MUNI
ECON

SUPTECH WORKSHOP III

Neural networks

Black-Litterman model

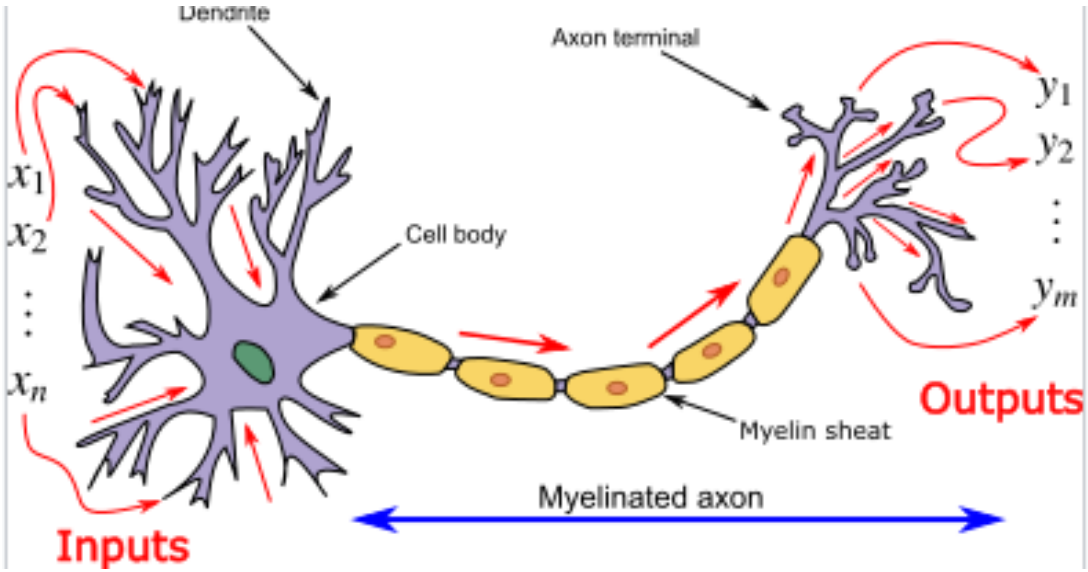
Let $\mathbf{r} = (r_1, r_2, \ldots, r_n)^T, n \in \mathbb{N}$ be asset returns. Assume $\mathbf{r} \sim \mathbf{N}(\mu, \mathbf{\Sigma})$. In turn, assume random mean returns $\mu \sim N(\pi, \tau\mathbf{\Sigma})$ where π is determined by the investor. The investor may formulate linear views, such as

$$p_{i1}\mu_1 + p_{i2}\mu_2 + \ldots + p_{in}\mu_n = q_i + \varepsilon_i$$

where $\varepsilon_i \sim N(0, \sigma_i^2)$, with σ_i^2 controlling for confidence. Collecting views into a matrix gives $\mathbf{P}_\mu \sim \mathbf{N}(\mu, \mathbf{\Omega})$ where $\mathbf{\Omega}$ is a diagonal matrix of $(\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2)$. Then

$$\begin{aligned}
&\mu | q, \mathbf{\Omega} \sim N(\mu_{BL}, \mathbf{\Sigma}_{BL}^{\mu}) \\
&\mu_{BL} = ((\tau\mathbf{\Sigma})^{-1} + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}((\tau\mathbf{\Sigma})^{-1}\pi + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{q}) \\
&\Sigma_{BL}^{\mu} = ((\tau\mathbf{\Sigma})^{-1} + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P})^{-1}
\end{aligned}$$

Neurons


Dendrite
Axon terminal
x_1
x_2
x_n
Cell body
y_1
y_2
y_m
Outputs
Myelin sheat
Myelinated axon
Inputs

Artificial Neural networks (ANNs)

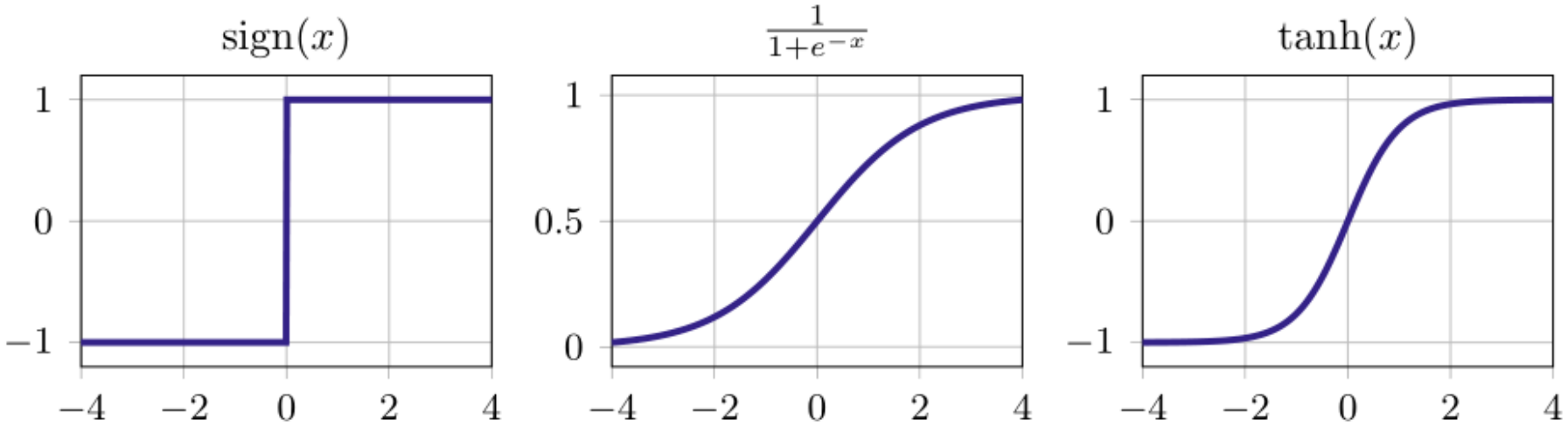
- multiple real-valued inputs $\mathbf{X} = (X_1, ..., X_r)^T$
- single output **Y** .
- The connection is indicated by weight β_i .
- The output is obtained by computing the activation value U as the sum of $\mathbf{X}$, with their respective weights in the vector $\beta = (\beta_1, ..., \beta_r)$, and a bias term β_0:

$$U = \beta_0 + \sum_{i=1}^{r} \beta_i X_i = \beta_0 + \mathbf{X}^T \beta$$

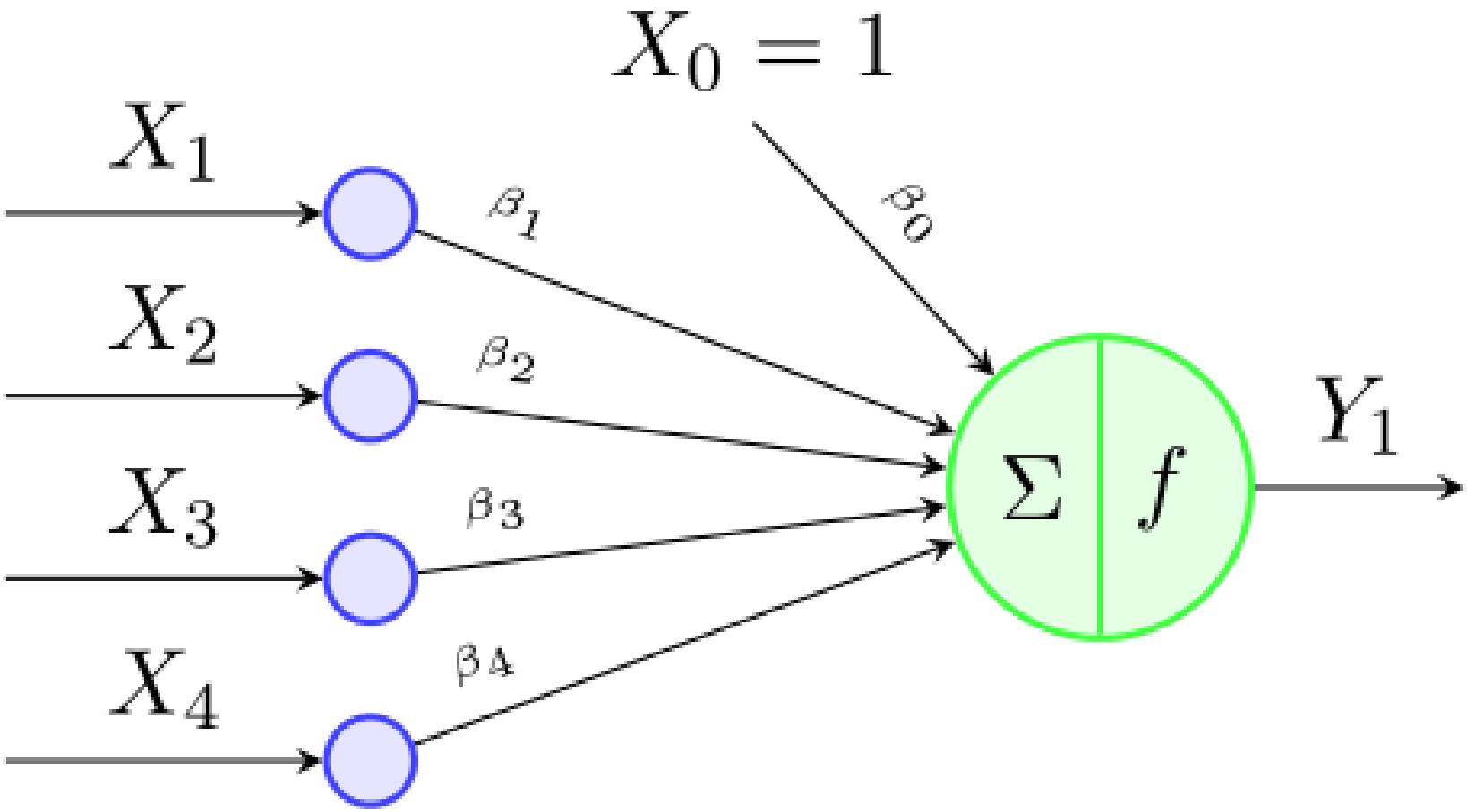
- the result is passed through an activation function f,

$$Y = f(U) = f(\beta_0 + \mathbf{X}^T \beta)$$

Activation functions



Single-layer perceptron



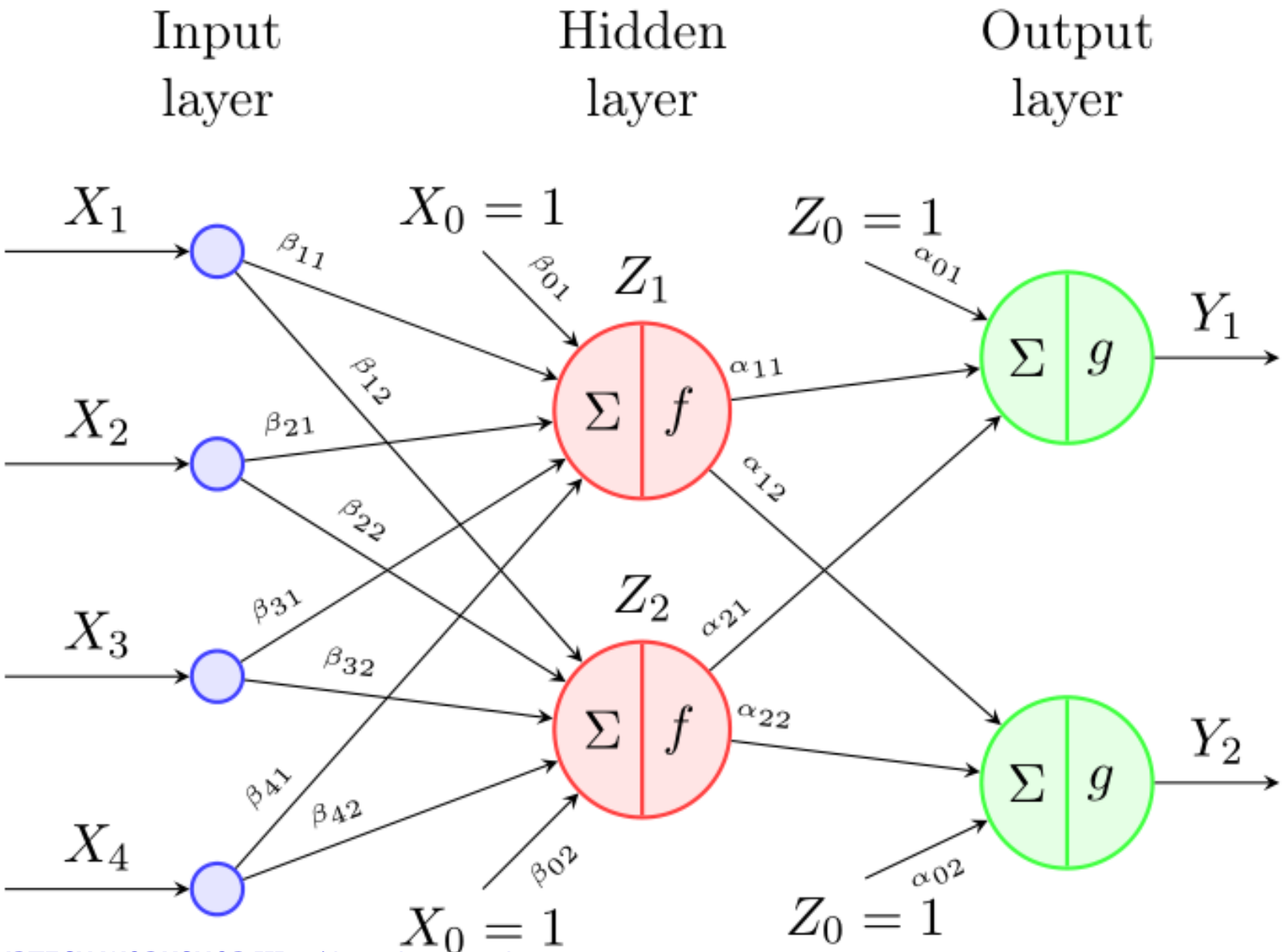
Learning methods

- Supervised learning: minimizing error function (regression).
- Unsupervised learning: finding features without teacher (clustering).
- Reinforcement learning: It differs from supervised learning in that labelled input/output pairs need not be presented, and sub-optimal actions need not be explicitly corrected. Instead the focus is finding a balance between exploration (of uncharted territory) and exploitation (of current knowledge).

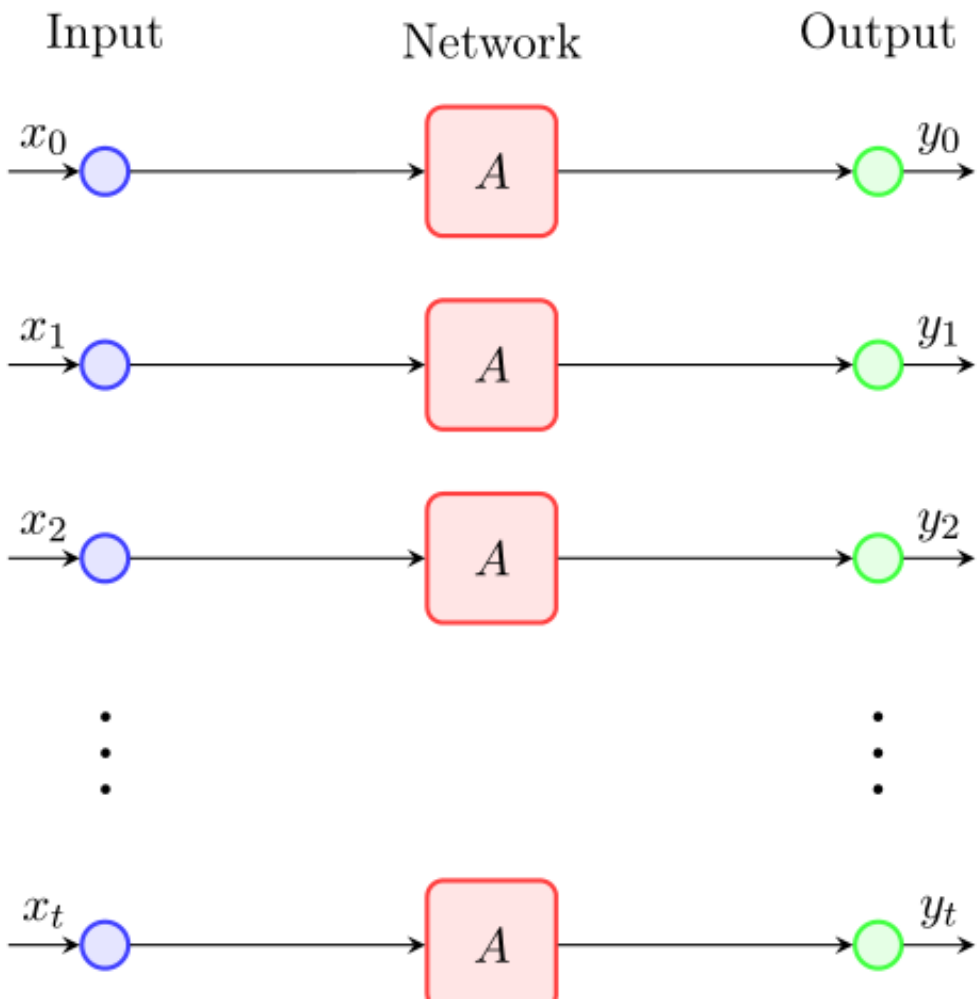
Feed-forward Neural Network (FNN)

- neurons are connected only from input to output
- no connections within a layer
- hidden layer - neurons not input and output
- most common - multi-layer perceptron (MLP)

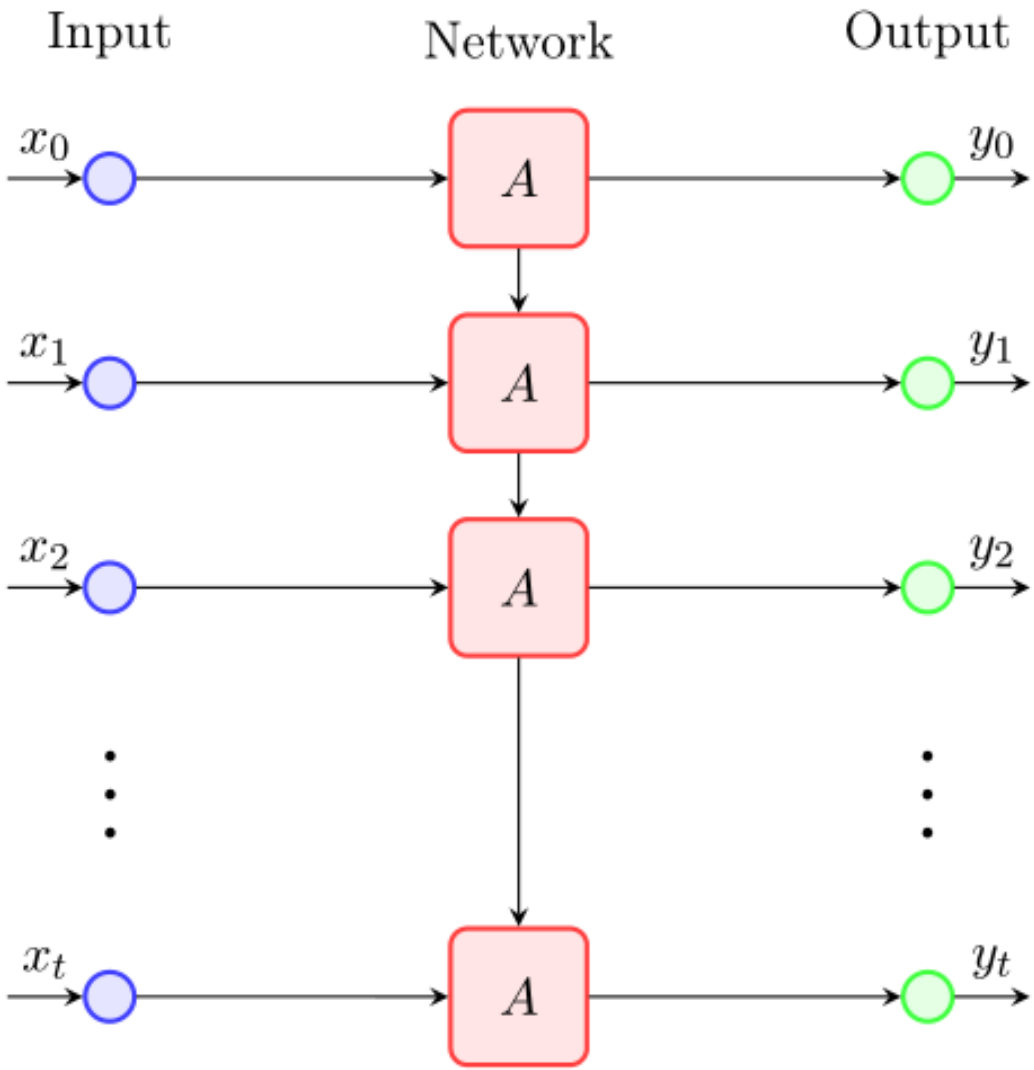
One hidden layer



Parallel MLPs



Unfolded recurrent neural network



Recurrent neural network