Seminář 10

$with(plots) : with(DEtools) :$

Příklad 2:Ověřte, že je rovnice exaktní a vyřešte ji postupnou integrací dlejednotlivých proměnných a) (2xt-2x-1)dx+ (x^2+ 2t+ 1)dt= 0

$Q := (x, t) \to 2\,x \cdot t - 2\,x - 1;\ P := (x, t) \to x\text{^}2 + 2\,t + 1;$

$$(x, t) \to 2\,x\,t - 2\,x - 1$$

$$(x, t) \to x^2 + 2\,t + 1$$

$diff\,(Q(x, t), t);\ diff\,(P(x, t), x);$

$$2\,x$$

$$2\,x$$

$\mathrm{U} := \mathrm{int}(P(x, t), \mathrm{t});$

$$x^2\,t + t^2 + t$$

Derivace se shodují, rovnice je exaktní.

$K := int(2\,x \cdot t - 2\,x - 1 - diff\,(\mathrm{int}(P(x, t), \mathrm{t}), x), x);$

$$-x^2 - x$$

$F = U + K;$

$$F = x^2\,t + t^2 + t - x^2 - x$$

b) $\left(\frac{\ln x}{t^2} - t\right)dt = \frac{1}{xt}\,dx$

$P := (x, t) \to \frac{\log(x)}{t^2} - t\,;\ Q := (x, t) \to \frac{-1}{x \cdot t}\,;$

$$(x, t) \to \frac{\log(x)}{t^2} - t$$

$$(x, t) \to -\frac{1}{x\,t}$$

$diff\,(Q(x, t), t);\ diff\,(P(x, t), x);$

$$\frac{1}{x\,t^2}$$

$$\frac{1}{x\,t^2}$$

Derivace se shodují, rovnice je exaktní

$\mathrm{U} := \mathrm{int}(P(x, t), \mathrm{t});$

$$-\frac{\ln(x)}{t} - \frac{1}{2}\,t^2$$

$K := int\left(-\frac{1}{(x \cdot t)} - diff\,(\mathrm{int}(P(x, t), \mathrm{t}), x), x\right);$

$$0$$

$F = U + K;$

$$F = -\frac{\ln(x)}{t} - \frac{1}{2}\,t^2$$

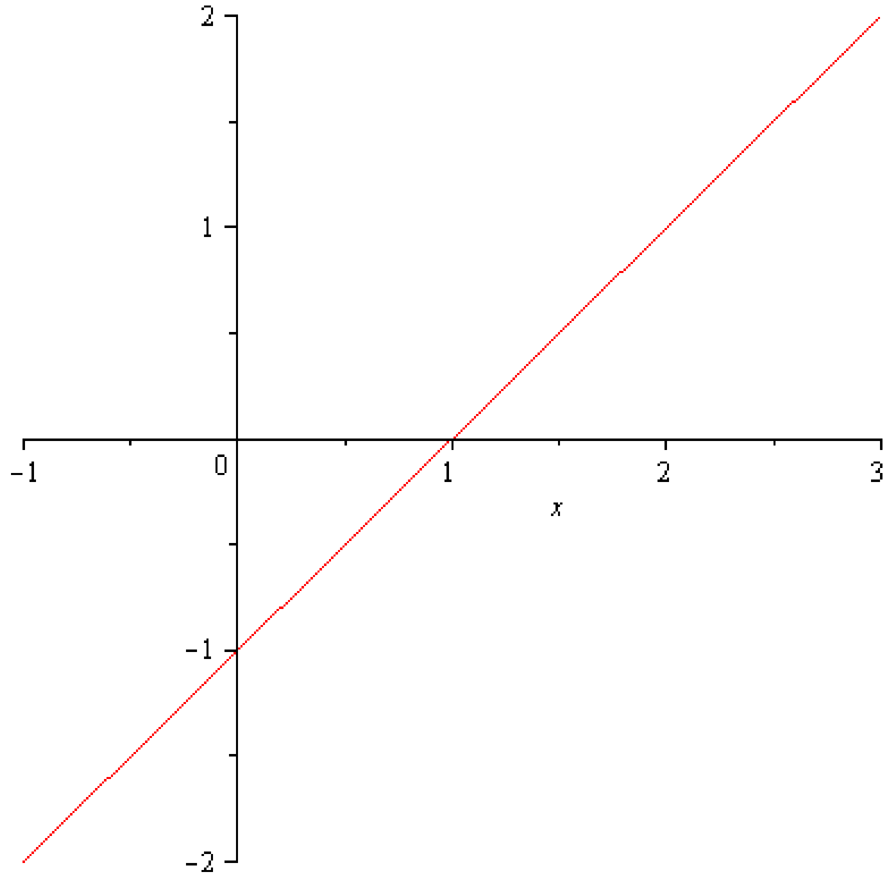
Příklad 3: Načrtněte fázový diagram spojený s diferenciální rovnicí a určete rovnovážný stav.

a)

$P := x \rightarrow x - 1;$

$$x \rightarrow x - 1$$

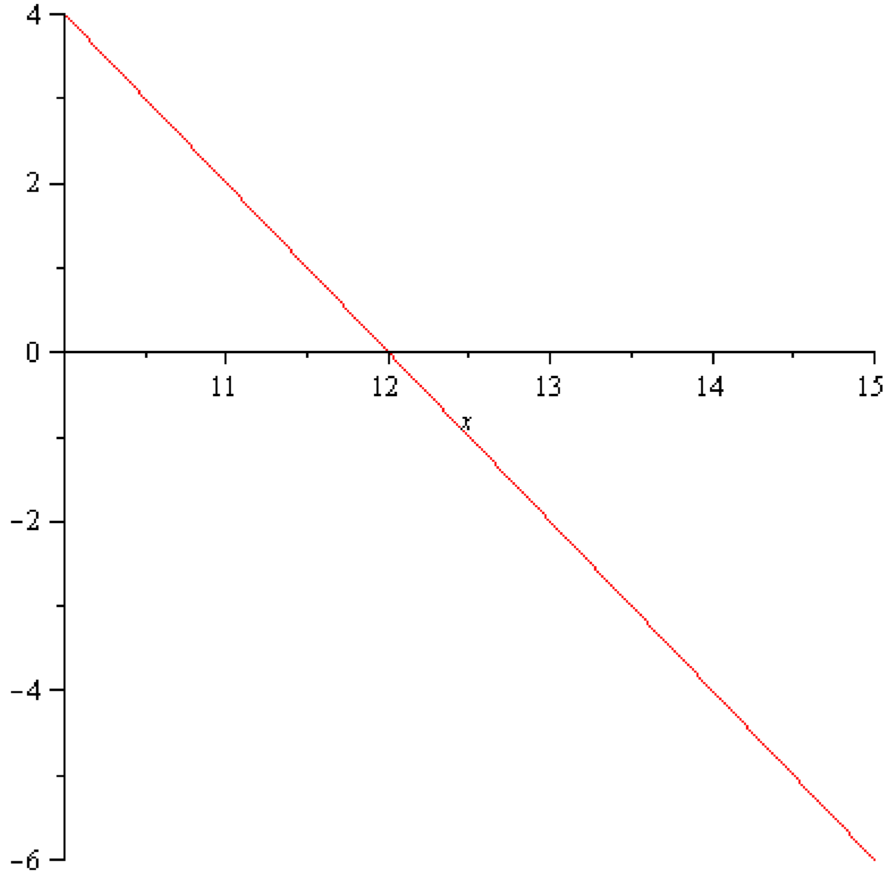
$plot(P(x), x = -1..3);$



b) $P := x \rightarrow 24 - 2x;$

$$x \rightarrow 24 - 2x$$

$plot(P(x), x = 10..15);$

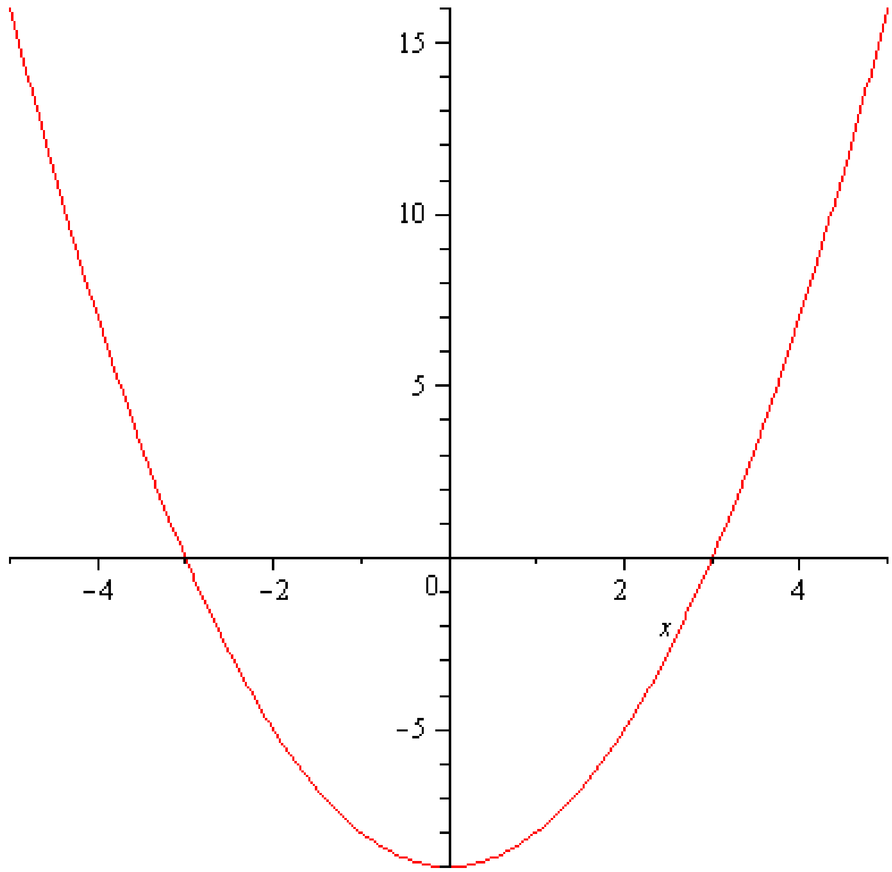


c)

$P := x \to x^2 - 9;$

$$x \to x^2 - 9$$

$plot(P(x), x = -5..5);$



Příklad 4: Určete typ rovnovážných bodů pro:

a) $solve\left(x^4 - 1 = 0, x\right);$

$$1, -1, I, -I$$

$Q := diff\left(x^4 - 1, x\right);$

$$4\,x^3$$

$eval(Q, x = 1);$

$$4$$

$eval(Q, x = -1)\ ;$

$$-4$$

b) $solve\left(3\,x^2 - 1 = 0, x\right);$

$$\frac{1}{3}\sqrt{3}, -\frac{1}{3}\sqrt{3}$$

$Q := diff\left(3\,x^2 - 1, x\right);$

$$6\,x$$

$eval\left(Q, x = \frac{1}{3}\sqrt{3}\right);$

$$2\sqrt{3}$$

$eval\left(Q, x = -\frac{1}{3}\sqrt{3}\right)\ ;$

$$-2\sqrt{3}$$

c) $solve(\mathrm{x}\cdot\exp(\mathrm{x}) = 0, x);$

$$0$$

$Q := diff(\mathrm{x}\cdot\exp(\mathrm{x}), x);$

$$\mathrm{e}^{x} + x\,\mathrm{e}^{x}$$

$eval(Q, x = 0);$

$$1$$

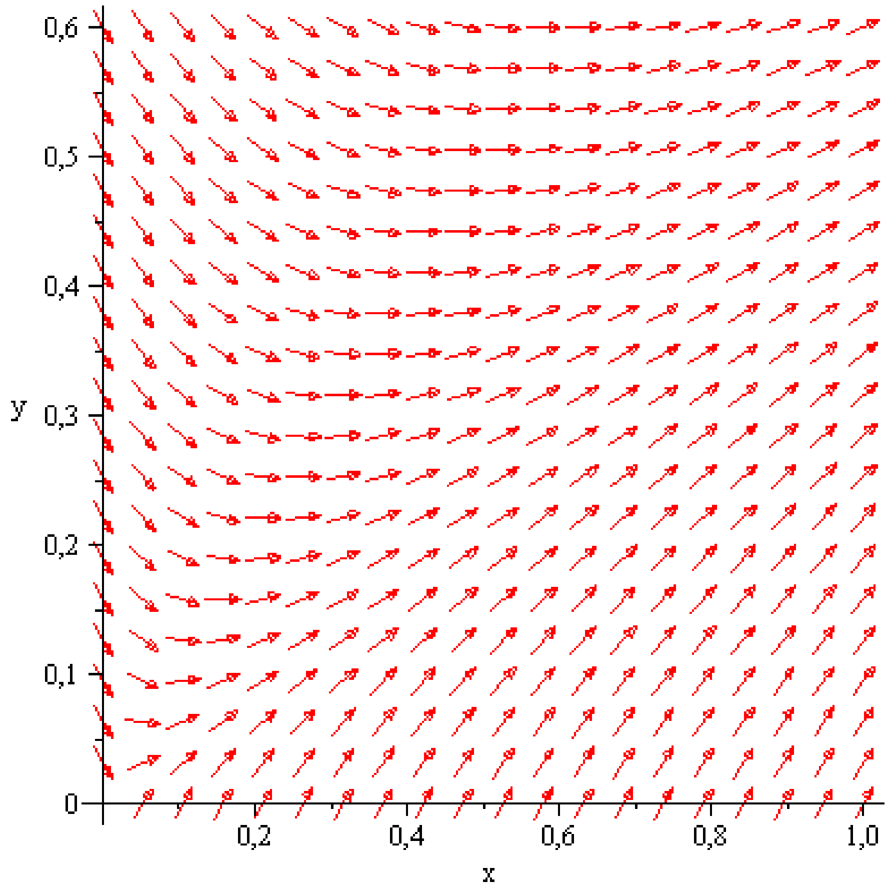
Příklad 5: Najděte obecné řešení systému použitím metody vlastních čísel arozhodněte, zda je systém globálně asymptoticky stabilní.

a) $de1 := diff(x(t), t) = x(t) + y(t);\ de2 := diff(y(t), t) = x(t) - y(t);$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,x(t) = x(t) + y(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,y(t) = x(t) - y(t)$$

$DEplot([de1, de2], [x(t), y(t)], t = 0\,..40, x = 0\,..1, y = 0\,..0.6,$
$arrows = medium);$



dsolve([*de1*, *de2*]);

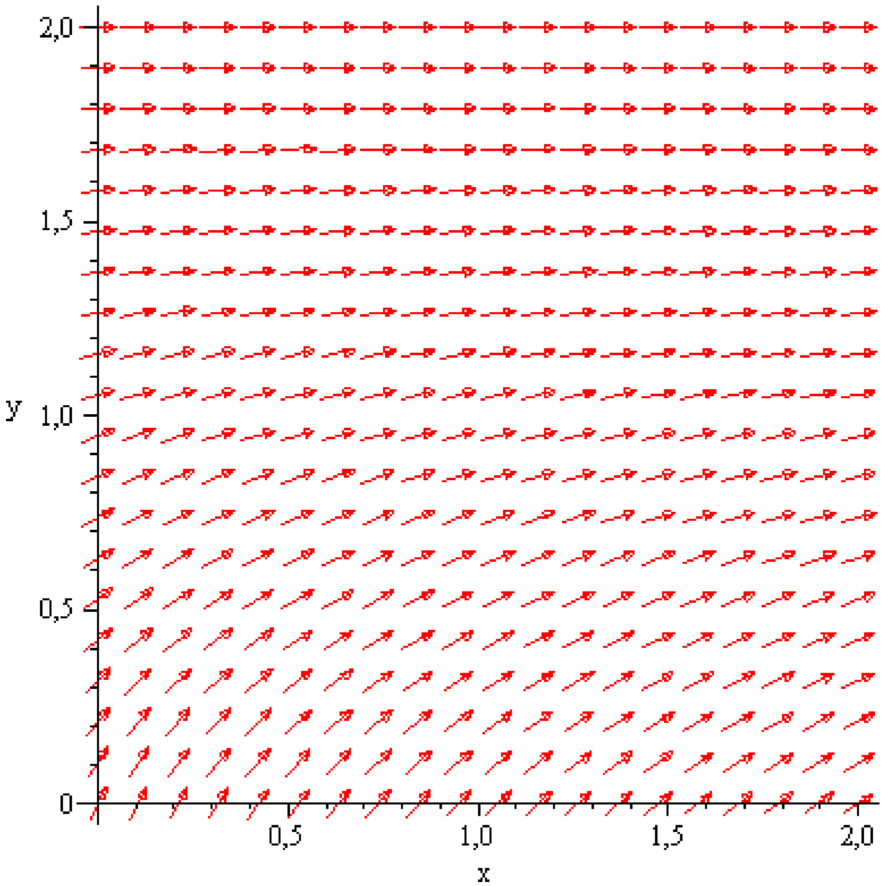
$$\left\{x(t) = \_C1\,\sqrt{2}\,e^{\sqrt{2}\,t} - \_C2\,\sqrt{2}\,e^{-\sqrt{2}\,t} + \_C1\,e^{\sqrt{2}\,t} + \_C2\,e^{-\sqrt{2}\,t}, y(t) = \_C1\,e^{\sqrt{2}\,t} + \_C2\,e^{-\sqrt{2}\,t}\right\}$$

b) *de1* := *diff*(*x*(*t*), *t*) = *x*(*t*) + 2 *y*(*t*) + 1;
de2 := *diff*(*y*(*t*), *t*) = 2 − *y*(*t*);

$$\frac{d}{dt}\,x(t) = x(t) + 2\,y(t) + 1$$

$$\frac{d}{dt}\,y(t) = 2 - y(t)$$

DEplot([*de1*, *de2*], [*x*(*t*), *y*(*t*)], *t* = 0 ..40, *x* = 0 ..2, *y* = 0 ..2,
arrows = *medium*);



dsolve([*de1*, *de2*]);

$$\left\{x(t) = -5 - e^{-t}\_C2 + e^{t}\_C1, y(t) = 2 + e^{-t}\_C2\right\}$$

Příklad 6: Najděte řešení systému s počáteční podmínkou.

a) *de1* := *diff*(*x*(*t*), *t*) = *x*(*t*) − 8 *y*(*t*);
de2 := *diff*(*y*(*t*), *t*) = 2 *x*(*t*) − 9·*y*(*t*);

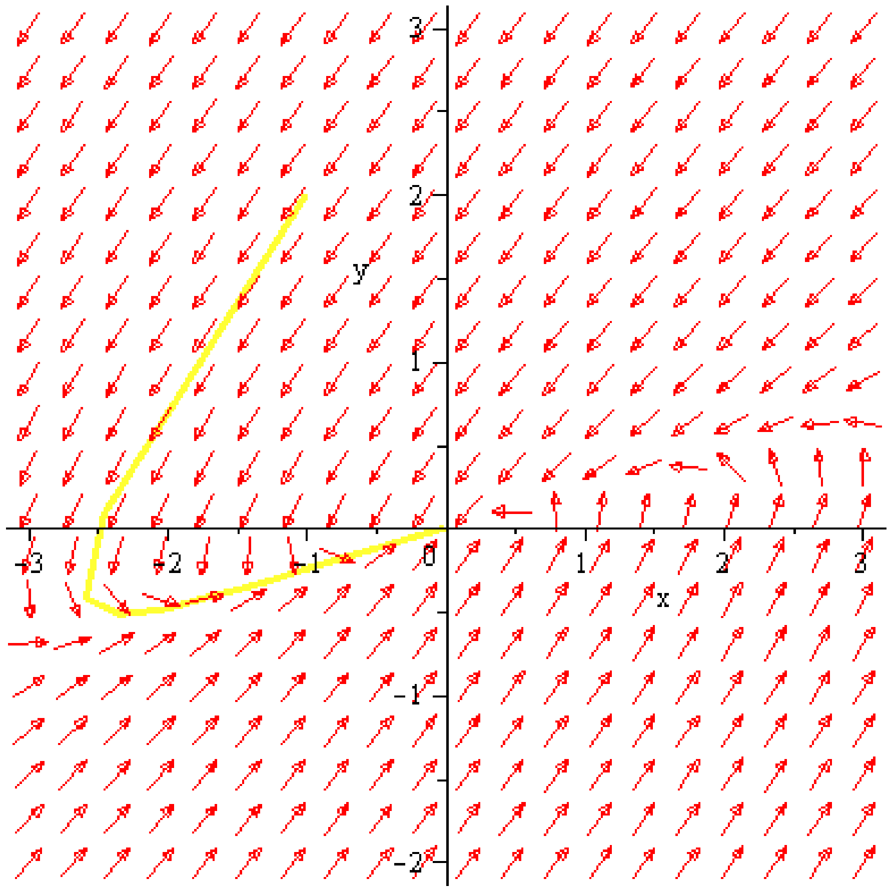
$$\frac{d}{dt}x(t) = x(t) - 8\,y(t)$$

$$\frac{d}{dt}y(t) = 2\,x(t) - 9\,y(t)$$

dsolve({*de1*, *de2*, *x*(0) =−1, *y*(0) = 2}) ;

$$\left\{y(t) = -e^{-t} + 3\,e^{-7t}, x(t) = -4\,e^{-t} + 3\,e^{-7t}\right\}$$

DEplot([*de1*, *de2*], [*x*(*t*), *y*(*t*)], *t* = 0 ..8, *x* =−3 ..3, *y* =−2 ..3,
arrows = *medium*, [[*x*(0) =−1, *y*(0) = 2]], *animatecurves* = *true*);



b) $de1 := diff(x(t), t) = -2\,x(t) - 3\,y(t);$
$de2 := diff(y(t), t) = -x(t) - 4 \cdot y(t);$

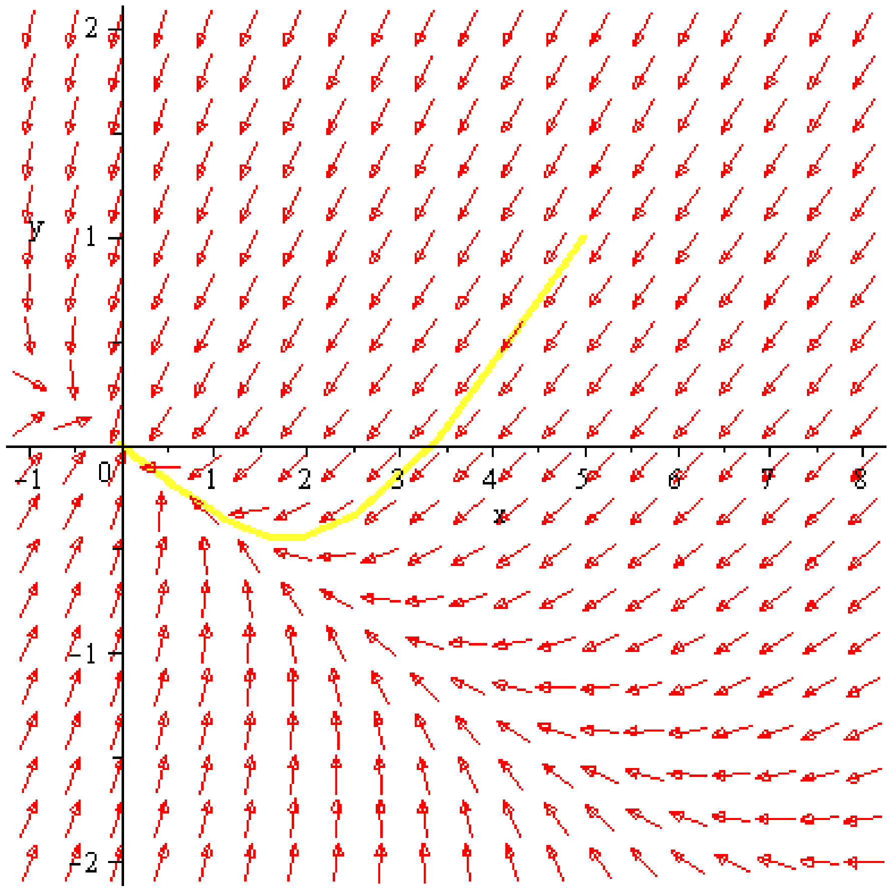
$$\frac{d}{dt}\,x(t) = -2\,x(t) - 3\,y(t)$$

$$\frac{d}{dt}\,y(t) = -x(t) - 4\,y(t)$$

$dsolve(\{de1, de2, x(0) = 5, y(0) = 1\})\;;$

$$\{x(t) = 2\,e^{-5\,t} + 3\,e^{-t}, y(t) = 2\,e^{-5\,t} - e^{-t}\}$$

$DEplot(\;[de1, de2], [x(t), y(t)], t = 0..8, x = -1..8, y = -2..2,$
$arrows = medium, [[x(0) = 5, y(0) = 1]], animatecurves = true\;);$


y
x
2
1
–1
–2
–1
0
1
2
3
4
5
6
7
8