Econometrics - Lecture 3

# Regression Models: Interpretation and Comparison

# Contents


- The Linear Model: Interpretation of Coefficients
- Selection of Regressors
- Selection Criteria
- Comparison of Competing Models
- Specification of the Functional Form
- Structural Break

# Economic Models

Describe economic relationships (not only a set of observations), have an economic interpretation

Linear regression model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + \varepsilon_i = x_i'\beta + \varepsilon_i$$

- Variables $Y$, $X_2$, …, $X_K$: observable
- Observations: $y_i$, $x_{i2}$, …, $x_{iK}$, $i = 1, \ldots, N$
- Error term $\varepsilon_i$ (disturbance term) contains all influences that are not included explicitly in the model; unobservable
- Assumption (A1), i.e., $E\{\varepsilon_i \mid X\} = 0$ or $E\{\varepsilon_i \mid x_i\} = 0$, gives

  $$E\{y_i \mid x_i\} = x_i'\beta$$

  the model describes the expected value of $y_i$ given $x_i$ (conditional expectation)

# Example: Wage Equation

Wage equation (Verbeek's dataset "wages1")

$$wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, school_i + \beta_4\, exper_i + \varepsilon_i$$

Answers questions like:

- What wage p.h. can be expected for a female with 12 years of education and 10 years of experience?

Wage equation fitted to all 3294 observations

$$wage_i = -3.38 + 1.34 * male_i + 0.64 * school_i + 0.12 * exper_i$$

- Expected wage p.h. of a female with 12 years of education and 10 years of experience: 5.50 USD

$$wage_i = -3.38 + 1.34 * 0 + 0.64 * 12 + 0.12 * 10 = 5.50$$

# Regression Coefficients

Linear regression model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + \varepsilon_i = x_i'\beta + \varepsilon_i$$

Coefficient $\beta_k$ measures the change of $Y$ if $X_k$ changes by one unit

$$\frac{\Delta E\{y_i \mid x_i\}}{\Delta x_k} = \beta_k \quad \text{for } \Delta x_k = 1$$

- For continuous regressors

$$\frac{\partial E\{y_i \mid x_i\}}{\partial x_{ik}} = \beta_k$$

Marginal effect of changing $X_k$ on $Y$

- Ceteris paribus condition: measuring the effect of a change of $Y$ due to a change $\Delta x_k = 1$ by $\beta_k$ implies
  - knowledge which other $X_i$, $i \neq k$, are in the model
  - that all other $X_i$, $i \neq k$, remain unchanged

# Example: Coefficients of Wage Equation

Wage equation

$$wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, school_i + \beta_4\, exper_i + \varepsilon_i$$

$\beta_3$ measures the impact of one additional year at school upon a person's wage, keeping gender and years of experience fixed

$$\frac{\partial E\{wage_i \mid male_i, school_i, exper_i\}}{\partial\, school_i} = \beta_3$$

Wage equation fitted to all 3294 observations

$$wage_i = -3.38 + 1.34 * male_i + 0.64 * school_i + 0.12 * exper_i$$

- One extra year at school, e.g., at the university, results in an increase of 64 cents; a 4-year study results in an increase of 2.56 USD of the wage p.h.
- This is true for otherwise (gender, experience) identical people

# Regression Coefficients, cont'd

- The marginal effect of a changing regressor may depend on other variables

Examples

- Wage equation: $wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, age_i + \beta_4\, age_i^2 + \varepsilon_i$

  the impact of changing age depends on age:

$$\frac{\partial E\{y_i|x_i\}}{\partial age_i} = \beta_3 + 2\beta_4 age_i$$

- Wage equation may contain $\beta_3\, age_i + \beta_4\, age_i\, male_i$: marginal effect of age depends upon gender

$$\frac{\partial E\{y_i|x_i\}}{\partial age_i} = \beta_3 + \beta_4 male_i$$

# Elasticities

Elasticity: measures the *relative* change in the dependent variable *Y* due to a *relative* change in $X_k$

- For a linear regression, the elasticity of *Y* with respect to $X_k$ is

$$\frac{\partial E\{y_i \mid x_i\} / E\{y_i \mid x_i\}}{\partial x_{ik} / x_{ik}} = \frac{\partial E\{y_i \mid x_i\}}{\partial x_{ik}} \frac{x_{ik}}{E\{y_i \mid x_i\}} = \frac{x_{ik}}{x_i'\beta} \beta_k$$

- For a log-linear model with $(\log x_i)' = (1, \log x_{i2}, \ldots, \log x_{ik})$

  $\log y_i = (\log x_i)' \beta + \varepsilon_i$

  elasticities are the coefficients β (see slide 10)

$$\frac{\partial E\{y_i \mid x_i\} / E\{y_i \mid x_i\}}{\partial x_{ik} / x_{ik}} = \beta_k$$

# Example: Wage Elasticity

Wage equation, fitted to all 3294 observations:

$$\log(wage_i) = 1.09 + 0.20\ male_i + 0.19 \log(exper_i)$$

The coefficient of $\log(exper_i)$ measures the elasticity of wages with respect to experience:

- 100% more years of experience result in an increase of wage by 0.19 or a 19% higher wage
- 10% more years of experience result in a 1.9% higher wage

# Elasticities, continues slide 8

This follows – for log $y_i$ = (log $x_i$)' $\beta$ + $\varepsilon_i$ – from

$$\frac{\partial E\{\log y_i \,|\, x_i\}}{\partial x_{ik}} = \frac{\partial E\{\log y_i \,|\, x_i\}}{\partial E\{y_i \,|\, x_i\}} \frac{\partial E\{y_i \,|\, x_i\}}{\partial x_{ik}}$$

$$\approx \frac{\partial \log E\{y_i \,|\, x_i\}}{\partial E\{y_i \,|\, x_i\}} \frac{\partial E\{y_i \,|\, x_i\}}{\partial x_{ik}} = \frac{1}{E\{y_i \,|\, x_i\}} \frac{\partial E\{y_i \,|\, x_i\}}{\partial x_{ik}}$$

$$\frac{\partial E\{\log y_i \,|\, x_i\}}{\partial x_{ik}} = \frac{\beta_k}{x_{ik}}$$

and

$$\frac{\partial E\{y_i \,|\, x_i\}}{\partial x_{ik}} \frac{x_{ik}}{E\{y_i \,|\, x_i\}} = \frac{\partial E\{\log y_i \,|\, x_i\}}{\partial x_{ik}} E\{y_i \,|\, x_i\} \frac{x_{ik}}{E\{y_i \,|\, x_i\}}$$

$$= \frac{\beta_k}{x_{ik}} x_{ik} = \beta_k$$

# Semi-Elasticities

Semi-elasticity: measures the *relative* change in the dependent variable *Y* due to an (absolute) one-unit-change in $X_k$

- Linear regression for

  $$\log y_i = x_i' \beta + \varepsilon_i$$

  the elasticity of *Y* with respect to $X_k$ is

  $$\frac{\partial E\{y_i \mid x_i\} / E\{y_i \mid x_i\}}{\partial x_{ik} / x_{ik}} = \beta_k x_{ik}$$

  $\beta_k$ measures the relative change in *Y* due to a change in $X_k$ by one unit
- $\beta_k$ is called semi-elasticity of *Y* with respect to $X_k$

# Example: Wage Differential

Wage equation, fitted to all 3294 observations:

$$log(wage_i) = 1.09 + 0.20\ male_i + 0.19 \log(exper_i)$$

- The semi-elasticity of the wages with respect to gender, i.e., the relative wage differential between males and females, is the coefficient of $male_i$: 0.20 or 20%
- The wage differential between males ($male_i$ =1) and females is obtained from $wage_f = \exp\{1.09 + 0.19 \log(exper_i)\}$ and $wage_m = wage_f \exp\{0.20\} = 1.22\ wage_f$; the wage differential is 0.22 or 22%; the coefficient 0.20[1)] is a good approximation.

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[1)] For small $x$, $\exp\{x\} = \Sigma_k x^k/k! \approx 1+x$

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# Selection of Regressors

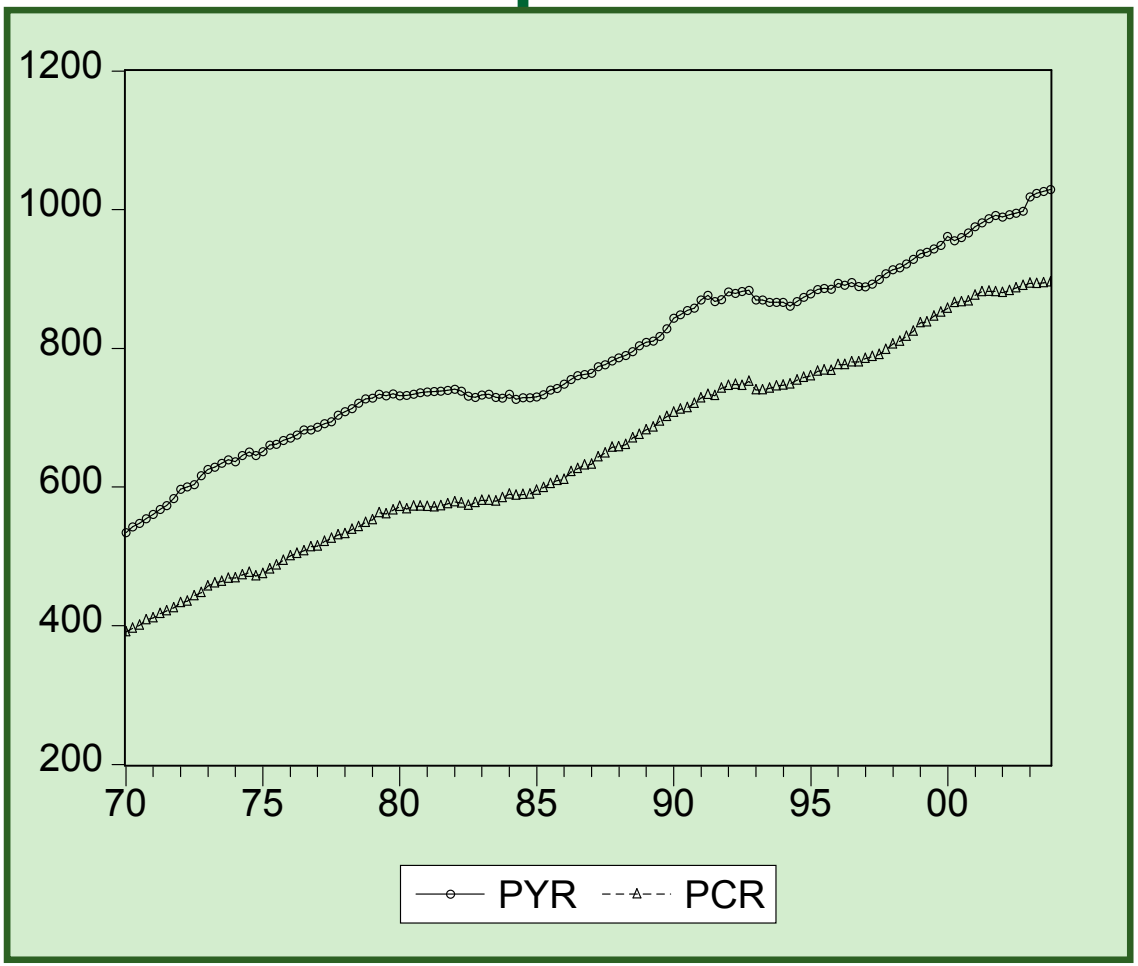
Specification errors:

- Omission of a relevant variable
- Inclusion of an irrelevant variable

Questions:

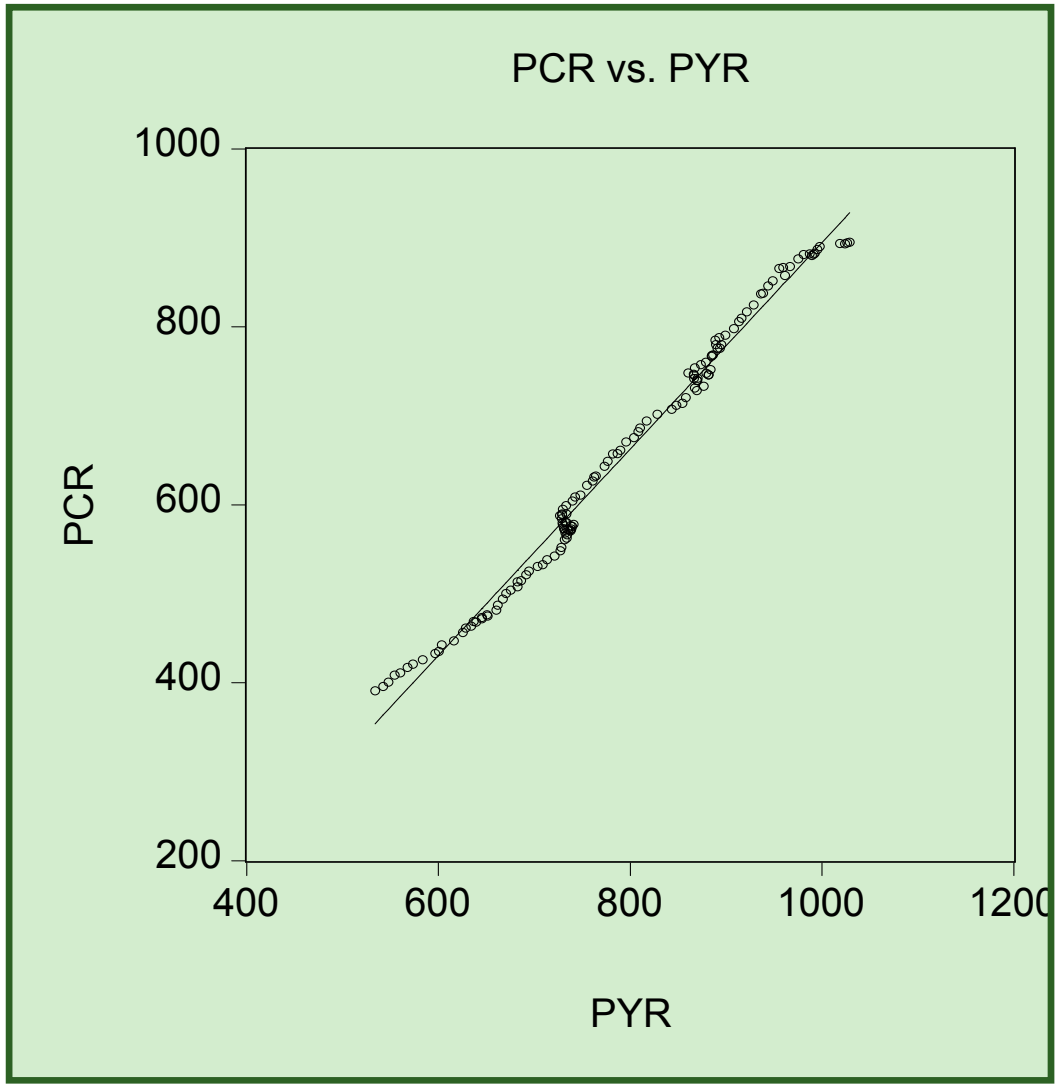
- What are the consequences of a specification error?
- How to avoid specification errors?
- How to detect an erroneous specification?

# Example: Income and Consumption



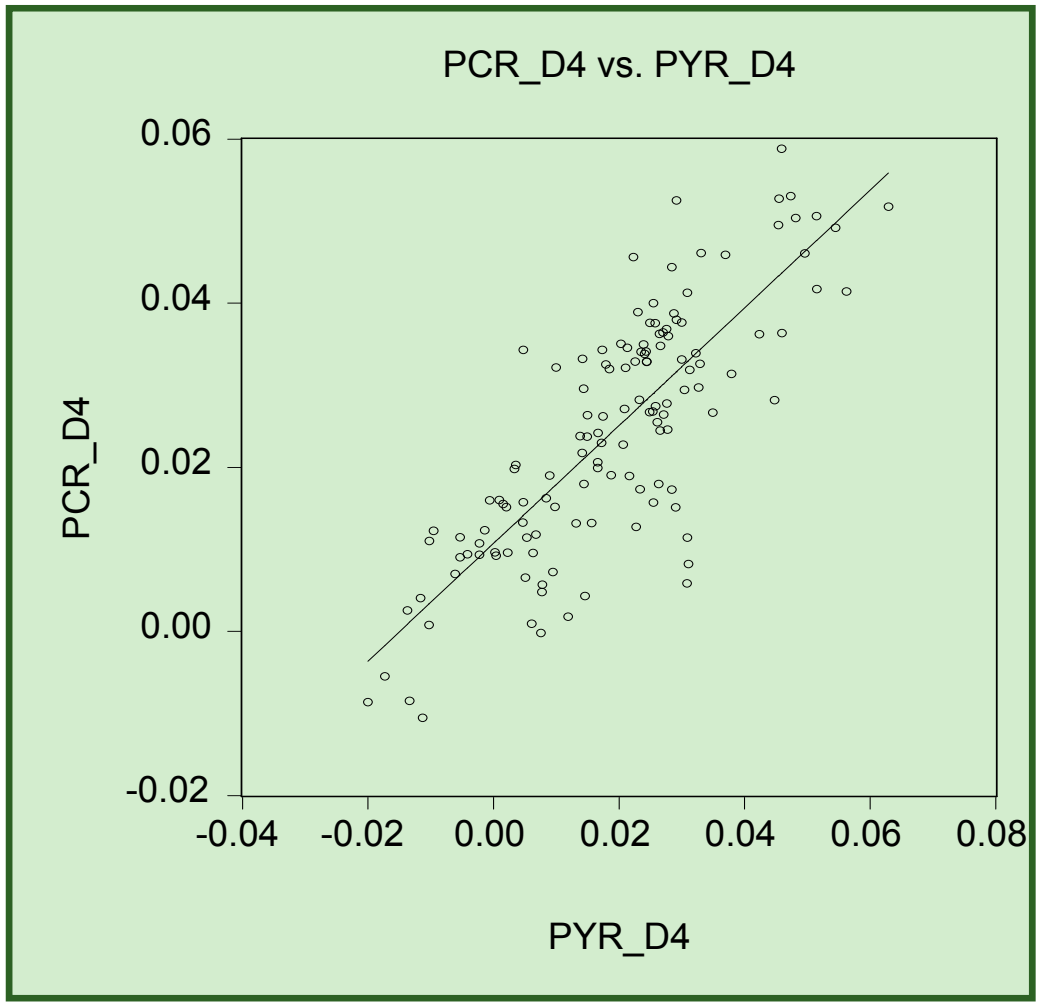

PCR: Private Consumption, real, in bn. EUROs
PYR: Household's Disposable Income, real, in bn. EUROs
1970:1-2003:4
Basis: 1995
Source: AWM-Database

# Income and Consumption




PCR: Private Consumption, real, in bn. EUROs
PYR: Household's Disposable Income, real, in bn. EUROs
1970:1-2003:4
Basis: 1995
Source: AWM-Database

# Income and Consumption: Growth Rates




PCR_D4: Private Consumption, real, yearly growth rate
PYR_D4: Household's Disposable Income, real, yearly growth rate
1970:1-2003:4
Basis: 1995
Source: AWM-Database

# Consumption Function

C: Private Consumption, real, yearly growth rate (PCR_D4)

Y: Household's Disposable Income, real, yearly growth rate (PYR_D4)

*T*: Trend ($T_i$ = *i*/1000)

$$\hat{C} = 0.011 + 0.761Y, \quad adj\,R^2 = 0.717$$

Consumption function with trend $T_i$ = i/1000:

$$\hat{C} = 0.016 + 0.708Y - 0.068T, \quad adj\,R^2 = 0.741$$

# Consumption Function, cont'd

## OLS estimated consumption function: Output from GRETL

Dependent variable : PCR_D4

| | coefficient | std. error | t-ratio | p-value |
|---|---|---|---|---|
| const | 0,0162489 | 0,00187868 | 8,649 | 1,76e-014 *** |
| PYR_D4 | 0,707963 | 0,0424086 | 16,69 | 4,94e-034 *** |
| T | -0,0682847 | 0,0188182 | -3,629 | 0,0004 *** |

| | | | |
|---|---|---|---|
| Mean dependent var | 0,024911 | S.D. dependent var | 0,015222 |
| Sum squared resid | 0,007726 | S.E. of regression | 0,007739 |
| R- squared | 0,745445 | Adjusted R-squared | 0,741498 |
| F(2, 129) | 188,8830 | P-value (F) | 4,71e-39 |
| Log-likelihood | 455,9302 | Akaike criterion | -905,8603 |
| Schwarz criterion | -897,2119 | Hannan-Quinn | -902,3460 |
| rho | 0,701126 | Durbin-Watson | 0,601668 |

# Misspecification: Two Models

Two models:

$$y_i = x_i'\beta + z_i'\gamma + \varepsilon_i \qquad \text{(A)}$$

$$y_i = x_i'\beta + v_i \qquad \text{(B)}$$

with $J$-vector $z_i$

# Misspecification: Omitted Regressor

Specified model is (B), but true model is (A)

$$y_i = x_i'\beta + z_i'\gamma + \varepsilon_i \qquad \text{(A)}$$

$$y_i = x_i'\beta + v_i \qquad \text{(B)}$$

OLS estimates $b_B$ of β from (B) can be written with $y_i$ from (A):

$$b_B = \beta + \left(\sum_i x_i x_i'\right)^{-1} \sum_i x_i z_i' \gamma + \left(\sum_i x_i x_i'\right)^{-1} \sum_i x_i \varepsilon_i$$

If (A) is the true model but (B) is specified, i.e., $J$ relevant regressors $z_i$ are omitted, $b_B$ is biased by

$$E\left\{\left(\sum_i x_i x_i'\right)^{-1} \sum_i x_i z_i' \gamma\right\}$$

Omitted variable bias!

No bias if (a) γ = 0 or if (b) variables in $x_i$ and $z_i$ are orthogonal

# Misspecification: Irrelevant Regressor

Specified model is (A), but true model is (B):

$$y_i = x_i'\beta + z_i'\gamma + \varepsilon_i \qquad \text{(A)}$$

$$y_i = x_i'\beta + v_i \qquad \text{(B)}$$

If (B) is the true model but (A) is specified, i.e., the model contains irrelevant regressors $z_i$

The OLS estimates $b_A$

- are unbiased
- have higher variances and standard errors than the OLS estimate $b_B$ obtained from fitting model (B)

# Consequences

Consequences of specification errors:

- Omission of a relevant variable
- Inclusion of a irrelevant variable

# Specification Search

*General-to-specific* modeling:

1. List all potential regressors, based on, e.g.,
   - economic theory
   - empirical research
   - availability of data
2. Specify the most general model: include all potential regressors
3. Iteratively, test which variables have to be dropped, re-estimate
4. Stop if no more variable has to be dropped

The procedure is known as the LSE (London School of Economics) method

# Specification Search, cont'd

Alternative procedures

- Specific-to-general modeling: start with a small model and add variables as long as they contribute to explaining *Y*
- Stepwise regression

Specification search can be subsumed under *data mining*

# Practice of Specification Search

Applied research

- Starts with a – in terms of economic theory – plausible specification
- Tests whether imposed restrictions are correct, such as
  - Test for omitted regressors
  - Test for autocorrelation of residuals
  - Test for heteroskedasticity
- Tests whether further restrictions need to be imposed
  - Test for irrelevant regressors

Obstacles for good specification

- Complexity of economic theory
- Limited availability of data

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# Regressor Selection Criteria

Criteria for adding and deleting regressors

- *t*-statistic, *F*-statistic
- Adjusted $R^2$
- Information Criteria: penalty for increasing number of regressors
  - Akaike's Information Criterion

$$AIC = \log \frac{1}{N} \sum_i e_i^2 + \frac{2K}{N}$$

  - Alternative criteria are
    - Schwarz's Bayesian Information Criterion (BIC)
    - Hannan-Quinn Information Criterion

The model with relevant regressors, with higher adj $R^2$, the smaller AIC is preferred

# Information Criteria

The most popular information criteria are

- Akaike's Information Criterion

$$AIC = \log \tfrac{1}{N} \sum_i e_i^2 + \tfrac{2K}{N}$$

- Schwarz's Bayesian Information Criterion

$$BIC = \log \tfrac{1}{N} \sum_i e_i^2 + \tfrac{K}{N} \log N$$

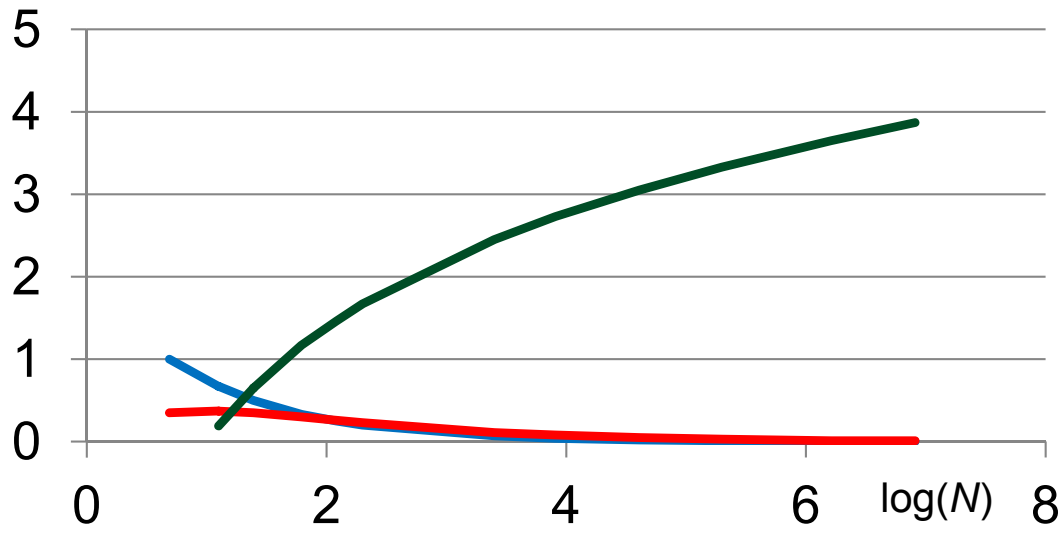
- Hannan-Quinn Information Criterion

$$HQIC = \log \tfrac{1}{N} \sum_i e_i^2 + 2K \log \log N$$

Decide in favour of the model with the *lowest* value of the information criterion

# Information Criteria: Penalties

- Akaike
  $2/N$
- Schwarz
  $\log(N)/N$
- Hannan-Quinn
  $2 \log(\log(N))$




| *N* | log(*N*) | *AIC* | *BIC* | *HQC* |
|---|---|---|---|---|
| 2 | 0,69 | 1,00 | 0,35 | -0,73 |
| 3 | 1,10 | 0,67 | 0,37 | 0,19 |
| 4 | 1,39 | 0,50 | 0,35 | 0,65 |
| 6 | 1,79 | 0,33 | 0,30 | 1,17 |
| 8 | 2,08 | 0,25 | 0,26 | 1,46 |
| 10 | 2,30 | 0,20 | 0,23 | 1,67 |
| 30 | 3,40 | 0,07 | 0,11 | 2,45 |
| 50 | 3,91 | 0,04 | 0,08 | 2,73 |
| 100 | 4,61 | 0,02 | 0,05 | 3,05 |
| 200 | 5,30 | 0,01 | 0,03 | 3,33 |
| 500 | 6,21 | 0,00 | 0,01 | 3,65 |
| 1000 | 6,91 | 0,00 | 0,01 | 3,87 |

# Wages: Which Regressors?

Are *school* and *exper* relevant regressors in

$$wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, school_i + \beta_4\, exper_i + \varepsilon_i$$

or shall they be omitted?

- *t*-test: *p*-values are 4.62E-80 (*school*) and 1.59E-7 (*exper*)
- *F*-test: *F* = [(0.1326-0.0317)/2]/[(1-0.1326)/(3294-4)] = 191.24, with *p*-value 2.68E-79
- adj $R^2$: 0.1318 for the wider model, much higher than 0.0315
- AIC: the wider model (AIC = 16690.2) is preferable; for the smaller model: AIC = 17048.5
- BIC: the wider model (BIC = 16714.6) is preferable; for the smaller model: BIC = 17060.7

All criteria suggest the wider model

# Wages, cont'd

OLS estimated smaller wage equation (Table 2.1, Verbeek)

| Dependent variable: *wage* | | |
|---|---|---|
| Variable | Estimate | Standard error |
| constant | 5.1469 | 0.0812 |
| *male* | 1.1661 | 0.1122 |
| $s = 3.2174$ | $R^2 = 0.0317$ | $F = 107.93$ |

with AIC = 17048.46, BIC = 17060.66

# Wages, cont'd

OLS estimated wider wage equation (Table 2.2, Verbeek)

**Table 2.2** OLS results wage equation

Dependent variable: *wage*

| Variable | Estimate | Standard error | *t*-ratio |
|---|---|---|---|
| constant | −3.3800 | 0.4650 | −7.2692 |
| *male* | 1.3444 | 0.1077 | 12.4853 |
| *school* | 0.6388 | 0.0328 | 19.4780 |
| *exper* | 0.1248 | 0.0238 | 5.2530 |

$s = 3.0462 \quad R^2 = 0.1326 \quad \overline{R}^2 = 0.1318 \quad F = 167.63$

with AIC = 16690.18, BIC = 16714.58

# The AIC Criterion

Various versions in literature

- Verbeek, also Greene:

$$AIC_V = \log \tfrac{1}{N} \sum_i e_i^2 + \tfrac{2K}{N} = \log(s^2) + 2K / N$$

- Akaike's original formula is

$$AIC_A = -2\, l(b)/N + 2K/N = AIC_V + 1 + \log(2\pi)$$

with the log-likelihood function

$$\ell(b) = -\frac{N}{2}\left(1 + \log(2\pi) + \log(s^2)\right)$$

- GRETL:

$$AIC_G = N \log(s^2) + 2K + N\left(1 + \log(2\pi)\right) = N\, AIC_A$$

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# Nested Models: Comparison

Model (B), $y_i = x_i'\beta + v_i$, see slide 21, is nested in model

$$y_i = x_i'\beta + z_i'\gamma + \varepsilon_i \qquad \text{(A)}$$

i.e., (A) is extended by $J$ additional regressors $z_i$

Do the $J$ added regressors contribute to explaining $Y$?

- $F$-test ($t$-test when $J = 1$) for testing $H_0$: all coefficients of added regressors are zero

$$F = \frac{(R_A^2 - R_B^2)/J}{(1 - R_A^2)/(N - K)}$$

  $R_B^2$ and $R_A^2$ are the $R^2$ of the models without (B) and with (A) the $J$ additional regressors, respectively

- Adjusted $R^2$: adj $R_A^2$ > adj $R_B^2$ equivalent to $F > 1$
- Information Criteria: choose the model with the smaller value of the information criterion

# Comparison of Non-nested Models

Non-nested models:

$$y_i = x_i'\beta + \varepsilon_i \qquad \text{(A)}$$

$$y_i = z_i'\gamma + v_i \qquad \text{(B)}$$

at least one component in $z_i$ that is not in $x_i$

- Non-nested or encompassing *F*-test: compares by *F*-tests artificially nested models

  $y_i = x_i'\beta + z_{2i}'\delta_B + \varepsilon^*_i$ with $z_{2i}$: regressors from $z_i$ not in $x_i$

  $y_i = z_i'\gamma + x_{2i}'\delta_A + v^*_i$ with $x_{2i}$: regressors from $x_i$ not in $z_i$

  - Test validity of model A by testing $H_0$: $\delta_B = 0$
  - Analogously, test validity of model B by testing $H_0$: $\delta_A = 0$
  - Possible results: A or B is valid, both models are valid, none is valid
- Other procedures: *J*-test, PE-test (see below)

# Wages: Which Model?

Which of the models is adequate?

$$\log(wage_i) = 0.119 + 0.260\ male_i + 0.115\ school_i \quad \text{(A)}$$

adj $R^2 = 0.121$, BIC = 5824.90,

$$\log(wage_i) = 0.119 + 0.064\ age_i \quad \text{(B)}$$

adj $R^2 = 0.069$, BIC = 6004.60

- Artificially nested model

$$\log(wage_i) =$$

$$= -0.472 + 0.243\ male_i + 0.088\ school_i + 0.035\ age_i$$

- Test of model validity
  - model A: *t*-test for *age*, *p*-value 5.79E-15; model A is not adequate
  - model B: *F*-test for *male* and *school*: model B is not adequate

# *J*-Test: Comparison of Non-nested Models

Non-nested models: (A) $y_i = x_i'\beta + \varepsilon_i$, (B) $y_i = z_i'\gamma + v_i$ with components of $z_i$ that are not in $x_i$

- Combined model

  $$y_i = (1 - \delta)\, x_i'\beta + \delta\, z_i'\gamma + u_i$$

  with $0 < \delta < 1$; $\delta$ indicates model adequacy

- Transformed model

  $$y_i = x_i'\beta^* + \delta z_i'c + u_i = x_i'\beta^* + \delta \hat{y}_{iB} + u^*_i$$

  with OLS estimate $c$ for $\gamma$ and predicted values $\hat{y}_{iB} = z_i'c$ obtained from fitting model B; $\beta^* = (1-\delta)\beta$

- *J*-test for validity of model A by testing $H_0$: $\delta = 0$
- Less computational effort than the encompassing *F*-test

# Wages: Which Model?

Which of the models is adequate?

$$\log(wage_i) = 0.119 + 0.260\ male_i + 0.115\ school_i \quad \text{(A)}$$

adj $R^2$ = 0.121, BIC = 5824.90,

$$\log(wage_i) = 0.119 + 0.064\ age_i \quad \text{(B)}$$

adj $R^2$ = 0.069, BIC = 6004.60

Test the validity of model B by means of the *J*-test

- Extend the model B to

  $$\log(wage_i) = -0.587 + 0.034\ age_i + 0.826\ \hat{y}_{iA}$$

  with values $\hat{y}_{iA}$ predicted for $\log(wage_i)$ from model A
- Test of model validity: *t*-test for coefficient of $\hat{y}_{iA}$, $t$ = 15.96, $p$-value 2.65E-55
- Model B is not a valid model

# Linear vs. Log-linear Model

Choice between linear and log-linear functional form

$$y_i = x_i'\beta + \varepsilon_i \qquad \text{(A)}$$

$$\log y_i = (\log x_i)'\beta + v_i \qquad \text{(B)}$$

- In terms of economic interpretation: Are effects additive or multiplicative?
- Log-transformation stabilizes variance, particularly if the dependent variable has a skewed distribution (wages, income, production, firm size, sales,…)
- Log-linear models are easily interpretable in terms of elasticities

# PE-Test: Linear vs. Log-linear Model

Choice between linear and log-linear functional form

- Estimate both models

$$y_i = x_i'\beta + \varepsilon_i \qquad \text{(A)}$$

$$\log y_i = (\log x_i)'\beta + v_i \qquad \text{(B)}$$

calculate the fitted values $y\_f$ (from model A) and $\log y\_f$ (from B)

- Test $H_0$: $\delta_{LIN} = 0$ in

$$y_i = x_i'\beta + \delta_{LIN}\,(\log (y\_f_i) - \log y\_f_i) + u_i$$

not rejecting $H_0$: $\delta_{LIN} = 0$ favors the model A

- Test $H_0$: $\delta_{LOG} = 0$ in

$$\log y_i = (\log x_i)'\beta + \delta_{LOG}\,(y\_f_i - \exp\{\log y\_f_i\}) + u_i$$

not rejecting $H_0$: $\delta_{LOG} = 0$ favors the model B

- Both null hypotheses are rejected: find a more adequate model

# Wages: Which Model?

Test of validity of models by means of the PE-test

The fitted models are (with *l_x* for log(*x*))

$$wage_i = -2.046 + 1.406\ male_i + 0.608\ school_i \qquad (A)$$

$$l\_wage_i = 0.119 + 0.260\ male_i + 0.115\ l\_school_i \qquad (B)$$

- *x_f*: predicted value of *x*: *d_log* = log(*wage_f*) – *l_wage_f*, *d_lin* = *wage_f* – exp(*l_wage_f*)
- Test of validity of model A:

$$wage_i = -1.708 + 1.379\ male_i + 0.637\ school_i - 4.731\ d\_log_i$$

with *p*-value 0.013 for *d_log*; validity of model A in doubt

- Test of model validity, model B:

$$l\_wage_i = -1.132 + 0.240\ male_i + 1.008\ l\_school_i + 0.171\ d\_lin_i$$

with *p*-value 0.076 for *d_lin*; model B to be preferred

# The PE-Test

Choice between linear and log-linear functional form

- The auxiliary regressions are estimated for testing purposes
- If the linear model is not rejected: accept the linear model
- If the log-linear model is not rejected: accept the log-linear model
- If both are rejected, neither model is appropriate, a more adequate model should be considered
- In case of the Individual Wages example:
  - Linear model (A): *t*-statistic is – 4.731, *p*-value 0.013: the model is rejected
  - Log-linear model (B): *t*-statistic is 0.171, *p*-value 0.076 : the model is not rejected

# Contents


- The Linear Model: Interpretation of Coefficients
- Selection of Regressors
- Selection Criteria
- Comparison of Competing Models
- Specification of the Functional Form
- Structural Break

# Non-linear Functional Forms

Model specification

$$y_i = g(x_i, \beta) + \varepsilon_i$$

substitution of $g(x_i, \beta)$ for $x_i'\beta$: allows for two types on non-linearity

- $g(x_i, \beta)$ non-linear in regressors (but linear in coefficients)
  - Powers of regressors, e.g., $g(x_i, \beta) = \beta_1 + \beta_2\, age_i + \beta_3\, age_i^2$
  - Interactions of regressors, e.g., $g(x_i, \beta) = \beta_1 + \beta_2\, age_i + \beta_3\, age_i * male_i$

  OLS technique still works; *t*-test, *F*-test for specification check
- $g(x_i, \beta)$ non-linear in regression coefficients, e.g.,
  - $g(x_i, \beta) = \beta_1\, x_{i1}^{\beta 2}\, x_{i2}^{\beta 3}$

    logarithmic transformation: $\log g(x_i, \beta) = \log \beta_1 + \beta_2 \log x_{i1} + \beta_3 \log x_{i2}$
  - $g(x_i, \beta) = \beta_1 + \beta_2\, x_i^{\beta 3}$

    non-linear least squares estimation, numerical procedures

  Various specification test procedures, e.g., RESET test, Chow test

# Individual Wages: Effect of Gender and Education

Effect of gender may be depending of education level

- Separate models for males and females
- Interaction terms between dummies for education level and male

Example: Belgian Household Panel, 1994 ("bwages", *N*=1472)

- Five education levels
- Model for log(*wage*) with education dummies; see next slide
- Model with interaction terms between education dummies and gender dummy; see slide 49
- *F*-statistic for interaction terms:

$$F(5, 1460) = \{(0.4032-0.3976)/5\}/\{(1-0.4032)/(1472-12)\}$$
$$= 2.74$$

with a *p*-value of 0.018

# Wages: Model with Education Dummies

Model with education dummies: Verbeek, Table 3.11

**Table 3.11** OLS results specification 5

Dependent variable: log(*wage*)

| Variable | Estimate | Standard error | *t*-ratio |
|---|---|---|---|
| constant | 1.272 | 0.045 | 28.369 |
| *male* | 0.118 | 0.015 | 7.610 |
| *educ* = 2 | 0.144 | 0.033 | 4.306 |
| *educ* = 3 | 0.305 | 0.032 | 9.521 |
| *educ* = 4 | 0.474 | 0.033 | 14.366 |
| *educ* = 5 | 0.639 | 0.033 | 19.237 |
| log(*exper*) | 0.230 | 0.011 | 21.804 |

$s = 0.282$ $R^2 = 0.3976$ $\bar{R}^2 = 0.3951$ $F = 161.14$ $S = 116.47$

# Wages: Model with Gender Interactions

Wage equation with interactions *educ*male*

**Table 3.12** OLS results specification 6

Dependent variable: log(*wage*)

| Variable | Estimate | Standard error | *t*-ratio |
|---|---|---|---|
| constant | 1.216 | 0.078 | 15.653 |
| *male* | 0.154 | 0.095 | 1.615 |
| *educ* = 2 | 0.224 | 0.068 | 3.316 |
| *educ* = 3 | 0.433 | 0.063 | 6.851 |
| *educ* = 4 | 0.602 | 0.063 | 9.585 |
| *educ* = 5 | 0.755 | 0.065 | 11.673 |
| log(*exper*) | 0.207 | 0.017 | 12.535 |
| *educ* = 2 × *male* | −0.097 | 0.078 | −1.242 |
| *educ* = 3 × *male* | −0.167 | 0.073 | −2.272 |
| *educ* = 4 × *male* | −0.172 | 0.074 | −2.317 |
| *educ* = 5 × *male* | −0.146 | 0.076 | −1.935 |
| log(*exper*) × *male* | 0.041 | 0.021 | 1.891 |

$s = 0.281$ $R^2 = 0.4032$ $\bar{R}^2 = 0.3988$ $F = 89.69$ $S = 115.37$

# RESET Test

Test of the linear model $E\{y_i \mid x_i\} = x_i'\beta$ against misspecification of the functional form:

- Null hypothesis: linear model is correct functional form
- Test of $H_0$: RESET test (Regression Specification Error Test), Ramsey (1969)
- Test idea: linear model is extended by adding $\hat{y}_i^2$, $\hat{y}_i^3$, ..., where $\hat{y}_i$ is the fitted values from the linear model; extension does not improve model fit under $H_0$
  - $\hat{y}_i^2$ is a function of squares (and interactions) of the regressor variables; analogously for $\hat{y}_i^3$, ...
  - If the *F*-test indicates that the additional regressor $\hat{y}_i^2$ contributes to explaining *Y*: the linear relation is not adequate, another functional form is more appropriate

# The RESET Test Procedure

Test of the linear model $E\{y_i | x_i\} = x_i'\beta$ against misspecification of the functional form:

- Linear model extended by adding $\hat{y}_i^2$, ..., $\hat{y}_i^Q$
- *F*- (or *t*-) test to decide whether $\hat{y}_i^2$, ..., $\hat{y}_i^Q$ contribute as additional regressors to explaining *Y*
- Maximal power *Q* of fitted values: typical choice is *Q* = 2 or *Q* = 3

In **GRETL**: Ordinary Least Squares… => Tests => Ramsey's RESET, input of *Q*

# Wages: RESET Test

The fitted models are (with *l_x* for log(*x*))

$$wage_i = -2.046 + 1.406\ male_i + 0.608\ school_i \qquad (A)$$

$$l\_wage_i = 0.119 + 0.260\ male_i + 0.115\ l\_school_i \qquad (B)$$

Test of specification of the functional form with $Q = 3$

- Model A: Test statistic: $F(2, 3288) = 10.23$, $p$-value = 3.723e-005
- Model B: Test statistic: $F(2, 3288) = 4.52$, $p$-value = 0.011

For both models the adequacy of the functional form is in doubt

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# Structural Break: Chow Test

In time-series context, coefficients of a model may change due to a major policy change, e.g., the oil price shock

- Modeling a process with structural break

$$E\{y_i | x_i\} = x_i'\beta + g_i x_i'\gamma$$

with dummy variable $g_i$=0 before the break, $g_i$=1 after the break

- Regressors $x_i$, coefficients β before, β+γ after the break
- Null hypothesis: no structural break, γ=0
- Test procedure: fitting the extended model, *F*- (or *t*-) test of γ=0

$$F = \frac{S_r - S_u}{S_u} \frac{N - 2K}{K}$$

with $S_r$ ($S_u$): sum of squared residuals of the (un)restricted model

- Chow test for structural break or structural change, Chow (1960)

# Chow Test: The Practice

Test procedure is performed in the following steps

- Fit the restricted model: $S_r$
- Fit the extended model: $S_u$
- Calculate *F* and the *p*-value from the *F*-distribution with *K* and *N-2K* d.f.

Needs knowledge of break point

In **GRETL**: Ordinary Least Squares… => Tests => Chow test

input of the first observation period after the break point

# Your Homework

1. Use the data set “bwages” of Verbeek for the following analyses:
   a) Estimate the model where the hourly wages (*wage*) are explained by *exper*, *male*, and *educ*; interpret the results.
   b) Educ represents the level of education; what does that mean for the estimated coefficient for *educ* in task a).
   c) Repeat task a) using dummy variables for the education levels, *d2* for *educ* = 2, …, *d5* for *educ* = 5 instead of the variable *educ*; compare the models from this and task a) by using (i) the non-nested *F*-test and (ii) the *J*-test; interpret the results.
   d) Use the PE-test to decide whether the model of a) (where hourly wages, *wage,* are explained) or the same model but with *lnwage,* log hourly wages, as explained variable is to be preferred; interpret the result.
   e) Estimate the model for log hourly wages (*lnwage*) with regressors *lnexper*, *male*, *educ,* and the interaction *male*lnexper* as additional regressor; interpret the results.

# Your Homework, cont'd

2. OLS is used to estimate β from $y_i = x_i'\beta + \varepsilon_i$, but a relevant regressor $z_i$ is neglected: $y_i = x_i'\beta + z_i'\gamma + \varepsilon_i$. (a) Show that the estimate *b* is biased, and derive an expression for the bias; (b) what test statistic can be used for testing $H_0$: $\gamma = 0$?
3. The linear regression is specified as

   $$\log y_i = x_i'\beta + \varepsilon_i$$

   Show that the elasticity of *Y* with respect to $X_k$ is $\beta_k x_{ik}$.