Financial Mathematic

Seminar

Luděk Benada, Dagmar Linnertova

Department of Finance - 402, benada.esf@gmail.com

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The Study materials prepared by

Mikhail Dmitrievich Balyka

Exercise 1

You put **333$** into a bank account 4 times per year (**at the beginning** of each quarter) for 25 years at **4,5% p.a. Interest** is calculated **monthly**.
‼BUT, to avoid inflation, every quarter you deposit 0,5% more (i.e $annuity_2/annuity_1=1{,}005$).
S_{25yrs} - ?

Exercise 1 cont'd

Inputs:

a=333 ahead PP=4t/y IP=1m

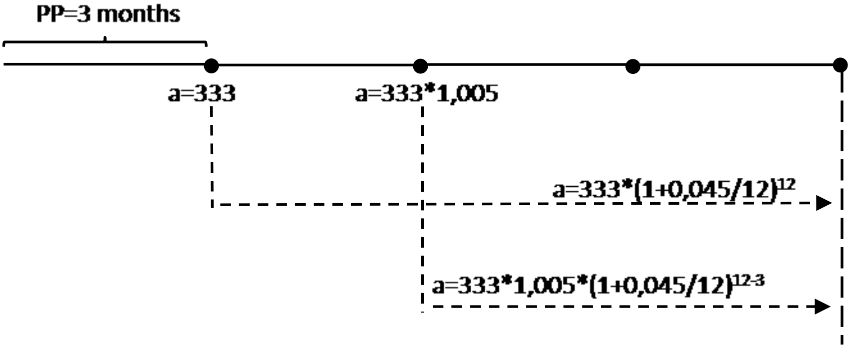
T=25yrs r=4,5%p.a. $a_2 = a_1 * 1{,}005$

S_{25yrs} - ?

PP>IP $\rightarrow S = a \times q \times \frac{q^n - 1}{q - 1}$

Exercise 1 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$



Annuities

Exercise 1 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

$$q = \frac{333*(1+0{,}045/12)^{12}}{333*1{,}005*(1+0{,}045/12)^{12\text{-}3}} = \frac{(1+\frac{0{,}045}{12})^3}{1{,}005}$$

$$S = 333 \times \frac{(1+\frac{0{,}045}{12})^3}{1{,}005} \times \frac{\left[\frac{\left(1+\frac{0{,}045}{12}\right)^3}{1{,}005}\right]^{4\times 25} - 1}{\frac{(1+\frac{0{,}045}{12})^3}{1{,}005} - 1} = 46382{,}69$$

Exercise 2

You put **200 000$** into a bank account at **2% p.q.** You pay **5 000$** to open this account and also **at the end of each month** you pay a bank fee **200$. Interest** is calculated **quarterly**.
Find the average annual return on this investment

Exercise 2 cont'd

Inputs:

PV=200 000 Initial costs=5000 Monthly costs (i.e. annuities)=200

PP=1month IP=3months r=2%p.q. T=5yrs After payment

Here we just need to respect time value of money and to calculate FVs of all cash flows:

$$FV = 200000 \times (1+0{,}02)^{20} - 5000 \times (1+0{,}02)^{20} - 200 \times 3 \times \left(1 + \frac{4}{6} \times 0{,}02\right) \times \frac{(1+0{,}02)^{20} - 1}{0{,}02} = 275084{,}1$$

Exercise 2 cont'd

$$FV = 200000 \times (1+0{,}02)^{20} - 5000 \times (1+0{,}02)^{20} - 200 \times 3 \times \left(1 + \frac{4}{6} \times 0{,}02\right) \times \frac{(1+0{,}02)^{20}-1}{0{,}02} = 275084{,}1$$

$$FV = PV \times (1+r)^{n}$$

$$r = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

$$r = \left(\frac{275084{,}1}{200000}\right)^{\frac{1}{5}} - 1 = 6{,}58\%$$

Exercise 3

You put **15 000 $** into a bank account **at the end** of each **quarter** at **3,7% p.a. for 10 years. Interest** is calculated **2 times per year**. **Tax rate is 15%** and it's calculated at the end of investment's period

S_{tax} -?

Exercise 3 cont'd

Inputs:

a=15 000 PP=3m(after) IP=6m r=3,7% p.a.

T=10 yrs tax=15% TP=10yrs

S_{tax} -?

$$S = a \times m \times (1 + \frac{m-1}{2m} \times r) \times \frac{(1+r)^n}{r}$$

To calculate the after tax's amount of money we need to deduct from our FV a sum of annuities made in one TP

Exercise 3 cont'd

$$S = a \times m \times \left(1 + \frac{m-1}{2m} \times r\right) \times \frac{(1+r)^n - 1}{r}$$

To calculate the after tax's amount of money we need to deduct from our FV a sum of annuities made in one TP

we deduct our annu-s (# of 'a' in one TP)

$$S = 15000 \times \left\{ \left[3 \times \left(1 + \frac{3-1}{2 \times 3} \times \frac{0{,}037}{2}\right) \times \frac{\left(1 + \frac{0{,}037}{2}\right)^{20} - 1}{\frac{0{,}037}{2}} - 40 \right] \times 0{,}85 + 40 \right\}$$

mult-ing by the % retained after tax and adding back our 'a' we will get 'S' after tax

Exercise 4

a=40 000 PP=3m ahead IP=1m TP=1m

r=3,9% p.a. tax=15% T=10years

S_{tax} -?

PP>IP → $S = a \times q \times \frac{q^n - 1}{q - 1}$

Now we just need to define q

Exercise 4 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

As we know tax is calculated from interest

How we can find it:

$$I = PV \times [(1 + r)^n - 1]$$

But in our case *n=1*, so we can rearrange the formula and find our interest after tax:

$$I = PV \times r \times (1 - tax)$$

Since, our $q = (1 + \frac{0{,}039}{12} \times 0{,}85)^3$

Exercise 5 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

$$S = 40000 \times (1 + \frac{0{,}039}{12} \times 0{,}85)^3 \times \frac{[(1 + \frac{0{,}039}{12} \times 0{,}85)^3]^{4\times 10} - 1}{(1 + \frac{0{,}039}{12} \times 0{,}85)^3 - 1}$$

Exercise 5

a=40 000 PP=3m ahead IP=1m TP=1year

r=3,9% p.a. tax=15% T=10 years

S_{tax} -?

PP>IP $\rightarrow S = a \times q \times \frac{q^n - 1}{q - 1}$

Exercise 5 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

$$S = 40000 \times [(\left(1+\frac{0{,}039}{12}\right)^3 \times \frac{\left(1+\frac{0{,}039}{12}\right)^{3\times 4} - 1}{\left(1+\frac{0{,}039}{12}\right)^3 - 1} - 4) \times 0{,}85 + 4]$$

$$\times \frac{((1+\frac{0{,}039}{12})^{12} - 1) \times 0{,}85 + 1)^{10} - 1}{(1+\frac{0{,}039}{12})^{12} - 1) \times 0{,}85}$$

Exercise 6

a=500 PP=4m ahead IP=2m TP=1year

r=4,7% p.a. tax=10% T=7 years

S_{tax} for continuous interest -?

$$q = e^{ft}$$

$f = \ln(1 + \frac{0,047}{6})^2 = 0,0156$ - interest intensity for 4 months

Exercise 6

a=500 PP=4m ahead IP=2m TP=1year

r=4,7% p.a. tax=10% T=7 years

$$q = e^{ft}$$

$f = \ln(1 + \frac{0,047}{6})^2 = 0,0156$ - interest intensity for 4 months

$$S = 500 \times [(\frac{e^{0,0156x3} - 1}{e^{0,0156} - 1} - 3) \times 0,9 + 3] \times \frac{((e^{0,0156x3} - 1) \times 0,9 + 1)^7}{(e^{0,0156x3} - 1) \times 0,9}$$