



WILEY

British and American Exports: A Study Suggested by the Theory of Comparative Costs.

Part II

Author(s): G. D. A. MacDougall

Source: *The Economic Journal*, Vol. 62, No. 247 (Sep., 1952), pp. 487-521

Published by: Wiley on behalf of the Royal Economic Society

Stable URL: <http://www.jstor.org/stable/2226897>

Accessed: 23-03-2017 10:45 UTC

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://about.jstor.org/terms>



Royal Economic Society, *Wiley* are collaborating with JSTOR to digitize, preserve and extend access to *The Economic Journal*

THE ECONOMIC JOURNAL

SEPTEMBER 1952

BRITISH AND AMERICAN EXPORTS: A STUDY SUGGESTED BY THE THEORY OF COMPARATIVE COSTS. PART II.¹

C. ELASTICITY OF SUBSTITUTION

IN Section B of this paper we dealt with observed facts about the prices and quantities of British and American exports of a large number of manufactures in a given period of one or five years, and made some speculations about the possible effects of errors in the data used. We reached the conclusion that, in the years 1934–38 for example, where the relative price of product A was 1% lower than the relative price of product B, the relative quantity of product A tended to be at least $3\frac{1}{2}\%$ greater than that of product B, and quite possibly $4-4\frac{1}{2}\%$ greater when allowance is made for bias due to errors of observation. We made no examination of changes over time. It would, therefore, be a bold step to conclude from the facts observed that, in pre-war conditions, a 1% *fall* in a typical product's relative price would tend to lead to a $4-4\frac{1}{2}\%$ (or even to a $3\frac{1}{2}\%$) *rise* in its relative quantity. In view, however, of the difficulties involved in most methods hitherto used for measuring elasticities in international trade,² it seems that this possible alternative approach, which might perhaps be called the "commodity comparison" method, should be further explored. A similar type of cross-section analysis is, after all, used when conclusions are drawn about income elasticity of demand from family budgets observed in a given period; although admittedly one set of consumers may be more likely to behave like another set, given the conditions of the latter set, than one manufactured export is to behave like another.

¹ Part I appeared in the ECONOMIC JOURNAL for December 1951. The whole article was largely completed before the author took up a temporary appointment in government service.

² See Orcutt, *op. cit.*

The "Commodity Comparison" Method versus the "Time Series" Method

Although the commodity comparison method of estimating elasticities in international trade is open to special objections, it avoids certain other difficulties involved in the use of time series.

(1) The latter method often suffers from a paucity of observations. For example, Mr. Chang's estimate of the elasticity of substitution between British and American exports of manufactures in the inter-war years¹ was based on fifteen observations (for the years 1924–38) compared with between 86 and 171 observations in each of the calculations described above.²

(2) In the time series method the range of relevant price variation (after allowance has been made for income changes) is often small in relation to the errors of observation—in many cases only 5% or 10%.³ For example, the standard deviation of the ratio of the U.S. to the U.K. export price indices of manufactures over the period 1922–38 was only 7–8% of the mean, even before allowing for trend or for world income, which, according to Professor A. J. Brown,⁴ was highly correlated with relative prices. In the calculations described above, the errors may be greater but the range of price variation is much larger. For example, the standard deviation of the 109 relative prices (U.S. : U.K.) used in the calculation for 1934–38 was 40% of the mean.⁵

(3) Mr. Orcutt has shown⁶ that most time series analyses

¹ Tse Chun Chang, "A Statistical Note on World Demand for Exports," *Review of Economics and Statistics*, May 1948.

² Mr. Kubinski's method ("The Elasticity of Substitution between Sources of British Imports, 1921–38," *Yorkshire Bulletin of Economic and Social Research*, January 1950) is based on an examination of 289 commodities imported into the U.K., but the number of observations in each correlation calculation is never more than eighteen (for each of the years 1921–38).

Messrs. D. J. Morgan and W. J. Corlett ("The Influence of Price in International Trade: A Study in Method," *Journal of the Royal Statistical Society*, 1951) give the results of a few calculations with as many as forty-five observations (for the years 1870 to 1914), but the great majority of their calculations are based on less than twenty observations.

See also Orcutt, *op. cit.*, footnote 7, where it is argued that, in the time series method, the effective may be substantially less than the actual number of observations.

³ See Orcutt, *op. cit.*, p. 121.

⁴ *Oxford Studies in the Price Mechanism*, p. 95.

⁵ The standard deviation of the 109 relative prices, U.K. : U.S., was 57% of the mean.

⁶ *Op. cit.*, pp. 125–6.

of inter-war data can at best show the consequences of relatively small changes in prices since the range of relative price variation was small. He has also given reasons for supposing that elasticities are greater for large than they are for small price changes, mainly because of the cost and trouble to the buyer of shifting from one source of supply to another. The commodity comparison method described above shows the extent to which relative quantities may vary when there are substantial differences in relative prices (at least as between different commodities at one time).

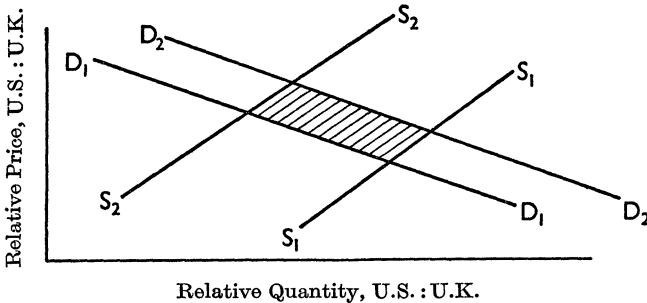
(4) According to Mr. Orcutt, the time series method may tend to give estimates of relatively short-run elasticities and underestimate the long-run elasticities. If this is so, the commodity comparison method may give a closer approximation to long-run elasticities, *i.e.*, to probable changes in relative quantities resulting from changes in relative prices if time were allowed for the consequences to work themselves out. This is, of course, an advantage only if it is desired to estimate long-run elasticities. Since the general pattern of relative export prices of British and American manufactures in our sample did not change greatly between 1934 and 1938, it may be that the regression coefficients for the later thirties give a fair idea of relatively long-run elasticities of substitution.

These apparent advantages of the commodity comparison method may justify its use, despite its limitations, at least as a complement to the time series method.¹ The last three of the four advantages just mentioned would lead one to expect higher elasticities than are obtained by the time series method. But, like the latter, the commodity comparison method may also tend to underestimate elasticities for a number of reasons. That connected with errors of observation was discussed in the last section, and there are two others that must now be explained.

¹ Mr. Orcutt has also shown (*op. cit.*, pp. 122-3) that, over time, supply and demand schedules are likely to move up and down together, and that this probably leads to an underestimate of elasticities of demand calculated from observed time series. We shall see later that, in the method described in the present article, there are schedules somewhat similar to demand and supply schedules and that these are not the same for each product, but there is no obvious reason why they should vary up and down together. This cannot, however, be counted as an advantage of the commodity comparison over the time series method, since Mr. Orcutt's criticism does not seem to apply to time series calculations using *relative* price and quantity indices. (See also footnote 1 to p. 491.)

Further Possible Bias in the Commodity Comparison Method

If we take the plunge and attempt to use the results of the commodity comparison method as some indication of what might happen to the relative demand for British and American exports of a particular product when relative prices change, we must assume that, for each product, there is a relationship between relative price and relative quantity demanded. Using our diagram measuring relative prices vertically and relative quantities horizontally on a double logarithmic scale, we have "demand substitution curves" for each product. Let us assume that these are straight lines and, for the moment, that they all have the same slope, which we wish to discover. There is, however, no reason why they should coincide. When, for example, British and American prices are equal (even after adjustment for "quality" differences) the relative quantities demanded may vary; Britain may have an advantage in one imperfect market, America in another. We thus get a series of demand substitution curves lying between D_1D_1 and D_2D_2 in the following diagram :



We can also think of "supply substitution curves" for each product, showing the relative quantities supplied at various relative prices.¹ These, likewise, are unlikely to coincide, because of differences in national factor endowments. When, for example,

¹ This is, of course, a simplification, since there may be an infinite number of pairs of prices that will give the same price ratio, and these different pairs of prices will normally lead to different ratios of quantity supplied unless the two national supply curves are related in a special way, *e.g.*, if they are parallel straight lines on a double logarithmic scale, *i.e.*, with equations of the type :

$$\log q = \log a_1 + b \log p$$

$$\log q = \log a_2 + b \log p$$

where p and q are relative price and relative quantity, and a_1 , a_2 and b are constants. The two curves in this case have the same (constant) elasticity.

British and American prices are equal, Britain may supply much more of product A than the U.S., and much less of product B. We thus get a series of supply substitution curves lying between S_1S_1 and S_2S_2 which we assume for the time being to be upward sloping.

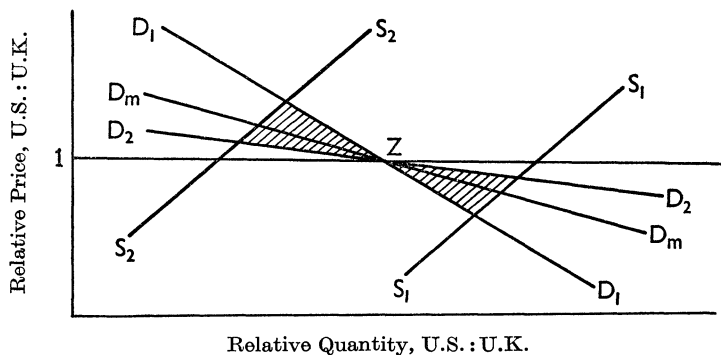
The observed points will lie at the intersection of the supply and demand substitution curves for each product, *i.e.*, within the shaded area. It is fairly clear (and this is demonstrated in Appendix B) that a regression line fitted to such points, so as to minimise the sum of the squares of the deviations in a horizontal direction, will tend to have a slope steeper than D_1D_1 , and will thus underestimate the true slope of the demand substitution curves for each commodity.¹ The fact that negative rather than positive slopes are obtained for the regression lines means, presumably, that the position of the demand substitution curves varies less from product to product than the position of the supply substitution curves, *i.e.*, broadly, that differences in comparative national advantages in the imperfect markets for the various commodities have a less important effect than differences in national factor endowments on the position of the curves. This does not seem altogether unreasonable.

We must now remove the assumption that the demand substitution curves are parallel. It seems almost certain that some will be flatter than others, *e.g.*, where the market is more perfect or the product less heterogeneous. To isolate this point, let us suppose that, where British and American prices are equal, the ratio of the quantities demanded is the same for each product; *i.e.*, if either country has an advantage, it is the same in the market for each product. The various demand substitution

¹ According to our conventions the nearer the curve to the horizontal the greater the slope (see footnote 1 to p. 711 of Part I).

When a supply substitution curve is well to the right, *i.e.*, when national factor endowments are such that America supplies far more of the product than Britain at the same price, it is possible that the demand substitution curve will also tend to be well to the right, *i.e.*, that the demand for the American product will be far more than that for the British product at the same price. This might be so, because America's inevitably large share in the market has made her products better known to importers. Similarly, supply substitution curves towards the left may tend to be associated with demand substitution curves towards the left. In other words, high demand substitution curves may tend to be associated with low supply substitution curves and vice versa. If this is the case, the calculated regression line may tend to overestimate the true slope of the demand substitution curves for each commodity for reasons similar to those given by Mr. Orcutt (*op. cit.*, p. 123), where he shows that, in so far as demand and supply curves tend to move up and down together, the calculated regression line will tend to underestimate the true slope.

curves will all pass through the point Z in the following diagram :



The steepest is D_1D_1 , the flattest D_2D_2 , the mean slope is that of D_mD_m .¹

The observed points will lie within the shaded area. It is fairly clear (and this is also demonstrated in Appendix B) that the regression line fitted to such points will tend to have a slope steeper than D_mD_m , and will thus underestimate the true average slope of the demand substitution curves for the various commodities.

The various demand substitution curves will, in fact, both have different slopes and cut the equal price axis at different points. The calculated slope will thus be steeper than the true average slope for both of these reasons.

This conclusion depends, however, on the assumption that the supply substitution curves slope upwards. If they are horizontal there is no bias, and if they are downward sloping the bias is in the other direction. This would necessitate downward sloping export-supply curves for individual British and American manufactures. It might be argued that, in the thirties at least, these curves did not slope appreciably upwards, since, *inter alia*, there was generally excess capacity and since exports were only a fraction of total output. For similar reasons it might be argued that the curves did not slope downwards and that, even in the long run, this would be unlikely where exports were not a large part of total output. The reader may make his private guess, but, for the purpose of this article, it seems safer to make no adjustment to our figures for the possible bias just discussed.

¹ The slope of D_mD_m is the *arithmetic* mean of the individual slopes. For reasons for choosing this form of average see footnote 2 to p. 496.

“*Product*” and “*Total*” *Elasticities of Substitution*

A “product” elasticity of substitution of, say, -4 between British and American exports of individual manufactures would not necessarily mean, of course, that the elasticity of substitution between the two countries’ total exports of manufactures was as high as -4 , because of the different patterns of exports. Suppose, for example, that the total market for British and American exports together were fixed for each product, and that there were only two products, the U.K. exporting 99 yards of cloth and 1 radio, the U.S. 99 radios and 1 yard of cloth. A 1% fall in the British price for each product relative to the American would then increase the *ratio* of British to American exports of cloth by 4% (from 99 to about 103 times), but this would raise the *quantity* of Britain’s cloth exports to only about 99.04 yards, while American exports fell to approximately 0.96 yards ($\frac{99.04}{0.96} = 103$). Britain’s exports of cloth would then rise by only 0.04% and, since the weight given to her radio exports is very small, her total volume index of exports would also rise only slightly. Similarly, America’s volume index would fall only slightly, and the ratio of the two volume indices would change by only a small fraction of 1%.

To get some idea of the importance of this factor, a calculation was made, covering the 109 products mentioned above for the years 1934–38, of the change that would result in the ratio of the two countries’ export-volume indices for the 109 products if there were a small uniform percentage change in the price ratio for all products caused either by a uniform change in all the prices of one country or by this combined with a different uniform change in all the prices of the other. It was assumed that the total market for each good, for U.S. and U.K. exports taken together, remained the same or changed in the same proportion. It was found that, assuming a uniform “product” elasticity of substitution for each commodity, the corresponding “total” elasticity of substitution (relating to price and volume indices) would be 0.614 times as large. Thus, if the product elasticity of substitution is taken as (a) -3.6 (the calculated regression coefficient), or (b) -4 to $-4\frac{1}{2}$ (allowing for bias due to errors of observation), the total elasticity would be (a) -2.2 or (b) -2.5 to -2.8 .

Similar calculations can be made for other years or groups of years, or for other pairs of countries. To find the total elasticity of substitution on the assumptions stated it is necessary to

multiply the product elasticity of substitution by $\frac{\Sigma V_a B}{\Sigma V_a} + \frac{\Sigma V_b A}{\Sigma V_b}$, where V_a and V_b are the values of the two countries' exports of each product, and A and B the proportions of the market (in terms of quantity) held by each of them ($A + B = 1$). The formula may perhaps be called an index of similarity of exports. Where the pattern of exports is exactly the same in the two countries the index is unity, where the exports are entirely different it is zero. Details are given in Appendix C.

Indices of similarity were calculated for the 109 British and American exports of manufactures for each of the years 1928 to 1938. These are shown in column (2) of Table VIII, together with the results of multiplying the indices by the corresponding regression coefficients (called "product elasticities of substitution" in the table). The resulting "total elasticities of substitution" are in column (3). Apart from year-to-year fluctuations, they show a steady increase throughout the period, rising from under $1\frac{1}{2}$ at the beginning to over 2 at the end.

TABLE VIII

British and American Exports of 109 Manufactures

	Product elasticity of substitution. ¹	Index of similarity.	Total elasticity of substitution (1) × (2).
	(1)	(2)	(3)
1928	2.501	0.472	1.2
1929	2.601	0.572	1.5
1930	2.599	0.542	1.4
1931	2.713	0.656	1.8
1932	2.602	0.647	1.7
1933	2.826	0.683	1.9
1934	3.241	0.561	1.8
1935	2.958	0.679	2.0
1936	2.934	0.611	1.8
1937	3.128	0.701	2.2
1938	3.134	0.660	2.1
1934-38	3.624	0.614	2.2

¹ As in column (3) of Table V. Before allowing for bias due to errors of observation.

The assumption that the total market for each product remains constant, or changes in the same proportion, tends, however, to understate the total elasticity of substitution. Suppose, for example, that America had roughly the same share in the Anglo-American export market for both silk and rayon

stockings (two products that are, in fact, included separately in the calculations described above). A relative fall in all American prices would induce foreign buyers to substitute American for British silk stockings, and American for British rayon stockings. It would also, probably, induce them to substitute American silk for British rayon stockings and American rayon for British silk stockings. But there is no obvious reason why the total market for either silk or rayon stockings should increase relatively to the other. If, however, America had a larger share in one market than in the other, the matter would be different. If, for example, America had the bulk of the market for silk stockings, and Britain the bulk of the market for rayon stockings, there would be much more scope for substitution of American silk for British rayon stockings than for the substitution of American rayon for British silk stockings, since exports of both these last two products would be small. It seems likely, therefore, that the total market for silk stockings (America's speciality) would increase in relation to the total market for rayon stockings (Britain's speciality). The increase in America's total exports relative to Britain's would then be greater than it would have been had the total silk-stocking market not expanded in relation to the total market for rayon stockings. This means that the total elasticity of substitution will tend to be a greater fraction of the product elasticity than that given by the formula described above.

This formula showed that if, as had been suggested earlier, the product elasticity of substitution were -4 to $-4\frac{1}{2}$, the total elasticity would be -2.5 to -2.8 . After what has just been said, we may perhaps hazard a guess that the total elasticity might be of an order approaching -3 .¹ This is about ten times as high as the figure of -0.3 obtained by Mr. Chang for the pre-

¹ It is perhaps not necessary to remind the reader that we are talking of elasticity of substitution and that this may differ from the elasticity of demand for British or American exports. If we assume that the world import market for manufactures and the prices charged by our competitors remain constant, a given elasticity of substitution between British and *all other* exports of manufactures would mean an elasticity of demand for British exports four-fifths as great, since Britain supplied about one-fifth of the world's exports of manufactures before the war. The elasticity of demand would, however, be greater in so far as the world import market for manufactures was increased as a result of lower British prices, British manufactures being substituted both for manufactures produced in importing countries and for non-manufactures. Any changes in the definition of elasticity of demand so as to allow, for example, for price reactions by our competitors or for income changes would give correspondingly different results.

war elasticity of substitution between British and American exports of manufactures.¹

The assumptions are, of course, still unrealistic. Allowance should be made for elasticities of demand by third countries for the exports of America and Britain together that vary from product to product; this also would lead to varying changes in the size of the total markets. Allowance should also be made for product elasticities of substitution that vary from product to product,² for varying changes in price according to elasticity of supply and so on. Further adjustments of this kind might tend to increase or to decrease the value of the total elasticity, but the author has been unable to think of any obvious reasons for such bias.

Changes over Time

The calculations described so far have all referred to one year or to an average of years. Some calculations have also been made for changes over time using the figures obtained for the 109 products. These may be of some use in attempting to assess short-run elasticities of substitution in particular periods. Suppose, for example, we wish to examine the effects on relative quantities of British and American exports of the large changes in relative prices that took place after Britain left the gold standard in September 1931. The published price and volume indices of exports of manufactures by themselves tell us relatively little. We find, for example, that, between 1930 and 1932, the American price index for exports of manufactures rose by 23% in relation to the British (in terms of dollars), while the American volume index fell relatively by 42%.³ This suggests an elasticity of substitution of over $-2\frac{1}{2}$.⁴ Similarly, between

¹ Chang, *op. cit.*, p. 112. Based on the years 1924-38. Mr. Chang appears to compare total U.K. exports (which were mainly manufactures and semi-manufactures) with U.S. exports of semi-manufactures and manufactured goods only. Professor A. J. Brown (*op. cit.*, p. 95) has given reasons for the low figure obtained by Mr. Chang.

² Some reasons are given in Appendix C for thinking that the assumption that each product elasticity of substitution is equal to the arithmetic mean of the product elasticities does not introduce any bias into the derivation of the total elasticity. This provides some justification for taking the arithmetic rather than any other mean of the product elasticities as the magnitude we wish to find. (See footnote 1 to p. 492.)

³ Using the indices for U.S. exports of "finished manufactures" and U.K. exports of "articles wholly or mainly manufactured." These are not wholly comparable, but this does not matter, since the figures are used here for illustration only.

⁴ $\frac{\log 0.58}{\log 1.23} = \frac{-0.2366}{0.0899} = -2.63.$

1932 and 1934, America's price index fell relatively by 27%, while her volume index rose relatively by 23%; this suggests an elasticity of substitution of only $-\frac{2}{3}$.¹ But it would be rash to rely on single observations of this type. We have reason to think, moreover, that American exports of manufactures are more sensitive than the British to cyclical fluctuations, since they contain a higher proportion of capital goods and of durable consumer goods for which the income elasticity of demand is high. This may account for part of the relative fall in American exports between 1930 and 1932, when activity was rapidly declining throughout the world. Then again, there might have been changes, between 1932 and 1934, in the imperfection of the world market that worked to the disadvantage of U.S. exports, for example, the Ottawa Agreements and the general spread of discriminatory practices. This might account in part for the comparatively small relative rise in American exports between 1932 and 1934.

More useful results may perhaps be obtained if we examine changes in individual products. A comparison was therefore made of changes, (a) between 1930 and 1932 and (b) between 1932 and 1934, in the relative quantities and relative prices of the 109 products mentioned above. In this way we can obtain 109 observations of short-period changes instead of only one (and there is also a substantial range of variation in relative prices). We can largely eliminate the complications that arise from the different patterns of British and American exports of manufactures, except in so far as there are differences within the commodity classifications used. We can isolate, in a general way, the effects of changes in the imperfections of the world market that work to the disadvantage of one country. Thus, in comparing 1932 and 1934, we plot the relative price index for each product in 1934 (1932 = 1) against the relative quantity index on a double logarithmic scale. The regression line cuts the horizontal axis, indicating no change in relative price, at a point where the relative quantity index is 0.72 (A in Fig. 3). This suggests that, where the relative price of a commodity remained unchanged between 1932 and 1934, there was a fall of 28% in the American quantity relative to the British quantity. This may give some indication of the extent to which imperfections of the markets changed to the disadvantage of the U.S.;²

$$^1 \frac{\log 1.23}{\log 0.73} = \frac{0.0899}{-0.1367} = -0.66.$$

² In the case of nearly one-third of the 109 products, the American relative quantity fell despite a fall in the American relative price.

although the later analysis (pp. 500 *et seq.* and Appendix D) shows the need for caution in drawing such a conclusion. The correlation coefficient is -0.62 , and the slope of the regression line is -2.0 . This suggests that, after allowing for the shift to America's disadvantage, a change of 1% in relative price of a product was accompanied by a change of 2% in relative quantity; in other words, that the product elasticity of substitution was -2.1 . A similar calculation for 1930-32 gave a correlation coefficient of -0.57 and an elasticity of -1.5 .

*Changes in Relative Prices and Quantities
of U.S. and U.K. Exports of 109 Manufactures
1932-34*

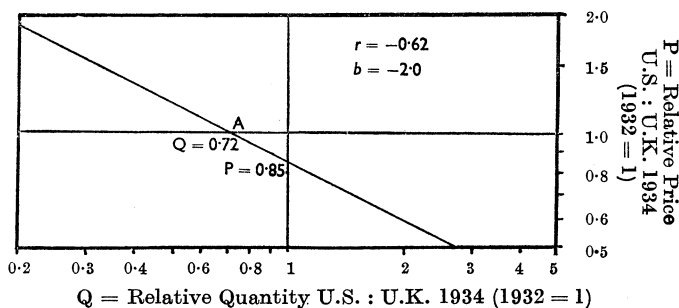


FIG. 3

These elasticities are considerably lower than those obtained by the other method, before allowing for downward bias. This may be partly because they can only show the comparatively short-run consequences of changes in relative prices, whereas the other method may give a better idea of long-run consequences. Even, however, when similar comparisons were made of changes over longer periods, relatively low figures were obtained. Thus, a comparison of 1929 and 1937, again for the 109 commodities, gave a correlation coefficient of only -0.45 and an elasticity of -1.8 . A comparison of the four years 1928-31 against the five years 1934-38 gave a correlation coefficient of -0.48 and an elasticity of -1.7 . These lower elasticities cannot be explained by the difference between long-run and short-run elasticities, unless it is thought that more than eight years are required for the full effects of price changes to be felt. It may be that they are biased towards zero more than the results of the earlier calculations, one important reason being that the range of varia-

¹ The equation of the regression line is :

$$\log q = -2.0 \log p + \log 0.72,$$

where q and p are indices of relative quantity and price (1934 : 1932)

tion in relative prices is much smaller. In general, one is hardly surprised to find rather a poor relation between relative price and relative quantity changes over a period of eight years, since so many things have happened, other than price changes, to affect the relative quantities demanded. The implications of the type of analysis just described probably deserve further examination, but this is not attempted here, as the present article is concerned primarily with the cross-section type of analysis.

Conclusions on Elasticity of Substitution

This concludes the discussion of elasticity of substitution.¹ It cannot be too strongly emphasised that any numerical results that may be obtained by this or by other methods cannot be used to make precise forecasts. It is, however, useful to gather together all evidence that may provide some background for those who have to make forecasts and decisions, to consider as carefully as possible what the evidence means and what it does not mean, to examine possible biases and so on. Practical judgments about, for example, the effects of exchange-rate variations or the degree of difficulty involved in restoring international equilibrium do not depend upon a knowledge of whether elasticities of substitution are, say, -2.3 or -2.7 , but they do require a knowledge of whether they are of the order of -0.03 , or -0.3 , or -3 or -30 . Such judgments also, of course, require a knowledge of, or at least a view on, many other things (which will vary from time to time), such as the flexibility of supply, the reactions of producers and of their governments to changes in the prices charged by their competitors, the extent to which demand for the goods of competing nations is determined by supply of currency, speed of delivery, etc., rather than by price, and so on. Practical judgments of this type inevitably involve an act of faith; no one can work out the full implications on the world economy of an important change in the economic policy of a major country; no one can foretell the actions and reactions of millions of producers and consumers and of governments throughout the world. But this does not render futile the attempt to establish the likely order of magnitude of, for example, the elasticity of substitution between the exports of various countries, nor does it rob such a concept of all meaning.

¹ Another type of analysis of the figures for the 109 products would be the correlation, for each product, of relative prices and relative quantities in the eleven years 1928-38, *i.e.*, 109 correlations with eleven observations each. This analysis, which would be similar to that made by Mr. Kubinski for imports into the U.K., has not been attempted.

This paper has suggested one method by which some relevant evidence can be collected and applied it to the particular case of American and British exports of manufactures, mainly in the years between the wars. There are doubtless difficulties involved in the method described other than those mentioned in this paper, and it is hoped that these will be explored by others. But it does seem fairly clear that there was some inverse relation between relative price and relative quantity and that variations in the former were associated with comparatively large variations in the latter. While it is dangerous to apply the results obtained to future changes, I believe that the evidence gives some grounds for thinking that, at least in conditions resembling those of the later thirties (an important proviso), changes in the relative prices of British and American exports of manufactures should lead to comparatively large changes in the relative quantities demanded in third markets, at least after a period of years.

D. EXPORTS AND FACTORS OTHER THAN PRICE

Apparent American Disadvantage in the Imperfect World Market

It will have been noticed that, according to the regression line in Fig. 2,¹ which referred to the years 1934–38, the U.S. tended to export little more than half as much of a commodity as Britain when she charged the same price and that she exported as much only when her price was 16% below the British. As Table IX shows, this was rather typical of the individual years 1933–38. When prices were equal, the U.S. tended to export between one-half and two-thirds as much as Britain and exported as much only when her price was 10–20% lower. For the earlier years, 1928–32, the regression lines show the U.S. in a more favourable relative position; when prices were equal she tended to export roughly 80–90% as much as Britain, and she sold as much when her price was some 5–10% lower. For the years before 1928, allowing for lack of comparability, the figures tell roughly the same story, at least from 1924 onwards. (The results for 1948 show a striking improvement in the relative position of the U.S. compared with 1937 and, though the comparison is dangerous, there seems to have been a similar improvement between 1913 and the early 1920s.)

The figures for the years between the wars suggest that, in the period 1933–38 and, to a lesser extent, between 1924 and 1932, the U.S. had, on balance, a comparative disadvantage in the imperfect world market as the result of such factors as Imperial

¹ In Part I.

Preference, discrimination against dollar goods, differences in transport costs, stronger commercial ties between Britain and overseas markets, greater selling efforts by British exporters, etc. But before jumping to this conclusion we must consider other possible reasons for the low figures in Table IX.

TABLE IX
Particular Values shown by Regression Lines

	No. of manufactures.	Relative quantity (U.S. : U.K.) when price equal.	Relative price (U.S. : U.K.) when quantity equal.
1913 ¹	(1) 32	(2) 0.48	(3) 0.80
1922	86	1.01	1.01
1923	86	0.95	0.97
1924	86	0.82	0.90
1925	86	0.87	0.94
1925	97	0.81	0.91
1926	97	0.83	0.92
1927	97	0.78	0.90
1928	97	0.85	0.93
1928	109	0.90	0.96
1929	109	0.87	0.95
1930	109	0.79	0.91
1931	109	0.86	0.94
1932	109	0.84	0.93
1933	109	0.57	0.82
1934	109	0.51	0.81
1935	109	0.69	0.88
1936	109	0.59	0.83
1937	109	0.67	0.88
1938	109	0.61	0.85
1934-38	109	0.53	0.84
1937	118	0.50	0.79
1948	118	1.10	1.06
Commonwealth countries 1937	115	0.22	0.54
Non-Commonwealth coun- tries 1937	115	1.36	1.16

¹ Mid-1912 to mid-1913 for U.S.

(1) One possible explanation is that different methods of valuing exports in the two countries led to an artificial relative under-valuation of American exports. If this was so, regression lines based on the corrected figures would be above the lines we have obtained. They would show the U.S. exporting a higher fraction of the British quantity when prices were equal. There are two obvious corrections

that should be made. First, U.S. exports are relatively under-valued because freight charges up to the Canadian border are not included in the recorded value of certain exports to Canada. This under-valuation probably represented about 1% of the value of U.S. exports to all countries before the war,¹ but may have been a smaller percentage of the value of manufactured exports, which tend to have a relatively high value per ton. Secondly, U.S. exports are in general valued f.a.s. (free along side), while U.K. exports are valued f.o.b. (free on board), the difference being the cost of loading. It is unlikely that this normally involved a difference of more than 1% of the value of manufactured exports. It is hard to discover any other important reason for American under-valuation, but the possibility should not be ruled out.

(2) A second possibility is that British exports were, on average, of a higher quality than the American, and that appropriate corrections would therefore raise the regression line.² This is a matter on which it is difficult to express an opinion.

(3) It has been shown above that the true average slopes of the lines are probably flatter than the calculated slopes because of errors of observation (other than a general tendency for the quality of one country's exports to be higher) and possibly for other reasons. The true regression lines would probably cut the equal-price and equal-quantity lines at different points, and this might reduce or eliminate the apparent American disadvantage. (This is discussed in Appendix D.)

For these reasons it seems unwise to conclude from the figures that, at least before 1933, the U.S. had a comparative disadvantage in the imperfect world market. For the later pre-war years the presumption is stronger that this was the case, even allowing for possible errors. According to the regression line for the years 1934–38, the U.S. exported as much as the U.K. when her price was 84% of the British. It is hard to make allowance for the errors under (3) above, but it is suggested

¹ *Balance of International Payments of U.S.* (Department of Commerce). It is pointed out in *Foreign Commerce and Navigation of the United States for 1946* that similar inaccuracies may occur in connection with ocean shipments.

² The correction would shift the observed points, on a double logarithmic scale, upwards and to the left at an angle of 45° to the horizontal. Since the regression lines obtained all have a slope flatter than -1, they would clearly be shifted upwards.

in Appendix D that this might well raise the figure to 86½%. A 2% allowance for under-valuation of U.S. exports (as in (1) above) would raise the figure farther to 88%. Thus we are left with a price margin of some 12%. If this cannot be explained by a higher average British quality ((2) above) and by immeasurable errors under (1) and (3) above, we may conclude that the U.S. had a comparative disadvantage in the later period.

That this was so is suggested in a more simple manner by the figures in columns (2) and (3) of Table X. These are, in fact, unweighted quantity and price index numbers for the products

TABLE X
Geometric Means and Medians

	No. of manufactures.	Unweighted geometric means.		Medians.	
		Relative quantity (U.S. : U.K.).	Relative price (U.S. : U.K.).	Relative quantity (U.S. : U.K.).	Relative price (U.S. : U.K.).
		(1)	(2)	(3)	(4)
1913 ¹	32	0.69	0.90	0.55	0.89
1922	86	1.42	0.84	1.075	0.88
1923	86	1.14	0.90	1.04	0.94
1924	86	0.88	0.96	0.85	1.035
1925	86	1.03	0.93	0.89	0.98
1925	97	1.02	0.90	0.89	0.95
1926	97	1.00	0.92	0.99	0.98
1927	97	0.98	0.91	0.85	0.98
1928	97	1.05	0.92	1.01	0.99
1928	109	1.18	0.90	1.17	0.96
1929	109	1.13	0.90	1.10	0.96
1930	109	1.07	0.89	0.92	0.925
1931	109	0.94	0.97	0.98	0.99
1932	109	0.56	1.17	0.56	1.17
1933	109	0.57	1.00	0.45	1.03
1934	109	0.62	0.94	0.59	0.995
1935	109	0.59	1.05	0.50	1.02
1936	109	0.64	0.97	0.55	1.02
1937	109	0.76	0.96	0.58	0.99
1938	109	0.78	0.92	0.63	0.95
1934-38	109	0.69	0.93	0.58	1.00
1937	118	0.65	0.91	0.58	0.97
1948	118	1.37	0.87	1.33	0.91
Commonwealth countries 1937	115	0.235	0.97	0.24	0.96
Non-Commonwealth countries 1937	115	1.61	0.92	1.53	0.95

¹ Mid-1912 to mid-1913 for the U.S.

in our sample. But, instead of comparing one year with another, they show for each year, or period of years, indices for the U.S., the U.K. being taken as unity. For the years 1934-38 taken as a whole, the recorded U.S. price was, on average, 93% of the British, but the U.S., instead of exporting more than Britain, as might have been expected, exported, on average, only 69%

as much.¹ If we assumed that a 2% correction is adequate for under-valuation of U.S. exports ((1) above) this would raise the 93% to 95%; the general picture would not be materially altered. If we further assumed that British products were as much as, say, 10% better in quality than the American, this would further raise the average U.S. price to around 105% of the British, but it would also reduce the average U.S. quantity to around 63% of the British. Such a low American export could be explained by 5% higher prices only if there were an elasticity of substitution of more than -9,² and this seems improbable.

If median relative quantities and prices are used (columns (4) and (5) of Table X), the result is less convincing, but, despite all the uncertainties, it seems not unlikely that, in the later thirties, the U.S. tended to export less of a commodity than Britain when the price was equal. It also seems likely that, in the earlier years, any American comparative disadvantage in the imperfect world market was smaller or non-existent. These tentative conclusions may not, however, apply to the whole range of manufactures; it should be remembered in particular that, for statistical reasons, the products in our sample include very few in the important field of machinery.

The figures in Table X confirm the striking improvement in the relative position of the U.S. between 1937 and 1948, though it is not easy to assess the relative importance of various possible causes. While British exporters were out of the market during

¹ These are geometric means. The arithmetic means are, of course, larger, and the harmonic means (*i.e.*, the reciprocals of the arithmetic means of relative price and quantity, U.K. : U.S. instead of U.S. : U.K.) are smaller. The various means are as follows :

	Arithmetic.	Geometric.	Harmonic.
Relative quantity, U.S. : U.K.	4.75	0.69	0.03
Relative price, U.S. : U.K.	1.03	0.93	0.86

The arithmetic and harmonic means of relative *quantity* are greatly influenced by a few extreme cases which have a much smaller effect on the geometric mean. Nevertheless, the large difference between the arithmetic and harmonic means shows the need for caution in the use of the geometric mean. The differences between the various means of relative *price* are much less marked because the range of variation is much smaller in relative price than it is in relative quantity.

As a form of check on the geometric means, the medians were also calculated and these are shown in columns (4) and (5) of Table X. The general picture is not very different, and the use of medians instead of geometric means does not greatly affect the arguments in the text.

² $\frac{\log 0.63}{\log 1.05} = -9.5.$

the war and early post-war years, American manufacturers doubtless built up goodwill and a preference for American models and qualities, and at the same time gained a lead in the development of new types of products. They were handicapped after the War by discrimination caused by dollar shortage, but this was limited by U.S. government loans and grants. On the other hand, the large change in the figures may reflect in part merely the inability of British exporters, in 1948, to meet demand at a given price.

Imperial Preference

It is tempting to try to discover how far the apparent disadvantage of the U.S. in the imperfect world market in the later thirties can be accounted for by Imperial Preference and how far the increase in preferences resulting from the Ottawa Agree-

Relative Prices and Quantities of U.S. and U.K. Exports of 115 Manufactures in 1937 :

- (a) to Commonwealth Countries
(b) to Non-Commonwealth Countries

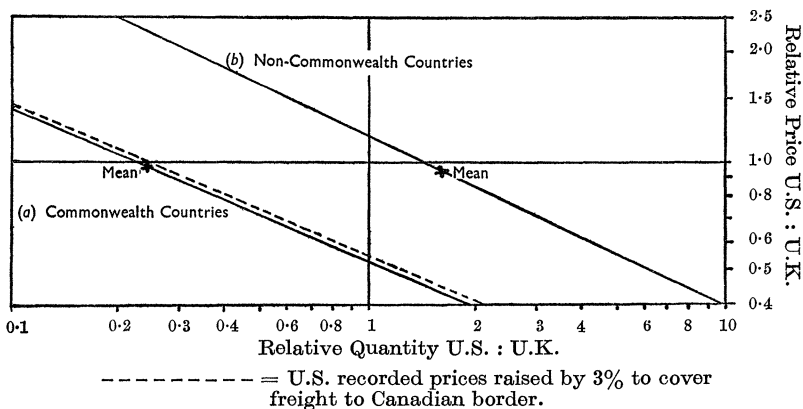


FIG. 4

ments accounts for the apparent change to America's disadvantage after 1932. It is shown in Appendix E that, after Ottawa, the margin of preference on U.K. goods entering British Commonwealth markets was, on average, of the order of 10%, so that, as roughly one-half of British exports went to Commonwealth countries, the margin represented about 5% of the total value of British exports. The increase in the margin resulting from the Ottawa Agreements must have been considerably less than 5%. The reader may care to make his private deductions about the effects of Imperial Preference, but, in view of the difficulties described above, it does not seem that trustworthy conclusions can be drawn from the figures so far given.

A more fruitful approach is to consider separately British and American exports (*a*) to Commonwealth countries (excluding the U.K.) and (*b*) to non-Commonwealth countries (excluding the U.S.). This has been done, for 115 commodities, for the year 1937.¹ Owing to the labour involved, similar calculations were not made for other years. The two regression lines are shown in Fig. 4, and the main results are given in Tables IX and X.

It will be seen from Table X that American prices were, on average, 92% of the British for exports to non-Commonwealth countries and 97% for exports to the Commonwealth.² This latter figure should perhaps be raised to about 100% to cover under-valuation of U.S. exports to Canada.³ Other possible corrections for relative under-valuation or quality differences need not concern us here, since they would probably apply in more or less equal degree to exports both to Commonwealth and to non-Commonwealth countries. We are concerned only with the fact that average relative prices were some 8% lower in trade with non-Commonwealth countries than they were in trade with the Commonwealth. There was, on the other hand, a very large difference in relative quantities. The U.S. exported, on average, 161% as much to non-Commonwealth countries as Britain and only 23½% as much as Britain to Commonwealth countries. In case these figures are biased by quality differences, it is safer to say that the relative quantities (U.S. : U.K.) were, on average, about 6.9 times as great for non-Commonwealth countries as they were for Commonwealth countries.⁴ Such a large difference could be explained by the 8% difference in relative prices only if we assumed an "elasticity of substitution" as high as -23.⁵ This is unlikely, and it therefore seems almost

¹ For statistical reasons, it was impossible to obtain figures for the 171 or for the 109 commodities covered by the previous analyses.

² The geometric means are used in this paragraph and the next, but the argument is unaffected if the medians are used instead.

³ Railway freight to the Canadian frontier not included in the recorded value of U.S. exports has been estimated at \$34 million in 1937 (*The Balance of International Payments of the United States in 1937*, p. 28). This is 4% of U.S. exports to the British Commonwealth (excluding the U.K.), which totalled \$827 million. Since the percentage for manufactures may have been smaller, and since there may have been offsetting factors, it seems reasonable to round up our observed figure of 97% to 100%.

⁴ A comparison of the values of British and American exports of all manufactures to Commonwealth and non-Commonwealth countries shows a much smaller discrepancy.

⁵ $\frac{\log 6.9}{\log 0.92} = -23.$

This is a sort of "spatial" elasticity of substitution. Questions of bias are not discussed, as the figure is purely illustrative.

certain that a large part of the difference is to be explained by factors other than price.

Of these, Imperial Preference seems to provide only a partial explanation. The margin of preference was about 10% of the value of British exports to the Commonwealth, so that if, as the figures suggest, the prices of British and American exports to the Commonwealth were, on average, the same excluding import duty, then American prices, including duty, were some 110% of the British. In non-Commonwealth countries they were 92% of the British, *i.e.*, relative prices (U.S. : U.K.) were about 16% lower in non-Commonwealth countries than they were in Commonwealth countries.¹ But, even allowing for this, the large difference in relative quantities could be explained solely by prices, including duty, only if we assumed an elasticity of substitution of at least -11.² It therefore seems likely that factors other than preferential duties were important.

An examination of the two regression lines may give some indication of the relative importance of Imperial Preference and other factors. It will be seen from Fig. 4 that the line for Commonwealth countries was far to the left of that for non-Commonwealth countries. Whatever corrections may be necessary,

¹ $0.92 \div 1.10 = 0.84$. It is assumed that the rate of duty on British and American goods in non-Commonwealth markets was the same. See also next footnote.

$$^2 \frac{\log 6.9}{\log 0.84} = -11.$$

This is an understatement for two reasons :

(a) Where duties are *ad valorem*, relative prices to Commonwealth countries, including duty (but excluding transport costs, the difference in which is small for most manufactures), would be :

$$\frac{1 + t_c + t_a}{1 + t_c} = 1 + \frac{t_a}{1 + t_c}$$

where t_c is the rate of common tariff and t_a the rate of additional duty on U.S. goods. Where t_a is 0.1 (10%), the relative price is thus less than 1.1, where t_c is not zero. Where duties are specific, relative prices, including duty, would be :

$$\frac{p + T_c + T_a}{p + T_c} = \frac{1 + \frac{T_c}{p} + \frac{T_a}{p}}{1 + \frac{T_c}{p}}$$

where T_c and T_a are the common and additional specific duties and p the American and British price. Where $\frac{T_c}{p} = 0.1$ this will again be less than 1.1.

(b) Relative prices of exports to non-Commonwealth countries, including duty, would be greater than 92% in so far as there are specific duties.

Relative prices to non-Commonwealth countries would thus be less than 16% lower than relative prices to Commonwealth countries, and the elasticity of substitution required to explain the difference in relative quantity would be higher than -11.

this seems to indicate a substantial difference in the relative advantages of America and Britain in these two markets at a given f.o.b. price ratio. According to the regression lines, U.S. exports to non-Commonwealth countries were 136% of the British when recorded prices were equal while her exports to the Commonwealth were only 22% of the British. It is suggested in Appendix D that, even allowing for corrections, it may still be fairly safe to say that, where British and American prices were equal in both markets, relative exports (U.S. : U.K.) were four times as great for non-Commonwealth as for Commonwealth countries.¹

Now the margin of Imperial Preference was about 10% of the value of British exports to Commonwealth countries. Where British and American prices in Commonwealth markets were equal, including duty, the American price, excluding duty, was, therefore, on average, 9% less than the British price, excluding duty.² Assuming a product elasticity of substitution as high as -5 , the relative quantity (U.S. : U.K.) was about 60% higher when prices were equal, including duty, than it was when prices were equal excluding duty.³ If, therefore, the relative quantities were about four times as great in non-Commonwealth markets as they were in Commonwealth markets when prices, excluding duty, were equal, they were still about $\frac{4}{1.6} = 2\frac{1}{2}$ times as great when prices, including duty, were equal. There is thus still a margin of this order to be explained by factors other than Imperial Preference. The latter, in fact, explains directly only a comparatively small part of the relative advantage of the U.K. in Commonwealth as compared with non-Commonwealth markets.

¹ According to the regression lines the U.S. exported the same amount as Britain to non-Commonwealth countries even when her price was 116% of the British, but the same amount to Commonwealth countries only when her price was 54% of the British. It is argued in Appendix D that the last figure may well be too low and the first too high.

² When duties are *ad valorem*, and prices are equal, including duty :

$$p_a(1 + t_c + t_a) = p_b(1 + t_c)$$

where p_a , p_b are the American and British prices, excluding duty, t_c the common rate of tariff and t_a the additional rate on U.S. goods (transport costs are ignored). *I.e.*,

$$\frac{p_a}{p_b} = \frac{1 + t_c}{1 + t_c + t_a} = \frac{1 + t_c}{1.1 + t_c}$$

which is greater than $\frac{1}{1.1}$, *i.e.*, greater than 0.91. In so far as the true figure exceeds 0.91, the argument in the text is strengthened.

³ Antilog $(-5 \log 0.91) = 1.6$.

It would seem that factors such as commercial ties of Britain with the Commonwealth and of the U.S. with certain non-Commonwealth countries were considerably more important; although Britain's ties with the Commonwealth may, in part, have been indirectly fostered by the preferential duties. If the elasticity of substitution was less than -5 , the importance of preferential duties was, of course, smaller.

It is interesting to speculate about the effect of preferential duties on the volume of Britain's export trade. Before the war, Commonwealth countries probably took around 37–38% of their imports of manufactures from Britain,¹ *i.e.*, the ratio of British to foreign exports of manufactures to Commonwealth countries was about 0.6.² If the preferences had been removed and prices, excluding duty, had remained the same, the ratio of British to foreign prices, including duty, would have risen by something like 10%. Assuming a *total* elasticity of substitution of -3 (the highest figure we dared to suggest for the *total* elasticity between British and U.S. exports of manufactures to all countries) the ratio of 0.6 would have fallen by about one-quarter,³ *i.e.*, to 0.45, so that Britain's proportionate share would have fallen from about 37½% to about 31%.⁴ If the total market had remained the same, the quantity of Britain's exports to Commonwealth countries would thus have fallen in the ratio 37½ : 31, *i.e.*, by about 17%, and, if her trade with other countries had been unaffected, her exports to all countries would have fallen by about 9%. Assuming a total elasticity of substitution of -2 , the figure would be about 6%. In view, however, of the many indirect repercussions and of the danger, emphasised above, of using calculated elasticities, too much significance should not be attached to these results, which suggest that preferential duties safeguarded less than 10% of Britain's total exports.⁵ It should be remembered, too, that they take no account of other preferential arrangements such as preferential import quotas.

¹ This is a rough calculation based on value; it is assumed that the figure for volume is the same.

² $\frac{37\frac{1}{2}}{63\frac{1}{2}} = 0.6$.

³ $-3 \log 1.1 = -0.1242$, the antilog of which is 0.75.

⁴ $\frac{31}{69} = 0.45$.

⁵ This is not, of course, the same as the percentage of U.K. exports enjoying preference, which was considerably higher. Table XI shows that more than one-half of U.K. exports to Commonwealth countries enjoyed preference, *i.e.*, over one-quarter of all U.K. exports.

E. GENERAL CONCLUSIONS

The studies described in this article (including Part I which appeared in the *ECONOMIC JOURNAL* for December 1951) have ranged over a rather wide field; one thing led to another. It is hoped that they have illustrated, among other things, the usefulness of tackling problems of international trade through a study of individual commodities. The article is intended to be partly a study of method, but some of the results may be of more general interest.

It seems that the labour theory of value, crude as it is, does help to provide some explanation of British and American export trade in manufactures in an imperfect world market and to illustrate the importance of tariffs in limiting international commerce. It reminds us that a country can compete in certain lines, even with a rival whose general level of productivity is much higher. (This does not, of course, mean that a high and rapidly rising level of productivity in, say, the U.S. may not aggravate the problem of international disequilibrium. For example, knowledge of the high American standard of living may increase any tendency of other countries to try to live beyond their means; high productivity, reflected partly in the ability to supply the latest products, creates a large demand for a country's exports; a more rapid growth of productivity than in other countries may mean continuous pressure on the international exchanges unless money wages also increase at an abnormally high rate; and so on.)

The fact that American weekly wages in manufacturing, which were about twice the British before the war, are now about $3\frac{1}{2}$ times as high suggests that, despite changes in relative productivity, there may be important opportunities for British exporters in many new lines. Figures in Tables IX and X, however, are consistent with the common view that the war resulted in a substantial improvement in the relative position of the U.S. in the imperfect world market. Some allowance must be made for this fact in any estimate of British export prospects unless it is offset by special selling efforts by British exporters, by discrimination against American goods or in other ways.

Leaving labour costs and turning to prices, a similar picture emerged. There was a fairly clear inverse relation between relative prices of British and American exports and the relative quantities exported, variations in the former being associated with comparatively large variations in the latter. The range

of relative prices showed, incidentally, the doubtful value of such statements as: "before devaluation . . . British goods had been 25% higher in price than American."¹ It seems likely that some British prices are usually higher than the American, some lower. The important question is how many are higher, how many lower, and by how much. Even if British and American prices were, according to some form of average, equal, this would not necessarily result in international equilibrium.

The results obtained suggested a possible method of estimating the elasticity of substitution between the exports of two countries by using data for a number of commodities in one period instead of data for one commodity (or group of commodities) in a number of periods. Although open to certain objections, this approach seems to avoid some of the difficulties connected with the more normal time series method.

A method was also suggested of using the "product" elasticity of substitution so obtained to estimate the "total" elasticity of substitution between groups of exports, which will normally be a lower figure. The calculations, allowing for possible bias, suggested a very much higher elasticity of substitution between British and American exports of manufactures than that obtained by Mr. Chang. An alternative method was also suggested of estimating short-period elasticities at particular times of rapid change.

Finally, the data for individual products were used in an attempt to estimate the importance of factors other than price in determining relative exports. The tentative conclusion was reached that, in the later thirties, the U.S. had on balance a comparative disadvantage in the imperfect world market, at least in the products studied. There was, however, a striking difference between Commonwealth and non-Commonwealth markets, the U.S. having a substantial disadvantage in the former and a substantial advantage in the latter. It seemed unlikely that more than a small part of this difference could be directly attributed to Imperial Preference, leaving the major part to be explained by commercial ties and other non-price factors. The importance of such factors in determining relative shares in export markets is not, however, inconsistent with a high price elasticity of substitution. A purchaser may be prepared to buy from A rather than B, even though A's price is, say, 5% higher; but if the margin rises to, say, 7% or 8% he may quickly switch the bulk of his custom to B.

¹ *The Times*, September 23, 1949.

It is hoped that the validity of the methods suggested in this article will be examined by others and that they may possibly be of some use in the study of other problems of international trade.

G. D. A. MACDOUGALL

*Nuffield College,
Oxford.*

APPENDIX B

Other Possible Bias in the Calculated Slopes of the Regression Lines

Let the true demand substitution curve for product R be

$$X_1 = A_r + B_r X_2 \quad . \quad . \quad . \quad . \quad (1)$$

where X_1 and X_2 are the logarithms of relative quantity and relative price respectively. (It is assumed in this Appendix that there are no errors of observation.) A_r and B_r are not the same for each product. For product R , B_r is the slope of the demand substitution curve, A_r the value of X_1 when $X_2 = 0$, *i.e.*, the logarithm of the relative quantity when the British and American prices are equal.

Equation (1) may be re-written

$$(x_1 + \bar{X}_1) = (a_r + \bar{A}) + (b_r + \bar{B})(x_2 + \bar{X}_2) \quad . \quad (2)$$

where \bar{X}_1 , \bar{X}_2 , \bar{A} and \bar{B} are means and x_1 , x_2 , a_r , b_r deviations from them.

$$i.e., \quad x_1 = \bar{B}x_2 + a_r + b_r x_2 + \bar{X}_2 b_r + (\bar{A} - \bar{X}_1 + \bar{B}\bar{X}_2) \quad (3)$$

Multiplying through by x_2 , and summing, we get

$$\Sigma x_1 x_2 = \bar{B} \Sigma x_2^2 + \Sigma a_r x_2 + \Sigma b_r x_2^2 + \bar{X}_2 \Sigma b_r x_2 \quad . \quad (4)$$

(Σx_2 multiplied by the constant expression in brackets in (3) is zero)

Let b be the regression coefficient obtained by the method of least squares from the observed data (minimising the sum of the squares of the deviations of the logarithms of relative quantity). Then

$$b = \frac{\Sigma x_1 x_2}{\Sigma x_2^2} = \bar{B} + \frac{\Sigma a_r x_2}{\Sigma x_2^2} + \frac{\Sigma b_r x_2^2}{\Sigma x_2^2} + \bar{X}_2 \frac{\Sigma b_r x_2}{\Sigma x_2^2} \quad . \quad (5)$$

The second, third and fourth terms on the right-hand side of (5) will tend to be positive if the supply substitution curves slope upwards.

The second will tend to be positive because a positive value of a_r denotes a demand substitution curve towards the right, and this will tend to mean a relatively high relative price (since the supply substitution curves slope upwards), *i.e.*, a positive value of x_2 . Similarly, negative values of a_r and x_2 will tend to go together.

The third term is a weighted average of the b_r 's, the weights being the x_2 's. A positive b_r denotes a steeper than average slope, and it is with these commodities that the large x_2 's will tend to be found, since a large x_2 denotes a relative price which is well below, or well above, the mean. Similarly, a negative b_r will tend to be associated with a small x_2 . Thus, while the unweighted average of the b_r 's is zero, the weighted average will tend to be positive.

The fourth term will tend to be positive for the following reasons. When $\bar{X}_2 > 0$, *i.e.*; broadly, when the majority of the relative prices are greater than unity, a positive b_r (*i.e.*, a steeper than average slope) will tend to mean a positive x_2 (*i.e.*, a higher than average relative price) and $\Sigma b_r x_2$ will tend to be positive. The fourth term will thus tend to be positive. When $\bar{X}_2 < 0$, a positive b_r will tend to be associated with a negative x_2 , and $\Sigma b_r x_2$ will tend to be negative. Again, the fourth term will tend to be positive.

Thus, from (5), b will tend to be greater than \bar{B} , *i.e.*, the calculated slope will tend to be steeper than the mean of the true slopes of the demand substitution curves if (i) the true curves are parallel but not coincident, or (ii) the true curves are not parallel but all cross the equal price line at the same point, or (iii) the true curves are not parallel and do not cross the equal price line at the same point.

If, however, the supply substitution curves are horizontal, it will readily be seen that these sources of bias disappear, and if they are downward sloping the bias is in the other direction.

APPENDIX C

Relation between the "Product" and the "Total" Elasticities of Substitution

(on the assumptions given on p. 493.)

If the quantitative *shares* of America and Britain in the exports of a commodity are A and B respectively ($A + B = 1$),

the price ratio for each product will change in the ratio $1 : \frac{m_a}{m_b}$. It is evident that the ratio of the price indices of the two countries will also change in the ratio $1 : \frac{m_a}{m_b}$. In other words, the proportionate change in the price ratio for each product and in the price-index ratio will be $\frac{m_a}{m_b} - 1$. Let us call this P .

The product elasticity of substitution will then be $\frac{K}{P}$, and the total elasticity of substitution will be

$$\frac{K}{P} \left[\frac{\Sigma V_a B}{\Sigma V_a} + \frac{\Sigma V_b A}{\Sigma V_b} \right].$$

The term in brackets has been called the index of similarity of exports. If the two countries have no exports in common it is zero. The first term is zero because, when America exports a product, Britain does not; hence, when V_a is not zero, B is zero. The second term is zero for similar reasons.

When the (value) pattern of exports is exactly the same, the index is unity. $\frac{V_a}{V_b}$ is constant, and the index may be re-written

$$\frac{\Sigma V_b(1 - A)}{\Sigma V_b} + \frac{\Sigma V_b A}{\Sigma V_b} = 1.$$

Where K is not the same for each product, the proportionate change in the ratio of the American to the British volume indices will be

$$\begin{aligned} & \frac{\Sigma V_a K B}{\Sigma V_a} + \frac{\Sigma V_b K A}{\Sigma V_b} \\ &= \bar{K} \left[\frac{\Sigma V_a B}{\Sigma V_a} + \frac{\Sigma V_b A}{\Sigma V_b} \right] + \left[\frac{\Sigma V_a B k}{\Sigma V_a} + \frac{\Sigma V_b A k}{\Sigma V_b} \right] \end{aligned}$$

where \bar{K} is the (arithmetic) mean of K , and k the deviation from it. The first set of terms is the same as (2) with \bar{K} substituted for K . An examination of the second set of terms suggests no obvious reason why they should be biased away from zero, so that the assumption that K is the same, for each commodity, as the arithmetic mean of the individual K 's does not seem to introduce any bias into the derivation of the total elasticity.

APPENDIX D

Errors in the Intercepts of the Regression Lines on the Equal Quantity and Equal Price Axes

Using the notation of Appendices A and B, let the true demand substitution schedule for commodity *R* be

$$X_1' = A_r + B_r X_2'$$

i.e., $(\bar{X}_1 + x_1 - \bar{E}_1 - e_1) = (\bar{A} + a_r) + (\bar{B} + b_r)(\bar{X}_2 + x_2 - \bar{E}_2 - e_2)$.

Summing and dividing by *N* (the number of observations), we get

$$\bar{X}_1 = \bar{A} + \bar{B}\bar{X}_2 + \frac{\Sigma b_r x_2}{N} + \frac{1}{N}(\Sigma a_r + \bar{B}\Sigma x_2 - \bar{B}\Sigma e_2 + \bar{X}_2 \Sigma b_r - \bar{E}_2 \Sigma b_r - \Sigma x_1 + \Sigma e_1) + (\bar{E}_1 - \bar{B}\bar{E}_2) - \frac{\Sigma b_r e_2}{N}.$$

The terms in the first bracket are necessarily zero. Those in the second are zero, assuming that the errors of observation are unbiased (the case of biased errors is dealt with on pp. 501-2). The last term tends to zero, since there seems no reason to expect correlation between *b_r* and *e₂*. We are thus left with

$$\bar{A} = \bar{X}_1 - \bar{B}\bar{X}_2 - \frac{\Sigma b_r x_2}{N} \quad . \quad . \quad . \quad (1)$$

\bar{A} is the average intercept on the equal price axis (*i.e.*, the value of *X₁* when *X₂* = 0) that we wish to discover. The equation of the regression line we find by the method of least squares is

$$(X_1 - \bar{X}_1) = b(X_2 - \bar{X}_2)$$

i.e., $X_1 = (\bar{X}_1 - b\bar{X}_2) + bX_2$.

The intercept on the equal-price axis that we find is thus

$$a = \bar{X}_1 - b\bar{X}_2 \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$\bar{A} - a = \bar{X}_2(b - \bar{B}) - \frac{\Sigma b_r x_2}{N} \quad . \quad . \quad . \quad (3)$$

Since we assume that the true average slope is flatter than the observed slope, and since both *b* and \bar{B} are negative, $(b - \bar{B}) > 0$. When $\bar{X}_2 > 0$, the first term on the right-hand side of (3) is thus positive and, when $\bar{X}_2 < 0$, it is negative. But, assuming upward sloping supply substitution curves, when $\bar{X}_2 > 0$, steep slopes of the demand substitution curves (*i.e.*,

positive b 's) will tend to be associated with high relative prices (*i.e.*, positive x_2 's), and therefore the second term will tend to be negative. Similarly, when $\bar{X}_2 < 0$, the second term will tend to be positive. We therefore cannot say whether the calculated intercept on the equal-price axis is biased to left or to right. If, however, the supply substitution curves are horizontal or downward sloping, we can say that the calculated intercept will be biased to the left when $\bar{X}_2 > 0$, and to the right when $\bar{X}_2 < 0$. In any case it will be unbiased when $\bar{X}_2 = 0$, *i.e.*, when average relative price is unity.

For exports to Commonwealth and non-Commonwealth countries, \bar{X}_2 equals -0.0128 and -0.0357 respectively. If the supply substitution curves are horizontal, the last term in (3) tends to zero, so that

$$\bar{A} = a + \bar{X}_2(b - \bar{B}) \quad . \quad . \quad . \quad (4)$$

Assuming \bar{B} 's of -3 , -4 or -5 , the intercept for exports to Commonwealth countries falls from the calculated figure of 0.22 to 0.21 , 0.20 or 0.19 , while that for exports to non-Commonwealth countries falls from the calculated figure of 1.36 to 1.26 , 1.16 or 1.07 . The ratio of the intercepts falls from the calculated figure of 6.2 to not less than $5\frac{1}{2}$.

If the supply substitution curves are downward sloping, both intercepts will tend to fall further, and the ratio may also fall further, but there seems to be no way of estimating the probable amount.

If the supply substitution curves are upward sloping, the last term in (3) tends to be positive, since \bar{X}_2 is negative in both cases. A lower limit for the true value of the intercept for exports to non-Commonwealth countries is thus given by (4), *i.e.*, 1.07 for a true \bar{B} of -5 . The corresponding figure of 0.19 for exports to Commonwealth countries will tend to be raised by the positive last term in (3), but, as \bar{X}_2 is not greatly below zero, we may perhaps guess that the true intercept is not greatly in excess of, say, 0.25 , even after allowing for the correction necessitated by underestimation of U.S. exports to Canada. If this is correct, the ratio of the true intercepts is thus at least 4 .

There may have been quality differences and differences in methods of valuation that applied equally to exports to each group of countries, but, provided these were not too large, and provided the true slopes for the two groups of countries were not very different, it does not seem unreasonable to assume that, when price (corrected where necessary) was equal, relative

quantity (U.S. : U.K.) was probably at least four times as great for non-Commonwealth as it was for Commonwealth countries.

We now turn to the intercept on the equal quantity axis, *i.e.*, the value of X_2 when $X_1 = 0$. The intercept we wish to find is

$$I = \frac{-\bar{A}}{\bar{B}} = \frac{-\bar{X}_1}{\bar{B}} + \bar{X}_2 + \frac{\Sigma b_r x_2}{N\bar{B}}.$$

The calculated intercept is

$$i = -\frac{a}{b} = -\frac{\bar{X}_1}{\bar{b}} + \bar{X}_2.$$

Hence
$$I - i = \bar{X}_1 \left(\frac{1}{\bar{b}} - \frac{1}{\bar{B}} \right) + \frac{\Sigma b_r x_2}{N\bar{B}}.$$

Now $\left(\frac{1}{\bar{b}} - \frac{1}{\bar{B}} \right) < 0$ and $\frac{\Sigma b_r x_2}{N\bar{B}}$ tends to have the opposite sign from \bar{X}_2 if the supply substitution curves slope upwards, the same sign if they slope downwards and to be zero if they are horizontal.

For exports to the Commonwealth, $\bar{X}_1 < 0$ and $\bar{X}_2 < 0$. Hence $I > i$ if the supply substitution curves are upward sloping or horizontal, while if they are downward sloping no conclusion can be drawn. For exports to non-Commonwealth countries, $\bar{X}_1 > 0$ and $\bar{X}_2 < 0$. Hence $I < i$ if the curves are downward sloping or horizontal, while if they are upward sloping no conclusion can be drawn. Hence the statement, in the footnote to p. 508, that the figure obtained for relative price where the quantity of exports was equal may well be too low for exports to Commonwealth countries and too high for exports to non-Commonwealth countries.

For exports to all countries in 1934-38, $\bar{X}_1 < 0$ and $\bar{X}_2 < 0$. If the supply substitution curves are horizontal, and $\bar{B} = -5$, the antilogarithm of I works out at 0.864. If the supply substitution curves slope downwards, the true figure is lower, but if they slope upwards it is higher. Hence the figure of 86½% on p. 503.

APPENDIX E

*Imperial Preference*¹

It was stated in the text that the average margin of preference enjoyed by U.K. exports to British Commonwealth countries

¹ This appendix is almost wholly the work of Miss Rosemary Orton. Much valuable help was obtained from London representatives of Dominion Governments.

before the war, but after Ottawa, was of the order of 10%. This was based on a calculation for 1937 (or the financial year 1937-38) covering countries that took 97% of U.K. exports to the Commonwealth.¹ The main sources used were the trade returns and, where necessary, the tariff lists, of the various countries.

For each product imported into a Commonwealth country from the U.K., the percentage margin of preference is the difference between the duty that would have been paid had the product been American and the duty actually paid on the U.K. product, expressed as a percentage of the value of the imports from the U.K., excluding duty. The average percentage margin of preference on all imports from the U.K. is the average of the percentage margins of preference on the various products weighted according to the value of imports of each product from the U.K. This is the same as the difference between the duty that would have been paid on the total of imports from the U.K. had the goods been American and the duty actually paid, expressed as a percentage of the total value of imports from the U.K.

The average percentage margin of preference for each country is shown in column (5) of Table XI. It will be seen that the average of these figures, weighted according to U.K. exports to each country in 1937 (column (1)), was 9-10%. This represents the difference between the duty that would have been paid on all U.K. exports to Commonwealth countries had they been American and the duty actually paid, expressed as a percentage of the value of all U.K. exports to Commonwealth countries.

The method of calculation used tends to overstate the average margin of preference in so far as the existence of preference presumably tends to increase the proportion of U.K. exports enjoying higher preferences and to reduce the proportion enjoying lower preferences or none at all. Likewise, it might be expected to increase the proportion of U.K. exports sent to countries granting the higher average preferences. If the various margins of preference were weighted according to the pattern that exports would have taken without preference, or according to the value of U.S., rather than U.K., exports to Commonwealth countries, one would expect the final figure to be lower, though this tendency might, of course, be offset by other factors.

The notes to Table XI describe only some of the difficulties involved in the calculations. The main purpose of the table is to help to show how the final figure of 10% was reached. The

¹ Defined for this purpose as countries classified as "British" in the U.K. trade returns.

results obtained for the various countries, although shown for convenience with some degree of precision, are subject to a considerable margin of error. The following broad picture is, however, probably correct and may be of some interest :

Of U.K. exports to Commonwealth countries, one-third went to Canada, Australia and New Zealand, which gave an average preference of around 20%; one-third went to India and South Africa, which gave an average preference of around 5%; one-sixth went to Eire, which on balance gave little preference, or to territories giving no preference at all; one-sixth went to miscellaneous countries, which gave various rates of preference averaging about 10%.

It is hoped to publish more details at a later date.

No account has been taken of preference given through import quotas or by any method other than import duties.

TABLE XI
Margins of Preference 1937

Country.	(1) % of total U.K. ex- ports to British countries.	(2) % of trade covered.	(3) % of imports from U.K. given pre- ference. ⁴	(4) % average margin of preference on goods given pre- ference.	(5) % average margin of preference on all im- ports from U.K.
New Zealand	8.0	100.0	88-90	25-27	23-24
Canada	10.9	99.7	88	23	20
Southern Rhodesia	1.2	99.5	96	19-20	19
Australia	14.9	100.0	90 ¹	19 ¹	17 ¹
British Guiana	0.5	99.8	93	18-19	17
Channel Islands	2.2	59.1	73-86	18-39	15-29
Jamaica	0.8	99.9	83	15	12-13
Trinidad and Tobago	1.0	99.4	64	19	12
Burma	1.3	98.6	62-93	10-21	9-13
Northern Rhodesia	0.4	99.8	93-97	9-11	9-10
Malta	0.5	100.0	71	12	8
Ceylon	1.6	97.2	70-86	8-21	7-8
Federated Malay States	1.2	97.9	44-58	11-19	6-8
India	14.2	96.0	50	12	6
Union of South Africa	16.5	99.3	39-43	5-8	2-4
Straits Settlements	3.4	100.0	24	13	3
Gibraltar	0.4	100.0	5-7	24-48	2
Hong Kong	1.3	100.0	5 ²	15	1
Eire	8.6	99.3	10	13-31 ³	1-2 ³
Territories granting no preference ⁶	8.2	100.0	0	—	0
Total of above countries	97.1	98.6 ⁵	55-57 ⁵	15-18 ⁵	9-10 ⁵

¹ Taken directly from the official Year Book.

² Takes no account of additional duty of 10% on certain U.K. goods.

³ Allows for additional 10% duty on certain U.K. goods.

⁴ Refers to imports from the U.K. that were covered in this calculation.

⁵ Weighted average.

⁶ Nigeria (including British Cameroons), Gold Coast (including Togoland) most of British East Africa, Anglo-Egyptian Sudan, Palestine, Aden, New Guinea and Papua.

GENERAL NOTES ON TABLE XI

Column (1).

Calculated from *Annual Statement of Trade of the U.K.*

Columns (2)-(5)

Calendar year 1937 for New Zealand, Northern and Southern Rhodesia, Trinidad and Tobago, South Africa, Eire, British Guiana, Malta, Jamaica, Hong Kong; financial year 1937-38 for the rest.

In calculating the preference margins it was sometimes necessary to give upper and lower limits for a small percentage of items for which it was impossible to reconcile the classifications in the trade returns with those in the tariff lists. Several trade returns contained, for example, items such as "other foods" and "other textile manufactures". It was impossible to find either the exact amounts given preference or the amount of preference, if any, conceded. This kind of difficulty accounts both for many of the percentages in Column 2 being less than 100 and for the outside limits in columns 3, 4 and 5 which take the place of precise figures. It also, together with rounding, accounts for the apparent slight inconsistencies between the last three columns.

WEIGHTING

A. To obtain average figures for each country.

The percentage margin of preference for each product was weighted by the value of imports from the U.K. as follows :

(a) Australia, Canada, Jamaica, Trinidad and Tobago, South Africa, India. Value of imports on which preference was granted.

(b) New Zealand, Northern and Southern Rhodesia, British Guiana, Malta, Eire, Hong Kong. Value of imports originating in the U.K. Since the qualification for preferential rates is that a certain percentage of an article's value should be the result of British labour (the percentage varying with the importing country), the figures given in columns (3) and (5) may be slightly above their true value.

(c) Channel Islands, Burma, Federated Malay States, Gibraltar, Ceylon, Straits Settlements. Value of U.K. exports as given in U.K. *Annual Statement of Trade*, Vol. IV. This will also give an upward bias to the figures in columns (3) and (5), since the U.K. figure includes all goods which in the U.K. have "undergone operations" which do not "leave them essentially unchanged."

In the Canadian trade returns imports are attributed to the country from which they are consigned. The margins of preference on the separate items were weighted by the value of imports under the preferential tariff and then expressed as a percentage of the total value of imports *consigned from* U.K. (as opposed to the total value of imports *originating in* U.K. for other countries). The figures for Canada in columns (3) and (5) are therefore, relatively, underestimates.

B. To obtain average figures for all British countries.

For consistency the figures for each country were weighted by total exports of U.K. produce and manufactures to each country (column (1)).