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International Factor-Price Equalisation Once Again

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INTERNATIONAL FACTOR-PRICE EQUALISATION ONCE AGAIN

1. INTRODUCTION

My recent paper ¹ attempting to show that free commodity trade will, under certain specified conditions, inevitably lead to complete factor-price equalisation appears to be in need of further amplification. I propose therefore (1) to restate the principal theorem, (2) to expand upon its intuitive demonstration, (3) to settle the matter definitively by a brief but rigorous mathematical demonstration, (4) to make a few extensions to the case of many commodities and factors, and finally (5) to comment briefly upon some realistic qualifications to its simplified assumptions.

I cannot pretend to present a balanced appraisal of the bearing of this analysis upon interpreting the actual world, because my own mind is not made up on this question: on the one hand, I think it would be folly to come to any startling conclusions on the basis of so simplified a model and such abstract reasoning; but on the other hand, strong simple cases often point the way to an element of truth present in a complex situation. Still, at the least, we ought to be clear in our deductive reasoning; and the elucidation of this side of the problem plus the qualifying discussion may contribute towards an ultimate appraisal of the theorem's realism and relevance.

2. STATEMENT OF THE THEOREM

My hypotheses are as follows:—

1. There are but two countries, America and Europe.
2. They produce but two commodities, food and clothing.
3. Each commodity is produced with two factors of production, land and labour. The production functions of each commodity show "constant returns to scale," in the sense that changing all inputs in the same proportion changes output in that same proportion, leaving all "productivities"

¹ "International Trade and the Equalisation of Factor Prices," *Economic Journal*, Vol. LVIII, June, 1948, pp. 163-184. I learn from Professor Lionel Robbins that A. P. Lerner, while a student at L.S.E., dealt with this problem. I have had a chance to look over Lerner's mimeographed report, dated December 1933, and it is a masterly, definitive treatment of the question, difficulties and all.

essentially unchanged. In short, all production functions are mathematically “homogeneous of the first order” and subject to Euler’s theorem.

4. The law of diminishing marginal productivity holds: as any one input is increased relative to other inputs, its marginal productivity diminishes.

The commodities differ in their “labour and land intensities.” Thus, food is relatively “land using” or “land-intensive,” while clothing is relatively “labour-intensive.” This means that whatever the prevailing ratio of wages to rents, the optimal proportion of labour to land is greater in clothing than in food.

6. Land and labour are assumed to be qualitatively identical inputs in the two countries and the technological production functions are assumed to be the same in the two countries.

7. All commodities move perfectly freely in international trade, without encountering tariffs or transport costs, and with competition effectively equalising the market price-ratio of food and clothing. No factors of production can move between the countries.

8. Something is being produced in both countries of both commodities with both factors of production. Each country may have moved in the direction of specialising on the commodity for which it has a comparative advantage, but it has not moved so far as to be specialising completely on one commodity.¹

All of this constitutes the hypothesis of the theorem. The conclusion states:—

Under these conditions, real factor prices must be exactly the same in both countries (and indeed the proportion of inputs used in food production in America must equal that in Europe, and similarly for clothing production).

Our problem is from now on a purely logical one. Is “If H , then inevitably C ” a correct statement? The issue is not whether C (factor-price equalisation) will actually hold; nor even whether H (the hypothesis) is a valid empirical generalisation. It is whether C can fail to be true when H is assumed true. Being a

¹ Actually we may admit the limiting case of “incipient specialisation,” where nothing is being produced of one of the commodities, but where it is a matter of indifference whether an infinitesimal amount is or is not being produced, so that price and marginal costs are equal.

logical question, it admits of only one answer : either the theorem is true or it is false.

One may wonder why such a definite problem could have given rise to misunderstanding. The answer perhaps lies in the fact that even so simple a set-up as this one involves more than a dozen economic variables : at least four inputs for each country, four marginal productivities for each country (marginal productivity of American labour in food, of American land in food . . .), two outputs for each country, the prices of the two commodities, the price in each country of the two inputs, the proportions of the inputs in different lines of production, and so forth. It is not always easy for the intellect to move purposefully in a hyper-space of many dimensions.

And the problem is made worse by the fact, insufficiently realised, that constant returns to scale is a very serious limitation on the production functions. As soon as one knows a single "curve" on such a surface, all other magnitudes are frozen into exact quantitative shapes and cannot be chosen at will. Thus, if one knows the returns of total product to labour working on one acre of land, then one already knows everything : the marginal productivity schedule of land, all the iso-product curves, the marginal-rate-of-substitution schedules, etc. This means one must use a carefully graduated ruler in drawing the different economic functions, making sure that they are numerically consistent in addition to their having plausible qualitative shapes.

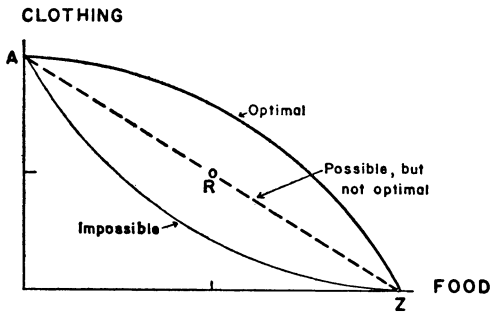
3. INTUITIVE PROOF

In each country there is assumed to be given totals of labour and land. If all resources are devoted to clothing, we get a certain maximum amount of clothing. If all are devoted to food production, we get a certain maximum amount of food. But what will happen if we are willing to devote only part of all land and part of total labour to the production of food, the rest being used in clothing production? Obviously, then we are in effect sacrificing some food in order to get some clothing. The iron law of scarcity tells us that we cannot have all we want of both goods, but must ultimately give up something of one good in getting some of another.

In short there is a best "production-possibility," or "transformation" curve showing us the maximum obtainable amount of one commodity for each amount of the other. Such a production-possibility schedule was drawn up for each country in Figure 1

of my earlier article. And in each case it was made to be a curve *convex* from above, so that the more you want of any good the greater is the cost, at the margin, in terms of the other good. This convexity property is very important and is related to the law of diminishing marginal productivity. Few readers had any qualms about accepting convexity, but perhaps some did not realise its far-reaching implications in showing why the factor-price equalisation theorem had to be true. I propose, therefore, to show why the production-possibility curve must obviously be convex (looked at from above).¹

To show that convexity, or increasing relative marginal costs must hold, it is sufficient for the present purpose to show that concavity, or decreasing marginal costs, involves an impossible contradiction. Now at the very worst, it is easily shown we can move along a straight-line opportunity cost line between the



two axes. For suppose we agree to give up half of the maximum obtainable amount of food. How much clothing can we be sure of getting? If we send out the crudest type of order: "Half of all labour and half of all land is to be shifted to clothing production," we will (because of the assumption of constant returns to scale) *exactly halve* food production; and we will acquire *exactly half* of the maximum amount of clothing produceable with all resources. Therefore, we end up at a point, *R*, exactly half-way between the limiting points *A* and *Z*. Similarly, if we decide to give up 10, 20, 30 or 90% of the maximum amount of food produceable, we can give out crude orders to transfer exactly 10, 20, 30 or 90% of *both* inputs from food to clothing. Because of constant returns to scale, it follows that we can be sure of getting 90, 80, 70 or 10% of maximum clothing.

¹ I am indebted for this line of reasoning to my colleague at M.I.T., Professor Robert L. Bishop, who for some years has been using it on beginning students in economics, with no noticeable disastrous effects. This proof is suggestive only, but it could easily be made rigorous.

In short, by giving such crude down-the-line orders that transfer both resources *always in the same proportion*, we can at worst travel along a straight line between the two limiting intercepts. Any concave curve would necessarily lie inside such a constant-cost straight line and can therefore be ruled out: hence decreasing (marginal, opportunity) costs are incompatible with the assumption of constant returns to scale.

But of course we can usually do even better than the straight-line case. A neophyte bureaucrat might be satisfied to give crude down-the-line orders, but there exist more efficient ways of giving up food for clothing. This is where social-economist (or "welfare economist") can supplement the talents of the mere technician who knows how best to use inputs in the production of any one good and nothing else. There are an infinity of ways of giving up, say, 50% of food: we may simply give up labour, or simply give up land, or give up constant percentages of labour and land, or still other proportions. But there will be only one best way to do so, only one best combination of labour and land that is to be transferred. Best in what sense? Best in the sense of getting for us the maximum obtainable amount of clothing, compatible with our pre-assigned decision to sacrifice a given amount of food.

Intuition tells us that, qualitatively, we should transfer a larger proportion of labour than of land to clothing production. This is because clothing is the labour-intensive commodity, by our original hypothesis. This means that the proportion of labour to land is actually declining in the food line as its production declines. What about the proportion of labour to land in clothing production? At first we were able to be generous in sparing labour, which after all was not "too well adapted" for food production. But now, when we come to give up still more food, there is less labour left in food production relative to land; hence, we cannot contrive to be quite so generous in transferring further labour to clothing production. As we expand clothing production further, the proportion of labour to land must also be falling in that line; but the labour-land ratio never falls to as low as the level appropriate for food, the land-intensive commodity.¹

¹ Some readers may find it paradoxical that—with a fixed ratio of total labour to total land—we nevertheless lower the ratio of labour to land *in both industries* as a result of producing more of the labour-intensive good and less of the other. Such readers find it hard to believe that men's wages and women's wages can both go up at the same time that average wages are going down. They forget that there is an inevitable shift in the industries' weights used to compute the

Intuition tells us that by following an optimal pattern which recognises the difference in factor intensities of the two goods, we can end up on a production possibility curve that is bloated out beyond a constant-cost straight line : in short, on a production possibility curve that is convex, obeying the law of increasing marginal costs of one good as it is expanded at the expense of the other good. Or to put the same thing in the language of the market-place : as the production of clothing expands, upward pressure is put on the price of the factor it uses most intensively, on wages relative to land rent. An increase in the ratio of wages to rent must in a competitive market press up the price of the labour-intensive commodity relative to the land-intensive commodity.

This one-directional relationship between relative factor prices and relative commodity prices is an absolute necessity, and it is vital for the recognition of the truth in the main theorem. Let me elaborate therefore upon the market mechanism bringing it about. Under perfect competition, everywhere within a domestic market there will be set up a uniform ratio of wages to rents. In the food industry, there will be one, and only one, optimal proportion of labour to land ; any attempt to combine productive factors in proportions that deviate from the optimum will be penalised by losses, and there will be set up a process of corrective adaptation. The same competitive forces will force an adaptation of the input proportion in clothing production, with equilibrium being attained only when the input proportions are such as to equate exactly the ratio of the physical marginal productivities of the factors (the "marginal rate of substitution" of labour for land in clothing production) to the ratio of factor prices prevailing in the market. The price mechanism has an unconscious wisdom. As if led by an invisible hand, it causes the economic system to move out to the optimal production-possibility curve. Through the intermediary of a common market factor-price ratio, the marginal rates of substitution of the factors become the same in both industries. And it is this marginal condition which intuition (as well as geometry and mathematics) tells us prescribes the optimal allocation of resources so as to yield maximum output. Not only does expanding clothing production result in the earlier described

average-factor ratio. Really to understand all this the reader must be referred to the Edgeworth box-diagram depicted in W. F. Stolper and P. A. Samuelson, "Protection and Real Wages," *Review of Economic Studies*, Vol. IX (1941), pp. 58-73.

qualitative pattern of dilution of the ratio of labour to land in both occupations; more than that, a price system is one way of achieving the exactly optimal quantitative degree of change in proportions.

I have established unequivocally the following facts :

Within any country : (a) a high ratio of wages to rents will cause a definite decrease in the proportion of labour to land in both industries ; (b) to each determinate state of factor proportion in the two industries there will correspond one, and only one, commodity price ratio and a unique configuration of wages and rent ; and finally, (c) that the change in factor proportions incident to an increase in $\frac{\text{wages}}{\text{rents}}$ must be followed by a one-directional increase in clothing prices relative to food prices.

An acute reader may try to run ahead of the argument and may be tempted to assert : “ But all this holds for one country, as of a given total factor endowment. Your established chain of causation is only from factor prices (and factor proportions) to commodity prices. Are you entitled to reverse the causation and to argue that the same commodity-price ratio must—even in countries of quite different total factor endowments—lead back to a common unique factor-price ratio, a common unique way of combining the inputs in the food and clothing industries, and a common set of absolute factor prices and marginal productivities ? ”

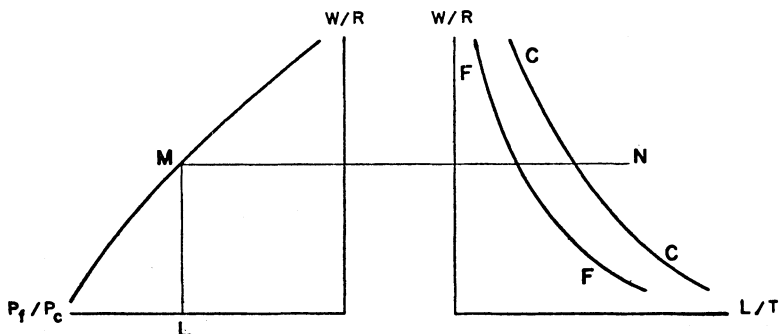
My answer is yes. This line of reasoning is absolutely rigorous. It is only proportions that matter, not scale. In such a perfectly competitive market each small association of factors (or firm, if one prefers that word) feels free to hire as many or as few extra factors as it likes. It neither knows nor cares anything about the totals for society. It is like a group of molecules in a perfect gas which is everywhere in thermal equilibrium. The molecules in any one small region behave in the same way regardless of the size of the room around them. A sample observed in the middle of a huge spherical room would act in the same way as a similar sample observed within a small rectangular room. Similarly, if we observe the behaviour of a representative firm ¹ in one country it will be exactly the same in all essentials as a representative firm taken from some other country—regardless of

¹ The representative firm concept is in the case of homogeneous production functions not subject to the usual difficulties associated with the Marshallian concept; in this case, it should be added, the “ scale ” of the firm is indeterminate and, fortunately, irrelevant.

the difference in total factor amounts and relative industrial concentration—provided only that factor-price ratios are really the same in the two markets.

All this follows from the italicised conclusion reached just above, especially from (c) taken in conjunction with (a) and (b).

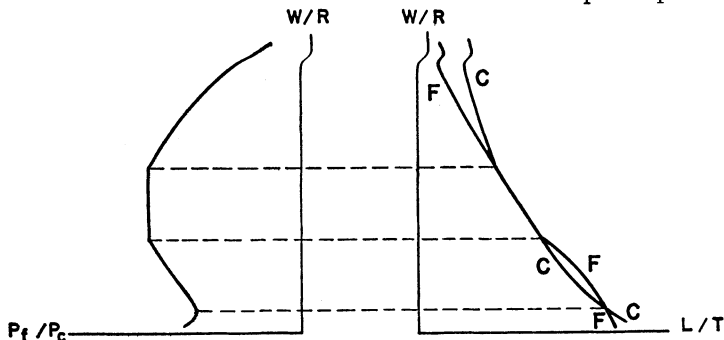
This really completes the intuitive demonstration of the theorem. The same international commodity-price ratio, must—so long as both commodities are being produced and priced at marginal costs—enable us to infer backwards a unique factor-price ratio, a unique set of factor proportions, and even a unique set of absolute wages and rents.



All this is summarised in the accompanying chart. On the right-hand side I have simply duplicated Figure 2 of my earlier paper. On the left-hand side I have added a chart showing the one-directional relation of commodity prices to factor prices.¹ As

¹ The left-hand curve is drawn in a qualitatively correct fashion. Actually its exact quantitative shape is determined by the two right-hand curves; but the chart is *not* exact in its quantitative details.

We may easily illustrate the importance of point (5) of our hypothesis, which insists on differences in factor intensities. Consider the depicted pathological



case which does not meet the requirements of our hypothesis, and in which factor intensities are for a range identical, and in still other regions food becomes the labour-intensive good. The resulting pattern of commodity prices does *not* necessarily result in factor-price equalisation. Cf. p. 175, n. 1 of my earlier article.

wages fall relative to rents the price of food is shown to rise relative to clothing in a monotonic fashion. The accompanying chart applies to either country and—so long as neither country is specialising completely—its validity is independent of their differing factor endowments. It follows that when we specify a common price ratio (say at L), we can move backward unambiguously (from M to N , etc.) to a common factor-price ratio and to a common factor proportion set-up in the two countries.

4. MATHEMATICAL PROOF

Now that the theorem has been demonstrated by common-sense reasoning, let me confirm it by more rigorous mathematical proof. The condition of equilibrium can be written in a variety of ways, and can be framed so as to involve more than a dozen equations. For example, let me call America's four marginal physical productivities—of labour in food, of land in food, of labour in clothing, of land in clothing— a , b , c and d . I use Greek letters— α , β , γ , δ —to designate the corresponding marginal productivities in Europe. Then we can end up with a number of equilibrium expressions of the form

$$\frac{a}{b} = \frac{c}{d}, \quad \frac{\alpha}{\beta} = \frac{\gamma}{\delta}, \quad \frac{a}{c} = \frac{\alpha}{\gamma}, \quad \dots \text{ etc.}$$

A number of economists have tortured themselves trying to manipulate these expressions so as to result in $a = \alpha$, etc., or at least in $\frac{a}{b} = \frac{\alpha}{\beta}$, etc. No proof of this kind is possible. The essential thing is that these numerous marginal productivities are by no means independent. Because proportions rather than scale are important, knowledge of the behaviour of the marginal productivity of labour tells us exactly what to expect of the marginal-productivity schedule of land. This is because increasing the amount of labour with land held constant is equivalent to reducing land with labour held constant.¹

¹ J. B. Clark recognised in his *Distribution of Wealth* that the "upper triangle" of his labour-marginal-productivity diagram must correspond to the "rectangle" of his other-factors diagram. But his draughtsman did not draw the curve accordingly! This is a mistake that Philip Wicksteed in his *Co-ordination of the Laws of Distribution* (London School of Economics Reprint) could not have made. Clark, a believer in Providence, was unaware of the blessing—in the form of Euler's theorem on homogeneous functions—that made his theory possible. Wicksteed, a man of the cloth, appreciated and interpreted the generosity of Nature. Cf. also F. H. Knight, *Risk, Uncertainty and Profit*, ch. IV, for a partial treatment of these reciprocal relations. G. J. Stigler, *Production and Distribution Theories: the Formative Period*, gives a valuable treatment of Wicksteed's theory as expounded by Flux and others.

Mathematically, instead of writing food production, F , as any joint function of labour devoted to it, L_f , and of land, T_f , we can write it as

$$F = F(L_f, T_f) = T_f f\left(\frac{L_f}{T_f}\right) \quad . \quad . \quad . \quad (1)$$

where the function f can be thought of as the returns of food on one unit of land, and where the number of units of land enters as a scale factor. The form of this function is the same for both countries; and there is, of course, a similar type of function holding for cloth production, C , in terms of L_c and T_c namely

$$C = C(L_c, T_c) = T_c c\left(\frac{L_c}{T_c}\right) \quad . \quad . \quad . \quad (2)$$

It is easy to show mathematically, by simple partial differentiation of (1), the following relations among marginal physical productivities

$$\text{M.P.P. labour in food} = \frac{\partial F}{\partial L_f} = f'\left(\frac{L_f}{T_f}\right)$$

where f' represents the derivative of f and depicts the schedule of marginal product of labour (working on one unit of land). This must be a declining schedule according to our hypothesis of diminishing returns, so that we must have

$$f''\left(\frac{L_f}{T_f}\right) < 0.$$

By direct differentiation of (1), or by use of Euler's theorem, or by use of the fact that the marginal product of land can also be identified as a rent residual, we easily find that

$$\text{M.P.P. land in food} = \frac{\partial F}{\partial T_f} = f\left(\frac{L_f}{T_f}\right) - \frac{L_f}{T_f} f'\left(\frac{L_f}{T_f}\right) = g\left(\frac{L_f}{T_f}\right)$$

where g is the name for the rent residual. It is easy to show that

$$g'\left(\frac{L_f}{T_f}\right) = -\frac{L_f}{T_f} f''\left(\frac{L_f}{T_f}\right).$$

By similar reasoning, we may write the marginal productivity of land in clothing production in its proper relation to that of labour

$$\text{M.P.P. labour in clothing} = \frac{\partial C}{\partial L_c} = c'\left(\frac{L_c}{T_c}\right)$$

$$\begin{aligned} \text{M.P.P. land in clothing} &= \frac{\partial C}{\partial T_c} = c\left(\frac{L_c}{T_c}\right) - \frac{L_c}{T_c} c'\left(\frac{L_c}{T_c}\right) = h\left(\frac{L_c}{T_c}\right) \\ h'\left(\frac{L_c}{T_c}\right) &= -\frac{L_c}{T_c} c''\left(\frac{L_c}{T_c}\right) \end{aligned}$$

The art of analysis in these problems is to select out the essential variables so as to reduce our equilibrium equations to the simplest form. Without specifying which country we are talking about, we certainly can infer from the fact that something of both goods is being produced with both factors the following conditions :—

Real wages (or labour marginal “ value ” productivities) must be the same in food and clothing production when expressed in terms of a common *measure*, such as clothing; the same is true of real rents (or land marginal “ value ” productivities). Or

$$\begin{aligned} (\text{food price}) (\text{M.P.P. labour in food}) \\ = (\text{clothing price}) (\text{M.P.P. labour in clothing}) \end{aligned}$$

$$\begin{aligned} (\text{food price}) (\text{M.P.P. land in food}) \\ = (\text{clothing price}) (\text{M.P.P. land in clothing}) \end{aligned}$$

which can be written in terms of previous notation ¹ as

$$\left(\frac{P_f}{P_c}\right) f' \left(\frac{L_f}{T_f}\right) - c' \left(\frac{L_c}{T_c}\right) = 0$$

$$\left(\frac{P_f}{P_c}\right) \left[f \left(\frac{L_f}{T_f}\right) - \frac{L_f}{T_f} f' \left(\frac{L_f}{T_f}\right) \right] - \left[c \left(\frac{L_c}{T_c}\right) - \frac{L_c}{T_c} c' \left(\frac{L_c}{T_c}\right) \right] = 0.$$

Now these are two equations in the three variables $\frac{L_f}{T_f}$, $\frac{L_c}{T_c}$, and $\frac{P_f}{P_c}$. If we take the latter price ratio as given to us by international-demand conditions, we are left with *two* equations to determine the *two* unknown factor proportions. This is a solvent situation, and we should normally expect the result to be determinate.

But a purist might still have doubts : “ How do you know that these two equations or schedules might not twist around and intersect in multiple equilibria ? ” Fortunately, the answer is simple and definite. On our hypothesis, any equilibrium configuration turns out to be absolutely unique. We may leave to a technical footnote the detailed mathematical proof of this fact.²

¹ In terms of our earlier $a, b, \dots, \alpha, \beta \dots$, these equations are of the form

$$\frac{P_f}{P_c} a = c, \frac{P_f}{P_c} b = d, \text{ etc.}$$

² The Implicit Function Theorem tells us that two suitably continuous

5. MULTIPLE COMMODITIES AND FACTORS

Adding a third or further commodities does not alter our analysis much. If anything, it increases the likelihood of complete factor-price equalisation. For all that we require is that at least *two* commodities are simultaneously being produced in both countries and then our previous conclusion follows. If we add a third commodity which is very much like either of our present commodities, we are not changing the situation materially. But if we add new commodities which are more extreme in their labour-land intensities, then we greatly increase the chance that two regions with very different factor endowments can still come into complete factor-price equalisation. A “queer” region is not penalised for being queer if there is queer work that needs doing.

I do not wish at this time to go into the technical mathematics of the *n* commodity, and *r* factor case. But it can be said that : (1) so long as the two regions are sufficiently close together in factor proportions, (2) so long as the goods differ in factor intensities, and (3) so long as the number of goods, *n*, is greater than the number of factors, *r*, we can hope to experience complete factor-price equalisation. On the other hand, if complete specialisation takes place it will do so for a whole collection of goods, the dividing line between exports and imports being a variable one depending upon reciprocal international demand (acting on factor prices) as in the classical theory of comparative advantage with multiple commodities.¹

equations of the form $W_1(y_1, y_2) = 0 = W_2(y_1, y_2)$, possessing a solution (y_1^0, y_2^0) , cannot have any other solution provided

$$\Delta = \begin{vmatrix} \frac{\partial W_1}{\partial y_1} & \frac{\partial W_1}{\partial y_2} \\ \frac{\partial W_2}{\partial y_1} & \frac{\partial W_2}{\partial y_2} \end{vmatrix} \neq 0$$

In this case, where $y_1 = L_f/T_f$, etc., it is easy to show that

$$\Delta = \begin{vmatrix} \frac{P_f f''}{\bar{P}_e} & -c'' \\ -\frac{P_f}{\bar{P}_e} \frac{L_f}{\bar{T}_f} f'' & +\frac{L_e}{\bar{T}_e} c'' \end{vmatrix} = \frac{P_f f'' c''}{\bar{P}_e} \left[\frac{L_e}{\bar{T}_e} - \frac{L_f}{\bar{T}_f} \right]$$

By hypothesis of diminishing returns f'' and c'' are negative, and the term in brackets (representing the respective labour intensities in food and clothing) cannot be equal to zero. Hence, the equilibrium is unique. As developed earlier, if the factor intensities become equal, or reverse themselves, the one-to-one relation between commodity and factor prices *must* be ruptured.

¹ The real wage of every resource must be the same in every place that it is used, when expressed in a common denominator. This gives us $r(n - 1)$

When we add a third productive factor and retain but two commodities, then the whole presumption towards factor-price equalisation disappears. Suppose American labour and American land have more capital to work with than does European labour and land. It is then quite possible that the marginal physical productivities of labour and land might be double that of Europe in both commodities. Obviously, commodity-price ratios would still be equal, production of both commodities will be taking place, but nonetheless absolute factor prices (or relative for that matter) need not be moved towards equality. This is our general expectation wherever the number of factors exceeds the number of commodities.

6. THE CONDITIONS OF COMPLETE SPECIALISATION

If complete specialisation takes place in one country, then our hypothesis is not fulfilled and the conclusion does not follow. How important is this empirically, and when can we expect complete specialisation to take place? As discussed earlier, the answer depends upon how disparate are the initial factor endowments of the two regions—how disparate in comparison with the differences in factor intensities of the two commodities.¹

Unless the two commodities differ extraordinarily in factor intensities, the production-possibility curve will be by no means so convex as it is usually drawn in the neo-classical literature of international trade, where it usually resembles a quarter circle whose slope ranges the spectrum from zero to infinity. It should rather have the crescent-like shape of the new moon. Opportunity costs tend to be more nearly constant than I had previously realised. This is a step in the direction of the older classical theory of comparative advantage. But with this important difference: the same causes that tend to produce *constant* costs also tend to produce *uniform* cost ratios between nations, which

independent equations involving the $(n - 1)$ commodity-price ratios and the $n(r - 1)$ factor proportions. If $n = r$, we have a determinate system once the goods price ratios are given. If $n > r$, we have the same result, but now the international price ratios cannot be presented arbitrarily as there are constant-cost paths on the production-possibility locus, with one blade of Marshall's scissors doing most of the cutting, so to speak. If $n < r$, it is quite possible for free commodity trade to exist alongside continuing factor-price differentials. It is never enough simply to count equations and unknowns. In addition we must make sure that there are not multiple solutions: that factor intensities in the different commodities and the laws of returns are such as to lead to a one-to-one relationship between commodity prices and factor prices.

¹ The reader may be referred to the earlier paper's discussion of Figures 1 and 2, with respect to "step-like formations" and overlap.

is not at all in the spirit of classical theory. (Undoubtedly much of the specialisation observed in the real world is due to something different from all this, namely decreasing-cost indivisibilities, tempered and counteracted by the existence of localised resources specifically adapted to particular lines of production.)

A parable may serve the double purpose of showing the range of factor endowment incompatible with complete specialisation and of removing any lingering element of paradox surrounding the view that commodity mobility may be a perfect substitute for factor mobility.

Let us suppose that in the beginning all factors were perfectly mobile, and nationalism had not yet reared its ugly head. Spatial transport costs being of no importance, there would be one world price of food and clothing, one real wage, one real rent, and the world's land and labour would be divided between food and clothing production in a determinate way, with uniform proportions of labour to land being used everywhere in clothing production, and with a much smaller—but uniform—proportion of labour to land being used in production of food.

Now suppose that an angel came down from heaven and notified some fraction of all the labour and land units producing clothing that they were to be called Americans, the rest to be called Europeans; and some different fraction of the food industry that henceforth they were to carry American passports. Obviously, just giving people and areas national labels does not alter anything: it does not change commodity or factor prices or production patterns.

But now turn a recording geographer loose, and what will he report? Two countries with quite different factor proportions, but with identical real wages and rents and identical modes of commodity production (but with different relative importances of food and clothing industries). Depending upon whether the angel makes up America by concentrating primarily on clothing units or on food units, the geographer will report a very high or a very low ratio of labour to land in the newly synthesised "country." But this he will never find: that the ratio of labour to land should ever exceed the proportions characteristic of the most labour-intensive industry (clothing) or ever fall short of the proportions of the least labour-intensive industry. Both countries *must* have factor proportions intermediate between the proportions in the two industries.

The angel can create a country with proportions *not* inter-

mediate between the factor intensities of food and clothing. But he cannot do so by following the above-described procedure, which was calculated to leave prices and production unchanged. If he wrests some labour in food production away from the land it has been working with, "sending" this labour to Europe and keeping it from working with the American land, then a substantive change in production and prices will have been introduced. Unless there are abnormal repercussions on the pattern of effective demand, we can expect one or both of the countries to specialise completely and real wages to fall in Europe relative to America in one or both commodities, with European real rents behaving in an opposite fashion. The extension of this parable to the many-commodities case may be left to the interested reader.

7. SOME QUALIFICATIONS

A number of qualifications to this theoretical argument are in order. In the first place, goods do not move without transport costs, and to the extent that commodity prices are not equalised it of course follows that factor prices will not tend to be fully equalised. Also, as I indicated in my earlier article, there are many reasons to doubt the usefulness of assuming identical production functions and categories of inputs in the two countries; and consequently, it is dangerous to draw sweeping practical conclusions concerning factor-price equalisation.

What about the propriety of assuming constant returns to scale? In justice to Ohlin, it should be pointed out that he, more than almost any other writer, has followed up the lead of Adam Smith and made *increasing returns* an important cause for trade. It is true that increasing returns *may* at the same time create difficulties for the survival of perfect competition, difficulties which cannot always be sidestepped by pretending that the increasing returns are due primarily to *external* rather than internal economies. But these difficulties do not give us the right to deny or neglect the importance of scale factors.¹ Where

¹ Statical increasing returns is related to, but analytically distinct from, these irreversible cost economies induced by expansion and experimentation and which provide the justification for "infant industry" protection. Statical increasing returns might justify permanent judicious protection but not protection all around, since our purpose in bringing about large-scale production is to achieve profitable trade and consumption.

One other point needs stressing. For very small outputs, increasing returns to scale may take place without affecting the above analysis provided that total demand is large enough to carry production into the realm of constant returns to scale. Increasing the "extent of the market" not only increases specialisation, it also increases the possibility of viable pure competition.

scale is important it is obviously possible for real wages to differ greatly between large free-trade areas and small ones, even with the same relative endowments of productive factors. And while it may have been rash of me to draw a moral concerning the worth of emigration from Europe out of an abstract simplified model, I must still record the view that the more realistic deviations from constant returns to scale and the actual production functions encountered in practice are likely to reinforce rather than oppose the view that high standards of life are possible in densely populated areas such as the island of Manhattan or the United Kingdom.

There is no iron-clad *a priori* necessity for the law of diminishing marginal productivity to be valid for either or both commodities.¹ In such cases the usual marginal conditions of equilibrium are replaced by inequalities, and we have a boundary maximum in which we go the limit and use zero of one of the inputs in one industry. If it still could be shown that one commodity is always more labour intensive than the other, then the main theorem would probably still be true. But it is precisely in these pathological cases that factor intensities may become alike or reverse themselves, giving rise to the difficulties discussed in my earlier footnote on p. 188.

In conclusion, some of these qualifications help us to reconcile results of abstract analysis with the obvious facts of life concerning the extreme diversity of productivity and factor prices in different regions of the world. Men receive lower wages in some countries than in others for a variety of reasons : because they are different by birth or training ; because their effective know-how is limited and the manner of their being combined with other productive factors is not optimal ; because they are confined to areas too small to develop the full economies of scale ; because some goods and materials cannot be brought to them freely from other parts of the world, as a result of natural or man-made obstacles ; and finally because the technological diversity of commodities with respect to factor intensities is not so great in comparison with the diversity

¹ A "Pythagorean" production function of the form $F = \sqrt{L^2 + T^2}$ is an example of such a homogeneous function with increasing marginal productivity. So long as neither factor is to have a negative marginal productivity, *average* product must not be rising ; but this is quite another thing. Surprisingly enough, the production possibility curve may still be convex with increasing marginal productivity. I have been asked whether any essential difference would be introduced by the assumption that one of the commodities, such as clothing, uses no land at all, or negligible land. Diminishing returns would still affect food as more of the transferable factor is added to the now specific factor of land ; but no essential modifications in our conclusions are introduced.

of regional factor endowments to emancipate labourers from the penalty of being confined to regions lacking in natural resources. In the face of these hard facts it would be rash to consider the existing distribution of population to be optimal in any sense, or to regard free trade as a panacea for the present geographical inequalities.

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