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ES2

$$\begin{pmatrix} x & y \\ 5 & 10 \\ 3 & 6 \\ -1 & -4 \\ 6 & 8 \\ 2 & 5 \end{pmatrix}$$

find correlation

$$E(x) = \frac{5+3-1+6+2}{5} = 3$$

$$E(y) = \frac{10+6-4+8+5}{5} = 5$$

$$E(xy) = \frac{50+18+4+48+10}{5} = 26$$

$$cov(x,y) = 26 - 3 \cdot 5 = 11$$

$$E(x^2) = \frac{25+9+1+36+4}{5} = 15 \longrightarrow var(x) = 15 - 3^2 = 6$$

$$E(y^2) = \frac{100+36+16+64+25}{5} = 48.2 \longrightarrow var(y) = 48{,}2 - 5^2 = 23.2$$

$$\sigma_x = \sqrt{6} = 2{,}45$$

$$\sigma_y = \sqrt{23.2} = 4{,}82$$

$$corr(x,y) = \frac{11}{2{,}45 \cdot 4{,}82} = 0{,}93$$

in excel you could just type both variables

and then = correl ('column of I variable', 'column of second variable') derivations:

$\pi = q(p)\cdot(p-c)$

$q(p) = a - b\cdot p$

$$\max_{p} \pi = q(p)\cdot(p-c) = (a-b\cdot p)(p-c) = ap - ac - bp^2 + bcp$$

FOC: [p] $a - 2bp + bc = 0$

$2bp = a + bc$

$$p = \frac{a+bc}{2b}$$

$$q = a - b\left(\frac{a+bc}{2b}\right) = a - \frac{a+bc}{2} = \frac{a-bc}{2}$$

now we can express this as a regression model

$$q = \alpha_0 + \alpha_1 \cdot c$$

derivation $\hat{\beta}_0$; $\hat{\beta}_1$:

model: $y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$

$$\varepsilon_i = y_i - \beta_0 - \beta_1 X_i$$

$$\sum_{i=1}^{n} \varepsilon_i = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X_i)^2$$

$$\min_{\beta_0, \beta_1} \sum_{i=1}^{n} \varepsilon_i$$

FOC: $[\beta_0]$ $-2\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X_i) = 0$

$$-\cancel{2}\sum y_i + \cancel{2}\beta_0 \cdot n + \cancel{2}\beta_1 \sum X_i = 0$$

$$\sum y_i - \beta_1 \sum x_i = \beta_0 \cdot n$$

$$\hat{\beta}_0 = \frac{\sum y_i - \beta_1 \sum x_i}{n} = \bar{y} - \bar{x}\hat{\beta}_1$$

$[\beta_1]$ $\cancel{2}\sum (y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$

$$-\sum (y_i - \bar{y} + \bar{x}\beta_1 - \beta_1 x_i) x_i = 0$$

$$-\sum (y_i - \bar{y}) x_i + \beta_1 \sum (x_i - \bar{x}) x_i = 0$$

$$\hat{\beta}_1 = \frac{\sum (y_i - \bar{y}) x_i}{\sum (x_i - \bar{x}) x_i} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})(x_i - \bar{x})} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

check A7 and A8 in the appendix of wooldridge book

show $\underbrace{\sum (x_i - \bar{x}) x_i}_{LHS} = \underbrace{\sum (x_i - \bar{x})^2}_{RHS}$

$$LHS = \sum (x_i^2 - \bar{x} x_i) = \sum x_i^2 - \bar{x} \sum x_i = \sum x_i^2 - n\bar{x}^2$$

$$RHS = \sum (x_i^2 - 2x_i\bar{x} + \bar{x}^2) = \sum x_i^2 - 2\bar{x}\sum x_i + \sum \bar{x}^2 =$$

$$= \sum x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum x_i^2 - n\bar{x}^2 \quad \blacksquare$$

derivation from output:

Firm	1	2	3	4	5	6
q	15	32	52	14	37	27
c	294	247	153	350	173	218

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{6}(q_i-\bar{q})(c_i-\bar{c})}{\sum_{i=1}^{6}(c_i-\bar{c})^2} = \frac{cov(q_i, c_i)}{var(c_i)} =$$

$$= \frac{E(q_i c_i) - E(q_i)E(c_i)}{E(c_i^2) - (E(c_i))^2} = -\frac{812,583}{4600,472} = -0,1766$$

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supporting calculations:

$$E(q_i c_i) = \frac{4410 + 7904 + 7956 + 4900 + 6401 + 5886}{6} = 6242,8.$$

$$E(q_i) = 29.5$$

$$E(c_i) = 239,1667$$

$$cov(q_i, c_i) = 6242,8 - 29.5 \cdot 239,1667 = -812,583$$

$$E(c_i^2) = 61801,17$$

$$(E(c_i))^2 = 239,1667^2 = 57200,69$$

$$var(c_i) = 61801,17 - 57200.69 = 4600.472$$

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$$\hat{\beta}_0 = \bar{y} - \bar{x} \cdot \hat{\beta}_1 = \bar{q} - \bar{c} \cdot \hat{\beta}_1 = 29,5 - 239,1667 \cdot (-0,1766) = 71.74$$

now, from original economic model

$$\hat{\beta}_0 = \frac{a}{2} \Rightarrow a = 2 \cdot 71,74 = 143,48 \; ; \; \hat{\beta}_1 = -\frac{b}{2} \Rightarrow b = 2 \cdot 0,1766 =$$

$$= 0,35$$