The profit-maximizing output of a monopolist is found by solving for the output at which marginal revenue is equal to marginal cost. Having solved for this output, you find the monopolist's price by plugging the profit-maximizing output into the demand function. In general, the marginal revenue function can be found by taking the derivative of the total revenue function with respect to the quantity. But in the special case of linear demand, it is easy to find the marginal revenue curve graphically. With a linear inverse demand curve, $p(y) = a - by$, the marginal revenue curve always takes the form $MR(y) = a - 2by$.

24.1 (0) Professor Bong has just written the first textbook in Punk Economics. It is called *Up Your Isoquant*. Market research suggests that the demand curve for this book will be $Q = 2,000 - 100P$, where P is its price. It will cost \$1,000 to set the book in type. This setup cost is necessary before any copies can be printed. In addition to the setup cost, there is a marginal cost of \$4 per book for every book printed.

(a) The total revenue function for Professor Bong's book is $R(Q) =$_____

_____.

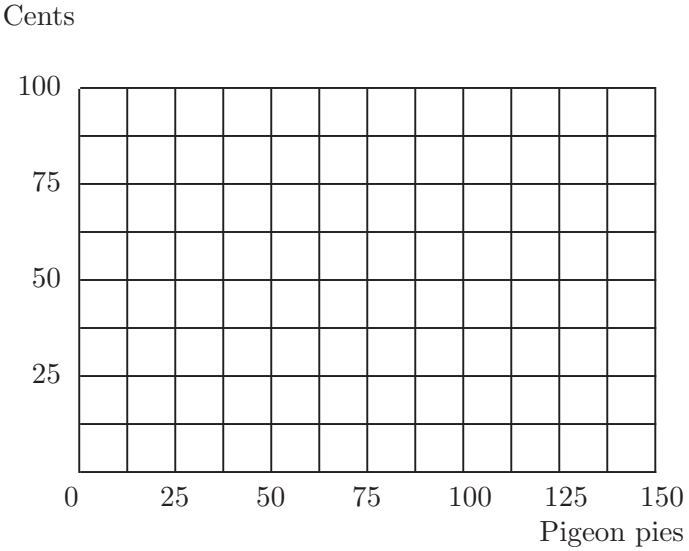
(b) The total cost function for producing Professor Bong's book is $C(Q) =$

_____.

(c) The marginal revenue function is $MR(Q) =$_____ and the marginal cost function is $MC(Q) =$_____ The profit-maximizing quantity of books for professor Bong to sell is $Q^* =$_____.

24.2 (0) Peter Morgan sells pigeon pies from a pushcart in Central Park. Morgan is the only supplier of this delicacy in Central Park. His costs are zero due to the abundant supplies of raw materials available in the park.

(a) When he first started his business, the inverse demand curve for pigeon pies was $p(y) = 100 - y$, where the price is measured in cents and y measures the number of pies sold. Use black ink to plot this curve in the graph below. On the same graph, use red ink to plot the marginal revenue curve.



(b) What level of output will maximize Peter's profits? _____ What price will Peter charge per pie?_____.

(c) After Peter had been in business for several months, he noticed that the demand curve had shifted to $p(y) = 75 - y/2$. Use blue ink to plot this curve in the graph above. Plot the new marginal revenue curve on the same graph with black ink.

(d) What is his profit-maximizing output at this new price? _____ What is the new profit-maximizing price?_____.

24.3 (0) Suppose that the demand function for Japanese cars in the United States is such that annual sales of cars (in thousands of cars) will be $250-2P$, where P is the price of Japanese cars in thousands of dollars.

(a) If the supply schedule is horizontal at a price of $5,000 what will be the equilibrium number of Japanese cars sold in the United States?

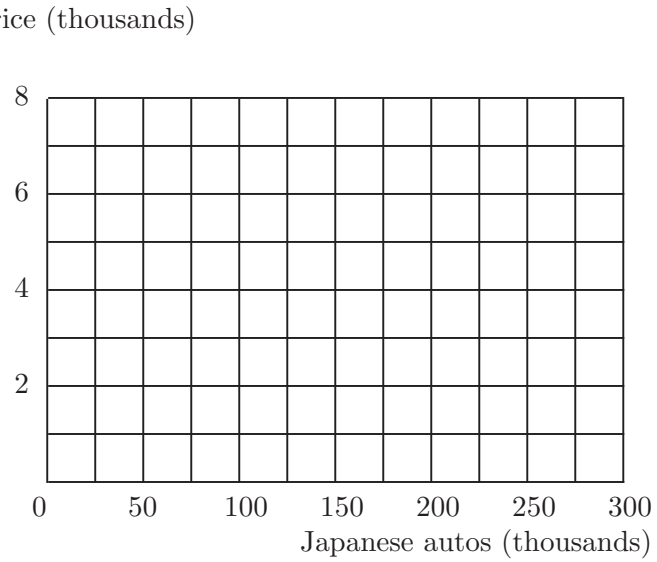
_____ thousand. How much money will Americans spend in total on Japanese cars? _____ billion dollars.

(b) Suppose that in response to pressure from American car manufacturers, the United States imposes an import duty on Japanese cars in such a way that for every car exported to the United States the Japanese manufacturers must pay a tax to the U.S. government of $2,000. How many Japanese automobiles will now be sold in the United States? _____ thousand. At what price will they be sold? _____ thousand dollars.

(c) How much revenue will the U.S. government collect with this tariff?

_____ million dollars.

(d) On the graph below, the price paid by American consumers is measured on the vertical axis. Use blue ink to show the demand and supply schedules before the import duty is imposed. After the import duty is imposed, the supply schedule shifts and the demand schedule stays as before. Use red ink to draw the new supply schedule.



(e) Suppose that instead of imposing an import duty, the U.S. government persuades the Japanese government to impose "voluntary export restrictions" on their exports of cars to the United States. Suppose that the Japanese agree to restrain their exports by requiring that every car exported to the United States must have an export license. Suppose further that the Japanese government agrees to issue only 236,000 export licenses and sells these licenses to the Japanese firms. If the Japanese firms know the American demand curve and if they know that only 236,000 Japanese cars will be sold in America, what price will they be able to charge in America for their cars? _____ thousand dollars.

(f) How much will a Japanese firm be willing to pay the Japanese government for an export license? _____ thousand dollars. (Hint: Think about what it costs to produce a car and how much it can be sold for if you have an export license.)

(g) How much will be the Japanese government's total revenue from the sale of export licenses? _____ million dollars.

(h) How much money will Americans spend on Japanese cars? _____ billion dollars.

(i) Why might the Japanese "voluntarily" submit to export controls?

_____

_____

_____.

24.4 (0) A monopolist has an inverse demand curve given by $p(y) = 12 - y$ and a cost curve given by $c(y) = y^2$.

(a) What will be its profit-maximizing level of output?_____.

(b) Suppose the government decides to put a tax on this monopolist so that for each unit it sells it has to pay the government \$2. What will be its output under this form of taxation?_____.

(c) Suppose now that the government puts a lump sum tax of \$10 on the profits of the monopolist. What will be its output?_____.

24.5 (1) In Gomorrah, New Jersey, there is only one newspaper, the *Daily Calumny*. The demand for the paper depends on the price and the amount of scandal reported. The demand function is $Q = 15S^{1/2}P^{-3}$, where Q is the number of issues sold per day, S is the number of column inches of scandal reported in the paper, and P is the price. Scandals are not a scarce commodity in Gomorrah. However, it takes resources to write, edit, and print stories of scandal. The cost of reporting S units of scandal is $\$10S$. These costs are independent of the number of papers sold. In addition it costs money to print and deliver the paper. These cost \$.10 per copy and the cost per unit is independent of the amount of scandal reported in the paper. Therefore the total cost of printing Q copies of the paper with S column inches of scandal is $\$10S + .10Q$.

(a) Calculate the price elasticity of demand for the *Daily Calumny*. _____ Does the price elasticity depend on the amount of scandal reported? _____ Is the price elasticity constant over all prices?_____

_____.

(b) Remember that $MR = P(1 + \frac{1}{\epsilon})$. To maximize profits, the *Daily Calumny* will set marginal revenue equal to marginal cost. Solve for the profit-maximizing price for the *Calumny* to charge per newspaper._____ When the newspaper charges this price, the difference between the price and the marginal cost of printing and delivering each newspaper is_____.

(c) If the *Daily Calumny* charges the profit-maximizing price and prints 100 column inches of scandal, how many copies would it sell? (Round to the nearest integer.) _____ Write a general expression for the number of copies sold as a function of S: $Q(S) =$_____.

(d) Assuming that the paper charges the profit-maximizing price, write an expression for profits as a function of Q and S. _____ Using the solution for $Q(S)$ that you found in the last section, substitute $Q(S)$ for Q to write an expression for profits as a function of S alone._____.

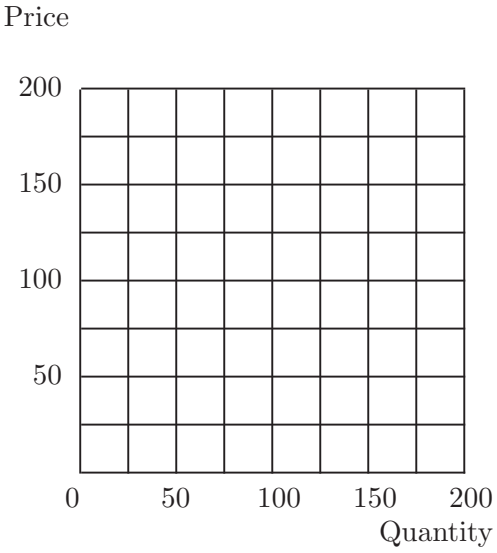
(e) If the *Daily Calumny* charges its profit-maximizing price, and prints the profit-maximizing amount of scandal, how many column inches of scandal should it print? _____ How many copies are sold _____ and what is the amount of profit for the *Daily Calumny* if it maximizes its profits?_____.

24.6 (0) In the graph below, use black ink to draw the inverse demand curve, $p_1(y) = 200 - y$.

(a) If the monopolist has zero costs, where on this curve will it choose to operate?_____.

(b) Now draw another demand curve that passes through the profit-maximizing point and is flatter than the original demand curve. Use a red pen to mark the part of this new demand curve on which the monopolist would choose to operate. (Hint: Remember the idea of revealed preference?)

(c) The monopolist would have (larger, smaller) profits at the new demand curve than it had at the original demand curve._____.



Problems in this chapter explore the possibilities of price discrimination by monopolists. There are also problems related to spatial markets, where transportation costs are accounted for and we show that lessons learned about spatial models give us a useful way of thinking about competition under product differentiation in economics and in politics.

Remember that a price discriminator wants the *marginal revenue* in each market to be equal to the marginal cost of production. Since he produces all of his output in one place, his marginal cost of production is the same for both markets and depends on his *total* output. The trick for solving these problems is to write marginal revenue in each market as a function of quantity sold in that market and to write marginal cost as a function of the sum of quantities sold in the two markets. The profit-maximizing conditions then become two equations that you can solve for the two unknown quantities sold in the two markets. Of course, if marginal cost is constant, your job is even easier, since all you have to do is find the quantities in each market for which marginal revenue equals the constant marginal cost.

A monopolist sells in two markets. The inverse demand curve in market 1 is $p_1 = 200 - q_1$. The inverse demand curve in market 2 is $p_2 = 300 - q_2$. The firm's total cost function is $C(q_1 + q_2) = (q_1 + q_2)^2$. The firm is able to price discriminate between the two markets. Let us find the prices that it will charge in each market. In market 1, the firm's marginal revenue is $200 - 2q_1$. In market 2, marginal revenue is $300 - 2q_2$. The firm's marginal costs are $2(q_1 + q_2)$. To maximize its profits, the firm sets marginal revenue in each market equal to marginal cost. This gives us the two equations $200 - 2q_1 = 2(q_1 + q_2)$ and $300 - 2q_2 = 2(q_1 + q_2)$. Solving these two equations in two unknowns for q_1 and q_2, we find $q_1 = 16.67$ and $q_2 = 66.67$. We can find the price charged in each market by plugging these quantities into the demand functions. The price charged in market 1 will be 183.33. The price charged in market 2 will be 233.33.

25.1 (0) Ferdinand Sludge has just written a disgusting new book, *Orgy in the Piggery*. His publisher, Graw McSwill, estimates that the demand for this book in the United States is $Q_1 = 50,000 - 2,000P_1$, where P_1 is the price in the U.S. measured in U.S. dollars. The demand for Sludge's opus in England is $Q_2 = 10,000 - 500P_2$, where P_2 is its price in England measured in U. S. dollars. His publisher has a cost function $C(Q) = \$50,000 + \$2Q$, where Q is the total number of copies of *Orgy* that it produces.

(a) If McSwill must charge the same price in both countries, how many copies should it sell? _____ What price should it charge to maximize

its profits _____ How much will those profits be?_____.

(b) If McSwill can charge a different price in each country and wants to maximize profits, how many copies should it sell in the United States? _____ What price should it charge in the United States? _____ _____ How many copies should it sell in England? _____ What price should it charge in England? _____ How much will its total profits be?_____.

25.2 (0) A monopoly faces an inverse demand curve, $p(y) = 100 - 2y$, and has constant marginal costs of 20.

(a) What is its profit-maximizing level of output? _____

(b) What is its profit-maximizing price?_____.

(c) What is the socially optimal price for this firm?_____.

(d) What is the socially optimal level of output for this firm?_____.

(e) What is the deadweight loss due to the monopolistic behavior of this firm?_____.

(f) Suppose this monopolist could operate as a perfectly discriminating monopolist and sell each unit of output at the highest price it would fetch. The deadweight loss in this case would be_____.

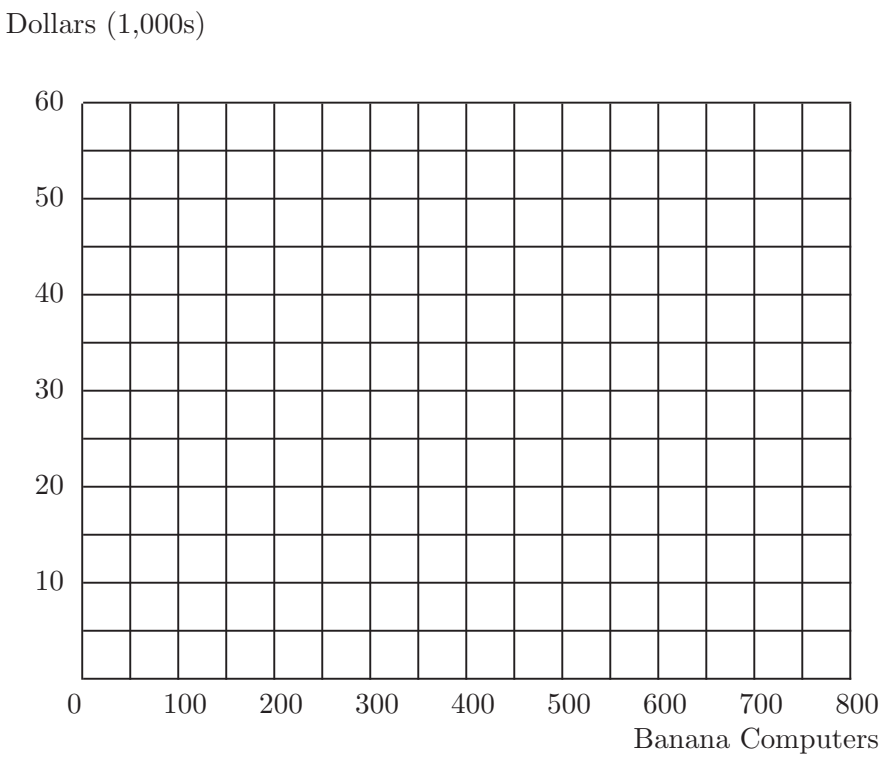
25.3 (1) Banana Computer Company sells Banana computers in both the domestic and foreign markets. Because of differences in the power supplies, a Banana purchased in one market cannot be used in the other market. The demand and marginal revenue curves associated with the two markets are as follows:

$$\begin{aligned} P_d &= 20,000 - 20Q \quad P_f = 25,000 - 50Q \\ MR_d &= 20,000 - 40Q \quad MR_f = 25,000 - 100Q. \end{aligned}$$

Banana's production process exhibits constant returns to scale and it takes $1,000,000 to produce 100 computers.

(a) Banana's long-run average cost function is $AC(Q) =$_____ and its long-run marginal cost function is $MC(Q) =$_____ (Hint: If there are constant returns to scale, does long-run average cost change as output changes?) Draw the average and marginal cost curves on the graph.

(b) Draw the demand curve for the domestic market in black ink and the marginal revenue curve for the domestic market in pencil. Draw the demand curve for the foreign market in red ink and the marginal revenue curve for the foreign market in blue ink.



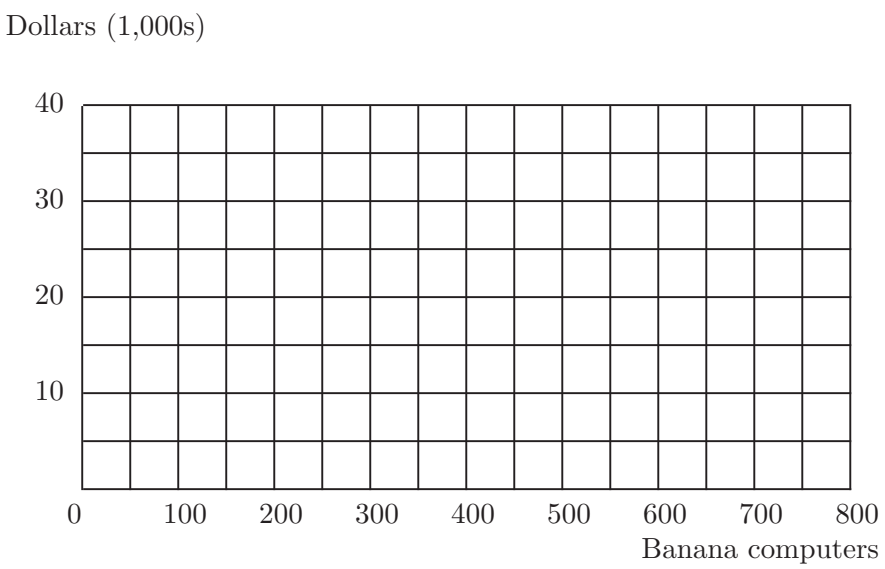
(c) If Banana is maximizing its profits, it will sell _____ computers in the domestic market at _____ dollars each and _____ computers in the foreign market at _____ dollars each. What are Banana's total profits?_____.

(d) At the profit-maximizing price and quantity, what is the price elasticity of demand in the domestic market? _____ What is the price

elasticity of demand in the foreign market? _____ Is demand more or less elastic in the market where the higher price is charged?_____

_____.

(e) Suppose that somebody figures out a wiring trick that allows a Banana computer built for either market to be costlessly converted to work in the other. (Ignore transportation costs.) On the graph below, draw the new inverse demand curve (with blue ink) and marginal revenue curve (with black ink) facing Banana.



(f) Given that costs haven't changed, how many Banana computers should Banana sell? _____ What price will it charge? _____ _____ How will Banana's profits change now that it can no longer practice price discrimination?_____.

25.4 (0) A monopolist has a cost function given by $c(y) = y^2$ and faces a demand curve given by $P(y) = 120 - y$.

(a) What is its profit-maximizing level of output? _____ What price will the monopolist charge?_____.

(b) If you put a lump sum tax of \$100 on this monopolist, what would its output be?_____.

(c) If you wanted to choose a price ceiling for this monopolist so as to maximize consumer plus producer surplus, what price ceiling should you choose?_____.

(d) How much output will the monopolist produce at this price ceiling? _____.

(e) Suppose that you put a specific tax on the monopolist of \$20 per unit output. What would its profit-maximizing level of output be?_____.

25.5 (1) The Grand Theater is a movie house in a medium-sized college town. This theater shows unusual films and treats early-arriving movie goers to live organ music and Bugs Bunny cartoons. If the theater is open, the owners have to pay a fixed nightly amount of \$500 for films, ushers, and so on, regardless of how many people come to the movie. For simplicity, assume that if the theater is closed, its costs are zero. The nightly demand for Grand Theater movies by students is $Q_S = 220 - 40P_S$, where Q_S is the number of movie tickets demanded by students at price P_S. The nightly demand for nonstudent moviegoers is $Q_N = 140 - 20P_N$.

(a) If the Grand Theater charges a single price, P_T, to everybody, then at prices between 0 and \$5.50, the aggregate demand function for movie tickets is $Q_T(P_T) =$ _____ Over this range of prices, the inverse demand function is then $P_T(Q_T) =$_____.

(b) What is the profit-maximizing number of tickets for the Grand Theater to sell if it charges one price to everybody? _____ At what price would this number of tickets be sold? _____ How much profits would the Grand make? _____ How many tickets would be sold to students? _____ To nonstudents?_____.

(c) Suppose that the cashier can accurately separate the students from the nonstudents at the door by making students show their school ID cards. Students cannot resell their tickets and nonstudents do not have access to student ID cards. Then the Grand can increase its profits by charging students and nonstudents different prices. What price will be charged to students? _____ How many student tickets will be sold?

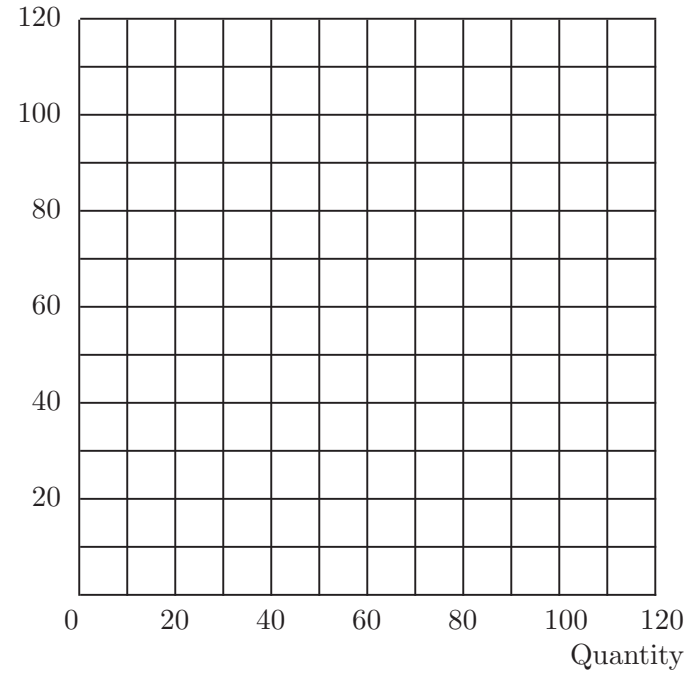
_____ What price will be charged to nonstudents? _____ How many nonstudent tickets will be sold? _____ How much profit will the Grand Theater make?_____.

(d) If you know calculus, see if you can do this part. Suppose that the Grand Theater can hold only 150 people and that the manager wants to maximize profits by charging separate prices to students and to nonstudents. If the capacity of the theater is 150 seats and Q_S tickets are sold to students, what is the maximum number of tickets that can be sold to nonstudents? $Q_N =$ _____ Write an expression for the price of *nonstudent* tickets as a function of the number of *student* tickets sold. (Hint: First find the inverse nonstudent demand function.)

_____ Write an expression for Grand Theater profits as a function of the number Q_S only. (Hint: Make substitutions using your previous answers.) _____

_____ How many student tickets should the Grand sell to maximize profits? _____ What price is charged to students? _____ How many nonstudent tickets are sold? _____ What price is charged to nonstudents? _____ How much profit does the Grand make under this arrangement?_____.

25.6 (2) The *Mall Street Journal* is considering offering a new service which will send news articles to readers by email. Their market research indicates that there are two types of potential users, impecunious students and high-level executives. Let x be the number of articles that a user requests per year. The executives have an inverse demand function $P_E(x) = 100 - x$ and the students have an inverse demand function $P_U(x) = 80 - x$. (Prices are measured in cents.) The *Journal* has a zero marginal cost of sending articles via email. Draw these demand functions in the graph below and label them.



(a) Suppose that the *Journal* can identify which users are students and which are executives. It offers each type of user a different all or nothing deal. A student can either buy access to 80 articles per year or to none at all. What is the maximum price a student will be willing to pay for access to 80 articles? _____ _____ (Hint: Recall the lesson on consumer's surplus and the area under the demand curve.) An executive can either buy access to 100 articles per year or to none at all. What is the maximum price an executive would be willing to pay for access to 100 articles?_____ _____.

(b) Suppose that the *Journal* can't tell which users are executives and which are undergraduates. Thus it can't be sure that executives wouldn't buy the student package if they found it to be a better deal for them. In this case, the *Journal* can still offer two packages, but it will have to let the users self-select the one that is optimal for them. Suppose that it offers two packages: one that allows up to 80 articles per year the other that allows up to 100 articles per year. What's the highest price that the undergraduates will pay for the 80-article subscription?_____.

(c) What is the total value to the executives of reading 80 articles per year?_____ (Hint: Look at the area under their demand curve and to the right of a vertical line at 80 articles.)

(d) What is the the maximum price that the *Journal* can charge for 100 articles per year if it wants executives to prefer this deal to buying 80 articles a year at the highest price the undergraduates are willing to pay for 80 articles?_____.

(e) Suppose that the *Mall Street Journal* decides to include only 60 articles in the student package. What is the most it could charge and still get student to buy this package?_____.

(f) If the *Mall Street Journal* offers a "student package" of 60 articles at this price, how much *net* consumer surplus would executives get from buying the student package?_____.

(g) What is the most that the *Mall Street Journal* could charge for a 100-article package and expect executives to buy this package rather than the student package?_____.

(h) If the number of executives in the population equals the number of students, would the *Mall Street Journal* make higher profits by offering a student package of 80 articles or a student package of 60 articles?_____.

25.7 (2) Bill Barriers, CEO of MightySoft software, is contemplating a new marketing strategy: bundling their best-selling wordprocessor and their spreadsheet together and selling the pair of software products for one price.

From the viewpoint of the company, bundling software and selling it at a discounted price has two effects on sales: (1) revenues go up due to to additional sales of the bundle; and (2) revenues go down since there is less of a demand for the individual components of the bundle.

The profitability of bundling depends on which of these two effects dominates. Suppose that MightySoft sells the wordprocessor for $200 and the spreadsheet for $250. A marketing survey of 100 people who purchased either of these packages in the last year turned up the following facts:

1) 20 people bought both.
2) 40 people bought only the wordprocessor. They would be willing to spend up to $120 more for the spreadsheet.
3) 40 people bought only the spreadsheet. They would be willing to spend up to $100 more for the wordprocessor.

In answering the following questions you may assume the following:

1) New purchasers of MightySoft products will have the same characteristics as this group.
2) There is a zero marginal cost to producing extra copies of either software package.
3) There is a zero marginal cost to creating a bundle.

(a) Let us assume that MightySoft also offers the products separately as well as bundled. In order to determine how to price the bundle, Bill Barriers asks himself the following questions. In order to sell the bundle to the wordprocessor purchasers, the price would have to be less than

_____.

(b) In order to sell the bundle to the spreadsheet users, the price would have to be less than_____.

(c) What would MightySoft's profits be on a group of 100 users if it priced the bundle at \$320?_____

_____.

(d) What would MightySoft's profits be on a group of 100 users if it priced the bundle at \$350?_____

_____

_____

_____.

(e) If MightySoft offers the bundle, what price should it set?_____

_____.

(f) What would profits be without offering the bundle?_____

_____.

(g) What would be the profits with the bundle?_____.

(h) Is it more profitable to bundle or not bundle?_____.

(i) Suppose that MightySoft worries about the reliability of their market survey and decides that they believe that without bundling t of the 100 people will buy both products, and $(100-t)/2$ will buy the wordprocessor only and $(100-t)/2$ will buy the spreadsheet only. Calculate profits as a function of t if there is no bundling._____.

(j) What are profits with the bundle?_____.

(k) At what values of t would it be unprofitable to offer the bundle?_____

_____

_____

_____.

(l) This analysis so far has been concerned only with customers who would purchase at least one of the programs at the original set of prices. Is there any additional source of demand for the bundle? What does this say about the calculations we have made about the profitability of bundling?_____

_____

_____

_____.

25.8 (0) Colonel Tom Barker is about to open his newest amusement park, Elvis World. Elvis World features a number of exciting attractions: you can ride the rapids in the Blue Suede Chutes, climb the Jailhouse Rock and eat dinner in the Heartburn Hotel. Colonel Tom figures that Elvis World will attract 1,000 people per day, and each person will take $x = 50 - 50p$ rides, where p is the price of a ride. Everyone who visits Elvis World is pretty much the same and negative rides are not allowed. The marginal cost of a ride is essentially zero.

(a) What is each person's inverse demand function for rides?_____

_____.

(b) If Colonel Tom sets the price to maximize profit, how many rides will be taken per day by a typical visitor?_____.

(c) What will the price of a ride be?_____.

(d) What will Colonel Tom's profits be per person?_____.

(e) What is the Pareto efficient price of a ride?_____.

(f) If Colonel Tom charged the Pareto efficient price for a ride, how many rides would be purchased?_____.

(g) How much consumers' surplus would be generated at this price and quantity?_____.

(h) If Colonel Tom decided to use a two-part tariff, he would set an admission fee of _____ and charge a price per ride of_____.

25.9 (1) The city of String Valley is squeezed between two mountains and is 36 miles long, running from north to south, and only about 1 block wide. Within the town, the population has a uniform density of 100 people per mile. Because of the rocky terrain, nobody lives outside the city limits on either the north or the south edge of town. Because of strict zoning regulations, the city has only three bowling alleys. One of these is located at the city limits on the north edge of town, one of them is located at the city limits on the south edge of town, and one is located at the exact center of town. Travel costs including time and gasoline are $1 per mile. All of the citizens of the town have the same preferences. They are willing to bowl once a week if the cost of bowling including travel costs and the price charged by the bowling alley does not exceed $15.

(a) Consider one of the bowling alleys at either edge of town. If it charges $10 for a night of bowling, how far will a citizen of String Valley be willing to travel to bowl there?_____ How many customers would this bowling alley have per week if it charged $10 per night of bowling?

_____.

(b) Write a formula for the number of customers that a bowling alley at the edge of town will have if it charges $\$p$ per night of bowling._____

_____.

(c) Write a formula for this bowling alley's inverse demand function.

_____.

(d) Suppose that the bowling alleys at the end of town have a marginal cost of \$3 per customer and set their prices to maximize profits. (For the time being assume that these bowling alleys face no competition from the other bowling alleys in town.) How many customers will they have? _____ What price will they charge?_____ How far away from the edge of town does their most distant customer live?_____.

(e) Now consider the bowling alley in the center of town. If it charges a price of \$$p$, how many customers will it have per week?_____.

(f) If the bowling alley in the center of town also has marginal costs of \$3 per customer and maximizes its profits, what price will it charge? _____ How many customers will it have per week?_____ How far away from the center of town will its most distant customers live? _____.

(g) Suppose that the city relaxes its zoning restrictions on where the bowling alleys can locate, but continues to issue operating licenses to only 3 bowling alleys. Both of the bowling alleys at the end of town are about to lose their leases and can locate anywhere in town that they like at about the same cost. The bowling alley in the center of town is committed to stay where it is. Would either of the alleys at the edge of town improve its profits by locating next to the existing bowling alley in the center of town?_____ What would be a profit-maximizing location for each of these two bowling alleys?_____ _____.

25.10 (1) In a congressional district somewhere in the U.S. West a new representative is being elected. The voters all have one-dimensional political views that can be neatly arrayed on a left-right spectrum. We can define the "location" of a citizen's political views in the following way. The citizen with the most extreme left-wing views is said to be at point 0 and the citizen with the most extreme right-wing views is said to be at point 1. If a citizen has views that are to the right of the views of the fraction x of the state's population, that citizen's views are said to be located at the point x. Candidates for office are forced to publically state their own political position on the zero-one left-right scale. Voters always vote for the candidate whose stated position is nearest to their own views. (If there is a tie for nearest candidate, voters flip a coin to decide which to vote for.)

(a) There are two candidates for the congressional seat. Suppose that each candidate cares only about getting as many votes as possible. Is there an equilibrium in which each candidate chooses the best position given the position of the other candidate? If so, describe this equilibrium._____.

25.11 (2) In the congressional district described by the previous problem, let us investigate what will happen if the two candidates do not care about the number of votes that they get but only about the amount of campaign contributions that they receive. Therefore each candidate chooses his ideological location in such a way as to maximize the amount of campaign contributions he receives, given the position of the other.*

Let us define a left-wing extremist as a voter whose political views lie to the left of the leftmost candidate, a right-wing extremist as a voter whose political views lie to the right of the rightmost candidate, and a moderate voter as one whose political views lie between the positions of the two candidates. Assume that each extremist voter contributes to the candidate whose position is closest to his or her own views and that moderate voters make no campaign contributions. The number of dollars that an extremist voter contributes to his or her favorite candidate is proportional to the distance between the two candidates. Specifically, we assume that there is some constant C such that if the left-wing candidate is located at x and the right-wing candidate is located at y, then total campaign contributions received by the left-wing candidate will be $\$Cx(y-x)$ and total campaign contributions received by the right-wing candidate will be $\$C(1-y)(y-x)$.

(a) If the right-wing candidate is located at y, the contribution-maximizing position for the left-wing candidate is $x =$ _____ If the left-wing candidate is located at x, the contribution-maximizing position for the right-wing candidate is $y =$_____ . (Hint: Take a derivative and set it equal to zero.)

(b) Solve for the unique pair of ideological positions for the two candidates such that each takes the position that maximizes his campaign contributions given the position of the other._____.

* This assumption is a bit extreme. Candidates typically spend at least some of their campaign contributions on advertising for votes, and this advertising affects the voting outcomes.

(c) Suppose that in addition to collecting contributions from extremists on their side, candidates can also collect campaign contributions from moderates whose views are closer to their position than to that of their rivals' position. Suppose that moderates, like extremists, contribute to their preferred candidate and that they contribute in proportion to the difference between their own ideological distance from their less-preferred candidate and their ideological distance from their more-preferred candidate. Show that in this case the unique positions in which the left- and right-wing candidates are each maximizing their campaign contributions, given the position of the other candidate, occurs where $x = 1/4$ and $y = 3/4$.