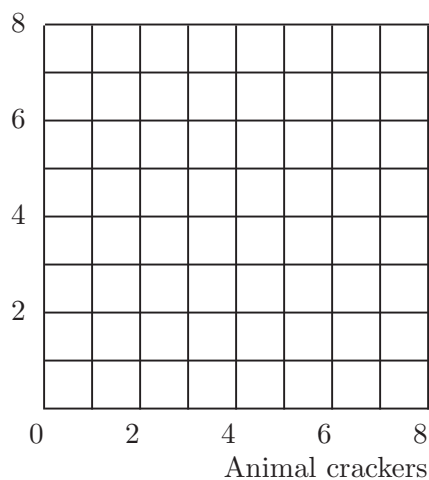
(a) On the graph below, sketch the locus of points that Martha finds indifferent to having 8 ounces of animal crackers and 2 ounces of beans. Also sketch the locus of points that she finds indifferent to having 6 ounces of animal crackers and 4 ounces of beans.

Beans



(b) Bertha Brassy has preferences represented by the utility function $V(a, b) = 1,000a^2b^2$, where a is the number of ounces of animal crackers that she consumes and b is the number of ounces of beans that she consumes. On the graph below, sketch the locus of points that Bertha finds indifferent to having 8 ounces of animal crackers and 2 ounces of beans. Also sketch the locus of points that she finds indifferent to having 6 ounces of animal crackers and 4 ounces of beans.

Beans



(c) Are Martha's preferences convex?_____ Are Bertha's?_____

_____.

(d) What can you say about the difference between the indifference curves you drew for Bertha and those you drew for Martha?_____

_____.

(e) How could you tell this was going to happen without having to draw the curves?_____

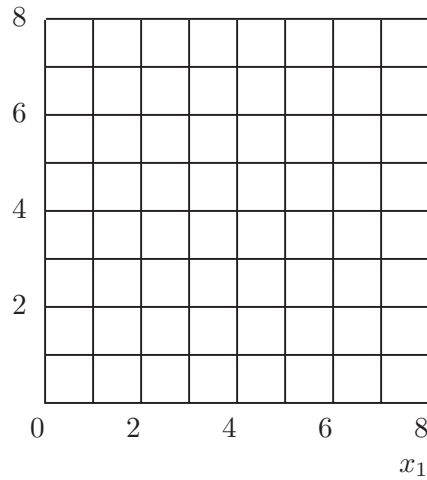
_____.

4.11 (0) Willy Wheeler's preferences over bundles that contain non-negative amounts of x_1 and x_2 are represented by the utility function $U(x_1, x_2) = x_1^2 + x_2^2$.

(a) Draw a few of his indifference curves. What kind of geometric figure are they?_____ Does Willy have

convex preferences?_____.

x_2



4.12 (0) Joe Bob has a utility function given by $u(x_1, x_2) = x_1^2 + 2x_1x_2 + x_2^2$.

(a) Compute Joe Bob's marginal rate of substitution: $MRS(x_1, x_2) =$

(b) Joe Bob's straight cousin, Al, has a utility function $v(x_1, x_2) = x_2 + x_1$.

Compute Al's marginal rate of substitution. $MRS(x_1, x_2) =$ _____.

(c) Do $u(x_1, x_2)$ and $v(x_1, x_2)$ represent the same preferences?_____

_____ Can you show that Joe Bob's utility function is a monotonic transformation of Al's? (Hint: Some have said that Joe Bob is square.)

4.13 (0) The idea of assigning numerical values to determine a preference ordering over a set of objects is not limited in application to commodity bundles. The *Bill James Baseball Abstract* argues that a baseball player's batting average is not an adequate measure of his offensive productivity. Batting averages treat singles just the same as extra base hits. Furthermore they do not give credit for "walks," although a walk is almost as good as a single. James argues that a double in two at-bats is better than a single, but not as good as two singles. To reflect these considerations, James proposes the following index, which he calls "runs created." Let A be the number of hits plus the number of walks that a batter gets in a season. Let B be the number of total bases that the batter gets in the season. (Thus, if a batter has S singles, W walks, D doubles, T triples, and H

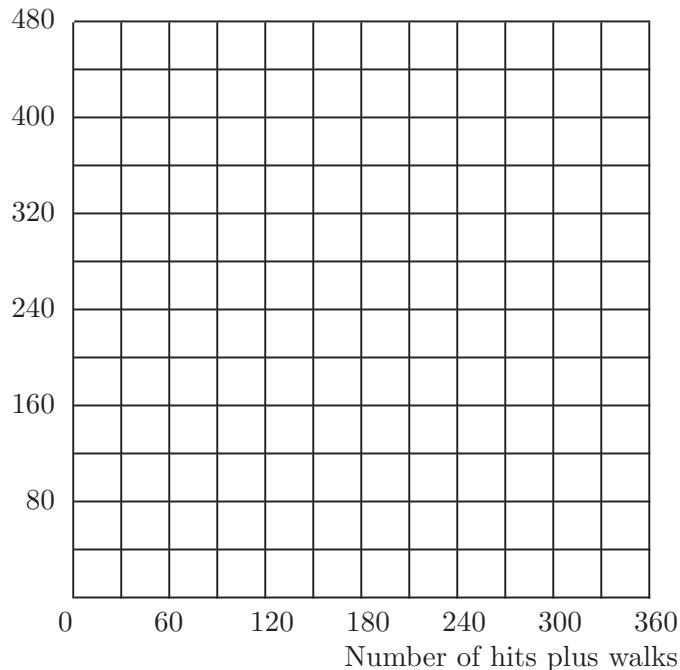
home runs, then $A = S + D + T + H + W$ and $B = S + W + 2D + 3T + 4H$.) Let N be the number of times the batter bats. Then his index of runs created in the season is defined to be AB/N and will be called his RC .

(a) In 1987, George Bell batted 649 times. He had 39 walks, 105 singles, 32 doubles, 4 triples, and 47 home runs. In 1987, Wade Boggs batted 656 times. He had 105 walks, 130 singles, 40 doubles, 6 triples, and 24 home runs. In 1987, Alan Trammell batted 657 times. He had 60 walks, 140 singles, 34 doubles, 3 triples, and 28 home runs. In 1987, Tony Gwynn batted 671 times. He had 82 walks, 162 singles, 36 doubles, 13 triples, and 7 home runs. We can calculate A , the number of hits plus walks, B the number of total bases, and RC , the runs created index for each of these players. For Bell, $A = 227$, $B = 408$, $RC = 143$. For Boggs, $A = 305$, $B = 429$, $RC = 199$. For Trammell, $A = 265$, $B = 389$, $RC = 157$. For Gwynn, $A = \underline{\hspace{2cm}}$, $B = \underline{\hspace{2cm}}$, $RC = \underline{\hspace{4cm}}$.

(b) If somebody has a preference ordering among these players, based only on the runs-created index, which player(s) would she prefer to Trammell?

(c) The differences in the number of times at bat for these players are small, and we will ignore them for simplicity of calculation. On the graph below, plot the combinations of A and B achieved by each of the players. Draw four “indifference curves,” one through each of the four points you have plotted. These indifference curves should represent combinations of A and B that lead to the same number of runs-created.

Number of total bases



4.14 (0) This problem concerns the runs-created index discussed in the preceding problem. Consider a batter who bats 100 times and always either makes an out, hits for a single, or hits a home run.

(a) Let x be the number of singles and y be the number of home runs in 100 at-bats. Suppose that the utility function $U(x, y)$ by which we evaluate alternative combinations of singles and home runs is the runs-created index. Then the formula for the utility function is $U(x, y) =$

(b) Let's try to find out about the shape of an indifference curve between singles and home runs. Hitting 10 home runs and no singles would give him the same runs-created index as hitting _____ singles and no home runs. Mark the points $(0, 10)$ and $(x, 0)$, where $U(x, 0) = U(0, 10)$.

(c) Where x is the number of singles you solved for in the previous part, mark the point $(x/2, 5)$ on your graph. Is $U(x/2, 5)$ greater than or less than or equal to $U(0, 10)$? _____ Is this consistent with the batter having convex preferences between singles and home runs? _____

Home runs

