OHLIN WAS RIGHT*

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I was originally led to study the problem of complete factor-price equalisation by the need to explain to a class in international trade Bertil Ohlin's seminal proposition that, although free mobility of factor inputs in international trade will equalise factor returns all the way, free mobility of goods can serve only to move factor-prices *toward* (but not all the way to) factor-price equalisation. As has been discussed elsewhere,[1] I found I could not quite prove the last part of the Ohlin proposition, that factor-price equalisation by trade would have to be necessarily partial and incomplete. Indeed, in the case where there are zero transport costs, no complete specialization in either country, and where two goods are strongly relative factor-intensive in their respective inputs of the two inputs available to society, with the same laws of knowledge operative everywhere, I ended up proving that Ohlin was wrong in the Pickwickian sense of being less than right: namely I proved (as was later learned Abba Lerner had done more than a decade earlier in an unpublished paper at the London School of Economics) that there would have to be more than partial factor-price equalisation—there would have to be *full* factor-price equalisation.[2]

I. Vindication

Recently in another connection I presented a simple, but rigorous, model of general equilibrium in international trade that could be expressed in terms of the two-dimensional diagrams of Marshall's partial equilibrium supply and

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My score of indebtedness, mounting over the years, to B. Ohlin, *Interregional and International Trade* (Harvard University Press, Cambridge, Mass., 1933) will be self-evident. Forty years have not aged this classic which sprang full-blown from the brow of its youthful author.

[1] P. A. Samuelson, "International Trade and the Equalisation of Factor Prices", *Economic Journal,* Vol. 58 (1948), pp. 163–184. The vast literature on this topic is surveyed in P. A. Samuelson, "Summary on Factor-Price Equalisation", *International Economic Review,* Vol. 58 (1967), pp. 286–295, where reference is made to the contributions of Lerner, Tinbergen, Meade, Pearce, McKenzie, Nikaido-Gale, and many others.

[2] To my knowledge only one support for the necessarily-incomplete equalisation thesis appeared in the literature. H. Uzawa, "Prices of the Factors of Production in International Trade", *Econometrica,* Vol. 27 (1959), pp. 448–468, sets forth in Section 6 a linear-demand model in which necessarily-incomplete-equalisation was deduced: unfortunately, the Uzawa functions were assumed to have single-valuedness properties which contradict the constant-returns-to-scale technologies presupposed in the discussions; hence the argument is not germane. Uzawa does quote Haberler's approval of the Ohlin thesis, but that approval may well have had reference to realistic transport costs for goods which, all are agreed, will present complete equalisation of either goods' prices or factors' prices.

demand.[1] The supply conditions of that model are of interest for their own sake since they portray what might be called the Ricardo–Viner case of pure rent.[2] They provide what this intricate subject can use to advantage, an alternative *simple* model that can free the discussion from the straight-jacket of the box-diagram analysis which Stolper and I imposed on the trade literature decades ago.[3]

My old classmate from Chicago days, Martin Bronfenbrenner, recently wrote to complain that I had not explained the implications of the new model for factor-price equalisation. Always game to try to fill any pointed-out vacuum, I proceeded to provide that analysis. The conclusion of the effort was this: After all, Bertil Ohlin's contention for partial but not total factor-price equalisation is essentially vindicated in this technological model.

It is the purpose of the present paper to describe these findings.

II. Graphical Resumé

Before turning to the new model, I show in Fig. 1 a self-contained summary of how free mobility in goods must compensate completely in the Lerner–Samuelson model for immobility of factors in equalising factor returns, provided two regions differ by not too much in geographical endowments. The correct post-trade situation, the heavy $SPEP'S'$ locus, is contrasted with the no-trade light NT locus and with the Ohlin-thesis broken line OH locus.

The horizontal axis portrays the labour/land endowment in Region A in ratio to that in Region B. Equality is at 1, the intersection of the axes. The vertical axis portrays the wage/rent outcome in Region A relative to that in B. The reader might, for simplicity, think of Region B as vastly greater than Region A; then he can imagine the relative endowment of Region A differing by more and more from unity in either direction. (If the goods come to inter-

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[1] P. A. Samuelson, "An Exact Hume–Ricardo–Marshall Model of International Trade", *Journal of International Economics*, Vol. 1 (1971), pp. 1–18. There is also my contribution to the Kindleberger *Festschrift* (ed. J. Bhagwadi et al.), *Trade, Balance of Payments and Growth* (North Holland Publishing Co, Amsterdam, 1971), Part 6, Chapter 15; "On the Trail of Conventional Beliefs About the Transfer Problem". Although the present paper adopts the industry-supply relations of these papers, it abandons their Marshallian partial-equilibrium demand relations.

[2] Viner's famous 1931 article, "Cost Curves and Supply Curves", in which the draftsman Wong will not draw the envelope of costs incorrectly despite Viner's insistence, develops the case. The original reference is to the 1931 *Zeitschrift für Nationalekonomie*, but the article has been reproduced in many anthologies and the reader is best advised to consult a version which includes a new appendix written a decade later. Cf. K. E. Boulding & G. J. Stigler, *Readings in Price Theory* (Richard D. Irwin, Inc., Chicago, 1952), Chapter 1, pp. 198–232.

[3] W. A. Stolper & P. A. Samuelson, "Protection and Real Wages", *Review of Economic Studies*, Vol. 9 (1941), pp. 58–73.

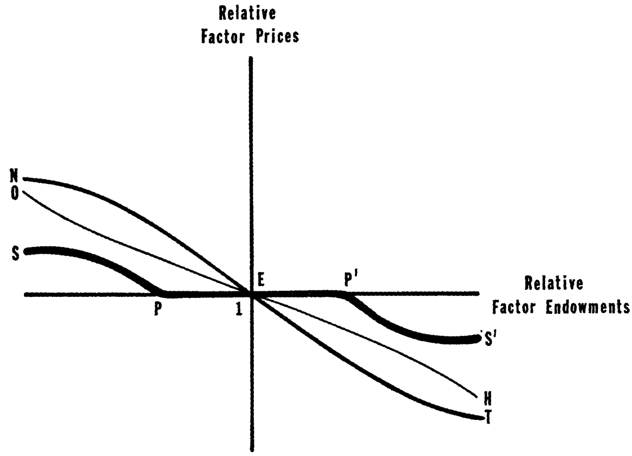

Fig. 1. As Region A's relative factor endowments diverge in ratio to Region B's from equality or unity, the No-Trade locus, *NT*, shows that inequality of relative factor prices can be expected under autarky. With trade serving to raise the demands for the cheap factor in each region, Ohlin claimed that the *OH* locus of partial but incomplete factor-price equalisation would result. However, the Lerner–Samuelson model is seen to have complete factor-price equalisation on the PEP′ horizontal branch of SPEP′S'. Once complete regional specialization on a single commodity is induced, the *SP* branch shows that the wage-rent ratios are only partially equalised in the Ohlin manner, even for the Lerner–Samuelson model; and partial transport costs would produce a similar effect.

change factor intensities at distant factor prices, the $S'P'$ branch could encounter another horizontal branch below the horizontal axis.)[1]

As Ohlin was the foremost to emphasize, difference in regional tastes can offset difference in regional factor endowments. Therefore, if both countries are of a comparable size, we can sidestep, or isolate, taste-difference complications by assuming the same tastes for all consumers all over the world no matter what their incomes ("uniform, homothetic preferences").[2]

III. The Ricardo–Viner case

This model assumes labour to be the only input transferable between industries. If labour worked alone at constant returns, this would give us the constant-cost case of classical comparative advantage. If, in addition, the laws

[1] Positive transport costs for the goods, of a constant percentage of price per unit, would cause the horizontal branches through P and P' to lie, respectively, above and below the horizontal axis, each terminating in the no-trade locus (which will, in the close neighborhood of E, be alone relevant).

[2] For derivation of the concept of social product in the simplifying case of homethetic, uniform tastes, see my contribution to the Hicks *Festschrift* (ed. J. N. Wolfe), *Value, Capital and Growth* (Edinburgh University Press, 1968), Ch. 19, "Two Generalizations of the Elasticity of Substitution", pp. 467–80, particularly Part II on homothetic general equilibrium and equation (20).

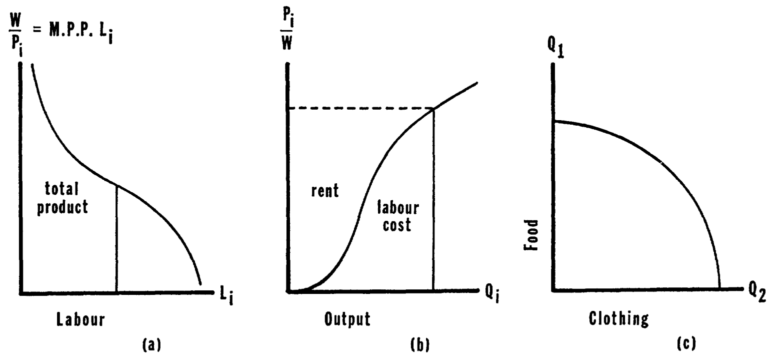

Fig. 2. For a typical industry i, the marginal-product and marginal-cost curves are shown in (a) and (b) respectively. If total L is successively divided into all for L_1 and none for L_2, half and half between L_1 and L_2, all for L_2 and none for L_1, ..., etc., we trace out the concave (from below) production-possibility frontier of (c). Diminishing returns to labour working in each industry with fixed specialized land makes concavity inevitable (save in the case where both lands are superabundant, with constant slope then set at relative labour costs only.)

of knowledge were everywhere the same, so that the simple labour production functions were everywhere the same, there would be no difference in production costs (no comparative advantages!) and no international trade would occur.

The kiss of Ohlin's analysis of increasing returns would bring to life the sleeping beauty of international trade even in a one factor world. However, in this paper we turn our backs on this aspect of Ohlin and Adam Smith and stay with the constant returns-to-scale assumption.

We go on to assume that there typically works with labour in each industry a non-transferable "land" specialized to that industry (food-land, clothing-land).[1]

Figs. $2a$ and $2b$ show the familiar marginal product and marginal cost curves. Fig. $2c$ shows the resulting regional bowed-out production-probability frontier. It looks like the similar frontiers of the Stolper–Samuelson model but now it could be a quarter circle with absolute slope running the gamut from 0 to infinity or 0° to 90°.

IV. One Region–Analysis

Within a single isolated region, the relative prices of goods (food and clothing) and relative factor prices (wages, food-land, clothing-land rents) will depend on the relative scarcity of the factors (labour, food-land, clothing-

[1] There could be more than one kind of specialized land, as we shall analyze.

land). The emerging general equilibrium will, of course, depend also on demand-tastes; but in the simplifying case where all tastes are uniform at all income levels, the system is in effect producing social product—i.e. food-clothing units whose constituent components depend only on the relative goods, prices once tastes are specified.

Labour abundance. Within a single region it is easy to see that an increasing abundance of any one factor—say, labour, first—will lower the real wage. Under our homothetic assumption, it must raise the output of both food and clothing, and hence by the law of diminishing returns to variable labour applied to fixed lands, the real wage will have to fall in terms of both goods and *a fortiori* in terms of social real product. By the same law, real rent of food-land must rise in terms of food; real rent of clothing-land must rise in terms of clothing.

But we have no way of knowing what increased labour abundance will do to the relative price of food and clothing. This could remain unchanged. Or, if food production happened to be more expandable by variable labour than is clothing production—as in the Cobb–Douglas case where labour's share in food costs exceeds its share in clothing costs—increased labour abundance must raise the ratio of clothing price to food price; hence the real food-land rent *in terms of clothing* need not necessarily be raised by labour abundance.

What about the effect of labour abundance on the real rent of clothing-land in terms of social product itself? No invariable result can be predicted. There is perhaps a presumption that labour abundance will be likely to raise any land's real rent—certainly all lands' rents together must be raised—but it is possible that the deterioration of food-land's real wage in terms of clothing could be so great as to make it drop in terms of social product.[1]

Summary. Labour abundance raises the real rents of each land in terms of its own products. Relative prices can move in either direction depending upon how strongly labour encounters diminishing returns in various industries. If a particular good's relative price is much raised, the other land may experience a drop in real rents relative to it (and even relative to social product).

A balanced increase in both lands is just like a reduction in labour alone. All rents together are lowered in terms of social product as the real wage in those terms rises (along a reversible two-variable "factor-price frontier").

Fig. 3*a* summarizes the effect of labour abundance in the production-possibility frontier. The new equilibrium must involve more of all goods, but otherwise there are no restrictions on the possibilities.[2]

[1] If labour and food-land are infinitely great substitutes and labour and clothing-land are infinitely-strong complements, the "perverse" result will follow at high labour supplies.

[2] To prove this, suppose the contrary that one good, say food, failed to increase. It's price would then rise relative to that of clothing. But how then could the extra supply of clothing have been coaxed out? Thus we are led to the proposition that both goods must increase.

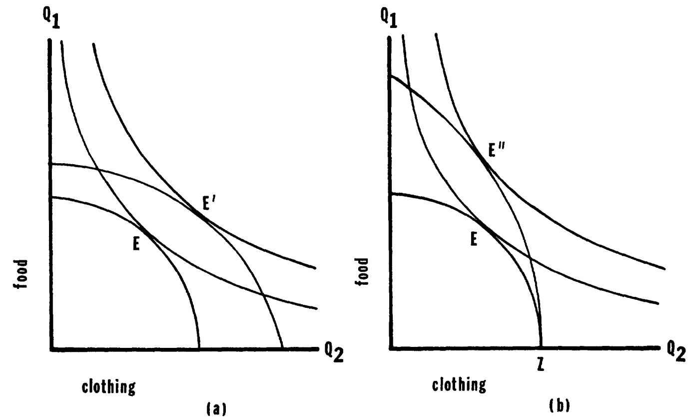

Fig. 3. (*a*) Labour abundance shifts us from E to E', lowering the real wage in terms of all goods and affecting food-clothing price ratio depending upon enhanced labour encounters diminishing returns in the respective industries. At least one real rent must rise in terms of social product; there is a weak presumption that both may rise. (*b*) Increased food-land pivots the production-possibility frontier vertically around unchanged maximum clothing intercept at Z. The quantity of food produced grows relative to that of clothing. The price of food will fall relative to that of clothing, and the real rent of food-land will fall in terms of all goods. If food-clothing demands are very low in elasticity of substitution, the real rent of the unchanged supply of clothing-land will rise and so will the real wage. If the food-clothing demand substitution is very elastic (as happens to be shown here), the real rent of clothing-land will fall in terms of clothing, social product, and even food.

Food-land-abundance. We may briefly describe the increase, in a closed economy with Ricardo–Viner technology and homothetic demand, of an increase in one land alone, say food-land. As Fig 2*a* shows, this tilts the production-possibility frontier vertically around the unchanged intercept of maximum clothing production. Hence food must be cheapened relative to clothing. The legend to Fig. 2*a* describes the reduction in food-land rent and probable[1] increase in the real wage and real clothing-land rents.

Summary. An increase in food-land lowers its real rent and the relative price of food. It will necessarily raise the real rent return of one other factor, labour- or clothing-land; it must raise their combined return. Elasticity-of-substitution of final demand tends to bring down clothing-land rent as food-land increases. Although food-land abundance always raises the real wage in terms of food, clothing might become so dear that the real wage in clothing could fall, and fall enough to reduce the real wage in terms of social product.

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[1] Real social product is a concave homogenous-first-degree function of labour, food-land, clothing-land, $q = q(V_0, V_1, V_2)$, with real factor prices in terms of social product given by $[\partial q/\partial V_i] = [w_0, w_1, w_2]$ and $[\partial w_i/\partial V_j] = [\partial^2 q/\partial V_i \partial V_j] = [\partial w_j/\partial V_i]$. For any i, one $\partial w_j/\partial V_i$ must be positive. The symmetry of this Hessian matrix tells us that the case described two footnotes back, in which an increase in labour reduces real food-land rent, must here invoke a reduction in food land.

Factor shares. So far I have been discussing effects of a factor change on relative-factor and good prices. What about relative factor shares? Classical economists of Ricardo's day were perhaps susceptible to the confusion that an increase in a factor-price, as e.g. land rent, also means an increased share of rent in national income. We know that shares can move in any direction, depending upon elasticities of productivity and on elasticities of substitution. How relative shares are affected by factor-augmenting substitution, in which one of a factor now does the work of more than one, will depend on those same elasticities.

Cobb–Douglas case. Before leaving the closed economy, I should describe the double Cobb–Douglas case in which the proportions of the consumer dollar spent on the different goods are constant and the proportions of each industry's costs going to labour are also constant. In this case social output is itself a simple Cobb–Douglas function of the three factors

$$q = bV_0^{k_0} V_1^{k_1} V_2^{k_2}, \quad \sum_0^2 k_j = 1$$

Thus, suppose labour always gets three-fourths of national income with food-land getting 0.15 and clothing-land 0.10. Then $[k_0, k_1, k_2] = [0.75, 0.15, 0.10]$. This would result from a 0.70 labour share in food, a 0.80 labour share in clothing, and fifty-fifty expenditure on the two goods. If $[\lambda_1, \lambda_2]$ are labour's shares in the two industries and $[\alpha_1, \alpha_2]$ are the good's share of consumption dollar,

$$k_0 = \alpha_1\lambda_1 + \alpha_2\lambda_2, \quad k_1 = \alpha_1(1-\lambda_1), \quad k_2 = \alpha_2(1-\lambda_2)$$

$$k_i = \alpha_i(1-\lambda_i)$$

In general, for any double Cobb–Douglas model, not necessarily Ricardo–Viner in technology, the factor shares in national income, $[k_i]$, are related to the factor shares in the jth industry, $[k_{ij}]$, and the shares of the j industry of the consumption dollar, $[\lambda_j]$, by the matrix identity,

$$[k_i] = [k_{ij}][\alpha_i]$$

The Cobb–Douglas case displays no perverse properties, as the following shows.

Summary. Increasing any factor lowers its real return, raises the real return of all other factors, and lowers the relative price of the good in which its factor-cost-share is relatively largest.

V. Various Geographical Endowments and Trade

Identical Endowments. If two regions have the same endowments of labour, food-land, and clothing-land, uniformity of tastes will produce identical factor-prices, commodity prices and of course no international trade.

If the two regions differ in scale, but all factor proportions are the same, the same absolute equalisation will result under our assumption of uniform (homothetic) tastes. Not only will there be no international trade in goods; even if factors could move between regions, there would be no incentive for them to do so.

Disproportionate endowments. Suppose Region A has relatively more food-land, Region B has relatively more clothing-land. Before trade, A will have relatively cheap food and B will have relatively cheap clothing. Real wages could be about equal in the two regions, the cheapness of one good just balancing the dearness of the other. Regional real outputs could also happen to be equal; but of course food-land rents would still be relatively low in A, and clothing-land rents relatively low in B.

If free trade is allowed, A will export food in exchange for B's exporting clothing. Both regions will be better off at the equalised commodity prices. With the international price ratio of clothing to food lower than Region A's autarky prices, A will shift labour from clothing to food thereby somewhat easing the redundancy and cheapness of its abundant factor, food-land. In B, trade has the opposite effect causing it to produce for export the good for which it has factor abundance: shifting labour from food to clothing tends to relieve the dearness of its scarce factor and relieve the cheapness of its plentiful factor.

All this leads, in the Ohlin manner, to partial but not complete factor-price equalisation.

To depict this Fig. 4 shows the autarky equilibrium for Region A at a, and for Region B at b. Region B happens to have the higher national product by virtue of its superabundance of, say, food-land. Perhaps its autarky real wage is also higher.

Fig. 4 also shows the effects of the free interregional trade. A's final equilibrium is at E_a and and a'; B's final equilibrium is at E_b and b'; the common international price ratio, which is intermediate between the two autarky prices, is found on that common ray-from-the-origin, $0E_aE_b$, at which the regions' trade vectors $a'E_a$ and E_bb' exactly match. Both regions get improved national products, GNP's, from trade. But in each region there is a shift in production toward greater specialization on the good which is relatively intensive in its abundant factor; these trade-induced shifts in production raise the relative factor prices of each region's relatively abundant factor from its autarky cheapness, thereby *tending* to equalise factor prices internationally.

If Region A ends up with a real wage higher than in autarky but still lower than that in Region B, the tendency toward equalisation will not have gone all the way; it will have been only partial, in vindication of Ohlin's original contention.

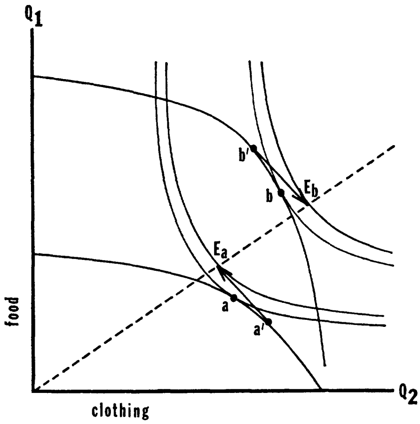

Fig. 4. Region B, above, has more food-land than Region A. In autarky, A at a has higher clothing-land price than B; at its autarky point b, Region B has relatively lower real rent of food-land. Free trade leads to equilibrium at E_a and E_b, where trade vectors $a'E_a$ and $E_b b'$ exactly match, and where the common international price ratio is between the autarky price ratios. Each region increases production of that good which has much of its relatively abundant factor: the move from b to b' involves shift of labour in B to food production, thereby relieving the cheapness of food-land and the dearness of clothing-land there; the move from a at a' has similar Ohlin influence, relieving the cheapness of A's relatively plentiful resource. Trade in goods partially equalises factor prices. [Alternate interpretation of diagram: suppose B has more labour than A, and food is more labour intensive than clothing. The production shifts due to trade then alleviate the dearness of A's labour and the cheapness of B's abundant labour.]

VI. Need for Factor Mobility

Mobility of goods has not been able to serve as a complete substitute for factor mobility in equalising all factor prices. With after-trade real wages lower in A than in B (in *all* goods!), labour has motivation to migrate from A to B if there are now no costs to such migration. As more and more labour migrates, A's real wage rises and B's falls. Finally, they must come into equality, at which point migration will cease. The present Viner–Ricardo technology has the remarkable property that none of the factors other than labour need migrate to achieve optimal world production and complete factor-price equalisation!

Theorem. In a general technology, when goods' prices are equalised by free trade, all the different factors may have unequal factor returns; factor-mobility *in all but one of the factors* will generally be needed to achieve full factor-price equalisation and maximal world production.

In a Viner–Ricardo technology, where labour is the only resource transferable between industries, it will always suffice for labour alone to be capable of migration to achieve full factor-price equalisation.

To prove this strong result, note that if the real wage at E_a is less (in every good) than at E_b, the fact that each region produces every good implies that *every* real rent is greater in Region A than in B. A glance back at Fig. 2*a* confirms that, within each and every industry, there is a unique tradeoff between its real wage and real rent.[1] By the same token, as migration of labour from A to B proceeds far enough to achieve real-wage equality, it must lower all of A's real rents into exact equality with B's rising real rents, *Q.E.D.*

The point is that if the mountains will not come to Mohammed, Mohammed can go to the mountains. It does not matter that there are now many kinds of mountains—food-land, clothing-land, etc. For, these mountains do not have to interact with each other, but each need only interact with labour. The ability of labour migration to compensate by itself for immobility of all the other factors will hold in a Viner–Ricardo technology for any number of goods, $n \geqslant 2$, provided labour works with one specialized resource in each industry. It fails to hold wherever one or more industries involve more than one non-labour factor, which are distributed in unequal proportions among the regions: e.g., suppose the food-industry in Region A involves a different ratio of (food-land)′ to (food-land)″ than the ratio prevailing in Region B; then complete factor-price equalisation would involve, if you can imagine it, migration of one of these food-lands as well as labour.

VII. The Singular Case of Complete Equalisation By Trade

The case in the previous section, in which Region B begins with relatively more of food-land but in which labour migrates from A to B to equalise all factor prices, alerts us to an interesting possibility. Evidently there *can* be situations in which free trade in goods will alone suffice to equlaise factors returns all the way.

Consider the geographical configuration after labour has migrated enough to equalise the free-trade real wage. Region B still has more of food-land than does Region A. Suppose trade in goods is now prohibited, then autarky regimes will involve lower food-land rents in B, lower clothing-land rents in A, and lower food-clothing price ratio in B than A.[2]

[1] This suggests a slight paradox. Region A began with *relatively* much clothing-land, and hence at autarky *a* presumably began with lower real clothing-land rent than at B's autarky point *b*. But free trade ended A with higher real clothing-land rent, r_2, than in B. Hence, goods' mobility caused on overshoot in which this factor-price went from divergence in one direction over to divergence in the opposite direction. On reflection, we perceive no reason why this should not occur.

[2] The autarky real wage in terms of clothing will presumably be higher in A; in B the autarky real wage will presumably be higher in terms of food. Which region will have the higher autarky real wage in terms of social product—that is, which region will have workers "better off"—we cannot say. Suppose the real wage, $w = W/P_Q$, happens to be the same in both regions. Then even if labour could migrate, it would not choose to do so. Why should it? Consequently, in the absence of goods' trade, the world will be stuck permanently in a geographical configuration which fails to maximize total world production of food and clothing. More precisely, we are not out on the world's maximal production-possibility frontier of $(Q_1 = Q_1^A + Q_2^B,\ Q_2 = Q_2^A + Q_2^B)$ production with world totals of $(L = L^A + L^B, = V_1^A + V_1^B,\ V_2 = V_2^A + V_2^B)$. But, if goods cannot be freely moved, what significance is there to a sum like $Q_i^A + Q_i^B$?

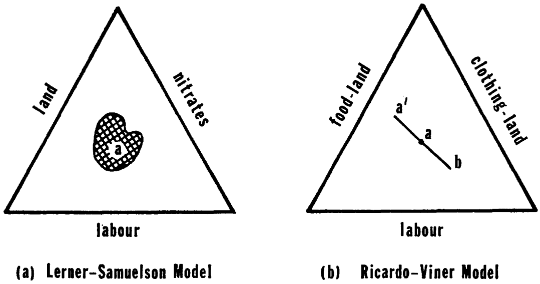

Fig. 5. In the 5(*a*) model, all economies nearly like a given economy that is at *a* will have complete factor-price-equalisation—as shown by the two-dimensional shaded area. In the present 5(*b*) model, only those economies near the point *a* that are on a linear razor's edge will have equalised prices.

These geographical divergences of factor returns imply that if, and only if, two out of three factors of the factors are now free to migrate internationally, would world total products, as measured by global GNP measures or by p–p frontiers, be maximized in the absence of trade.

In short, only with free trade in goods does it suffice to have labour alone migrate in order to maximize world production efficiency. The previous theorem about Ricardo–Viner technology does not apply to autarky situations: for them, some mountains must move (along with labour) to another mountain.

But now revert to the situation in which labour mobility did give rise to the complete equalisation of factor returns under free trade. Freeze all factor movements from this point on. Nonetheless, by hypothesis, introducing free trade will equalise all factor returns. Since free trade succeeds in equalising all factor returns, and since they are not equalised in autarky, clearly we have produced a case in which—contrary to the strong Ohlin dictum—*free trade happens to lead to complete rather than partial factor-price equalisation.*

This singular case resembles the Lerner–Samuelson model in the sense that it contradicts the Uzawa–Ohlin dictum of *necessarily*-partial-rather-than-complete factor-price equalisation. However, I do think that the reader who re-examines Ohlin's argumentations and my 1948 exegesis of them could interpret him to believe that only-partial factor-price equalisation is most likely rather than that complete factor-price equalisation is logically impossible.[1] Ohlin, as a follower of Heckscher, could hardly have thought otherwise.

Ohlin's weaker dictum, that partial rather than complete equalisation is most likely, is confirmed, not refuted, by the present singular case. Thus, the

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[1] It is Ellsworth's textbook, in its attempt to provide a proof for Ohlin, that does purport to demonstrate the logical impossibility of complete equalization. Cf. my 1948 discussion of P. T. Ellsworth, *International Economics* (MacMillan, London 1937). In a real sense, the present singular case does refute what might be called the Ellsworth–Uzawa contention.

present Ricardo–Viner example, precisely because it is singular, does differ from the Lerner–Samuelson model in which all-the-way equalisation is the rule rather than the exception. Fig. 5*b* shows how the present example differs from 5*a*'s Lerner–Samuelson three-good three-factor model. In both cases, three factors, (V_0, V_1, V_2), are represented by points inside the equilateral triangles: the amount of V_i is proportional to the distance from any point to the *i*th side; and for all points the sum of the distances add up to the same normalization constant.

Consider the point *a* in 5*a*. All other regions, that have geographical endowments "near to" those of Region A, in the sense of falling into the two dimensional shaded are around *a*, will come into complete factor-price equalisation with Region A.

By contrast, look at 5*b*. Here, only on a singular razor's edge, the locus *a′ab*, will there be complete factor-price equalisation from trade alone. Elsewhere near *a*, and that means for "almost all" nearby points, the factor-price equalisation will be at best partial.

How do we recognize this singular locus along which free trade can achieve full factor-price equalisation. It is easy. Imagine both countries initially alike, at *a*. Now, take some fraction of the labour and food-land that work together in A's food industry and, without altering their proportion, send them both in a dose to Region B to work along with that same labour/food-land ratio in B's food industry. And, if you like, take some fraction of the labour and clothing-land in Region B and send them in a dose to A. Then the "new B" will be at *b* in Fig. 5*b* and the "new A" will be at *a′*. But with free trade in goods, it will be the case that the final equilibrium for *a′* and *b* will involve the same world productions (and consumptions) as at *a*'s autarky; and exactly the same (equalised) factor returns; from *a′* clothing will be exported in return for an equal value of the food exported from *b*. It is no accident that the now-unbalanced productions of each region can be worked off by trade. By contrast, contemplate what happens at *a′* and *b* under autarky. At *a′* there would be too much clothing produced, if—as will actually happen under autarky—some labour were not shifted to the food-land there. Similarly, new Region B will, under autarky, shift some labour from food to clothing production. Hence, the pre-trade prices would, under uniform tastes, have been quite different at *a′* and *b*: the former has lower clothing and clothing-land prices, the latter, lower food and foodland prices. And it is free trade in goods that suceeds in restoring the complete equality of all relative prices that had prevailed at *a*. (The mathematical appendix explains why *a′ab* is linear.)[1]

[1]Ricardo–Viner technology aside, such a singular case can always be found. Proof: start Regions A and B alike, with *r* factors and *n* goods. Send from A to B doses of factors in the proportions of one (or more) industry. Under autarky, this will hurt the over all well-being of both regions as domestic productions are distorted toward a "more balanced" configuration. But with free trade in goods, all regional productions can take place with the same factor-proportions of the original equal-endowment configuration. *Q.E.D.*

VIII. Conclusions

1. The simple Ricardo–Viner model, involving n goods, will involve $r = n+1 > n$ factors, labour plus a specialised land for each good. We know from the standard analysis of factor-price equalisation that, when the number of factors exceeds the number of goods, no complete factor-price equalisation can be expected from trade alone. (E.g., with one good and two factors, corn produced by labour and land, no one expects regions of different labour/land endowments to end up with equal wages or rents in the absence of factor mobility.)

2. Nonetheless, taste differences aside, free trade in goods will benefit each region and *all* regions in the aggregate. Within this Ricardo–Viner model, the patterns of trade will, in the Ohlin fashion, involve each region's exporting the good whose input requirements it happens to have in special abundance. Production adaptations to trade will thus tend to raise the factor-prices of the most abundant inputs, which would otherwise be cheapest under autarky. The trade-induced movement of factor-prices toward equality, and away from geographically-induced diversity, will generally be only *partially* equalising. With labour's post-trade real wage ending up different in two regions, goods' trade falls short of permitting that maximal world production which migration of labour (of labour alone!) could effectuate.

3. If labour works with more than one immobile land, and if such lands do not occur in the same proportions geographically, we have $r > n+1$, and there is no useful sense in which we can say labour produces within the "same" production functions internationally. Hence, no factor-price equalisation is to be expected.[1] Also, in real life, taste differences must be expected to complicate the analysis, particularly when they are not random.

Mathematical Appendix

1. Let the $(i=1, 2, ..., n)$ outputs of the $(j=1, 2, ..., J)$ countries be denoted by $[Q_i^j]$. Each is produced by the inputs (L_i^j, V_i^j), according to the concave homogeneous-first-degree production functions

$$Q_i^j = F_i(L_i^j, V_i^j) = V_i^j Q_i(L_i^j / V_i^j)$$

The total factor endowments of the jth country are given by

$$(L^j, V_1^j, \ldots, V_n^j) = (\sum_i L_i^j, V_1^j, \ldots, V_n^j).$$

2. Tastes and demand are summarized by a uniform homothetic set of indifference contours in terms of the n goods consumed, either in a region or in the world,

[1] When labour works with more than one specialized land, we need the Inada conditions to rule out the shutting down of production of some goods in some regions. Such specializations are actually realistic.

$$u = u[C_1, ..., C_n]$$

where u is a homogeneous-first-degree concave function.

For simplicity, regularity conditions are placed on the u and F_i functions so that they are smooth, with positive partial derivatives for positive arguments, and satisfying so-called Inada conditions whereby the partial derivative with respect to any variable goes from $+\infty$ to 0 as that variable goes from 0 to $+\infty$ for any positive levels of the other variables.[1]

3. Autarky equilibrium for any region with $(L, V_1, ..., V_n)$ endowment is defined by

$$P_i/W = Q_i'(L_i/V_i)^{-1} = S_i(Q_i) \quad (i=1, \dots, n)$$

$$L_1 + \dots + L_n = L$$

$$\frac{P_i/W}{P_1/W} = \frac{\partial u[Q_1, \dots, Q_n]/\partial Q_i}{\partial u[Q_1, \dots, Q_n]/\partial Q_1} \quad (i=2, \dots, n) \tag{1}$$

Here W is the wage rate, $[P_i]$ the prices, $[W/P_i]$ the real wages in terms of the respective goods, and $S_i(Q_i)$ the rising marginal cost functions easily derivable from the production functions $Q_i(L_i/V_i)$, with $S_i(0)=0$ and $S_i(\infty)=\infty$. The $3n$ variables, $[Q_i, L_i, P_i/W]$ are uniquely defined by the $2n+1+(n-1)$ equations of (1).

The comparative statics of the equilibrium, as we change any or all of $(L, V_1, ..., V_n)$, can be largely summarized in terms of the derivable function of social product

$$U = q(L, V_1, \dots, V_n)$$
$$= \operatorname*{Max}_{L_i} u[V_1 Q_1(L_1/V_1), \dots, V_n Q_n(L_n/V_n)]$$

$$\text{subject to } \sum_{i=1}^{n} L_i = L \equiv V_0; \tag{2}$$

namely, by

$$w = r_0 = q_0(L, V_1, \dots, V_n) = \partial q/\partial L = \partial q/\partial V_0$$

$$r_i = q_i(L, V_1, \dots, V_n) = \partial q/\partial V_i$$

$$\partial r_i/\partial V_j = \partial^2 q/\partial V_i \partial V_j = q_{ij} = q_{ji} \quad (i, j = 0, 1, \dots, n) \tag{3}$$

Here w is the real wage in terms of social product, r_i the similar real rents, and, by convention, L and V_0 are used interchangeably. By concavity and homogeneity $(q_{ij}]$ is negative semi-definite.

[1] Inada conditions are more popular in the textbook than in the real world. If marginal productivities and marginal costs begin at positive intercepts, the equations below must be qualified by inequalities. When specialization causes some goods not to be produced at all in a particular region, that enhances Ohlin's case for partial rather than complete equalisation, just as in the Lerner–Samuelson model.

Continuing to use real social product, q, as numeraire, with $P_q \equiv 1$, the real prices $P_i/P_q = p_i$ are equal to

$$p_i = \partial u[Q_1, ..., Q_n]/\partial Q_i$$
$$= q_0(L, V_1, ..., V_n)/Q_i'(L_i/V_i) \quad (i=1, ..., n) \tag{4}$$

Also

$$w = W/P_q = r_0$$

$$r_i = R_i/P_q = (R_i/P_i)p_i = [Q_i(L_i/V_i) - (W/P_i)(L_i/V_i)]p_i$$

For $n=2$, it is not hard to show that

$$\partial(W/P_i)/\partial L < 0$$

$$\partial(W/P_i)/\partial V_i > 0. \quad (i=1, 2)$$

For the limiting cases where the indifference contours are respectively of ∞ and 0 elasticities of substitution, the matrix

$$[\partial r_i/\partial V_j] = \begin{bmatrix} q_{00} & q_{01} & q_{02} \\ q_{10} & q_{11} & q_{12} \\ q_{20} & q_{21} & q_{22} \end{bmatrix}$$

has sign patterns $\begin{bmatrix} - & + & + \\ + & - & - \\ + & - & - \end{bmatrix}$ and $\begin{bmatrix} - & + & + \\ + & - & + \\ + & + & - \end{bmatrix}$,

but I do not see that, for intermediate cases, it is forbidden to have the pattern

$$\begin{bmatrix} - & - & + \\ - & - & + \\ + & + & - \end{bmatrix}$$

4. Free trade in goods leads to equilibrium defined by

$$P_i/W^j = Q_i'(L_i^j/V_i^j)^{-1} = S_i(Q_i^j), \quad (i=1, \ldots, n; j=1, \ldots, J) \tag{5 a}$$

$$L_1^j + \ldots + L_n^j = L^j \tag{5 b}$$

$$P_1(C_1^j - Q_1^j) + \ldots + P_n(C_n^j - Q_n^j) = 0 \tag{5 c}$$

$$P_i/P_1 = u_i[\sum_j Q_1^j, \ldots, \sum_j Q_n^j]/u_1[\sum_j Q_1^j, \ldots, \sum_j Q_n^j], \quad (i=2, \ldots, n) \tag{5 d}$$

$$\frac{P_i/W^j}{P_1/W^j} = u_i[C_1^j, \ldots, C_n^j]/u_1[C_1^j, \ldots, C_n^j] \tag{5 e}$$

Here $[C_i^j]$ is the amount consumed of the ith good in the jth country and

$u_i[\ ]$ stands for $\partial u[\ ]/\partial C_i$. The P's denote prices in *any* common international unit.

It is of interest to note that, if one is interested only in the equilibrium of international prices and real wages, and not in the pattern of trade and of regional consumption breakdown, all the relations of (5) involving C's, namely (5c) and (5e) can be ignored in this homothetic case: the equations (5a), (5b), (5d), which are $2nJ+J+(n-1)$ in number, suffice to determine uniquely the nJ $[Q_i^j]$, the nJ $[L_i^j]$, the J $[P_1/W^j]$ and the $(n-1)$ $[P_i/P_1]$.

If we then add the J balance-of-payments equations of (5c) and the $J(n-1)$ domestic consumption-demand equations of (5e), we further determine uniquely the remaining Jn consumption unknowns $[C_i]$.

Heuristically, and for that matter rigorously, we can determine all the post-trade real wages and rents from the following maximum problem:

$$U^*(L^1, V_1^1, \ldots, V_n^1; \ldots; L^J, V_1^J, \ldots, V_n^J)$$

$$= \underset{L_i^j}{\mathrm{Max}}\, u\left[\sum_{j=1}^{J} V_1^j\, Q_1(L_1^j, V_1^j), \ldots, \sum_{j=1}^{J} V_n^j Q_n(L_n^j, V_n^j)\right]$$

$$\text{subject to } \sum_{i=1}^{n} L_i^j = L^j \quad (j=1, \ldots, J) \tag{6}$$

If all prices, wages, and rent are expressed in a single currency unit, one can prove

$$W^j/W^1 = (\partial U^*/\partial L^j)/(\partial U^*/\partial L^1) \quad (j=2, \ldots, J)$$

$$R_i^j/W^j = (\partial U^*/\partial V_i^j)/(\partial U^*/\partial L^j) \quad (j=1, \ldots, J; i=1, \ldots, n) \tag{7}$$

Here R_i denotes the rent of the ith land in the jth country.

5. Equilibrium with factor mobility, which the text has shown need involve only labour mobility in the Ricardo–Viner model, is defined by the same equations as (5), but with the allocation of total L among regions now to be determined by the additional equations involving geographically-equalised real wages. In a free-trade world, if the real wage in terms of any good, say the first, is equalised regionally, *all* real wages are equalised. Hence, we can adjoin to (5)

$$P_1/W^1 = P_1/W^2 = \ldots = P_1/W^J \tag{5f}$$

These are the $J-1$ new equations needed to determine the new $J-1$ interregional allocations $[L^j]$ of the given world labor supply to achieve complete equalisation and efficiency.

6. Again, heuristically, we can determine the equilibrium real wages and real rents when both factors and goods are mobile, without using (5a)–(5f), but merely from (1) applied to world totals

$$w = q_0(\sum_j L^j, \sum_j V_1^j, \ldots, \sum_j V_n^j)$$

$$r_i = q_i(\sum_j L^j, \sum_j V_1^j, \ldots, \sum_j V_n^j) \quad (i = 1, \ldots, n) \tag{8}$$

7. A valuable heuristic way of analysing the differences between full and partial factor-price equalisation has been provided by Uzawa in the cited 1959 paper. Full equalisation achieves a higher-order of welfare realisation than partial; partial achieves a higher-order than autarky; autarky under perfect competition achieves a higher order than other feasible autarky allocations. All these welfare concepts can be unambiguously handled by the device of production-possibility frontiers; or even better, in our case of uniform homothetic demands, by reckonings of achieved real GNP's.

The following equations characterise the three stages: complete factor-price equalisation, partial, and autarky.

$$q(\sum_j L^j, \sum_j V_1^j, \ldots, \sum_j V_r^j)$$

$$= \underset{L^j}{\text{Max}}\, U^*(L^1, V_1^1, \ldots, V_n^1; \ldots; L^J, V_1^J, \ldots, V_n^J) \tag{9 a}$$

labour mobility and free trade

$$\geqslant U^*(L^1, V_1^1, \ldots, V_n^1; \ldots; L^J, V_1^J, \ldots, V_n^J)$$

$$= \underset{L_i^j}{\text{Max}}\, u[\sum_j F_1(L_1^j, V_1^j), \ldots, \sum_j F_n(L_n^j, V_n^j)] \tag{9 b}$$

free trade

$$\geqslant \sum_j q(L^j, V_1^j, \ldots, V_r^j)$$

$$= \sum_j \underset{L_i^j}{\text{Max}}\, u[F_1(L_1^j, V_1^j, \ldots, F_n(L_n^j, V_n^j)] \tag{9 c}$$

autarky

Fig. 6 shows, symbolically, these relations. The outer frontier shows the situation when all factors are mobile, migrating to equalise all factor returns and give the world maximal production possibilities. The middle frontier shows the results of free trade in goods. As each nation is improved by trade, total world GNP (reckoned at the homothetic tastes) is higher than it is at autarky; however, if labour cannot move to wipe out any post-trade geographical differences in the real wage, the aggregate GNP under goods trade falls short of that under factor mobility.

These two frontiers are the productions that would be observed as the homothetic tastes changed their food-clothing intensities, running the gamut from one extreme to the other. What then is the inner frontier? It represents the world sums of all autarky productions that would be engendered by the

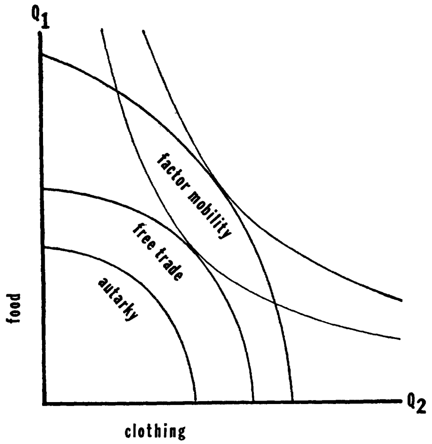

Fig. 6. The outer frontier shows world production possibilities when factors can move optimally—or, in the Ricardo–Viner case, when labour can move to equalise the post-trade real wage rate. The intermediate frontier shows world totals produced when goods can move freely in trade but factors are immobile. The inner curve shows what world production totals would be, as tastes changed uniformly in each county toward one good or the other, and when neither goods nor factors can move between regions. If resource endowments were the same in all regions, all three curves would coincide. In singular cases, the present Ricardo–Viner model could have the intermediate curve tangential to the outer frontier. This is in contrast to the Lerner–Samuelson model in which the two outer curves coincide for all regions that are near enough alike.

same change in tastes.[1] The fact that that this inner locus lies inside the middle one, represents the production inefficiency attributable to autarky. But, in a sense, there is a further consumption inefficiency as well: thus, suppose all countries under autarky have the same well-being. That "average level" will be less than the average level that would be read off the homothetic indifference contour going through the relevant point on the inner curve, even after proper allowance is made for the number of people: people are, so to speak, forced under autarky to consume "unbalanced" diets.

It is possible, as we have seen, for the singular case to occur in which free trade in goods is a full substitute for factor mobility. In such a case the middle frontier must touch the outer frontier in at least one place. However, save in the uninteresting case of identical geographical endowments—when all these curves are identical and no mobility will ever be used—the inner curve can never touch the intermediate frontier.

The mathematical condition for the singular case to occur can be written down briefly for the case of two regions, A and B. Suppose with balanced

[1] E.g., write a Cobb–Douglas $u = Q_1^k Q_2^{1-k}$ and let k go from zero to one. Or write a fixed-proportions $u = \text{Min}\,[Q_1/k,\ Q_2/(1-k)]$ with $0 < k < 1$. These two alternatives will generate the same two outer frontiers, for the reason that those frontiers each represent solutions to maximal production problems under specified constraints. But the precise shape of the inner locus need not be the same.

endowments, equilibrium would take place with uniform world prices proportional to $(W, P_1, ..., P_n, R_1, ..., R_n)$ and with industry i everywhere using, per unit of output, a_{0i} of labour and a_{ii} of V_i. Let the migration of the ith factor from A to B be written as ΔV_i. Then, full factor-price equalisation will be preserved by free trade provided

$$\Delta V_0 = \sum_{i=1}^{n} (a_{0i}/a_{ii}) \Delta V_i \tag{10}$$

even though factor endowments have now become relatively different. After such migration, the same equilibrium prices will prevail under free trade, and the same total world productions and consumptions. However, were all tastes now to change, it would be virtually impossible for free trade in goods to continue to keep real wages geographically the same. Thus, if A now has relatively much food production and tastes turn toward food rather than clothing, how can that help but give B lower real wages? So the intermediate locus is touched only at the one singular point.

Examination of (9c) shows that (1) expresses the necessary condition for its maximum condition. Similarly, for (9b), the conditions (5a, b, d) are necessary. To achieve (9a), (5f) must be satisfied as well.

8. If labour works with more than one specialized resource in any industry, V_i must be interpreted as a vector, being short for $(V_{i1}, V_{i2}, ..., V_{ik_i})$, where k_i is the number of non-labour factors in the ith industry. Then (5a) must be replaced by

$$P_i/W^j = \partial F(L_i^j, V_i^j)/\partial L_i^j = S_i(Q_i^j), \quad (i = 1, \ldots, n) \tag{11}$$

$$P_i/R_{ik}^j = \partial F(L_i^j, V_i^j)/\partial V_{ik}^j, \quad (k = 1, \ldots, k_i)$$

and a similar rewriting of (1) is needed.

But now, migration of labour alone will not suffice to achieve complete factor price equalisation. The single condition of (5f) must be augmented, so that the numerous following all hold:

$$\frac{P_i}{R_{ik}^1} = \frac{P_i}{R_{ik}^2} = \ldots = \frac{P_i}{R_{ik}^J}, \quad (i = 1, \ldots, n; k = 1, \ldots, k_i) \tag{12}$$

Only if all (or all but one) of the $r = 1 + k_1 + ... + k_n$ factors can migrate freely, singular cases aside, will these conditions be guaranteed.

9. What if goods involve transport costs? The simplest case is the following: as any good goes from Region A to B, or vice versa, only the fraction f_i arrives there. Clearly, as every $f_i \to 1$, the Ohlin rule of partial factor-price equalisation will prevail. But now two regions that differ only by a trifle in factor endowments will not be able to trade; and their factor returns will necessarily differ by a trifle permanently. Free labour migration will almost, but not quite, equalise factor returns. It will equalise the real wage reckoned

in terms of the homothetic tastes; but in regions of food-land abundance, the real wage in food will be compensatingly high and in clothing will be compensatingly low, with the rents unequal in the opposite directions. In real life, tastes are not uniform, and sun lovers migrate toward the sun.