

Review of Mathematics

Function

1. What is a function? What is a dependent and independent variable?
2. Draw graphs of the following functions (correspondences):
 - (a) $p = 2x - 5$
 - (b) $x_2 = 5 - \sqrt{x_1}$
 - (c) $x_2 = 5 - x_1^2$
 - (d) $x_2 = 4/x_1$
 - (e) $S(p) = s_1(p) + s_2(p)$,
where $s_1(p) = p$ a $s_2(p) = 2p$.
 - (f) $S(p) = s_1(p) + s_2(p)$,
where $s_1(p) = 2p$ a $s_2(p) = p - 1$ for $p \geq 1$
and $s_2(p) = 0$ for $p < 1$.
3. Draw curves that correspond to this equation:
 $10 = \min\{x_1, x_2\}$.
4. What is a continuous function? What is the opposite to the continuous function?
5. What is a monotonic function? What is the shape of a positive and negative monotonic function?
6. What is the shape of a convex and concave function?
7. What is the inverse function? Formulate the inverse function of:
 - (a) $y = ax + b$
 - (b) $y = 5/x$
 - (c) $y = e^x$

Logarithm

8. What is the logarithm? Draw the function
 $y = \ln x$.
9. Calculate the logarithm of $x_1^a x_2^b$.

Derivatives

10. What is the derivative? What is the relationship between the derivative and the slope of a tangent

line to a function? What is the relationship between derivatives and convexity (concavity) of a function?

11. What is the product rule? What is the chain rule?
12. Take the derivative of the following functions with respect to p :
 - (a) $D(p) = 50 - 2p$
 - (b) $D(p) = 30p^{-2}$
 - (c) $D(p) = (2p + a)^{-b}$
 - (d) $R(p) = pq(p)$
13. What is a partial derivative? Take a partial derivative of the following functions with respect to x_1 a x_2 :
 - (a) $U(x_1, x_2) = ax_1 + bx_2$
 - (b) $f(x_1, x_2) = x_1^a x_2^b$
 - (c) $U(x_1, x_2) = a \ln x_1 + bx_2$
 - (d) $U(x_1, x_2) = a\sqrt{x_1} + bx_2$
 - (e) $U(x_1, x_2) = (x_1^2 + x_2^2)^a$

Optimalization

14. Solve the following problem:

$$\max_{x_1, x_2} c \ln x_1 + d \ln x_2$$

$$\text{subject to } p_1 x_1 + p_2 x_2 = m,$$

where $0 < c < 1$, $0 < d < 1$, $p_1 > 0$, $p_2 > 0$
a $m > 0$ are constants.