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WELFARE ANALYSIS OF TAX REFORMS USING HOUSEHOLD DATA

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ABSTRACT

The paper discusses a methodology for calculating the distribution of gains and losses from a policy change using data for a large sample of households. Estimates are based on the equivalent income function, which is money metric utility defined over observable variables. This enables calculations to be standardised, and a computer program to compute the statistics presented in the paper is available for a general demand system. Equivalent income is related to measures of deadweight loss, and standard errors are computed for each of the welfare measures. An application to UK data for 5895 households is given which simulates a reform that involves eliminating housing subsidies.

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WELFARE ANALYSIS OF TAX REFORMS USING HOUSEHOLD DATA

by Mervyn A. King

1. Introduction

The use of cross-section data on individual households not only to estimate equations for labour supply and commodity demands but also to employ the estimates to simulate tax reforms is becoming widespread. In this paper we present a methodology for such simulations emphasising both the efficiency and the distributional consequences of the reform, and the design of welfare measures which can be computed for each household. The welfare measures are presented in terms of a series of "equivalent" statistics which provide information on both the distributional and efficiency effects of a reform, and can be used also to generate measures of social welfare (one example of which is an index of inequality). We are particularly concerned with the case in which prices vary between households.

The basic assumption is that there are available data for a cross-section of households and also econometric estimates of the relevant behavioural equations. With this information we then (a) define a reform (section 2), (b) compute various measures of the gain accruing to each household in the sample and of the overall efficiency gain

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(section 3), (c) calculate measures of the "social" gain and of inequality using an explicit social welfare function (section 4), and (d) use the estimated standard errors of the parameters of the behavioural equations to construct confidence intervals for our welfare measures (section 5). The theoretical concepts are illustrated by an empirical application to the simulation of the abolition of housing subsidies to both home-owners and tenants living in publicly subsidised accommodation using a data set for 5895 households in England and Wales (section 6). A computer programme is available on request to perform the computations described here for a general specification of a reform and given parameters and a functional form for an indirect utility function.

2. Defining a Reform

We assume that the data set contains observations on H households indexed by h. In the pre-reform or original position household h has (net of tax) exogenous income y_h^o and faces a vector of prices $\underline{p}_h^o$. After the reform the household faces a new vector of prices and income, $(\underline{p}_h^p, y_h^p)$. A reform is defined as a mapping from the original to the post-reform vector.

$$\{y_h^o, \underline{p}_h^o\} \rightarrow \{y_p^h, \underline{p}_h^p\} \qquad \forall\ h = 1...H \qquad (2.1)$$

We shall assume that a reform can be completely characterised in terms of its effects on household incomes and prices. This implies that prices are independent of household actions and is equivalent to supposing that each household faces a linear budget constraint. For many purposes (such as changing the rates of indirect taxes) this is a reasonable assumption, but for other applications budget constraints may be nonlinear. Examples of nonlinear budget constraints may be found in work on labour supply (Burtless and Hausman 1978, Wales and Woodland 1979). In these cases we may still define a reform by (2.1) provided that the income and price vectors are replaced by vectors of "virtual" incomes and prices which are defined such that if the budget constraint were linear in these prices households would choose the same consumption bundle as they select under the nonlinear budget constraint. Another example of a nonlinear budget constraint, albeit of a rather extreme form, is where households are rationed in the amount of particular commodities which they may consume; again this constraint may be modelled in terms of virtual prices, Rothbarth (1941) Neary and Roberts (1980) and Deaton and Muellbauer (1980a). In principle, therefore, nonlinear budget constraints present no problem; in practice, however, it may be computationally expensive to compute the virtual prices and income for each household in the sample when the budget constraint is very complex.

In general, we shall allow prices to be household-specific. This is of importance not only for nonlinear budget constraints but also for applications to labour supply and housing where the relevant prices vary across households.[1] It also allows household composition to enter the indirect utility function by making the effective prices which a household pays a function of household composition, as suggested by Gorman (1976).

We have observations on initial incomes and prices for each household. A reform is defined by specifying for each household a set of post-reform prices and an income level. Although this is straightforward in principle, the computation of the post-reform equilibrium may be complicated. We have already mentioned the problems caused by nonlinear budget constraints. Another difficulty is that the post-

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[1] In the sample of 5895 households used in this paper, the coefficient of variation of the price of housing services was of the same order of magnitude as that for earnings (King (1980)).

reform values cannot be chosen independently but must satisfy an overall revenue constraint to ensure that the reform is feasible. We shall examine reforms which are revenue-neutral. This is for two reasons. First, to ensure that the government's budget constraint is satisfied by the reform. Secondly, because for revenue-neutral reforms our measure of the distributional impact of the reform is related in a very simple manner to our measure of the efficiency gain (see section 3 below). It is crucial to our approach that we consider reforms which satisfy an overall production constraint. This enables us to produce a measure of the change in deadweight loss which is related in an intuitively appealing way to our measure of the distribution of gains and losses resulting from the reform. When government expenditure on goods and services remains unchanged the overall production constraint is satisfied by a revenue-neutral reform. In principle, it is straightforward to allow for reforms which involve changes in government expenditure. To do this we must specify household preferences in terms of an

indirect utility function which has as arguments the prices of private goods and the quantities of public goods. The difficulty with implementing this is, of course, that we do not have observations on the revealed preferences of households for public goods.[1] Consequently, we shall restrict our attention to revenue-neutral reforms.

If we consider only taxes on households, then a revenue-neutral reform must satisfy

[1] Though it may be possible to infer these by estimating demands for private goods as a function of prices of private goods and quantities of public goods.

$$\sum_h f_h R_h^o(y_{hG}^o, \underline{X}_h^o) = \sum_h f_h R_h^p(y_{hG}^p, \underline{X}_h^p) \tag{2.2}$$

where f_h is the sampling weight for each household

y_{hG} is pre-tax exogenous income

$\underline{X}_h$ is a vector of household commodity demands (including leisure)

R_h is the tax revenue function levied on household h.

The precise form of the revenue functions depends on the tax-subsidy system employed. For linear commodity taxes the overall revenue constraint becomes

$$\sum_h f_h t^o(y_{hG}^o) + \sum_h f_h \underline{t}_h^{o'} P_h^o \underline{X}_h^o = \sum_h f_h t^p(y_{hG}^p) + \sum_h f_h \underline{t}_h^{p'} P_h^p \underline{X}_h^p \tag{2.3}$$

where $\underline{t}_h$ is a vector of the <u>tax-inclusive</u> rates of household-specific commodity taxes or subsidies.

$t(.)$ is the tax function levied on exogenous income.

P_h is a diagonal matrix such that $P_h \underline{i} = \underline{p}_h$, where $\underline{i}$ is the unit vector.

The use of sampling weights (which reflect stratification in the sample design or estimated differential response) is to ensure an unbiassed estimate of tax revenue. If information is available also on the extent of under-reporting of particular items of expenditure (which occurs for alcohol and tobacco in most household surveys) the weights can be made commodity-specific. The extension to include producer taxes and subsidies is straight forward.

The specification of the reform itself will dictate which tax rate is the residual rate to be determined by the revenue-neutral constraint. Since post-reform household demands are functions of the vector of prices, and hence of tax rates, equation (2.3) is an implicit equation for the residual tax rate. To solve this we require estimates of household demand functions. For a particular specification of a household's indirect utility function (denoted by v), the vector of commodity demands is given by the Roy-Ville identity

$$\underline{X} = \frac{-\dfrac{\partial v}{\partial \underline{p}}}{\dfrac{\partial v}{\partial y}} \tag{2.4}$$

Given estimates of this demand system the value of the residual tax rate may be determined by solving (2.3) and (2.4).

Another factor to be considered is the change in producer or factor prices which may result from the change in equilibrium output levels. Producer prices (denoted by the vector $\underline{q}$) are assumed to be uniform across consumers and, in general, are a function of total supplies ($\underline{z}$). The net supply vector in the post-reform equilibrium is equal to

$$\underline{z} = \underline{z}(\underline{q}^{p}) = \sum_{h} \underline{X}_{h}^{p} \tag{2.5}$$

where the vector-valued function $\underline{z}$ is equal to the derivatives of the industry profit functions. Again the extension to incorporate producer taxes is straightforward.

Consumer prices are related to producer prices via the tax rates and, given functional forms for both demand and supply functions (or, equivalently, for indirect utility and profit functions), the post-reform equilibrium can be found by solving iteratively the system of equations given by (2.3), (2.4) and (2.5). Given this solution the reform as described by the mapping (2.1) is completely defined, and we may turn to the measurement of gains and losses for each household in the sample. The computation of the post-reform equilibrium is similar in spirit to the numerical general equilibrium models of Shoven and Whalley (1972, 1973, 1977).[1] These models have a detailed production side (with many commodities) and assume either one or a small number of representative consumers. The approach here is to use a simple model of the production sector but to use actual observations on a large sample of households. The two approaches are clearly complementary.

Finally, we must note that in what follows we shall assume the existence and uniqueness of equilibrium. In any application the former is not a problem because the definition of the reform will provide a constructive proof of existence. Uniqueness, however, is not guaranteed and must be investigated in each case. If there is more than one solution for the residual tax rate, we could choose between the equilibria according to either a social welfare function or a simple criterion such as select the equilibrium corresponding to the

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[1] Their work contributed a great deal also to the development of algorithms for the computation of general equilibrium models.

lowest tax rate (subsidy). The empirical significance of non-uniqueness is unclear: in the examples considered below there is a unique equilibrium.

3. The Measurement of Welfare and Deadweight Loss

In this section we describe measures of both the distributional and efficiency effects of a reform based upon ordinal utility functions. Measures of the social gain of a reform which are based on an explicit social welfare function will be discussed in Section 4. Both sets of calculations are derived from a single measure of the money value of a household's welfare in terms of its consumption possibility set. This is given by the "equivalent income function" which is defined below.

Household preferences may be represented by either the direct or indirect utility function which are denoted, respectively, by

$$u = u(\underline{X}) \quad (3.1)$$

$$v = v(\underline{p}, y) \quad (3.2)$$

We wish to compare the levels of a household's welfare when it faces different consumption possibility sets. To do this we choose a reference price vector, denoted by $\underline{p}^R$. The choice of the reference price vector is arbitrary, although as we shall argue below there are certain choices for this which allow a natural interpretation of equivalent income. For a given budget constraint $(\underline{p},y)$ "equivalent income" is defined as that level of income which, at the reference price vector, affords the same level of utility as can be attained under the given budget constraint. Formally,

$$v(\underline{p}^R, y_E) = v(\underline{p}, y) \tag{3.3}$$

Inverting the indirect utility function we obtain equivalent income in terms of the expenditure function

$$y_E = e(\underline{p}^R, v) \tag{3.4}$$

Combining (3.3) and (3.4)

$$y_E = f(\underline{p}^R, \underline{p}, y) \tag{3.5}$$

This definition of equivalent income has been suggested also by Varian (1980). It is very similar to the concept proposed by McKenzie (1956) which was later christened 'money metric utility' by Samuelson (1974), and is defined by [1]

$$\begin{aligned} m &\equiv e(\underline{p}^R, u) \\ &= g(\underline{p}^R, \underline{x}) \end{aligned} \tag{3.6}$$

The main advantage of using the equivalent income function is that it makes computations extremely easy because preferences enter only into the form of the function f and the arguments of the function depend only upon the reform under consideration and are completely independent of

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1 In unpublished notes, Varian (1980) derives an "indirect income compensation function" which is formally identical to our equivalent income function. Choice of terminology is left to the reader. For further discussion of money metric utility see Deaton (1980)

preferences.[1] In (3.6), however, preferences influence both the form and the arguments of the money metric utility function. With the equivalent income function preferences are described by the form of the function and opportunities by the values of the arguments of the function. As we shall show, the separation of the representation of preferences and opportunities enables us to evaluate both the efficiency and the distributional effects of a reform in terms of a single function. All of the welfare measures presented in this paper are calculated by evaluating the equivalent income function for different values of its arguments.

The properties of the equivalent income function may be derived from the well-known properties of indirect utility and expenditure functions (see, for example, Deaton and Muellbauer (1980a), and Diewert (1978)). These imply that f is increasing in p^R and y and decreasing in $\underline{p}$, is concave and homogeneous of degree one in reference prices, and is continuous with first and second derivatives in all arguments.[2] Commodity demands are given by

$$\underline{x}(\underline{p}, y) = \left.\frac{\partial f}{\partial \underline{p}^R}\right|_{\underline{p}^R = \underline{p}} = \frac{-\frac{\partial f}{\partial \underline{p}}}{\frac{\partial f}{\partial y}} \tag{3.7}$$

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[1] This is true only for linear budget constraints. For non-linear budget constraints (and rationing) the calculations of virtual prices requires knowledge of preferences. Even here, however, the equivalent income function reduces the calculation of welfare measures to a two-step procedure; first, the computation of virtual prices, and secondly, the calculation of equivalent income.

[2] Strictly speaking, first and second derivatives exist except possibly on a set of measure zero, and by increasing (decreasing) in $\underline{p}^R$ ($\underline{p}$) we mean nondecreasing (nonincreasing) in $\underline{p}^R$($\underline{p}$) and increasing (decreasing) in at least one element of $\underline{p}^R$($\underline{p}$).

Given a set of demand functions, the equivalent income function may be generated by solving the system of partial differential equations together with the boundary condition[1]

$$y = f(\underline{p}, \underline{p}, y) \tag{3.8}$$

As an example, consider the two-commodity Cobb-Douglas case with the indirect utility function

$$v(p_1, p_2, y) = \frac{p_1^a \, p_2^{1-a}}{y} \tag{3.9}$$

From (3.3) and (3.4) the equivalent income function is given by

$$y_E = \left(\frac{p_1^R}{p_1}\right)^a \left(\frac{p_2^R}{p_2}\right)^{1-a} \cdot y \tag{3.10}$$

In the appendix we give the equivalent income function for some common demand systems.

The first use to which we shall put the equivalent income function is to measure welfare gains and losses, and to relate these to the concept of the deadweight loss or excess burden of a tax. A natural measure of the value of a reform to a household is the change in equivalent income. This measure of welfare gain is given by

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[1]Provided that the estimated demand system has a symmetric negative semidefinite matrix (Samuelson 1950, Hurwicz and Uzawa, 1971) and the Lipschitz condition is satisfied.

$$WG_h = y_{Eh}^p - y_{Eh}^o$$
$$= f(\underline{p}^R, \underline{p}_h^p, y_h^p) - f(\underline{p}^R, \underline{p}_h^o, y_h^o) \tag{3.11}$$

The choice of reference price vector is arbitrary, although for any given household there are two natural choices, the pre- and post-reform prices vectors. Using initial prices the welfare gain is that sum of money which the household would have accepted in the initial position as equivalent to the impact of the reform, and we describe this as the equivalent gain. It is defined by

$$EG_h = f(\underline{p}_h^o, \underline{p}_h^p, y_h^p) - f(\underline{p}_h^o, \underline{p}_h^o, y_h^o)$$
$$= f(\underline{p}_h^o, \underline{p}_h^p, y_h^p) - y_h^o \tag{3.12}$$

Implementing the reform is equivalent to giving each household an addition to current income equal to the value of their equivalent gain.

If we now measure equivalent income at a reference price vector equal to the post-reform price vector, we may define "compensating gain" as

$$CG_h = f(\underline{p}_h^p, \underline{p}_h^p, y_h^p) - f(\underline{p}_h^p, \underline{p}_h^o, y_h^o)$$
$$= y_h^p - f(\underline{p}_h^p, \underline{p}_h^o, y_h^o) \tag{3.13}$$

Equivalent and compensating gains are two special cases of the general measure of welfare gain corresponding to two particular reference price vectors. In general, the welfare gain to a household of a tax reform is measured by the change in equivalent income. A natural definition, therefore, of the excess burden or deadweight loss of a tax is the change in equivalent income resulting from its introduction. In a one-person economy the change in deadweight loss (ΔDL) arising from a tax reform is

$$\Delta DL = y_E^p - y_E^o = WG_h \tag{3.14}$$

This definition brings out clearly the dependence of the measure of deadweight loss on the choice of reference price vector and provides a convenient framework within which to discuss existing measures of deadweight loss and the deficiences of some of these.

In a many-person economy an aggregate measure of deadweight loss could be defined as

$$\Delta DL = \sum_h WG_h \tag{3.15}$$

But the dependence of the measure on the choice of reference price vector means that adding welfare gains in this way to produce a measure of aggregate deadweight loss, as suggested by Harberger (1971), is not a procedure which is free of distributional implications because, as we show in section 4 below, the choice of reference price vector is equivalent to making distributional judgements. Measuring deadweight loss by (3.15) does not so much ignore distributional issues as imply the use of a specific set of distributional weights, which, moreover, is likely to alter from one empirical application to another as the reference price vector changes.

It is clear that welfare gain is defined only for a completely specified reform. If we consider the introduction of a distortionary tax financed by a reduction in lump-sum taxes, then the welfare gain (negative in this case) is equal to the deadweight loss imposed by the tax. When a reform involves changes in two or more distortionary taxes the welfare gain measures the net reduction in deadweight loss, and this cannot be attributed to one tax rather than another. Where the additional tax revenue is used to finance higher public expenditure, the welfare gain measures the net benefit of the public good. Both the definition and measurement of deadweight loss require a precise definition of the reform. If we consider a new tax the proceeds of which are used to make lump-sum payments, then, even if we are prepared to sum welfare gains over households, the value of deadweight loss will depend upon how the lum-sum payments are distributed among households.[1]

We turn now to the relationship between our measure of the welfare gain and conventional measures of the excess burden or deadweight loss from distortionary taxes. Approximations to consumers' surplus based on Harberger

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[1]Deadweight loss will depend upon the distribution of payments except when households face identical prices and preferences are such that they give rise to parallel linear Engel curves - see the discussion on aggregation below.

triangles may be replaced by exact measures of deadweight loss derived from explicit utility functions (Mohring 1971, Diamond and McFadden 1974, Rosen 1978, Kay 1980, Hausman 1981, Auerbach and Rosen 1980). Since estimated demand functions can be integrated to obtain the underlying utility function, there is no reason to use an approximation when an exact measure can be computed. It is easy to see the relationship between welfare gain and other exact measures of the deadweight loss of a tax. Consider the Hicksian measures of compensating and equivalent variation, CV and EV respectively (Hicks 1946, pps. 330-3), defined for a change in prices holding income constant. The compensating variation is the amount of money which the household would need to be given at the new set of prices in order to attain the pre-reform level of utility. In terms of the expenditure function it is defined by

$$\begin{aligned} CV &= e(\underline{p}^p, v^o) - e(\underline{p}^o, v^o) \\ &= f(\underline{p}^p, \underline{p}^o, y^o) - y^o \end{aligned} \qquad (3.16)$$

The equivalent variation is defined in terms of the post-reform level of utility by

$$\begin{aligned} EV &= e(\underline{p}^p, v^p) - e(\underline{p}^o, v^p) \\ &= y^p - f(\underline{p}^o, \underline{p}^p, y^p) \end{aligned} \qquad (3.17)$$

Hence from (3.12), (3.13), (3.16) and (3.17)

$$EG_h = y_h^p - y_h^o - EV_h \tag{3.18}$$

$$CG_h = y_h^p - y_h^o - CV_h \tag{3.19}$$

To obtain an estimate of deadweight loss the usual approach is to subtract from either the compensating or equivalent variation the change in tax revenue. Diamond and McFadden (1974) proposed a definition in terms of the compensating variation and Kay (1980) suggested the use of the equivalent variation. Assume that we are prepared to sum welfare gains across households. In terms of the equivalent variation the change in deadweight loss resulting from a reform is

$$\Delta DL = \sum_h EV_h - (R^p - R^o) \tag{3.20}$$

where R^o and R^p are original and post-reforms levels of total revenue respectively.

If we assume that producer prices are constant (i.e. pre-tax exogenous incomes are constant) then for a revenue-neutral reform

$$R^p - R^o = \sum_h (y_h^p - y_h^o) \tag{3.21}$$

Combining (3.18), (3.20) and (3.21) we have

$$\Delta DL = -\sum_h EG_h \tag{3.22}$$

The reduction in deadweight loss resulting from tax reform is equal to the sum of the equivalent gains over all households.

In contrast, Diamond and McFadden (1974) define the deadweight loss of a distortionary tax in terms of the compensating variation. Translated to the case of a move from one distortionary tax system to another this becomes[1]

$$\Delta DL = \sum_h CV_h - (R^P - R^O) \qquad (3.23)$$

From (3.19), (3.21) and (3.23)

$$\Delta DL = -\sum_h CG_h \qquad (3.24)$$

There is, however, a serious problem with the use of (3.24) to evaluate the relative efficiency losses of alternative tax reforms. A definition of deadweight loss based on compensating variation cannot be used to compare (or choose between) mutually exclusive tax reforms. The reason for this is that any such comparison must employ a common reference price vector. Measures based on equivalent gain or equivalent variation use the pre-reform price vector to evaluate all possible reforms. But measures based on

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[1] Diamond and McFadden (op. cit) define deadweight loss in terms of the value of the compensated tax revenue function rather than the actual level of post-reform revenue as here. But this does not affect the basic point that compensating variation implies the use of changing reference price vectors - see below.

compensating variation employ a different reference price vector for each reform. Hence the money value of the gain from reform A cannot be compared with the money value of the gain from reform B because these values are computed at difference reference price levels. It is for this reason that an unambiguous improvements in the tax system (an increase in utility for a given level of net revenue) may lead to an increase in deadweight loss as measured by (3.24) (Kay, 1980). Although choice of reference price vector is arbitrary, welfare comparisons of taxes can only be made with respect to a common reference price vector.[1]

The measures of deadweight loss proposed by Diamond and McFadden (1974) and Kay (1980)presuppose fixed producer prices. This is not true, however, of our measure in terms of welfare gains, provided that the effects of the change in producer prices on consumer prices and household exogenous incomes are correctly specified in the definition of the reform. The argument for measuring the efficiency gain of a reform as the sum of welfare gains does not require the assumption of fixed producer prices. An empirical example of changing producer prices in this framework is given in King (1981).

There is an infinite number of reference price

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The use of equivalent income function demonstrates this clearly because equivalent and compensating gains are defined with respect to a reference price level whereas equivalent and compensating variation are defined with respect to a reference utility level.

vectors which could be used to compute deadweight loss. The original or pre-reform price vector is a natural choice. Once the reform has been defined, we may calculate equivalent gain for each household in the sample. The distributional impact of the reform is measured by the distribution of equivalent gains, and the "efficiency gain" to the economy as a whole by the average value of equivalent gain. One of the advantages of using individual household data is to examine the distribution of gains and losses within particular sub-groups of the population, such as decile groups of the distribution of original equivalent income or for different types of household. It must be stressed that deadweight loss as used here means the welfare loss from raising a given amount of revenue from distortionary taxes rather than from nondistortionary taxes. It is <u>not</u> a measure of allocative efficiency. Thus a reform which involves a move from one Pareto-efficient allocation to another by means of lump-sum transfers does not, in general, yield a zero measure for the change in deadweight loss. The rason for this is as follows. Moving between Pareto-efficient allocations using lump-sum transfers implies a linear transformation curve in terms of exogenous incomes (endowments) such that

$$\sum_h y_h = \bar{Y} \tag{3.25}$$

A constant level of deadweight loss is represented by the contours of

$$\sum_h y_{Eh} = \bar{Y}_E \tag{3.26}$$

It follows that (3.25) implies (3.26) if and only if y_{Eh} is a linear function of y_h. It is easy to see from (3.3) and (3.4) that for any arbitrary reference price vector this requires expenditure and the level of indirect utility to be linearly related. In turn this implies the familiar condition for aggregation to be possible of parallel linear Engel curves (Gorman 1961). For more general preferences a move from one Pareto-efficient allocation to another will imply a non-zero value for the measure of deadweight loss.

To date the discussion has assumed the existence of data for individual households. But estimates of deadweight loss are often made when only aggregate demand responses are known. The errors implied by the use of aggregate data depend on the distribution over the population of incomes, prices, and preferences. Given a definition of change in deadweight loss as the difference between mean equivalent income before and after the reform, it is clear that, even if we assume identical preferences and prices, evaluating the welfare gain at the average income level will give the correct value only if the equivalent income function is linear in income. This condition implies parallel linear Engel curves. If the equivalent income function is evaluated not at mean household income but at an income level given by a specific function of household income, then it can be shown that for correct estimates of the welfare gain preferences must satisfy price-indenendent generalised linearity (Muellbauer 1975, 1976). When prices vary across the population then evaluating the equivalent income function at mean prices and incomes will produce biassed estimates unless y_E is linear in prices. This is implausible because it requires demand for the commodity whose price varies to be independent of income.

4. The Social Value of a Reform

In addition to information about the distribution of welfare gains among households, we may be interested also in calculations of the "social value" of a reform. By this we mean any measure the computation of which requires an explicit social welfare function. In other words, the social value of a reform depends upon an assumption about the cardinality of utility functions, whereas to measure welfare gain requires only an ordinal measure of utility.

When individual preferences are defined over more than one commodity, a social welfare function which respects individual preferences cannot be written simply as a function of household incomes but will depend also on the price vectors facing households.[1] The marginal social valuation of an additional unit of income for household h depends not only on the income of household h but also upon the price vector which it faces. In order to compare households we must choose a common reference price vector at which to make the comparison. This means that when individual preferences are defined over a vector of commodities, social welfare may be defined over the levels of equivalent income.[2]

$$W = W(y_{E1}, y_{E2}, \dots y_{EH}) \tag{4.1}$$

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[1] An alternative approach is to define social welfare directly on the quantities consumed by households with no regard paid to the implicit weighting of the different commodities implied by utility maximisation. Multi-dimensional measures of inequality based on this approach are discussed by Atkinson and Bourguignon (1981)

[2] To simplify the exposition we ignore sampling weights which would be taken into account in an empirical application.

Equivalent income provides a scalar measure of a household's welfare in money terms. Social welfare will take into account also differences in household size and composition. Suppose that preferences depend upon a vector of characteristics, $\underline{c}$, describing household composition so that the indirect utility function becomes $v(\underline{p}, \underline{c}, y)$. Equivalent income may now be defined in terms not only of a reference price vector but also of a reference household.

$$v(\underline{p}^R, \underline{c}^R, y_E) = v(\underline{p}, \underline{c}, y) \tag{4.2}$$

Hence, as before,

$$y_E = f(\underline{p}^R, \underline{c}^R, \underline{p}, \underline{c}, y) \tag{4.3}$$

Commodity demands are given by (3.7)[1]. There is no guarantee, however, that (4.3) can be recovered from observable demands. As Cramer (1969) and Muellbauer (1974b) show, identification of the parameters of (4.3) depends upon the particular model used to relate composition to demand behaviour and the data set on which the model is estimated.[2] Assuming (4.3) has been recovered, the final choice is whether to take the household or the individual as the unit. The former gives equal weight to each household's equivalent income (as given by (4.3)) in (4.1); the latter weights equivalent income by

[1] The partial derivatives are now evaluated at the reference compositi[on] vector as well as at reference prices. The boundary condition (3.8) becomes $f(\underline{p}, \underline{c}, \underline{p}, \underline{c}, y) = y$.

[2] For a discussion of household composition, equivalence scales and money metric utility see Deaton and Muellbauer (1980a).

the number of persons in the household.

When making welfare comparisons between households equivalent income must be computed using the same reference price vector for each household. Although the choice of the common reference price vector is arbitrary certain choices are easier to interpret than others. One natural choice is the average value of prices in the original position. It would seem easier to ask policy-makers to state their relative valuations of increments to equivalent income at different levels of equivalent income (a measure of inequality aversion on the part of the policy-maker) for the currently observed price level than for some other hypothetical price vector. Hence in the empirical estimates we shall use the mean values (over households) of pre-reform prices. With this choice of reference price vector, then in the particular case where all households face the same pre-reform price vector, original equivalent income is equal to original income and post-reform equivalent income is equal to original income plus the value of equivalent gain.

In general welfare comparisons between households depend upon the choice of reference price vector (Roberts, 1980). In one case, however, social choices are independent of the reference price vector. When household preferences are homothetic we have

$$v(\underline{p}, y) = g(\underline{p})y \qquad (4.4)$$

$$e(\underline{p}, v) = h(\underline{p})v \qquad (4.5)$$

This gives for the equivalent income function

$$y_E = h(\underline{p}^R)g(\underline{p})y \tag{4.6}$$

If the social welfare function is homothetic then it is obvious that social choices are independent of the reference price vector.[1]

The first measure we define is the "social (equivalent) gain", denoted by SG, which is an exact measure of the social value of a reform parallel to the equivalent gain for an individual household defined in Section 3. Consider an absolute increment to each household's original equivalent income. The social gain is the value of such an increment which would produce a level of social welfare equal to that obtaining in the post-reform equilibrium. It is defined by

$$W(y^o_{E1} + SG, \ldots y^o_{EH} + SG) = W(y^p_{E1} \ldots y^p_{EH}) \tag{4.7}$$

In the special case where (a) the reference price vector is taken to be the pre-reform price vector, and (b) all households face identical pre-reform prices, then the social gain is equal to the number of "uniformly distributed dollars" measure of the benefit of the reform used by Feldstein (1974) and Rosen (1976). This is defined as the sum of money which, if equally distributed to all households in the original position, would produce a level of social welfare equal to the post-reform

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[1]This derivation illustrates propositions 1 and 2 in Roberts (1980). It is clear also that homotheticity of W is a necessary condition.

level. Only in this special case is original equivalent income equal to original exogenous income and the two measures coincide. In general, the social gain is to be preferred because it takes into account differences in prices between households.

An alternative way of measuring social gain is to compute the equal proportionate increase in original equivalent incomes which would produce a level of welfare equal to that derived from the post-reform equilibrium. This value denoted by λ, is defined by

$$W(\lambda y^{o}_{E1}, \ . \ . \ . \ \lambda y^{o}_{EH}) = W(y^{p}_{E1}, \ldots y^{p}_{EH}) \tag{4.8}$$

Equivalent income may also be used to construct a scalar measure of inequality defined over the distribution of equivalent incomes rather than over simply incomes as in the usual case. Following Atkinson (1970) and Sen (1973) we define the"equally distributed equivalent level of equivalent income" as that level of equivalent income which, if shared equally by all households, would produce the same level of social welfare as that generated by the actual distribution of equivalent incomes. The original and post-reform values of this measure, y^{o}_{E} and y^{p}_{E}, are defined by

$$W(\tilde{y}^{o}_{E}, \ldots \tilde{y}^{o}_{E}) = W(y^{o}_{E1}, \ldots y^{o}_{EH}) \tag{4.9}$$

$$W(\tilde{y}^{p}_{E}, \ldots \tilde{y}^{p}_{E}) = W(y^{p}_{E1}, \ldots y^{p}_{EH}) \tag{4.10}$$

If we assume that W is symmetric and quasi-concave, then an index of inequality may be defined as

$$I = 1 - \frac{\tilde{y}_E}{\bar{y}_E} \tag{4.11}$$

where $\bar{y}_E$ is the mean level of equivalent income.[1] If we are prepared to make the further assumption that W is homothetic (i.e. that the measure of inequality is independent of the mean of the distribution) then there is a simple relationship between the proportionate social gain and the original and post-reform inequality measure. For a homothetic W

$$W(\lambda\tilde{y}_E^o, \ldots \lambda\tilde{y}_E^o) = W(\lambda y_{E1}^o, \ldots \lambda y_{EH}^o) \tag{4.12}$$

From (4.8), (4.10) and (4.12)

$$\tilde{y}_E^p = \lambda\tilde{y}_E^o \tag{4.13}$$

Hence from (4.11)

$$\lambda = \frac{\bar{y}_E^p(1 - I^p)}{\bar{y}_E^o(1 - I^o)} \tag{4.14}$$

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[1] It is clear from (4.6) that only if both W and household preferences are homothetic will the index of inequality be independent of the reference price vector (Muellbauer (1974a) Roberts (1980)). Even when this holds the index will depend upon actual prices unless all households face identical prices.

The proportionate social gain is equal to the increase in mean equivalent income adjusted for the change in inequality.

In order to calculate numerical values for either the social gain or an index of inequality we require a specific functional form for W. We shall assume that W is additively separable in equivalent incomes and is homothetic. As Atkinson (1970) has shown, this implies that

$$W = \sum \frac{y_{Eh}^{1-\varepsilon}}{1-\varepsilon} \qquad \varepsilon \geqslant 0, \neq 1$$

$$= \sum_{h} \log y_{Eh} \qquad \varepsilon = 1 \qquad (4.15)$$

where ε is the (relative) inequality aversion parameter. With this specification numerical values for social gain, proportionate social gain and the index of inequality may be computed from equations (4.7)-(4.11).

The concept of social gain is closely related to the use of distributional weights in cost-benefit analysis. The benefits of a project are measured by a weighted sum of individual equivalent incomes where the weights are proportional to the social marginal utility of income. The important point to note in this context is that, in general, the weights depend on prices (both actual and reference prices).[1]

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1 See Meade (1955) and Little and Mirrlees (1974).

When only the "efficiency" gains of a reform are valued ($\varepsilon = 0$) the social gain is given by

$$SG = \frac{1}{N} \sum_h \left\{ f(\bar{\underline{p}}^o, \underline{p}_h^p, y_h^p) - f(\bar{\underline{p}}^o, \underline{p}_h^o, y_h^o) \right\} \tag{4.16}$$

where equivalent income is computed at a common reference price vector equal to mean original prices

$$\bar{\underline{p}}^o = \frac{1}{N} \sum_h \underline{p}_h^o \tag{4.17}$$

This may be contrasted with the mean equivalent gain derived in Section 3 to measure deadweight loss which is given by

$$\frac{1}{N}\sum_h EG_h = \frac{1}{N} \sum_h \left\{ f(\underline{p}_h^o, \underline{p}_h^p, y_h^p) - f(\underline{p}_h^o, \underline{p}_h^o, y_h^o) \right\} \tag{4.18}$$

When all households face the same pre-reform price vector the two measures are identical. If prices vary across households, then the two measures differ although, as we shall see in Section 6, the difference may be small even where there is substantial variation in prices. The use of either (4.18) or (4.16) to measure deadweight loss implies equal weights for dollars of equivalent income regardless of the household to which they accrue. But except in the case of homothetic preferences, the distributional weights implicit in this measure vary with the reference price vector.

5. Confidence Intervals for Welfare Measures

In previous sections we have computed values for various statistics of the effects of a reform, such as the mean equivalent gain for the whole or sub-groups of the population, inequality indexes, and the social gain. These values depend upon the parameters of the equivalent income function which in turn are derived from estimation of a demand system. Because we do not know the true values of the parameters, the information set relevant to policy-makers will include confidence intervals for the welfare measures described in sections 3 and 4. Using the covariance matrix of the parameter estimates it is possible to derive formulae for the asymptotic standard errors for each of the statistics of a reform. When estimation is carried out using a large sample of households asymptotic results are not unduly restrictive.

As shown above, all of the statistics in which we are interested are based on the equivalent income function. Let this be a function of K parameters, $\beta_1 \ldots \beta_K$. We may write (3.5) as

$$y_{Eh} = f(\underline{p}_h^R, \underline{p}_h, y_h) = f_h(\beta_1, \ldots \beta_K) \tag{5.1}$$

Assume that we have maximum likelihood estimates of the parameters $\hat{\beta}_1, \ldots \hat{\beta}_K$, obtained from a sample of T households. In general T will differ from H although they are equal if the same sample is used for both estimation and simulation. We shall assume that the maximum likelihood estimators are asymptotically normally distributed such that

$\sqrt{T}(\beta-\beta) \overset{\rightarrow}{\sim} N(O,\Sigma)$.[1] The information required to produce standard errors for a statistic consists of the K partial derivatives f_{hi} ($i=1, \ldots, K$) and the $\frac{K(K+1)}{2}$ elements, $\frac{\sigma_{ij}}{T}$, of the estimated parameter covariance matrix.[2]

Suppose we are interested in some statistic s which may be written as

$$s = s(\beta_1, \ldots, \beta_k) \tag{5.2}$$

For the statistics in which we are interested this may be written as

$$s = s(f_1(\beta_1, \ldots, \beta_K), \ldots f_H(\beta_1, \ldots, \beta_K) \tag{5.3}$$

Since f_h is differentiable and the statistics derived above are differentiable with respect to f_h, we may expand (5.2) around the true parameter values using a Taylor series

$$s - \hat{s} = \sum_{i=1}^{k} \frac{\partial s}{\partial \beta_i} (\hat{\beta}_i - \beta) + R \tag{5.4}$$

where $\hat{s}$ is the value of the statistic obtained using the point estimate of the parameters and R contains second and higher

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[1] The exact conditions for normality depend upon whether the estimated demand system is nonlinear but they involve the boundedness of the exogenous variables (Amemiya 1977, Gomulka and Pemberton 1980).

[2] The user of the computer programme which calculates the statistics presented in this paper must supply functional forms for the derivatives (as a function of $\underline{p}^R$, $\underline{p}$ and y), and the covariance matrix.

order terms. Rao (1973, p. 387) shows that as $T \to \infty$, R converges in probability to zero. Hence from (5.4) s is a linear function of limiting normal variables which is asymptotically distributed with mean s* and variance

$$\sigma_s^2 = \sum_i \sum_j \sigma_{ij} \frac{\partial s}{\partial \beta_i} \frac{\partial s}{\partial \beta_j} \qquad i, j = 1 \ldots K \tag{5.5}$$

where the derivatives are evaluated at $\beta_i = \hat{\beta}_i$

The first statistic to examine is the mean equivalent gain which measures the reduction in deadweight loss resulting from the reform. For this statistic

$$s = \frac{1}{H} \sum_h f(p_h^o, p_h^p, y_h^p) - \frac{1}{N} \sum_h y_h^o \tag{5.6}$$

$$\frac{\partial s}{\partial \beta_i} = \frac{1}{H} \sum_h \frac{\partial f_h}{\partial \beta_i} \tag{5.7}$$

where the derivatives are evaluated at the appropriate reference price vectors and budget constraints. The standard error for the efficiency gain is

$$SE(s) = \frac{1}{H} \sqrt{\sum_i \sum_j \sigma_{ij} \sum_h \frac{\partial f_h}{\partial \beta_i} \sum_h \frac{\partial f_h}{\partial \beta_j}} \tag{5.8}$$

This is straightforward to compute, and the only difficulty in practice arises from the need, when deriving the partial derivatives, to take account of the fact that either post-reform income, or one of the post-reform prices, is endogenous to the model (because it depends on the residual tax rate) and hence a function of the parameter values.[1] An example of the calculation of the standard errors of the efficiency gain allowing for this uncertainty about the residual tax rate will be given in section 6.

If we compare the expression for social welfare given in (4.1) with (5.2) we see immediately that (5.5) is a general formula for the standard error of any measure of social welfare.[2] To use this formula we require only that the relevant statistic can be written as an explicit function of the parameter values. This is true for many of the statistics that we would like to compute. For example, it is true of the indexes of inequality and the proportionate social gain. But there are some measures for which only an implicit functional relationship exists

$$h(s, \beta_1, \ldots \beta_K) = 0 \quad (5.9)$$

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1 Provided the error terms in the demand equations are uncorrelated with tax rates then using (2.3) it is possible to show that in large samples we can ignore the residual variance of the demand equations when calculating this distribution of the residual tax rate. Essentially, this is because the latter depends upon aggregate revenue (and hence consumption) whereas the residual variance relates to the unexplained variation between households.

2 Provided that W is differentiable.

It is clear, for example, from (4.7) that the measure of (absolute) social gain comes into the category of statistics for which no explicit function can be derived. We can derive a standard error, however, as follows. If social preferences satisfy the Pareto condition (W is increasing in equivalent income), then from (4.7) it is evident that h is monotone in s for given β. Hence from the implicit function theorem there is a unique value for each β_i of

$$\frac{\partial s}{\partial \beta_i} = - \frac{\frac{\partial h}{\partial \beta_i}}{\frac{\partial h}{\partial s}} \tag{5.10}$$

Combining (5.5) and (5.10) gives

$$SE(s) = \left(\frac{\partial h}{\partial s}\right)^{-1} \sqrt{\sum_i \sum_j \sigma_{ij} \frac{\partial h}{\partial \beta_i} \frac{\partial h}{\partial \beta_j}} \tag{5.11}$$

The derivatives may be evaluated from (4.7) and (4.15) and we shall give an example in section 6.

Exactly the same approach can be used to construct confidence intervals for welfare measures when producer prices change and we wish to take into account uncertainty about the parameters of supply functions. The parameter vector is simply augmented to include the supply parameters and the formulae in (5.8) and (5.11) used with the augmented vector of parameters.

The computation of standard errors for a wide range of welfare measures is facilitated by the use of the equivalent income function. The basic information required is the derivatives of the equivalent income function with respect to the parameters and the covariance matrix of the parameters. The other inputs to the calculation of standard errors are common to all reforms and all demand systems, and are programmed into the package. The user of the package need specify only functional forms for the derivatives of the equivalent income function and values for the covariance matrix.

6. An Empirical Example

In this section we present an empirical application to the housing market for a 1973 sample of 5895 households in England and Wales taken from the Family Expenditure Survey. The example is by way of illustration and there are several aspects of the housing market which we ignore in order to focus attention on the measures described above. Preferences are assumed to be defined over two commodities, housing services (H) and a "composite" commodity of other goods and services (C), and the indirect utility function is assumed to be homothetic translog with the following demand equation for housing services estimated by King (1980)[1]

$$X_H = \frac{y}{p_H}\left[\beta_1 + \beta_2 \log\left(\frac{p_H}{p_C}\right)\right] \tag{6.1}$$

$$\beta_1 = \underset{(0.0008)}{0.1022} \qquad \beta_2 = \underset{(0.0009)}{0.0238}$$

The corresponding equivalent income function is

$$y_E = y\left(\frac{p_H^R}{p_H}\right)^{\beta_1}\left(\frac{p_C^R}{p_C}\right)^{1-\beta_1} \exp\left[\beta_2\left\{\left[\log\left(\frac{p_H^R}{p_C^R}\right)\right]^2 - \left[\log\left(\frac{p_H}{p_C}\right)\right]^2\right\}\right] \tag{6.2}$$

The reform which we simulate is the following. The price of housing services is increased for most households by eliminating the tax concessions to owner-occupation and raising rents in the local authority sector until the mean housing costs (gross of income-related rent and rate rebates)

[1] For a full description of the data and definitions of all the variables see King (1980).

in the two tenures are equal.[1] The additional revenue generated is handed back to households in the form of an equal lump-sum payment, and so the sum of equivalent gains is a measure of the reduction in deadweight loss from reducing housing subsidies. The value of the lump-sum payment which can be financed is given by solving iteratively the system of equations given by (2.3) and (2.4) for the assumed demand system. There is a unique solution for the value of the lump-sum payment. Two of the unrealistic features of the simulation are that we shall assume constant producer prices and we shall ignore household composition effects.[2]

There is both income and price variation in the sample. As reference price vector we use the mean values of pre-reform prices in the sample when computing equivalent incomes. The equivalent gain is computed at each household's original price vector. Summary statistics of the reform are shown in Table 1. This shows the values of prices and incomes before and after the reform together with the values of pre- and post-reform equivalent incomes, housing consumption, and equivalent gain. All monetary values are expressed in £1973 per week. The lump-sum subsidy which can be financed is £1.16 per week (the difference between mean y^o and mean y^P in Table 1). The efficiency gains of the reform as measured by the mean value of equivalent gain are 19.3p per

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[1] Controlled private rental accommodation is unaffected.

[2] For an investigation of the capitalisation effects which occur for a less than infinite elasticity of supply of housing services see King (1981). We ignore also the effects of the reform on choice of tenure, although since the choice of tenure in the UK appears to be principally determined by rationing rather than relative prices (King (1980)) this is unlikely to matter significantly.

week per household. This amounts to 16.6 per cent of the revenue generated by the reform, and is equal to 0.4 of 1 per cent of mean household income. We may compare the welfare gain with an alternative naive measure of the effect of the reform which ignores behavioural responses and is the sort of number typically presented in official analyses of tax changes. We call this the cash gain and is defined by

$$G = \hat{y}^p - y^o - (\underline{p}^p - \underline{p}^o)'\underline{x}^o \qquad (6.3)$$

where $\underline{x}^o$ is the original consumption vector and $\hat{y}^p$ is an estimate of post-reform income consistent with a revenue-neutral reform given unchanged behaviour. The true y^p will differ from $\hat{y}^p$ because of behavioural responses by households. By definition

$$\sum_h (\hat{y}^p_h - y^o_h) = \sum_h (\underline{p}^p_h - \underline{p}^o_h)'\underline{x}^o_h \qquad (6.4)$$

It is clear that by construction the mean value of cash gain for a revenue-neutral reform is zero. Although it ignores behavioural responses (and therefore may not represent a feasible equilibrium) cash gain measures the first-round or impact effect of the reform before households have had time to adjust their behaviour. From Table 1 we can see that cash gain underestimates the number of households who would benefit from the reform by 561, or 15 per cent of the true number.

The summary table conceals much of the redistribution which results from the reform, and one of the principal advantages of using individual household data is to discover "who gains, who loses?". The program first ranks all households by some specified variable Z_1, which in this application we take to be original equivalent income, and divides the sample into quantile groups according to Z_1.[1] Any quantile division may be chosen and here we examine decile groups. For each quantile group mean cash and equivalent gain (and the standard error of the latter) are calculated together with the number of households with positive and negative values for the two measures of gain. In addition we compute the proportionate cash gain (mean cash gain divided by mean original income) and proportionate equivalent gain PEG (mean equivalent gain divided by mean original equivalent income). The gains and losses for each decile group of original equivalent income are shown in Table 2.

Even Table 2, however, does not illustrate the main benefit from using household data which is to examine the distribution of gains and losses within each quantile group. To do this each quantile group is re-ranked by a variable Z_2 and divided into sub-quantile groups according to Z_2. A natural choice for Z_2 is equivalent gain, and we may, for example, construct decile groups of equivalent gain for each

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[1] The rankings are used also to compute an index of horizontal equity of the reform which is not discussed here, see King (forthcoming). The program allows the calculations described here to be repeated for sub-samples, such as particular types of household (e.g. owner-occupier) described by appropriate variables in the data set.

decile of the income distribution. In this way we obtain a picture of the distribution of gains and losses within each quantile group. Some of this information is shown in figure 1. For each decile of original equivalent income the vertical line connects the tenth and ninetieth percentiles of the distribution of equivalent gains, thus giving an impression of the spread of gains and losses within each decile group. The dotted line connects the mean gain in each decile group illustrating the amount of vertical redistribution between groups, and the popularity of the reform may be judged by looking at the median values for equivalent gain.

The remaining calculations concern measures of social welfare. Indices of inequality (as given by (4.11) and (4.15)) for both the pre- and post-reform distributions of equivalent income are shown in Table 3 for different values of the inequality aversion parameter. Corresponding values of the social gain, and their associated standard errors, are given in Table 4. It is clear that for the particular reform simulated here, its main impact is distributional, with the top decile, in particular, losing heavily from the reduction of housing subsidies.

7. Conclusions

We have presented a methodology for calculating the distribution of gains and losses from a reform using individual household data. The central concept is the equivalent income function. All of the measures which we have discussed are based on the equivalent income function which greatly simplifies computation. Preferences determine the functional form of the equivalent income function, and its arguments are dictated by the particular reform under consideration. This separation enables the calculations to be standardised and a computer package to compute the statistics presented above for a general demand system and specification of a reform is available from the author at marginal cost.

One use of equivalent income is to integrate the calculation of distributional and efficiency effects. The distribution of equivalent gains among households measures the distributional effects of the reform, and the average value of equivalent gain is an exact measure of the efficiency gain. Measuring the reduction in deadweight loss in this manner avoids the use of approximation formuale, shows very clearly the role of the reference price vector in the choice of index of deadweight loss, and should be easier to explain to policy-makers than a measure defined in terms of the difference between the areas of two triangles (for example, compensating variation minus change in tax revenue). In part this is because our measure of efficiency gain is closely related to the distributional effects and policy-makers are obviously very concerned with the answer to the question "who gains, who loses?". This question has often been answered with reference to hypothetical households, (for example, the well-known married couple with two children and a husband receiving average earnings), but it is clearly preferable to exploit data for the whole distribution. The comparison between cash gain and equivalent gain shows the importance of taking behavioural responses into account.

The equivalent income function may be used also to calculate various measures of social welfare (section 4), and, if we have the covariance matrix for the parameters of the estimated demand system then standard errors may be computed for each of the welfare measures.

APPENDIX : FUNCTIONAL FORMS FOR THE EQUIVALENT INCOME FUNCTION

We present some examples of the equivalent income function for several common models of demand. The general function is

$$y_E = f(\underline{p}^R, \underline{p}, y) \tag{A 1}$$

The first two examples are for single equation models in which demand for a commodity is a function only of income and own-price. The remaining three examples are for complete demand systems. These do not necessarily satisfy the axioms of consumer behaviour for all possible parameter values and this must be checked in each case before estimates are used to compute welfare measures. The properties which the equivalent income function must exhibit in order to satisfy the axioms of consumer behaviour are given in and above (3.7) in the main text.

The reader may verify the following results by checking that the demand functions are indeed given by differentiating equivalent income with respect to reference prices and evaluating the derivative at $\underline{p}^R = \underline{p}$.

(i) linear demand[1]

$$X = \alpha p + \beta y + Z\gamma \tag{A 2}$$

where $Z\gamma$ represents other variables affecting demand

[1] The indirect utility functions for cases (i) and (ii) have been derived by Hausman (1981).

$$y_E = e^{\beta(p^R - p)}\left\{y + \frac{\alpha}{\beta}p + \frac{\alpha}{\beta^2} + \frac{Z\gamma}{\beta}\right\} - \left\{\frac{\alpha}{\beta}p^R + \frac{\alpha}{\beta^2} + \frac{Z\gamma}{\beta}\right\} \qquad \text{(A 3)}$$

(ii) loglinear demand

$$\log X = \alpha \log p + \beta \log y + Z\gamma \qquad \text{(A 4)}$$

$$y_E = \left\{y^{1-\beta} + (1-\beta)q\right\}^{\frac{1}{1-\beta}} \qquad \beta \neq 1 \qquad \text{(A 5)}$$

$$ye^q \qquad \beta = 1$$

$$\text{where} \quad q = \frac{e^{Z\gamma}}{1+\alpha}\left\{(p^R)^{1+\alpha} - p^{1+\alpha}\right\}$$

(iii) linear expenditure system

$$p_k X_k = p_k \gamma_k + \beta_k\left\{y - \sum_j p_j \gamma_j\right\} \qquad k = 1 \ldots N \qquad \text{(A 6)}$$

$$y_E = \sum_k \gamma_k p_k^R + \prod_k \left(\frac{p_k^R}{p_k}\right)^{\beta_k}\left\{y - \sum_j \gamma_j p_j\right\} \qquad \text{(A 7)}$$

(iv) indirect translog demand system (Christensen, Jorgenson and Lau (1975))

The budget share for commodity i is given by

$$w_i = \frac{\alpha_i + \sum_j \beta_{ij} \log\left(\frac{p_j}{y}\right)}{\sum_i \alpha_i + \sum_{ij}\sum \beta_{ij} \log\left(\frac{p_j}{y}\right)} \qquad \text{(A 8)}$$

Equivalent income is given by the solution to the following quadratic equation

$$\tfrac{1}{2}(\log y_E)^2 \sum_i \beta_{ij} - \log y_E\left(\sum_i \alpha_i + \tfrac{1}{2}\sum_{ij}\sum \beta_{ij} \log p_i^R p_j^R\right) -$$

$$\tfrac{1}{2}(\log y)^2 \sum_i \beta ij + \sum_i \alpha_i \log\left(\frac{p_i^R}{p_i}\right) + \tfrac{1}{2}\sum_{ij}\sum \beta_{ij} (\log p_i^R \log p_j^R -$$

$$\log p_i \log p_j) + \log y \left(\sum_i \alpha_i + \tfrac{1}{2}\sum_{ij}\sum \beta_{ij} \log p_i\, p_j\right) = 0$$

(A 9)

We must now choose which of the two roots to use. The quadratic derives from an equation of the form $v(p^R, y_E) = v(p, y)$. Since indirect utility is increasing and quasi-concave in income, then equivalent income is equal to the smaller of the two roots which corresponds to the region in which the quadratic approximation satisfies the axioms of consumer behaviour.

(v) The AIDS form of the Working-Leser demand system (Deaton and Muellbauer (1980b)

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left(\frac{y}{p}\right) \qquad \text{(A 10)}$$

where $\log p = \alpha_o + \sum_k \alpha_k \log p_k + \tfrac{1}{2}\sum_{jk}\sum \gamma_{kj} \log p_k \log p_j$

Equivalent income is given by

$$\log y_E = \alpha_o + \sum_j \alpha_j \log p_j^R + \tfrac{1}{2}\sum_{kj}\sum \gamma_{kj} \log p_k^R \log p_j^R +$$

$$\prod_k \left(\frac{p_k^R}{p_k}\right)^{\beta_k} \{\log y - \alpha_o - \sum_j \alpha_j \log p_j -$$

$$\tfrac{1}{2}\sum_{k}\sum_{j}\gamma_{kj} \log p_k \log p_j \} \qquad \text{(A 11)}$$

REFERENCES

Amemiya, T. (1977), "The Maximum Likelihood and the Nonlinear Three-Stage Least Squares Estimator in the General Nonlinear Simultaneous Equation Model", Econometrica, Vol. 45, pps. 955-68.

Atkinson, A.B. (1970) "On the Measurement of Inequality", Journal of Economic Theory, Vol. 2, pps. 244-63.

Atkinson, A.B. and Bourguignon, F. (1981) "The Comparison of Multi-Dimensioned Distributions of Economic Status", mimeo.

Auerbach, A.J., and Rosen, H.S. (1980) "Will the Real Excess Burden Please Stand Up? (Or, Seven Measures in Search of a Concept)", National Bureau of Economic Research Working Paper No. 495.

Burtless, G. and Hausman, J.A. (1978) "The Effects of Taxation on Labor Supply: Evaluating the Gary Negative Income Tax Experiment", Journal of Political Economy, Vol. 86, pps. 1103-1130.

Christensen, L.R., Jorgenson, D.W., and Lau, L.J. (1975) "Transcendental Logarithmic Utility Functions", American Economic Review, Vol. 65, pps. 367-83.

Cramer, J.S. (1969) Emprical Econometrics, North Holland, Amsterdam.

Deaton, A.S. (1980) "Measurement of Welfare: Theory and Practical Guidelines", Living Standards Measurement Study Working Paper No. 7, World Bank, Washington,DC.

Deaton, A.S., and Muellbauer, J. (1980a) Economics and Consumer Behaviour, Cambridge University Press.

Deaton, A.S. and Muellbauer, J. (1980b) "An Almost Ideal Demand System", American Economic Review, Vol. 70, pps. 312-26.

Diamond, P.A. and McFadden, D.L. (1974) "Some Uses of the Expenditure Function in Public Finance", Journal of Public Economics, Vol. 3, pps. 3-21.

Diewert, W.E. (1978) "Duality Approaches to Microeconomic Theory", University of British Columbia Discussion Paper 78-09.

Feldstein, M.S. (1974) "Distributional Preferences in Public Expenditure Analysis" in (eds.) Hochman, H. and Peterson, G. Redistribution Through Public Choice, Columbia University Press, New York.

Gomulka, J. and Pemberton, M. (1980) "Asymptotic Theory of Regression Analysis: A Survey", SSRC Programme on Taxation, Incentives, and the Distribution of Income Discussion Paper No. 14, London, mimeo.

Gorman, W.M. (1961) "On a Class of Preference Fields", Metroeconomica, Vol. 13, pps. 53-6.

Gorman, W.M. (1976) "Tricks with Utility Functions" in (ed.)Artis, M.J. and Nobay, A.R. Essays in Economic Analysis, Cambridge University Press.

Harberger, A.C. (1971) "Three Basic Postulates for Applied Welfare Economics", Journal of Economic Literature, Vol. 9, pps. 785-97.

Hausman, J.A. (1981) "Exact Consumer's Surplus and Deadweight Loss", American Economic Review, forthcoming.

Hicks, J.R. (1946) Value and Capital, (second edition), Oxford University Press.

Hurwicz, L. and Uzawa, H. (1971) "On the Integrability of Demand Functions", in (ed.) Chipman, J., Hurwicz, L., Richter, M., and Sonnenschein, H. Preferences, Utility and Demand, Harcourt Brace Jovanovich, New York.

Kay, J.A. (1980) "The Deadweight Loss from a Tax System", Journal of Public Economics, Vol. 13, pps. 111-19.

King, M.A. (1980) "An Econometric Model of Tenure Choice and Demand for Housing as a Joint Decision", Journal of Public Economics, Vol. 14, pps. 137-59.

King, M.A. (1981) "The Distribution of Gains and Losses from Changes in the Tax Treatment of Housing", in (ed.) Feldstein, M.S. and Hartman, D. Behavioural Simulation in Tax Policy Analysis, Chicago University Press, Forthcoming.

King, M.A. (Forthcoming) "An Index of Inequality: With Applications to Horizontal Equity and Social Mobility, Econometrica.

Little, I.M.D., and Mirrlees, J.A. (1974) Project Appraisal and Planning for Developing Countries, Heinemann, London.

McKenzie, L. (1956) "Demand without a Utility Index", Review of Economic Studies, Vol. 24, pps. 185-9.

Meade, J.E. (1955) Trade and Welfare, Oxford University Press.

Mohring, H. (1971) "Alternative Gain and Loss Measures", Western Economic Journal, Vol. 9, pps. 349-68.

Muellbauer, J. (1974a) "Inequality Measures, Prices and Household Composition", Review of Economic Studies, Vol. 41, pps. 493-504.

Muellbauer, J. (1974b) "Household Composition, Engel Curves and Welfare Comparisons between Households", European Economic Review, Vol. 5, pps. 103-22.

Muellbauer, J. (1975) "Aggregation, Income Distribution and Consumer Demand", Review of Economic Studies, Vol. 42, pps. 525-43.

Muellbauer, J. (1976) "Community Preferences and the Representative Consumer", Econometrica, Vol. 44, pps. 972-99.

Neary, J.P. and Roberts, K.W.S. (1980) "The Theory of Household Behaviour under Rationing", European Economic Review, Vol. 13, pps. 25-42.

Rao, C.R. (1973) Linear Statistical Inference and Its Applications, (2nd edition). John Wiley and Sons, New York.

Roberts, K. (1980) "Price-Independent Welfare Prescriptions", Journal of Public Economics, Vol. 13, pps 277-297.

Rosen, H.S. (1976) "A Methodology for Evaluating Tax Reform Proposals", Journal of Public Economics, Vol. 6 pps. 105-22.

Rosen, H.S. (1978) "The measurement of excess burden with Explicit Utility Functions", Journal of Political Economy, Vol. 86 (supplement), pps, S121 - S135.

Rosen, H.S. (1979) "Housing Decisions and the US Income Tax: An Econometric Analysis", Journal of Public Economics, Vol. 11, pps 1-24.

Rosen, H.S. and Small, K.A. (1981) "Applied Welfare Economics with Discrete Choice Models", Econometrica, Vol. 49, forthcoming.

Samuelson, P.A. (1950) "The Problem of Inegrability in Utility Theory", Economica, Vol. 17, pps. 355-85.

Samuelson, P.A. (1974) "Complementarity - An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory", Journal of Economic Literature, Vol. 12, pps. 1255-89.

Sen, A. (1973) On Economic Inequality, Oxford University Press.

Shoven, J.B. and Whalley, J. (1972) "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the US", Journal of Public Economics, Vol. 1, pps. 281-321.

Rothbarth, E. (1941) "The Measurement of Changes in Real Income under Conditions of Rationing", Review of Economic Studies, Vol. 8, pps. 100-07.

Shoven, J.B. and Whalley, J. (1973) "General Equilibrium with Taxes: A Computational Procedure and an Existence Proof", Review of Economic Studies, Vol. 40, pps. 475-89.

Shoven, J.B. and Whalley, J. (1977) "Equal Yield Tax Alternatives: General Equilibrium Computational Techniques", Journal of Public Economics, Vol. 8, pps. 211-24.

Varian, H. (1980) "Notes on Cost-Benefit Analysis", mimeo, Michigan.

Wales, T.J. and Woodland, A.D. (1979) "Labour Supply and Progessive Taxes", Review of Economic Studies, Vol. 46, pps. 83-95.

	MIN	AVE	MAX	POS	ZERO	NEG	SD	CV
y^o	3.415	44.233	618.876	5895	0	0	29.070	0.657
p_h^o	0.150	0.982	7.572	5895	0	0	0.396	0.403
p_c^o	1.000	1.034	1.064	5895	0	0	0.022	0.021
y^p	4.579	45.396	620.040	5895	0	0	29.070	0.640
p_h^p	0.150	1.219	7.572	5895	0	0	0.356	0.292
p_c^p	1.000	1.034	1.064	5895	0	0	0.022	0.021
y_E^o	2.952	44.192	601.890	5895	0	0	28.845	0.653
y_E^p	3.957	44.381	603.022	5895	0	0	28.284	0.637
G	-12.080	0.000	6.678	3190	0	2705	1.039	
EG	-10.152	0.193	9.941	3751	0	2144	0.870	4.507
x^o	0.353	4.912	53.496	5895	0	0	2.342	0.477
x^p	0.235	4.121	61.514	5895	0	0	2.644	0.642

<u>TABLE 1 SUMMARY STATISTICS OF REFORM</u>

Decile	Mean Original Equivalent income	Mean Cash Gain	Mean Equivalent Gain	Standard Error of Mean EG	PEG (per cent)	% of households that gain*
1	11.08	-0.08	0.76	0.002	6.9	98
2	17.41	-0.03	0.64	0.003	3.7	87
3	24.38	0.16	0.62	0.002	2.5	86
4	31.38	0.25	0.51	0.003	1.6	90
5	37.60	0.22	0.40	0.004	1.1	91
6	43.58	0.17	0.27	0.005	0.6	88
7	49.27	0.08	0.15	0.006	0.3	46
8	56.92	-0.03	-0.05	0.007	-0.1	22
9	67.46	-0.16	-0.27	0.009	-0.4	18
10	102.80	-0.57	-1.09	0.015	-1.1	11
Overall	44.19	0	0.19	0.005	0.4	64

* A household gains if it has a positive equivalent gain

TABLE 2 THE DISTRIBUTION OF GAINS BY DECILES OF ORIGINAL EQUIVALENT INCOME

(£ per week, 1973 prices)

TABLE 3 INDEX OF INEQUALITY FOR THE DISTRIBUTIONS OF EQUIVALENT INCOME

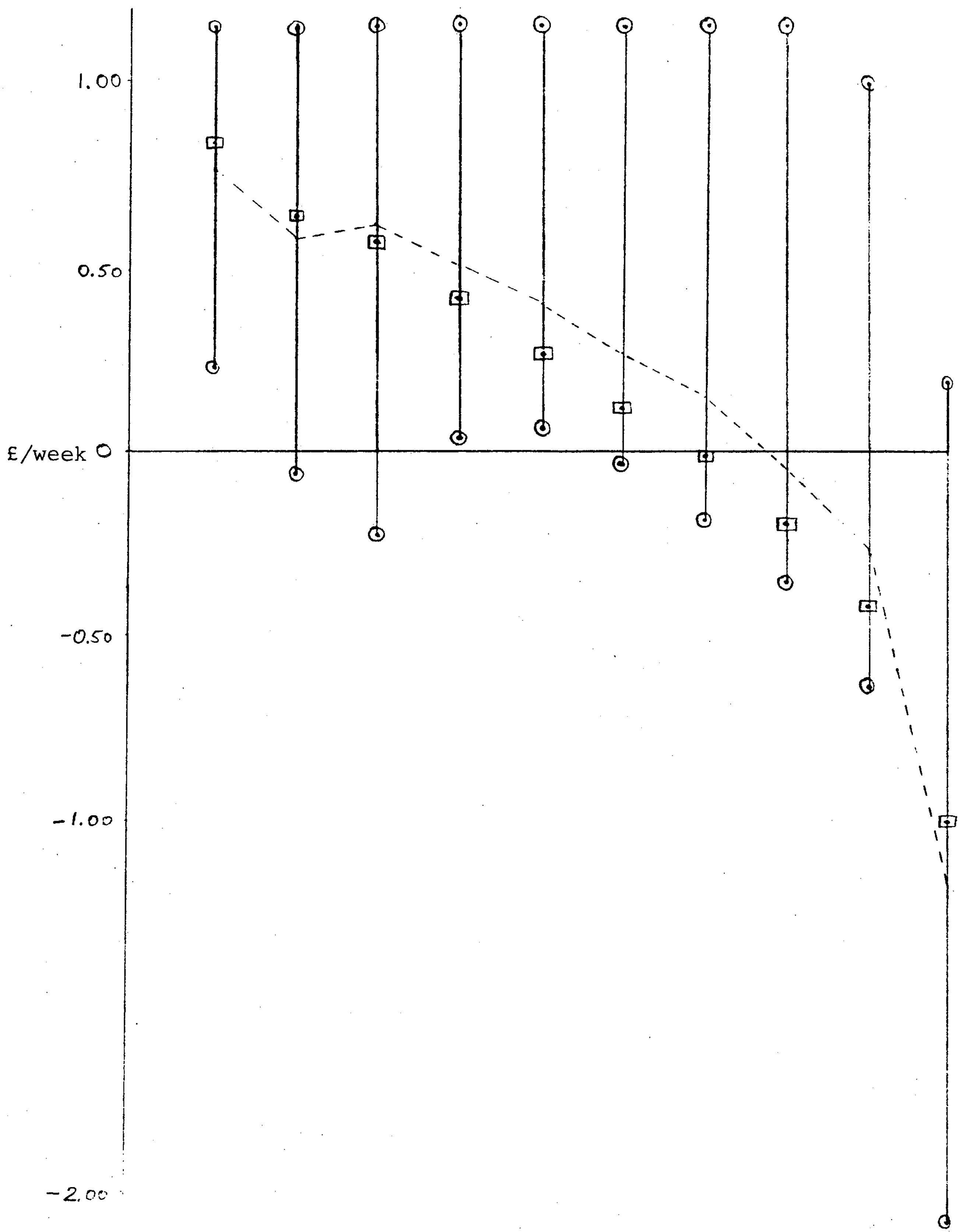
ε	Original	Post-Reform
0.5	0.086	0.082
1.0	0.171	0.162
2.0	0.330	0.311
5.0	0.635	0.593

TABLE 4 SOCIAL GAIN

(standard errors in brackets)

ε	£ per week per household
0	0.190 (0.007)
0.5	0.331 (0.006)
1.0	0.450 (0.005)
2.0	0.624 (0.004)
5.0	0.849 (0.003)

FIGURE 1 THE DISTRIBUTION OF EQUIVALENT GAINS BY DECILE OF ORIGINAL EQUIVALENT INCOME



Key
- ⊙ First and Ninth Deciles
- ⊡ Median
- --- Line of Means