Seminář 9

with(*plots*) : *with*(*DEtools*) :

Příklad 1: a) Ukažte, že x:=t->C*exp(-t)+exp(t)/2;

$$t \to C\,e^{-t} + \frac{1}{2}\,e^{t}$$

je pro libovolnou hodnotu konstanty C řešením diferenciální rovnice x'(t) + x(t) = exp(t).

$P = \exp(t);\ L = diff(x(t), t) + x(t);$

$$P = e^{t}$$

$$L = e^{t}$$

b)Ukažte, že $x := t \to C*t\hat{}2;$

$$t \to C\,t^{2}$$

je pro libovolnou hodnotu konstanty C řešením diferenciální rovnice t*x'(t) = 2*x(t).

$P = 2\cdot x(t);\ L = t\cdot diff(x(t), t);$

$$P = 2\,C\,t^{2}$$

$$L = 2\,C\,t^{2}$$

Najděte partikulární řešení procházející bodem (1; 2).

solve(*C* * 1^2 = 2, *C*);

Příklad 3: Řešte následující diferenciální rovnice:

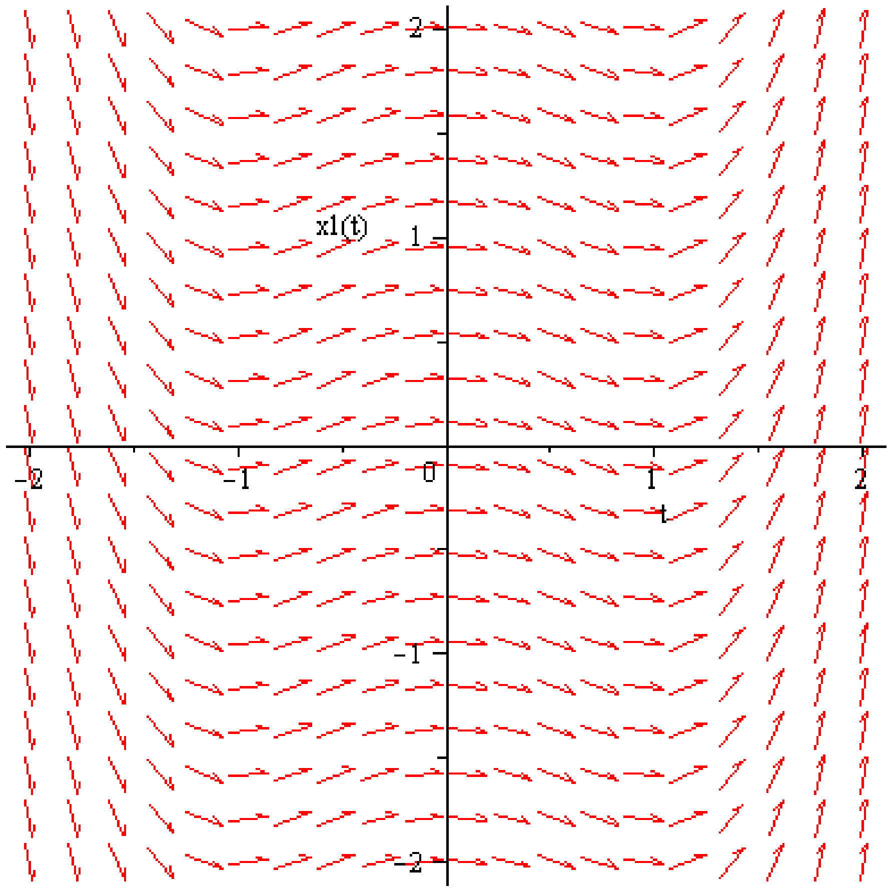
a) $ODE1 := diff(x1(t), t) = t^{3} - t;$

$$\frac{d}{dt}\,x1(t) = t^{3} - t$$

dsolve(*ODE1*, *x1*(*t*));

$$x1(t) = \frac{1}{4}\,t^{4} - \frac{1}{2}\,t^{2} + \_C1$$

dfieldplot(*ODE1*, *x1*(*t*), *t* = −2 ..2, *x1* = −2 ..2);



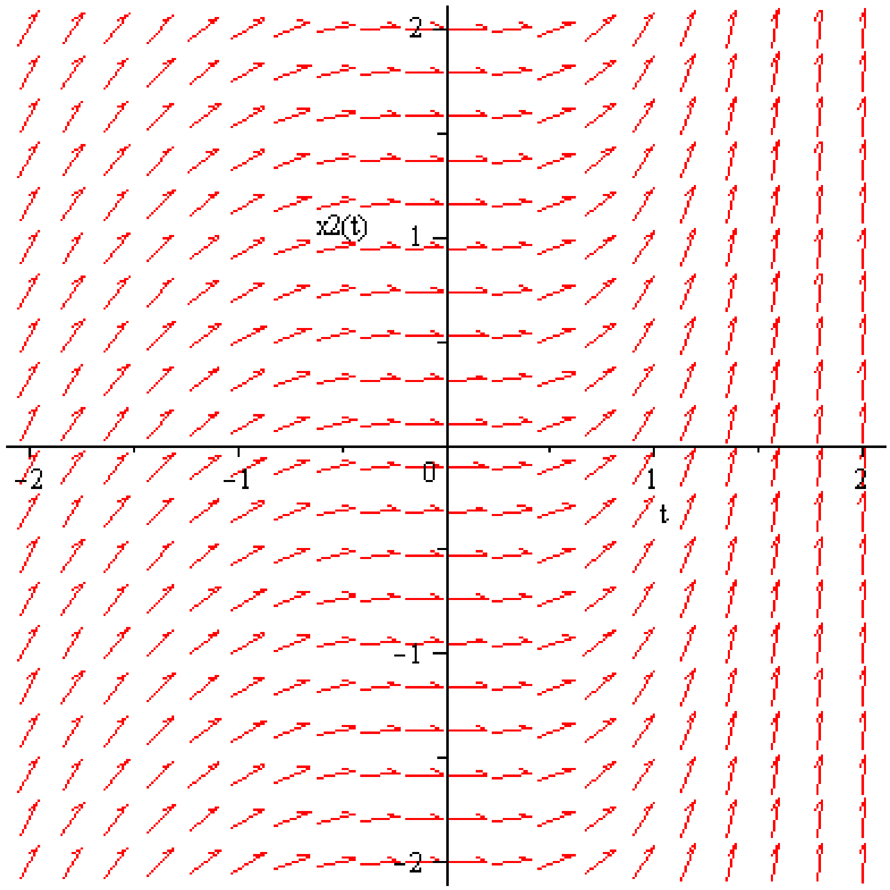
b) $ODE2 := diff(x2(t), t) = t \cdot \exp(t) - t;$

$$\frac{d}{dt} x2(t) = t\, e^{t} - t$$

$dsolve(ODE2, x2(t));$

$$x2(t) = t\, e^{t} - e^{t} - \frac{1}{2} t^2 + \_C1$$

$dfieldplot(ODE2, x2(t), t = -2..2, x2 = -2..2);$



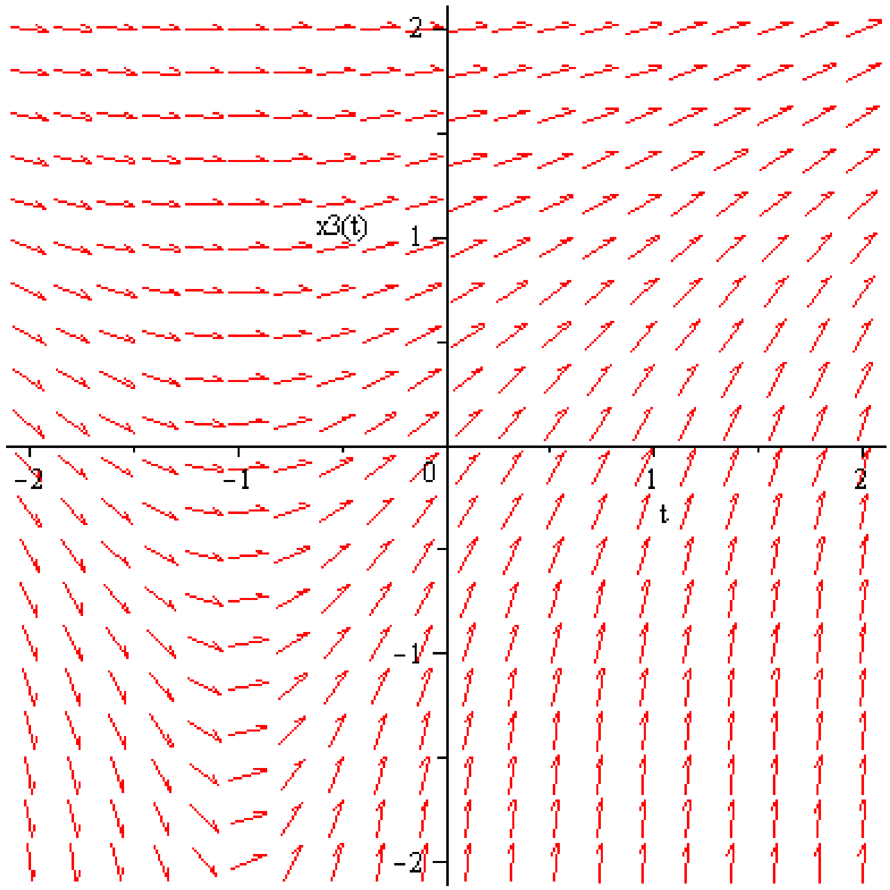
c) $ODE3 := \exp(x3(t)) \cdot diff(x3(t), t) = t + 1;$

$$e^{x3(t)} \left(\frac{d}{dt} x3(t)\right) = t + 1$$

$dsolve(ODE3, x3(t));$

$$x3(t) = \ln\left(\frac{1}{2} t^2 + t + \_C1\right)$$

$dfieldplot(ODE3, x3(t), t = -2..2, x3 = -2..2);$



Příklad 4: Najděte obecné řešení následujících diferenciálních rovnic. Také najděte partikulární řešení procházející zadanými body.

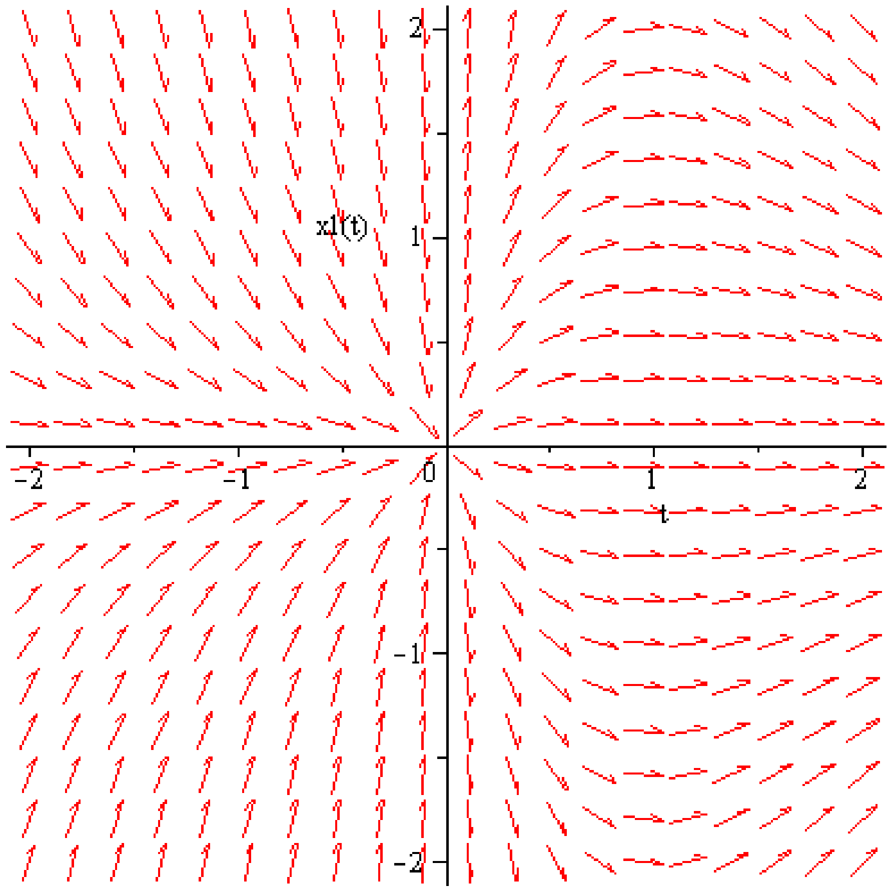
a) $ODE1 := t \cdot diff(x1(t), t) = x1(t) \cdot (1 - t);$

$$t \left(\frac{d}{dt} x1(t) \right) = x1(t)\ (1 - t)$$

$dsolve(ODE1, x1(t));$

$$x1(t) = \_C1\ t\ e^{-t}$$

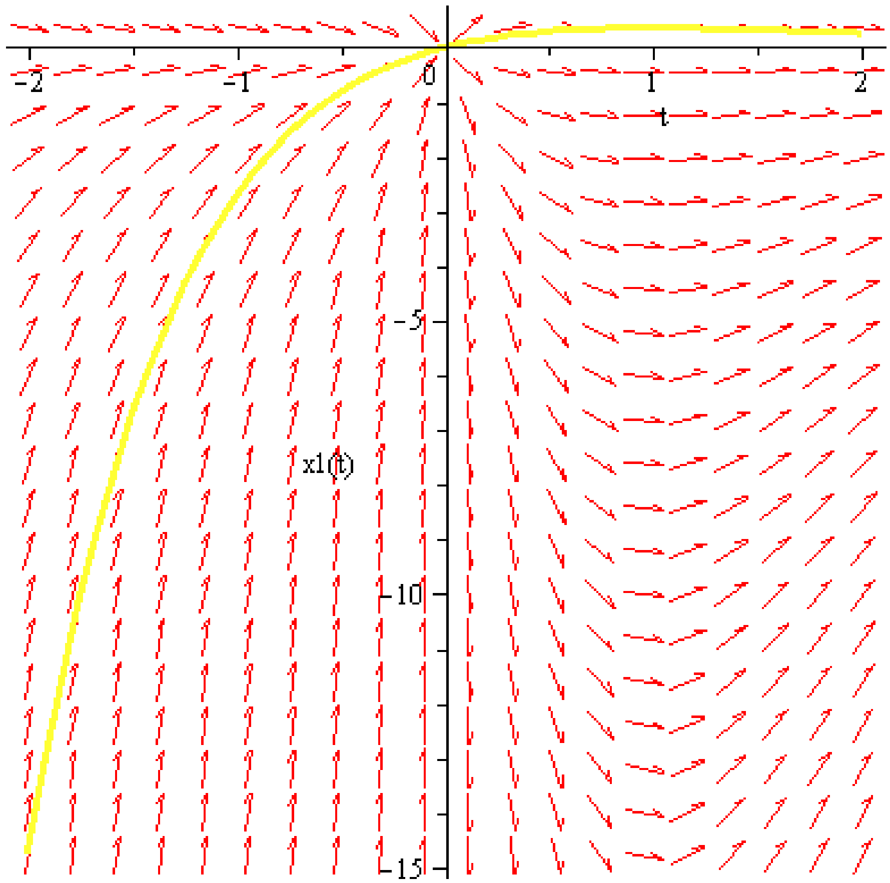
$dfieldplot(ODE1, x1(t), t = -2..2, x1 = -2..2);$



$dsolve(\{ODE1, x1(1) = \exp(-1)\}, x1(t));$

$$x1(t) = t\,e^{-t}$$

$DEplot(ODE1, x1(t), t=-2..2, [x1(1) = \exp(-1)]);$



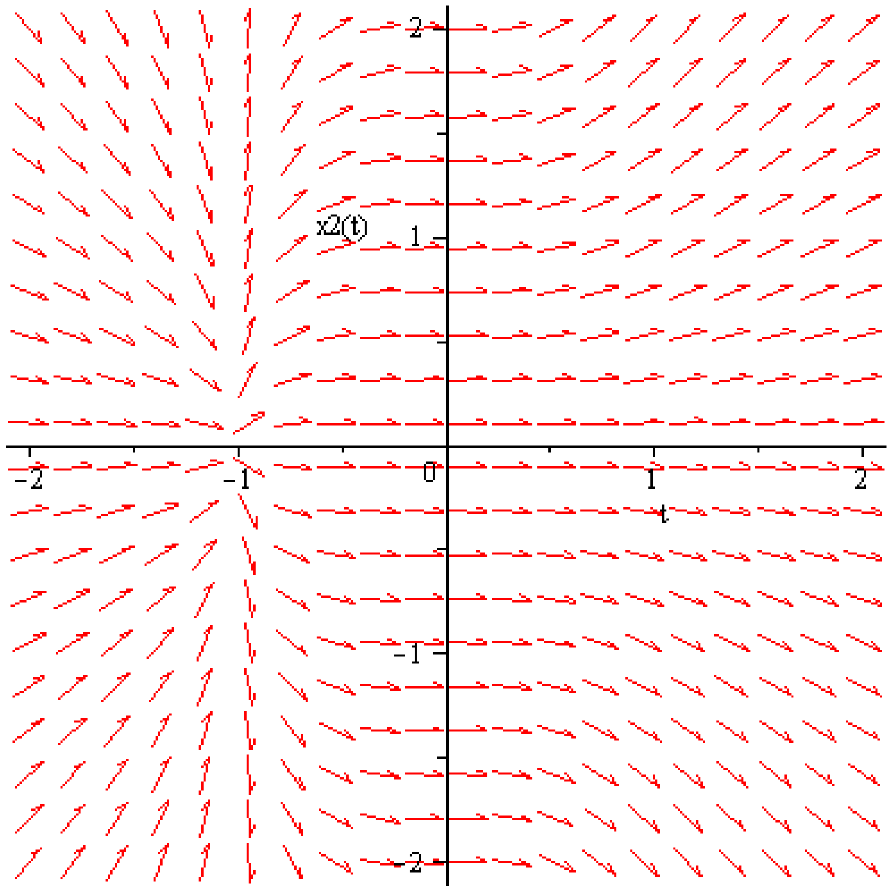
b) $ODE2 := \left(1 + t^3\right) \cdot diff\left(x2(t), t\right) = t^2 \cdot x2(t);$

$$\left(1 + t^3\right) \left(\frac{d}{dt} x2(t)\right) = t^2\, x2(t)$$

$dsolve\left(ODE2, x2(t)\right);$

$$x2(t) = \_C1\, \left(1 + t^3\right)^{1/3}$$

$dfieldplot\left(ODE2, x2(t), t = -2..2, x2 = -2..2\right);$



$dsolve(\{ODE2, x2(0) = 2\}, x2(t));$

$$x2(t) = 2\left(1 + t^3\right)^{1/3}$$

$DEplot(ODE2, x2(t), t = -2..2, [x2(0) = 2]);$

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Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of -.99999999, probably
a singularity
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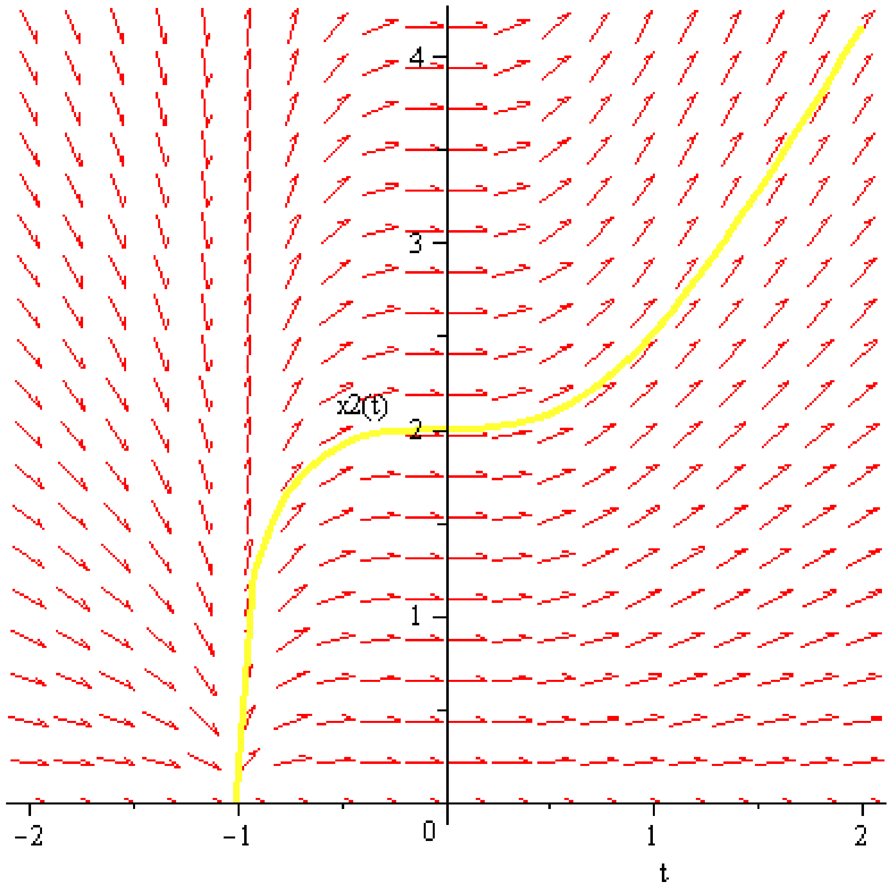


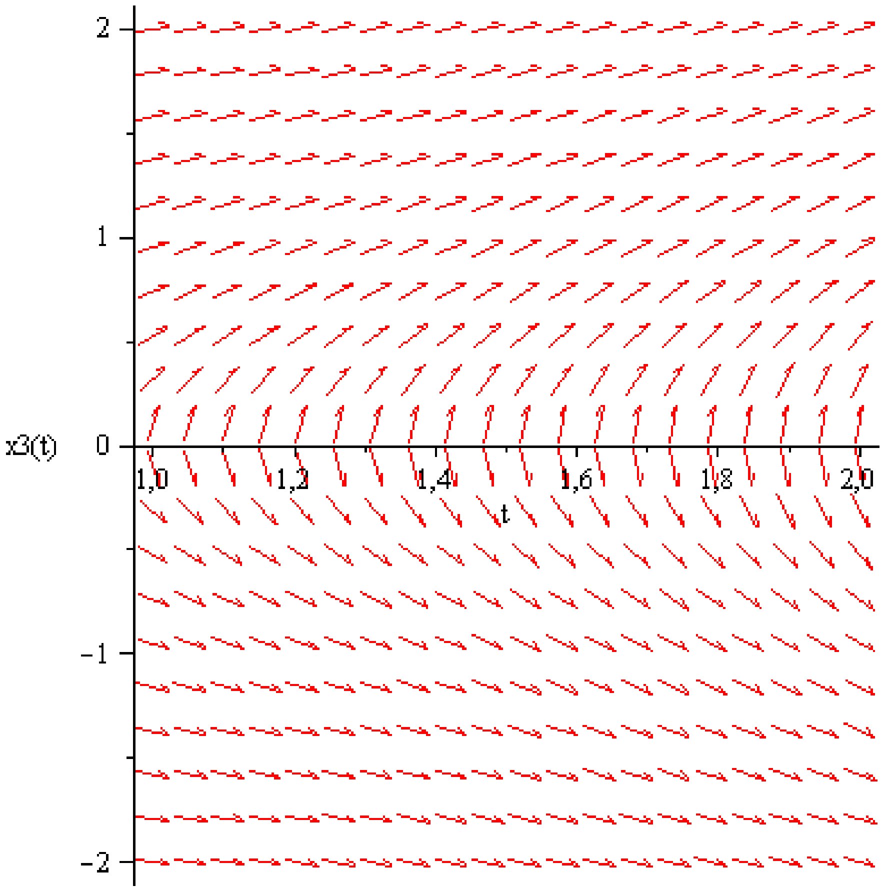
c) $ODE3 := x3(t) \cdot diff(x3(t), t) = t;$

$$x3(t) \left(\frac{d}{dt} x3(t) \right) = t$$

$dsolve(ODE3, x3(t));$

$$x3(t) = \sqrt{t^2 + \_C1}, x3(t) = -\sqrt{t^2 + \_C1}$$

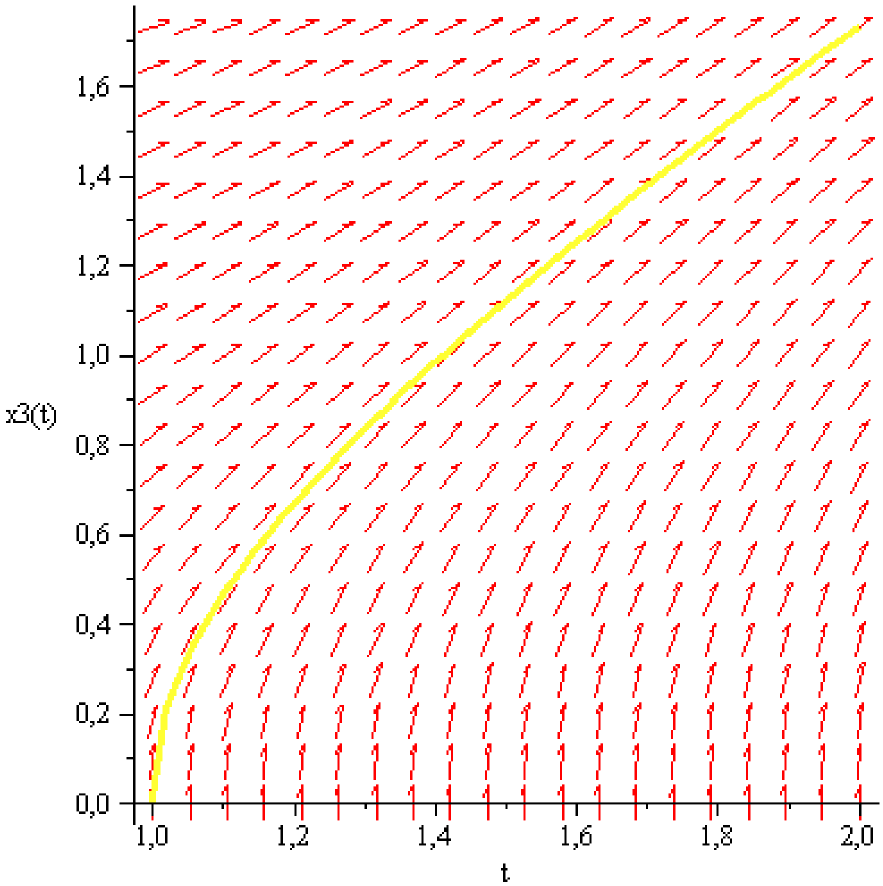
$dfieldplot(ODE3, x3(t), t = 1..2, x3 = -2..2);$



dsolve({*ODE3*, *x3*(sqrt(2)) = 1}, *x3*(*t*));

$$x3(t) = \sqrt{t^2 - 1}$$

DEplot(*ODE3*, *x3*(*t*), *t* = 1 ..2, [*x3*(sqrt(2)) = 1]);



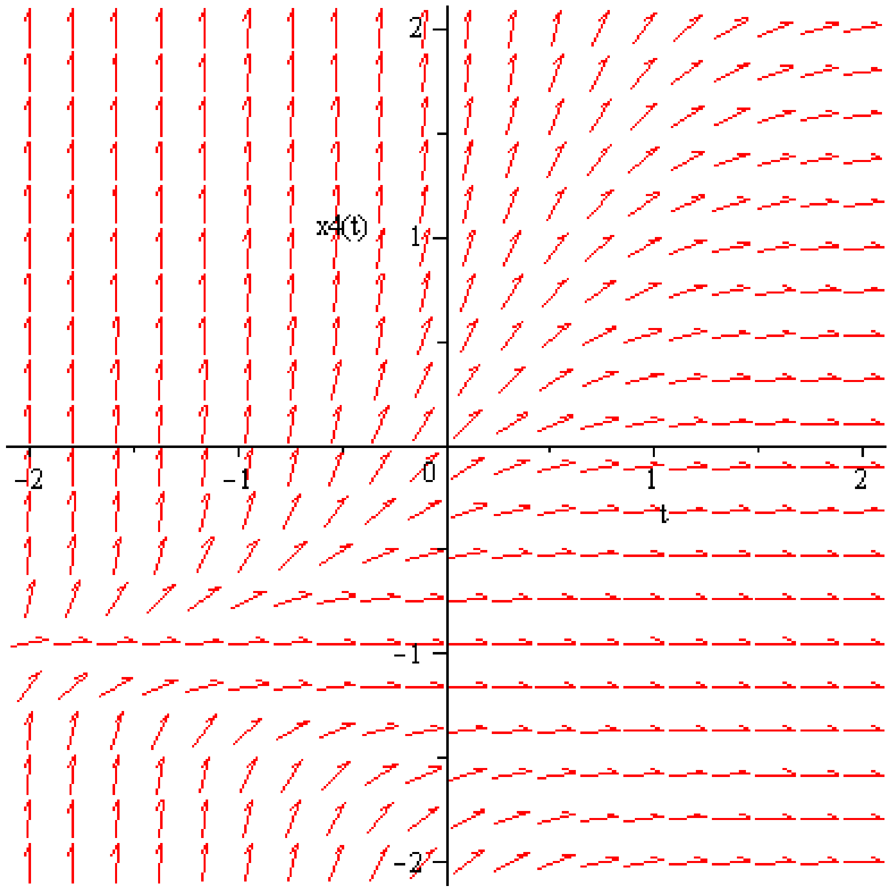
c) $ODE4 := \exp(2\cdot t)diff(x4(t), t) - x4(t)^2 - 2\cdot x4(t) = 1;$

$$e^{2t}\left(\frac{d}{dt}x4(t)\right) - x4(t)^2 - 2\,x4(t) = 1$$

$dsolve(ODE4, x4(t));$

$$x4(t) = -\frac{-e^{-2t} + 2 + 2\_C1}{-e^{-2t} + 2\_C1}$$

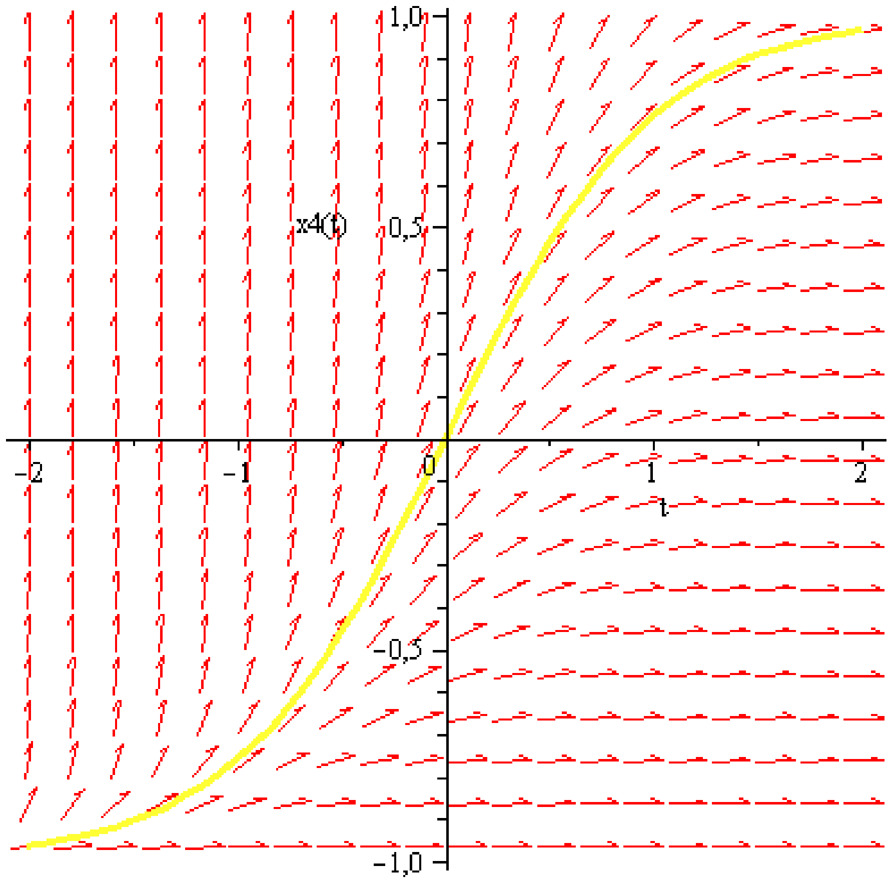
$dfieldplot(ODE4, x4(t), t = -2..2, x4 = -2..2);$



dsolve({*ODE4*, *x4*(0) = 0}, *x4*(*t*));

$$x4(t) = -\frac{e^{-2t} - 1}{1 + e^{-2t}}$$

DEplot(*ODE4*, *x4*(*t*), *t* = −2 ..2, [*x4*(0) = 0]);


x4(t)
1,0
0,5
−0,5
−1,0
−2
−1
0
1
2
t