In this chapter you will solve problems for firm and industry outcomes when the firms engage in Cournot competition, Stackelberg competition, and other sorts of oligopoly behavior. In Cournot competition, each firm chooses its own output to maximize its profits given the output that it expects the other firm to produce. The industry price depends on the industry output, say, $q_A + q_B$, where A and B are the firms. To maximize profits, firm A sets its marginal revenue (which depends on the output of firm A and the expected output of firm B since the expected industry price depends on the sum of these outputs) equal to its marginal cost. Solving this equation for firm A's output as a function of firm B's expected output gives you one reaction function; analogous steps give you firm B's reaction function. Solve these two equations simultaneously to get the Cournot equilibrium outputs of the two firms.

In Heifer's Breath, Wisconsin, there are two bakers, Anderson and Carlson. Anderson's bread tastes just like Carlson's—nobody can tell the difference. Anderson has constant marginal costs of \$1 per loaf of bread. Carlson has constant marginal costs of \$2 per loaf. Fixed costs are zero for both of them. The inverse demand function for bread in Heifer's Breath is $p(q) = 6 - .01q$, where q is the total number of loaves sold per day.

Let us find Anderson's Cournot reaction function. If Carlson bakes q_C loaves, then if Anderson bakes q_A loaves, total output will be $q_A + q_C$ and price will be $6 - .01(q_A + q_C)$. For Anderson, the total cost of producing q_A units of bread is just q_A, so his profits are

$$\begin{aligned} pq_A - q_A &= (6 - .01q_A - .01q_C)q_A - q_A \\ &= 6q_A - .01q_A^2 - .01q_C q_A - q_A. \end{aligned}$$

Therefore if Carlson is going to bake q_C units, then Anderson will choose q_A to maximize $6q_A - .01q_A^2 - .01q_C q_A - q_A$. This expression is maximized when $6 - .02q_A - .01q_C = 1$. (You can find this out either by setting A's marginal revenue equal to his marginal cost or directly by setting the derivative of profits with respect to q_A equal to zero.) Anderson's reaction function, $R_A(q_C)$ tells us Anderson's best output if he knows that Carlson is going to bake q_C. We solve from the previous equation to find $R_A(q_C) = (5 - .01q_C)/.02 = 250 - .5q_C$.

We can find Carlson's reaction function in the same way. If Carlson knows that Anderson is going to produce q_A units, then Carlson's profits will be $p(q_A + q_C) - 2q_C = (6 - .01q_A - .01q_C)q_C - 2q_C = 6q_C - .01q_A q_C - .01q_C^2 - 2q_C$. Carlson's profits will be maximized if he chooses q_C to satisfy the equation $6 - .01q_A - .02q_C = 2$. Therefore Carlson's reaction function is $R_C(q_A) = (4 - .01q_A)/.02 = 200 - .5q_A$.

Let us denote the Cournot equilibrium quantities by $\bar{q}_A$ and $\bar{q}_C$. The Cournot equilibrium conditions are that $\bar{q}_A = R_A(\bar{q}_C)$ and $\bar{q}_C = R_C(\bar{q}_A)$.

Solving these two equations in two unknowns we find that $\bar{q}_A = 200$ and $\bar{q}_C = 100$. Now we can also solve for the Cournot equilibrium price and for the profits of each baker. The Cournot equilibrium price is $6 - .01(200 + 100) = \$3$. Then in Cournot equilibrium, Anderson makes a profit of \$2 on each of 200 loaves and Carlson makes \$1 on each of 100 loaves.

In Stackelberg competition, the follower's profit-maximizing output choice depends on the amount of output that he expects the leader to produce. His reaction function, $R_F(q_L)$, is constructed in the same way as for a Cournot competitor. The leader knows the reaction function of the follower and gets to choose her own output, q_L, first. So the leader knows that the industry price depends on the *sum* of her own output and the follower's output, that is, on $q_L + R_F(q_L)$. Since the industry price can be expressed as a function of q_L only, so can the leader's marginal revenue. So once you get the follower's reaction function and substitute it into the inverse demand function, you can write down an expression that depends on just q_L and that says marginal revenue equals marginal cost for the leader. You can solve this expression for the leader's Stackelberg output and plug in to the follower's reaction function to get the follower's Stackelberg output.

Suppose that one of the bakers of Heifer's Breath plays the role of Stackelberg leader. Perhaps this is because Carlson always gets up an hour earlier than Anderson and has his bread in the oven before Anderson gets started. If Anderson always finds out how much bread Carlson has in his oven and if Carlson knows that Anderson knows this, then Carlson can act like a Stackelberg leader. Carlson knows that Anderson's reaction function is $R_A(q_C) = 250 - .5q_c$. Therefore Carlson knows that if he bakes q_C loaves of bread, then the total amount of bread that will be baked in Heifer's Breath will be $q_C + R_A(q_C) = q_C + 250 - .5q_C = 250 + .5q_C$. Since Carlson's production decision determines total production and hence the price of bread, we can write Carlson's profit simply as a function of his own output. Carlson will choose the quantity that maximizes this profit. If Carlson bakes q_C loaves, the price will be $p = 6 - .01(250 + .5q_C) = 3.5 - .005q_C$. Then Carlson's profits will be $pq_C - 2q_C = (3.5 - .005q_C)q_C - 2q_C = 1.5q_C - .005q_C^2$. His profits are maximized when $q_C = 150$. (Find this either by setting marginal revenue equal to marginal cost or directly by setting the derivative of profits to zero and solving for q_C.) If Carlson produces 150 loaves, then Anderson will produce $250 - .5 \times 150 = 175$ loaves. The price of bread will be $6 - .01(175 + 150) = 2.75$. Carlson will now make \$.75 per loaf on each of 150 loaves and Anderson will make \$1.75 on each of 175 loaves.

27.1 (0) Carl and Simon are two rival pumpkin growers who sell their pumpkins at the Farmers' Market in Lake Witchisit, Minnesota. They are the only sellers of pumpkins at the market, where the demand function for pumpkins is $q = 3,200 - 1,600p$. The total number of pumpkins sold at the market is $q = q_C + q_S$, where q_C is the number that Carl sells and q_S is the number that Simon sells. The cost of producing pumpkins for either farmer is \$.50 per pumpkin no matter how many pumpkins he produces.

____

(a) The inverse demand function for pumpkins at the Farmers' Market is $p = a - b(q_C + q_S)$, where $a =$ ________ and $b =$ ____________ The marginal cost of producing a pumpkin for either farmer is____________.

(b) Every spring, each of the farmers decides how many pumpkins to grow. They both know the local demand function and they each know how many pumpkins were sold by the other farmer last year. In fact, each farmer assumes that the other farmer will sell the same number this year as he sold last year. So, for example, if Simon sold 400 pumpkins last year, Carl believes that Simon will sell 400 pumpkins again this year. If Simon sold 400 pumpkins last year, what does Carl think the price of pumpkins will be if Carl sells 1,200 pumpkins this year? ________ If Simon sold q_S^{t-1} pumpkins in year $t-1$, then in the spring of year t, Carl thinks that if he, Carl, sells q_C^t pumpkins this year, the price of pumpkins this year will be____.

(c) If Simon sold 400 pumpkins last year, Carl believes that if he sells q_C^t pumpkins this year then the inverse demand function that he faces is $p = 2 - 400/1,600 - q_C^t/1,600 = 1.75 - q_C^t/1,600$. Therefore if Simon sold 400 pumpkins last year, Carl's marginal revenue this year will be $1.75 - q_C^t/800$. More generally, if Simon sold q_S^{t-1} pumpkins last year, then Carl believes that if he, himself, sells q_C^t pumpkins this year, his marginal revenue this year will be________________________________.

(d) Carl believes that Simon will never change the amount of pumpkins that he produces from the amount q_S^{t-1} that he sold last year. Therefore Carl plants enough pumpkins this year so that he can sell the amount that maximizes his profits this year. To maximize this profit, he chooses the output this year that sets his marginal revenue this year equal to his marginal cost. This means that to find Carl's output this year when Simon's output last year was q_S^{t-1}, Carl solves the following equation.

____.

(e) Carl's Cournot reaction function, $R_C^t(q_S^{t-1})$, is a function that tells us what Carl's profit-maximizing output this year would be as a function of Simon's output last year. Use the equation you wrote in the last answer to find Carl's reaction function, $R_C^t(q_S^{t-1}) =$ ______________ (Hint: This is a linear expression of the form $a - bq_S^{t-1}$. You have to find the constants a and b.)

(f) Suppose that Simon makes his decisions in the same way that Carl does. Notice that the problem is completely symmetric in the roles played by Carl and Simon. Therefore without even calculating it, we can guess that Simon's reaction function is $R_S^t(q_C^{t-1}) =$ ________________ (Of course, if you don't like to guess, you could work this out by following similar steps to the ones you used to find Carl's reaction function.)

(g) Suppose that in year 1, Carl produced 200 pumpkins and Simon produced 1,000 pumpkins. In year 2, how many would Carl produce? __________ How many would Simon produce? __________ In year 3, how many would Carl produce? __________ How many would Simon produce? __________ Use a calculator or pen and paper to work out several more terms in this series. To what level of output does Carl's output appear to be converging?_______ How about Simon's?________________.

(h) Write down two simultaneous equations that could be solved to find outputs q_S and q_C such that, if Carl is producing q_C and Simon is producing q_S, then they will both want to produce the same amount in the next period. (Hint: Use the reaction functions.)________________________

____.

(i) Solve the two equations you wrote down in the last part for an equilibrium output for each farmer. Each farmer, in Cournot equilibrium, produces _______ units of output. The total amount of pumpkins brought to the Farmers' Market in Lake Witchisit is __________ The price of pumpkins in that market is _______ How much profit does each farmer make?

____.

27.2 (0) Suppose that the pumpkin market in Lake Witchisit is as we described it in the last problem except for one detail. Every spring, the snow thaws off of Carl's pumpkin field a week before it thaws off of Simon's. Therefore Carl can plant his pumpkins one week earlier than Simon can. Now Simon lives just down the road from Carl, and he can tell by looking at Carl's fields how many pumpkins Carl planted and how many Carl will harvest in the fall. (Suppose also that Carl will sell every pumpkin that he produces.) Therefore instead of assuming that Carl will sell the same amount of pumpkins that he did last year, Simon sees how many Carl is actually going to sell this year. Simon has this information before he makes his own decision about how many to plant.

____

(a) If Carl plants enough pumpkins to yield q_C^t this year, then Simon knows that the profit-maximizing amount to produce this year is $q_S^t =$ Hint: Remember the reaction functions you found in the last problem.

____.

(b) When Carl plants his pumpkins, he understands how Simon will make his decision. Therefore Carl knows that the amount that Simon will produce this year will be determined by the amount that Carl produces. In particular, if Carl's output is q_C^t, then Simon will produce and sell

____________ and the total output of the two producers will be ______

____________ Therefore Carl knows that if his own output is q_C, the

price of pumpkins in the market will be____________________________.

(c) In the last part of the problem, you found how the price of pumpkins this year in the Farmers' Market is related to the number of pumpkins that Carl produces this year. Now write an expression for Carl's total

revenue in year t as a function of his own output, q_C^t. ____________

______ Write an expression for Carl's marginal revenue in year t as a

function of q_C^t.______________________________________.

(d) Find the profit-maximizing output for Carl. _________ Find the

profit-maximizing output for Simon. _________ Find the equilibrium

price of pumpkins in the Lake Witchisit Farmers' Market. _________

How much profit does Carl make? _________ How much profit does

Simon make? _________ An equilibrium of the type we discuss here is

known as a ____________ equilibrium.

(e) If he wanted to, it would be possible for Carl to delay his planting until the same time that Simon planted so that neither of them would know the other's plans for this year when he planted. Would it be in Carl's interest to do this? Explain. (Hint: What are Carl's profits in the equilibrium above? How do they compare with his profits

in Cournot equilibrium?)______________________________

____

____.

27.3 (0) Suppose that Carl and Simon sign a marketing agreement. They decide to determine their total output jointly and to each produce the same number of pumpkins. To maximize their joint profits, how many pumpkins should they produce in toto? ______ How much does each one of them produce? ______ How much profit does each one of them make?______________________________.

27.4 (0) The inverse market demand curve for bean sprouts is given by $P(Y) = 100 - 2Y$, and the total cost function for any firm in the industry is given by $TC(y) = 4y$.

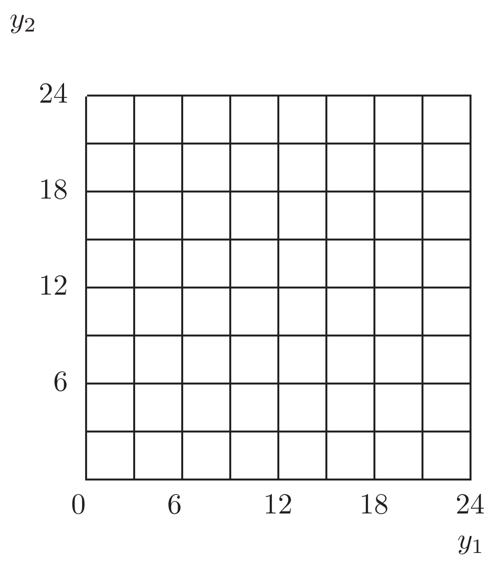
(a) The marginal cost for any firm in the industry is equal to ______ The change in price for a one-unit increase in output is equal to______.

(b) If the bean-sprout industry were perfectly competitive, the industry output would be ______ , and the industry price would be______.

(c) Suppose that two Cournot firms operated in the market. The reaction function for Firm 1 would be ______ (Reminder: Unlike the example in your textbook, the marginal cost is not zero here.) The reaction function of Firm 2 would be ______ If the firms were operating at the Cournot equilibrium point, industry output would be ______ , each firm would produce ______ , and the market price would be______________________________.

(d) For the Cournot case, draw the two reaction curves and indicate the equilibrium point on the graph below.

____



(e) If the two firms decided to collude, industry output would be ________

and the market price would equal________________________.

(f) Suppose both of the colluding firms are producing equal amounts of output. If one of the colluding firms assumes that the other firm would not react to a change in industry output, what would happen to a firm's own profits if it increased its output by one unit? ________________

____

(g) Suppose one firm acts as a Stackleberg leader and the other firm behaves as a follower. The maximization problem for the leader can be written as________________________________.

Solving this problem results in the leader producing an output of ________ and the follower producing ______ This implies an industry output of ________ and price of________________________.

27.5 (0) Grinch is the sole owner of a mineral water spring that costlessly burbles forth as much mineral water as Grinch cares to bottle. It costs Grinch \$2 per gallon to bottle this water. The inverse demand curve for Grinch's mineral water is $p = \$20 - .20q$, where p is the price per gallon and q is the number of gallons sold.

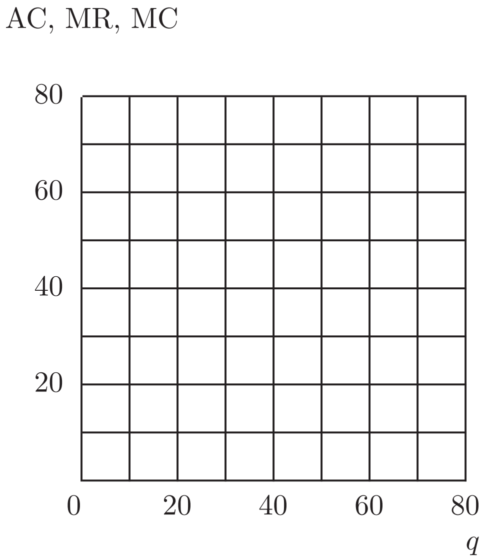
(a) Write down an expression for profits as a function of q: $\Pi(q) =$ ________________ Find the profit-maximizing choice of q for Grinch. ____.

(b) What price does Grinch get per gallon of mineral water if he produces the profit-maximizing quantity? __________ How much profit does he make?____.

(c) Suppose, now, that Grinch's neighbor, Grubb finds a mineral spring that produces mineral water that is just as good as Grinch's water, but that it costs Grubb \$6 a bottle to get his water out of the ground and bottle it. Total market demand for mineral water remains as before. Suppose that Grinch and Grubb each believe that the other's quantity decision is independent of his own. What is the Cournot equilibrium output for Grubb?__________ What is the price in the Cournot equilibrium? ____.

27.6 (1) Albatross Airlines has a monopoly on air travel between Peoria and Dubuque. If Albatross makes one trip in each direction per day, the demand schedule for round trips is $q = 160 - 2p$, where q is the number of passengers per day. (Assume that nobody makes one-way trips.) There is an "overhead" fixed cost of \$2,000 per day that is necessary to fly the airplane regardless of the number of passengers. In addition, there is a marginal cost of \$10 per passenger. Thus, total daily costs are $\$2,000+10q$ if the plane flies at all.

(a) On the graph below, sketch and label the marginal revenue curve, and the average and marginal cost curves.



____

(b) Calculate the profit-maximizing price and quantity and total daily profits for Albatross Airlines. $p =$________ , $q =$________ , $\pi =$______

____.

(c) If the interest rate is 10% per year, how much would someone be willing to pay to own Albatross Airlines's monopoly on the Dubuque-Peoria route. (Assuming that demand and cost conditions remain unchanged forever.)____.

(d) If another firm with the same costs as Albatross Airlines were to enter the Dubuque-Peoria market and if the industry then became a Cournot duopoly, would the new entrant make a profit?____________________

____.

(e) Suppose that the throbbing night life in Peoria and Dubuque becomes widely known and in consequence the population of both places doubles. As a result, the demand for airplane trips between the two places doubles to become $q = 320 - 4p$. Suppose that the original airplane had a capacity of 80 passengers. If AA must stick with this single plane and if no other airline enters the market, what price should it charge to maximize its output and how much profit would it make? $p =$______ , $\pi =$__________.

(f) Let us assume that the overhead costs per plane are constant regardless of the number of planes. If AA added a second plane with the same costs and capacity as the first plane, what price would it charge? __________ How many tickets would it sell? __________ How much would its profits be? __________ If AA could prevent entry by another competitor, would it choose to add a second plane?__________________________________.

(g) Suppose that AA stuck with one plane and another firm entered the market with a plane of its own. If the second firm has the same cost function as the first and if the two firms act as Cournot oligopolists, what will be the price, ______ , quantities, ________ , and profits?__________.

27.7 (0) Alex and Anna are the only sellers of kangaroos in Sydney, Australia. Anna chooses her profit-maximizing number of kangaroos to sell, q_1, based on the number of kangaroos that she expects Alex to sell. Alex knows how Anna will react and chooses the number of kangaroos that she herself will sell, q_2, after taking this information into account. The inverse demand function for kangaroos is $P(q_1 + q_2) = 2,000 - 2(q_1 + q_2)$. It costs \$400 to raise a kangaroo to sell.

(a) Alex and Anna are Stackelberg competitors. ________ is the leader and ________ is the follower.

(b) If Anna expects Alex to sell q_2 kangaroos, what will her own marginal revenue be if she herself sells q_1 kangaroos?________________________.

(c) What is Anna's reaction function, $R(q_2)$?________________________.

(d) Now if Alex sells q_2 kangaroos, what is the total number of kangaroos that will be sold? ____________ What will be the market price as a function of q_2 only?____.

(e) What is Alex's marginal revenue as a function of q_2 only? ________ ____________ How many kangaroos will Alex sell? ________ How many kangaroos will Anna sell? ________ What will the industry price be?____.

27.8 (0) Consider an industry with the following structure. There are 50 firms that behave in a competitive manner and have identical cost functions given by $c(y) = y^2/2$. There is one monopolist that has 0 marginal costs. The demand curve for the product is given by

$$D(p) = 1,000 - 50p.$$

(a) What is the supply curve of one of the competitive firms? ________ The total supply from the competitive sector at price p is $S(p) =$________ ____.

(b) If the monopolist sets a price p, the amount that it can sell is $D_m(p) =$ ____.

(c) The monopolist's profit-maximizing output is $y_m =$__________ What is the monopolist's profit-maximizing price?____________________.

(d) How much output will the competitive sector provide at this price? __________ What will be the total amount of output sold in this industry?____________________.

27.9 (0) Consider a market with one large firm and many small firms. The supply curve of the small firms taken together is

$$S(p) = 100 + p.$$

The demand curve for the product is

$$D(p) = 200 - p.$$

The cost function for the one large firm is

$$c(y) = 25y.$$

(a) Suppose that the large firm is forced to operate at a zero level of output. What will be the equilibrium price? ______ What will be the equilibrium quantity?____________________.

(b) Suppose now that the large firm attempts to exploit its market power and set a profit-maximizing price. In order to model this we assume that customers always go first to the competitive firms and buy as much as they are able to and then go to the large firm. In this situation, the equilibrium price will be __________ The quantity supplied by the large firm will be ______ and the equilibrium quantity supplied by the competitive firms will be____________________.

(c) What will be the large firm's profits?____________________.

(d) Finally suppose that the large firm could force the competitive firms out of the business and behave as a real monopolist. What will be the equilibrium price? __________ What will be the equilibrium quantity? __________ What will be the large firm's profits?____________________.

27.10 (2) In a remote area of the American Midwest before the railroads arrived, cast iron cookstoves were much desired, but people lived far apart, roads were poor, and heavy stoves were expensive to transport. Stoves could be shipped by river boat to the town of Bouncing Springs, Missouri. Ben Kinmore was the only stove dealer in Bouncing Springs. He could buy as many stoves as he wished for \$20 each, delivered to his store. Ben's only customers were farmers who lived along a road that ran east and west through town. There were no other stove dealers along the road in either direction. No farmers lived in Bouncing Springs, but along the road, in either direction, there was one farm every mile. The cost of hauling a stove was \$1 per mile. The owners of every farm had a reservation price of \$120 for a cast iron cookstove. That is, any of them would be willing to pay up to \$120 to have a stove rather than to not have one. Nobody had use for more than one stove. Ben Kinmore charged a base price of $\$p$ for stoves and added to the price the cost of delivery. For example, if the base price of stoves was \$40 and you lived 45 miles west of Bouncing Springs, you would have to pay \$85 to get a stove, \$40 base price plus a hauling charge of \$45. Since the reservation price of every farmer was \$120, it follows that if the base price were \$40, any farmer who lived within 80 miles of Bouncing Springs would be willing to pay \$40 plus the price of delivery to have a cookstove. Therefore at a base price of \$40, Ben could sell 80 cookstoves to the farmers living west of him. Similarly, if his base price is \$40, he could sell 80 cookstoves to the farmers living within 80 miles to his east, for a total of 160 cookstoves.

(a) If Ben set a base price of $\$p$ for cookstoves where $p < 120$, and if he charged \$1 a mile for delivering them, what would be the total number of cookstoves he could sell? __________ (Remember to count the ones he could sell to his east as well as to his west.) Assume that Ben has no other costs than buying the stoves and delivering them. Then Ben would make a profit of $p - 20$ per stove. Write Ben's total profit as a function of the base price, $\$p$, that he charges:____________________.

(b) Ben's profit-maximizing base price is ________ (Hint: You just wrote profits as a function of prices. Now differentiate this expression for profits with respect to p.) Ben's most distant customer would be located at a distance of ______ miles from him. Ben would sell _____ cookstoves and make a total profit of____________________.

(c) Suppose that instead of setting a single base price and making all buyers pay for the cost of transportation, Ben offers free delivery of cookstoves. He sets a price $\$p$ and promises to deliver for free to any farmer who lives within $p - 20$ miles of him. (He won't deliver to anyone who lives further than that, because it then costs him more than $\$p$ to buy a stove and deliver it.) If he is going to price in this way,

____

how high should he set p? ________ How many cookstoves would Ben deliver?________ How much would his total revenue be? ______ ______ How much would his total costs be, including the cost of deliveries and the cost of buying the stoves? __________ (Hint: For any n, the sum of the series $1 + 2 + \ldots + n$ is equal to $n(n + 1)/2$.) How much profit would he make? __________ Can you explain why it is more profitable for Ben to use this pricing scheme where he pays the cost of delivery himself rather than the scheme where the farmers pay for their own deliveries?______________________________

____

____

____.

27.11 (2) Perhaps you wondered what Ben Kinmore, who lives off in the woods quietly collecting his monopoly profits, is doing in this chapter on oligopoly. Well, unfortunately for Ben, before he got around to selling any stoves, the railroad built a track to the town of Deep Furrow, just 40 miles down the road, west of Bouncing Springs. The storekeeper in Deep Furrow, Huey Sunshine, was also able to get cookstoves delivered by train to his store for \$20 each. Huey and Ben were the only stove dealers on the road. Let us concentrate our attention on how they would compete for the customers who lived between them. We can do this, because Ben can charge different base prices for the cookstoves he ships east and the cookstoves he ships west. So can Huey.

Suppose that Ben sets a base price, p_B, for stoves he sends west and adds a charge of \$1 per mile for delivery. Suppose that Huey sets a base price, p_H, for stoves he sends east and adds a charge of \$1 per mile for delivery. Farmers who live between Ben and Huey would buy from the seller who is willing to deliver most cheaply to them (so long as the delivered price does not exceed \$120). If Ben's base price is p_B and Huey's base price is p_H, somebody who lives x miles west of Ben would have to pay a total of $p_B + x$ to have a stove delivered from Ben and $p_H + (40 - x)$ to have a stove delivered by Huey.

(a) If Ben's base price is p_B and Huey's is p_H, write down an equation that could be solved for the distance x^* to the west of Bouncing Springs that Ben's market extends. ________________ If Ben's base price is p_B and Huey's is p_H, then Ben will sell ____________ cookstoves and Huey will sell ____________ cookstoves.

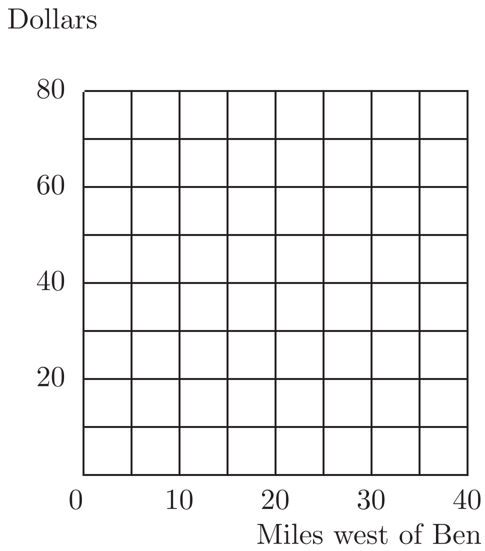
(b) Recalling that Ben makes a profit of $p_B - 20$ on every cookstove that he sells, Ben's profits can be expressed as the following function of p_B and p_H. ____.

(c) If Ben thinks that Huey's price will stay at p_H, no matter what price Ben chooses, what choice of p_B will maximize Ben's profits? ____________________ (Hint: Set the derivative of Ben's profits with respect to his price equal to zero.) Suppose that Huey thinks that Ben's price will stay at p_B, no matter what price Huey chooses, what choice of p_H will maximize Huey's profits? ________________ (Hint: Use the symmetry of the problem and the answer to the last question.)

(d) Can you find a base price for Ben and a base price for Huey such that each is a profit-maximizing choice given what the other guy is doing? (Hint: Find prices p_B and p_H that simultaneously solve the last two equations.) ________________ How many cookstoves does Ben sell to farmers living west of him? ______ How much profit does he make on these sales? ____.

(e) Suppose that Ben and Huey decided to compete for the customers who live between them by price discriminating. Suppose that Ben offers to deliver a stove to a farmer who lives x miles west of him for a price equal to the maximum of Ben's total cost of delivering a stove to that farmer and Huey's total cost of delivering to the same farmer less 1 penny. Suppose that Huey offers to deliver a stove to a farmer who lives x miles west of Ben for a price equal to the maximum of Huey's own total cost of delivering to this farmer and Ben's total cost of delivering to him less a penny. For example, if a farmer lives 10 miles west of Ben, Ben's total cost of delivering to him is \$30, \$20 to get the stove and \$10 for hauling it 10 miles west. Huey's total cost of delivering it to him is \$50, \$20 to get the stove and \$30 to haul it 30 miles east. Ben will charge the maximum of his own cost, which is \$30, and Huey's cost less a penny, which is \$49.99. The maximum of these two numbers is __________ . Huey will charge the maximum of his own total cost of delivering to this farmer, which is \$50, and Ben's cost less a penny, which is \$29.99. Therefore Huey will charge __________ to deliver to this farmer. This farmer will buy from ______ whose price to him is cheaper by one penny. When the two merchants have this pricing policy, all farmers who live within ________ miles of Ben will buy from Ben and all farmers who live within ________ miles of Huey will buy from Huey. A farmer who lives x miles west of Ben

and buys from Ben must pay ________ dollars to have a cookstove delivered to him. A farmer who lives x miles east of Huey and buys from Huey must pay ________ for delivery of a stove. On the graph below, use blue ink to graph the cost to Ben of delivering to a farmer who lives x miles west of him. Use red ink to graph the total cost to Huey of delivering a cookstove to a farmer who lives x miles west of Ben. Use pencil to mark the lowest price available to a farmer as a function of how far west he lives from Ben.



(f) With the pricing policies you just graphed, which farmers get stoves delivered most cheaply, those who live closest to the merchants or those who live midway between them? ________________
On the graph you made, shade in the area representing each merchant's profits. How much profits does each merchant make? ________ If Ben and Huey are pricing in this way, is there any way for either of them to increase his profits by changing the price he charges to some farmers?

________________________________.

In this introduction we offer two examples of two-person games. The first game has a dominant strategy equilibrium. The second game is a zero-sum game that has a Nash equilibrium in pure strategies that is not a dominant strategy equilibrium.

Albert and Victoria are roommates. Each of them prefers a clean room to a dirty room, but neither likes housecleaning. If both clean the room, they each get a payoff of 5. If one cleans and the other doesn't clean, the person who does the cleaning has a utility of 2, and the person who doesn't clean has a utility of 6. If neither cleans, the room stays a mess and each has a utility of 3. The payoffs from the strategies "Clean" and "Don't Clean" are shown in the box below.

Clean Room–Dirty Room

		Victoria: Clean	Victoria: Don't Clean
Albert	Clean	$5, 5$	$2, 6$
Albert	Don't Clean	$6, 2$	$3, 3$

In this game, whether or not Victoria chooses to clean, Albert will get a higher payoff if he doesn't clean than if he does clean. Therefore "Don't Clean" is a dominant strategy for Albert. Similar reasoning shows that no matter what Albert chooses to do, Victoria is better off if she chooses "Don't Clean." Therefore the outcome where both roommates choose "Don't Clean" is a dominant strategy equilibrium. This is true despite the fact that both persons would be better off if they both chose to clean the room.

This game is set in the South Pacific in 1943. Admiral Imamura must transport Japanese troops from the port of Rabaul in New Britain, across the Bismarck Sea to New Guinea. The Japanese fleet could either travel north of New Britain, where it is likely to be foggy, or south of New Britain, where the weather is likely to be clear. U.S. Admiral Kenney hopes to bomb the troop ships. Kenney has to choose whether to concentrate his reconnaissance aircraft on the Northern or the Southern route. Once he finds the convoy, he can bomb it until its arrival in New Guinea. Kenney's staff has estimated the number of days of bombing time for each of the outcomes. The payoffs to Kenney and Imamura from each outcome are shown in the box below. The game is modeled as a "zero-sum game:" for each outcome, Imamura's payoff is the negative of Kenney's payoff.

The Battle of the Bismarck Sea

		Imamura	
		North	South
Kenney	North	$2, -2$	$2, -2$
	South	$1, -1$	$3, -3$

This game does not have a dominant strategy equilibrium, since there is no dominant strategy for Kenney. His best choice depends on what Imamura does. The only Nash equilibrium for this game is where Imamura chooses the northern route and Kenney concentrates his search on the northern route. To check this, notice that if Imamura goes North, then Kenney gets an expected two days of bombing if he (Kenney) chooses North and only one day if he (Kenney) chooses South. Furthermore, if Kenney concentrates on the north, Imamura is indifferent between going north or south, since he can be expected to be bombed for two days either way. Therefore if both choose "North," then neither has an incentive to act differently. You can verify that for any other combination of choices, one admiral or the other would want to change. As things actually worked out, Imamura chose the Northern route and Kenney concentrated his search on the North. After about a day's search the Americans found the Japanese fleet and inflicted heavy damage on it.*

28.1 (0) This problem is designed to give you practice in reading a game matrix and to check that you understand the definition of a dominant strategy. Consider the following game matrix.

A Game Matrix

		Player B	
		Left	Right
Player A	Top	a, b	c, d
	Bottom	e, f	g, h

* This example is discussed in R. Duncan Luce and Howard Raiffa's *Games and Decisions*, John Wiley, 1957, or Dover, 1989. We recommend this book to anyone interested in reading more about game theory.

____

(a) If (top, left) is a dominant strategy equilibrium, then we know that

$a >$________ , $b >$________ , ________ $> g$, and ________ $> h$.

(b) If (top, left) is a Nash equilibrium, then which of the above inequalities

must be satisfied?____.

(c) If (top, left) is a dominant strategy equilibrium must it be a

Nash equilibrium? Why?____________________________________

____.

28.2 (0) In order to learn how people actually play in game situations, economists and other social scientists frequently conduct experiments in which subjects play games for money. One such game is known as the *voluntary public goods game*. This game is chosen to represent situations in which individuals can take actions that are costly to themselves but that are beneficial to an entire community.

In this problem we will deal with a two-player version of the voluntary public goods game. Two players are put in separate rooms. Each player is given \$10. The player can use this money in either of two ways. He can *keep* it or he can *contribute* it to a "public fund." Money that goes into the public fund gets multiplied by 1.6 and then divided equally between the two players. If both contribute their \$10, then each gets back $\$20 \times 1.6/2 = \$16$. If one contributes and the other does not, each gets back $\$10 \times 1.6/2 = \$8$ from the public fund so that the contributor has \$8 at the end of the game and the non-contributor has \$18–his original \$10 plus \$8 back from the public fund. If neither contributes, both have their original \$10. The payoff matrix for this game is:

Voluntary Public Goods Game

		Player B: Contribute	Player B: Keep
Player A	Contribute	\$16, \$16	\$8, \$18
	Keep	\$18, \$8	\$10, \$10

(a) If the other player keeps, what is your payoff if you keep? __________

If the other player keeps, what is your payoff if you contribute?________.

(b) If the other player contributes, what is your payoff if you keep? __________ If the other player contributes, what is your payoff if you contribute?____.

(c) Does this game have a dominant strategy equilibrium? __________ If so, what is it?____.

28.3 (1) Let us consider a more general version of the voluntary public goods game described in the previous question. This game has N players, each of whom can contribute either \$10 or nothing to the public fund. All money that is contributed to the public fund gets multiplied by some number $B > 1$ and then divided equally among all players in the game (including those who do not contribute.) Thus if all N players contribute \$10 to the fund, the amount of money available to be divided among the N players will be $\$10BN$ and each player will get $\$10BN/N = \$10B$ back from the public fund.

(a) If $B > 1$, which of the following outcomes gives the higher payoff to each player? *a)* All players contribute their \$10 or *b)* all players keep their \$10.____.

(b) Suppose that exactly K of the other players contribute. If you keep your \$10, you will have this \$10 plus your share of the public fund contributed by others. What will your payoff be in this case? __________________ . If you contribute your \$10, what will be the total number of contributors?__________ What will be your payoff?__________________.

(c) If $B = 3$ and $N = 5$, what is the dominant strategy equilibrium for this game?____________________ Explain your answer.____________________

____

____

____

____.

____

(d) In general, what relationship between B and N must hold for "Keep" to be a dominant strategy?___________________________________

____

____.

(e) Sometimes the action that maximizes a player's *absolute* payoff, does not maximize his *relative* payoff. Consider the example of a voluntary public goods game as described above, where $B = 6$ and $N = 5$. Suppose that four of the five players in the group contribute their \$10, while the fifth player keeps his \$10. What is the payoff of each of the four contributors? _______________ What is the payoff of the player who keeps his \$10? __________________ Who has the highest payoff in the group? ____________________________ What would be the payoff to the fifth player if instead of keeping his \$10, he contributes, so that all five players contribute.________________ If the other four players contribute, what should the fifth player to maximize his *absolute* payoff?____________ What should he do to maximize his payoff *relative* to that of the other players?____.

(f) If $B = 6$ and $N = 5$, what is the dominant strategy equilibrium for this game?_______________ Explain your answer.____________________

____

____

____

____.

28.4 (1) The Stag Hunt game is based on a story told by Jean Jacques Rousseau in his book *Discourses on the Origin and Foundation of Inequality Among Men* (1754). The story goes something like this: "Two hunters set out to kill a stag. One has agreed to drive the stag through the forest, and the other to post at a place where the stag must pass. If both faithfully perform their assigned stag-hunting tasks, they will surely kill the stag and each will get an equal share of this large animal. During the course of the hunt, each hunter has an opportunity to abandon the stag hunt and to pursue a hare. If a hunter pursues the hare instead of the stag he is certain to catch the hare and the stag is certain to escape. Each hunter would rather share half of a stag than have a hare to himself."

The matrix below shows payoffs in a stag hunt game. If both hunters hunt stag, each gets a payoff of 4. If both hunt hare, each gets 3. If one hunts stag and the other hunts hare, the stag hunter gets 0 and the hare hunter gets 3.

The Stag Hunt Game

		Hunter B	
		Hunt Stag	Hunt Hare
Hunter A	Hunt Stag	$4, 4$	$0, 3$
	Hunt Hare	$3, 0$	$3, 3$

(a) If you are sure that the other hunter will hunt stag, what is the best thing for you to do?____.

(b) If you are sure that the other hunter will hunt hare, what is the best thing for you to do?____.

(c) Does either hunter have a dominant strategy in this game?________ If so, what is it? If not explain why not.____________________________

____.

(d) This game has two pure strategy Nash equilibria. What are they?

____.

(e) Is one Nash equilibrium better for both hunters than the other?

________ If so, which is the better equilibrium?___________________________.

(f) If a hunter believes that with probability 1/2 the other hunter will hunt stag and with probability 1/2 he will hunt hare, what should this hunter do to maximize his expected payoff?______________________________.

28.5 (1) Evangeline and Gabriel met at a freshman mixer. They want desperately to meet each other again, but they forgot to exchange names or phone numbers when they met the first time. There are two possible strategies available for each of them. These are Go to the Big Party or Stay Home and Study. They will surely meet if they both go to the party, and they will surely not otherwise. The payoff to meeting is 1,000 for each of them. The payoff to not meeting is zero for both of them. The payoffs are described by the matrix below.

Close Encounters of the Second Kind

		Gabriel	
		Go to Party	Stay Home
Evangeline	Go to Party	$1000, 1000$	$0, 0$
	Stay Home	$0, 0$	$0, 0$

(a) A strategy is said to be a weakly dominant strategy for a player if the payoff from using this strategy is *at least as high* as the payoff from using any other strategy. Is there any outcome in this game where both players are using weakly dominant strategies?______________________

____.

(b) Find all of the pure-strategy Nash equilibria for this game.__________

____.

(c) Do any of the pure Nash equilibria that you found seem more reasonable than others? Why or why not?______________________________

____

____.

(d) Let us change the game a little bit. Evangeline and Gabriel are still desperate to find each other. But now there are two parties that they can go to. There is a little party at which they would be sure to meet if they both went there and a huge party at which they might never see each other. The expected payoff to each of them is 1,000 if they both go to the little party. Since there is only a 50-50 chance that they would find each other at the huge party, the expected payoff to each of them is only 500. If they go to different parties, the payoff to both of them is zero. The payoff matrix for this game is:

More Close Encounters

		Gabriel: Little Party	Gabriel: Big Party
Evangeline	Little Party	$1000, 1000$	$0, 0$
	Big Party	$0, 0$	$500, 500$

(e) Does this game have a dominant strategy equilibrium?_______ What are the two Nash equilibria in pure strategies?________________________

____.

(f) One of the Nash equilibria is Pareto superior to the other. Suppose that each person thought that there was some slight chance that the other would go to the little party. Would that be enough to convince them both to attend the little party?_______ Can you think of any reason why the Pareto superior equilibrium might emerge if both players understand the game matrix, if both know that the other understands it, and each knows that the other knows that he or she understands the game matrix?____

____.

28.6 (1) The introduction to this chapter of *Workouts*, recounted the sad tale of roommates Victoria and Albert and their dirty room. The payoff matrix for their relationship was given as follows.

Domestic Life with Victoria and Albert

		Victoria: Clean	Victoria: Don't Clean
Albert	Clean	$5, 5$	$2, 6$
	Don't Clean	$6, 2$	$3, 3$

Suppose that we add a second stage to this game in which Victoria and Albert each have a chance to punish the other. Imagine that at the end of the day, Victoria and Albert are each able to see whether the other has done any housecleaning. After seeing what the other has done, each has the option of starting a quarrel. A quarrel hurts both of them, regardless of who started it. Thus we will assume that if either or both of them starts a quarrel, the day's payoff for each of them is reduced by 2. (For example if Victoria cleans and Albert doesn't clean and if Victoria, on seeing this result, starts a quarrel, Albert's payoff will be $6 - 2 = 4$ and Victoria's will be $2 - 2 = 0$.)

(a) Suppose that it is evening and Victoria sees that Albert has chosen not to clean and she thinks that he will not start a quarrel. Which strategy will give her a higher payoff for the whole day, Quarrel or Not Quarrel?

______________________________.

(b) Suppose that Victoria and Albert each believe that the other will try to take the actions that will maximize his or her total payoff for the day.

Does either believe the other will start a quarrel?________ Assuming that each is trying to maximize his or her own payoff, given the actions of the other, what would you expect each of them to do in the first stage of the game, clean or not Clean?______________________________.

(c) Suppose that Victoria and Albert are governed by emotions that they cannot control. Neither can avoid getting angry if the other does not clean. And if either one is angry, they will quarrel so that the payoff of each is diminished by 2. Given that there is certain to be a quarrel if either does not clean, the payoff matrix for the game between Victoria and Albert becomes:

Vengeful Victoria and Angry Albert

		Victoria: Clean	Victoria: Don't Clean
Albert	Clean	$5, 5$	$0, 4$
	Don't Clean	$4, 0$	$1, 1$

(d) If the other player cleans, is it better to clean or not clean? __________

If the other player does not clean, is it better to clean or not clean.________

__________ Explain______________________________

____

____

____.

(e) Does this game have a dominant strategy? ______ Explain______

____.

(f) This game has two Nash equilibria. What are they?______________

____.

(g) Explain how it could happen that Albert and Victoria would both be better off if both are easy to anger than if they are rational about when to get angry, but it might also happen that they would both be worse off.____

____

____.

(h) Suppose that Albert and Victoria are both aware that Albert will get angry and start a quarrel if Victoria does not clean, but that Victoria is level-headed and will not start a quarrel. What would be the equilibrium outcome?______________________________________

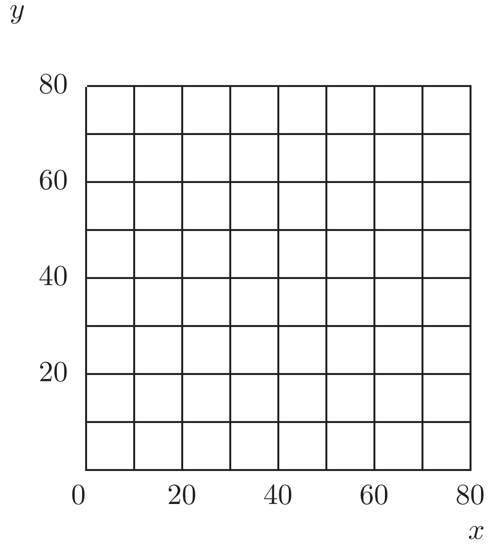
____.

28.7 (1) Maynard's Cross is a trendy bistro that specializes in carpaccio and other uncooked substances. Most people who come to Maynard's come to see and be seen by other people of the kind who come to Maynard's. There is, however, a hard core of 10 customers per evening who come for the carpaccio and don't care how many other people come. The number of additional customers who appear at Maynard's depends on how many people they expect to see. In particular, if people expect that the number of customers at Maynard's in an evening will be X, then the number of people who actually come to Maynard's is $Y = 10 + .8X$. In equilibrium, it must be true that the number of people who actually attend the restaurant is equal to the number who are expected to attend.

(a) What two simultaneous equations must you solve to find the equilibrium attendance at Maynard's?______________________________.

(b) What is the equilibrium nightly attendance?____________________.

(c) On the following axes, draw the lines that represent each of the two equations you mentioned in Part (a). Label the equilibrium attendance level.



(d) Suppose that one additional carpaccio enthusiast moves to the area. Like the other 10, he eats at Maynard's every night no matter how many others eat there. Write down the new equations determining attendance at Maynard's and solve for the new equilibrium number of customers.

____.

(e) Use a different color ink to draw a new line representing the equation that changed. How many additional customers did the new steady customer attract (besides himself)?____________________.

(f) Suppose that everyone bases expectations about tonight's attendance on last night's attendance and that last night's attendance is public knowledge. Then $X_t = Y_{t-1}$, where X_t is expected attendance on day t and Y_{t-1} is actual attendance on day $t-1$. At any time t, $Y_t = 10 + .8X_t$. Suppose that on the first night that Maynard's is open, attendance is 20.

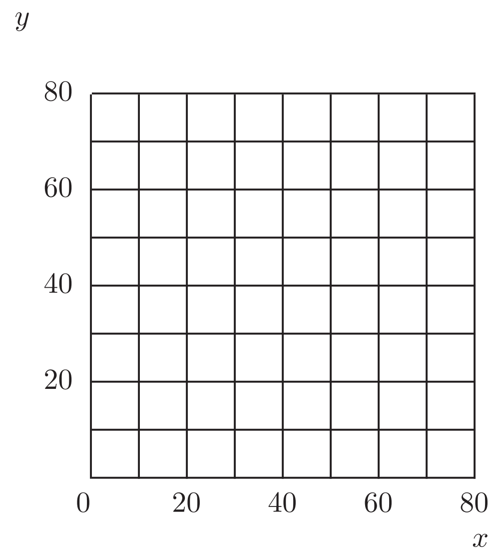
What will be attendance on the second night?____________________.

(g) What will be the attendance on the third night?____________________.

(h) Attendance will tend toward some limiting value. What is it?______.

28.8 (0) Yogi's Bar and Grill is frequented by unsociable types who hate crowds. If Yogi's regular customers expect that the crowd at Yogi's will be X, then the number of people who show up at Yogi's, Y, will be the larger of the two numbers, $120 - 2X$ and 0. Thus $Y = \max\{120 - 2X, 0\}$.

(a) Solve for the equilibrium attendance at Yogi's. Draw a diagram depicting this equilibrium on the axes below.



(b) Suppose that people expect the number of customers on any given night to be the same as the number on the previous night. Suppose that 50 customers show up at Yogi's on the first day of business. How many will show up on the second day? ______ The third day? ______ The fourth day? ______ The fifth day? ______ The sixth day? ______ The ninety-ninth day? ______ The hundredth day? ______.

(c) What would you say is wrong with this model if at least some of Yogi's customers have memory spans of more than a day or two?______

______

______

______.