Chapter 5

NAME______________

Choice

Introduction. You have studied budgets, and you have studied preferences. Now is the time to put these two ideas together and do something with them. In this chapter you study the commodity bundle chosen by a utility-maximizing consumer from a given budget.

Given prices and income, you know how to graph a consumer's budget. If you also know the consumer's preferences, you can graph some of his indifference curves. The consumer will choose the "best" indifference curve that he can reach given his budget. But when you try to do this, you have to ask yourself, "How do I find the most desirable indifference curve that the consumer can reach?" The answer to this question is "look in the likely places." Where are the likely places? As your textbook tells you, there are three kinds of likely places. These are: (*i*) a tangency between an indifference curve and the budget line; (*ii*) a kink in an indifference curve; (*iii*) a "corner" where the consumer specializes in consuming just one good.

Here is how you find a point of tangency if we are told the consumer's utility function, the prices of both goods, and the consumer's income. The budget line and an indifference curve are tangent at a point (x_1, x_2) if they have the same slope at that point. Now the slope of an indifference curve at (x_1, x_2) is the ratio $-MU_1(x_1, x_2)/MU_2(x_1, x_2)$. (This slope is also known as the marginal rate of substitution.) The slope of the budget line is $-p_1/p_2$. Therefore an indifference curve is tangent to the budget line at the point (x_1, x_2) when $MU_1(x_1, x_2)/MU_2(x_1, x_2) = p_1/p_2$. This gives us one equation in the two unknowns, x_1 and x_2. If we hope to solve for the x's, we need another equation. That other equation is the budget equation $p_1x_1 + p_2x_2 = m$. With these two equations you can solve for (x_1, x_2).*

Example: A consumer has the utility function $U(x_1, x_2) = x_1^2 x_2$. The price of good 1 is $p_1 = 1$, the price of good 2 is $p_2 = 3$, and his income is 180. Then, $MU_1(x_1, x_2) = 2x_1x_2$ and $MU_2(x_1, x_2) = x_1^2$. Therefore his marginal rate of substitution is $-MU_1(x_1, x_2)/MU_2(x_1, x_2) = -2x_1x_2/x_1^2 = -2x_2/x_1$. This implies that his indifference curve will be tangent to his budget line when $-2x_2/x_1 = -p_1/p_2 = -1/3$. Simplifying this expression, we have $6x_2 = x_1$. This is one of the two equations we need to solve for the two unknowns, x_1 and x_2. The other equation is the budget equation. In this case the budget equation is $x_1 + 3x_2 = 180$. Solving these two equations in two unknowns, we find $x_1 = 120$ and

* Some people have trouble remembering whether the marginal rate of substitution is $-MU_1/MU_2$ or $-MU_2/MU_1$. It isn't really crucial to remember which way this goes as long as you remember that a tangency happens when the marginal utilities of any two goods are in the same proportion as their prices.

$x_2 = 20$. Therefore we know that the consumer chooses the bundle $(x_1, x_2) = (120, 20)$.

For equilibrium at kinks or at corners, we don't need the slope of the indifference curves to equal the slope of the budget line. So we don't have the tangency equation to work with. But we still have the budget equation. The second equation that you can use is an equation that tells you that you are at one of the kinky points or at a corner. You will see exactly how this works when you work a few exercises.

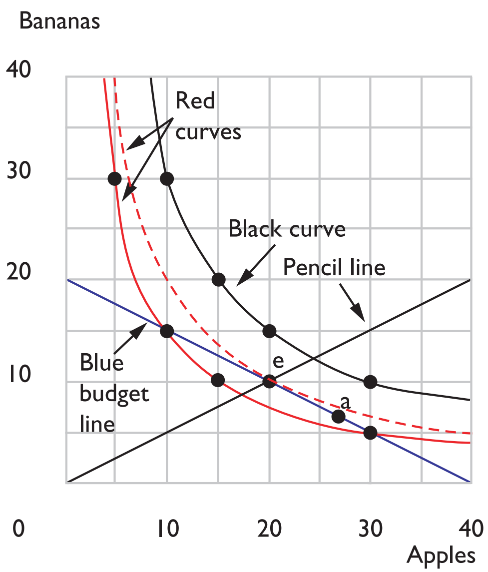
Example: A consumer has the utility function $U(x_1, x_2) = \min\{x_1, 3x_2\}$. The price of x_1 is 2, the price of x_2 is 1, and her income is 140. Her indifference curves are L-shaped. The corners of the L's all lie along the line $x_1 = 3x_2$. She will choose a combination at one of the corners, so this gives us one of the two equations we need for finding the unknowns x_1 and x_2. The second equation is her budget equation, which is $2x_1 + x_2 = 140$. Solve these two equations to find that $x_1 = 60$ and $x_2 = 20$. So we know that the consumer chooses the bundle $(x_1, x_2) = (60, 20)$.

When you have finished these exercises, we hope that you will be able to do the following:

- Calculate the best bundle a consumer can afford at given prices and income in the case of simple utility functions where the best affordable bundle happens at a point of tangency.
- Find the best affordable bundle, given prices and income for a consumer with kinked indifference curves.
- Recognize standard examples where the best bundle a consumer can afford happens at a corner of the budget set.
- Draw a diagram illustrating each of the above types of equilibrium.
- Apply the methods you have learned to choices made with some kinds of nonlinear budgets that arise in real-world situations.

5.1 (0) We begin again with Charlie of the apples and bananas. Recall that Charlie's utility function is $U(x_A, x_B) = x_Ax_B$. Suppose that the price of apples is 1, the price of bananas is 2, and Charlie's income is 40.

(a) On the graph below, use blue ink to draw Charlie's budget line. (Use a ruler and try to make this line accurate.) Plot a few points on the indifference curve that gives Charlie a utility of 150 and sketch this curve with red ink. Now plot a few points on the indifference curve that gives Charlie a utility of 300 and sketch this curve with black ink or pencil.



(b) Can Charlie afford any bundles that give him a utility of 150? Yes.

(c) Can Charlie afford any bundles that give him a utility of 300? No.

(d) On your graph, mark a point that Charlie can afford and that gives him a higher utility than 150. Label that point A.

(e) Neither of the indifference curves that you drew is tangent to Charlie's budget line. Let's try to find one that is. At any point, (x_A, x_B), Charlie's marginal rate of substitution is a function of x_A and x_B. In fact, if you calculate the ratio of marginal utilities for Charlie's utility function, you will find that Charlie's marginal rate of substitution is $MRS(x_A, x_B) = -x_B/x_A$. This is the slope of his indifference curve at (x_A, x_B). The slope of Charlie's budget line is $-1/2$ (give a numerical answer).

(f) Write an equation that implies that the budget line is tangent to an indifference curve at (x_A, x_B). $-x_B/x_A = -1/2$. There are many solutions to this equation. Each of these solutions corresponds to a point on a different indifference curve. Use pencil to draw a line that passes through all of these points.

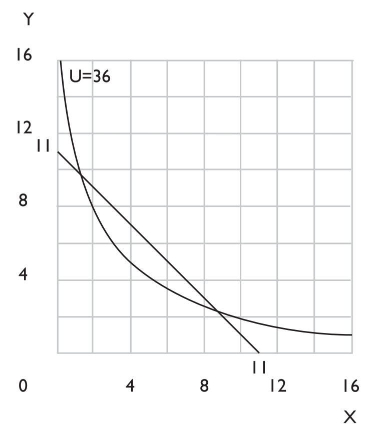
(g) The best bundle that Charlie can afford must lie somewhere on the line you just penciled in. It must also lie on his budget line. If the point is outside of his budget line, he can't afford it. If the point lies inside of his budget line, he can afford to do better by buying more of both goods. On your graph, label this best affordable bundle with an E. This happens where $x_A =$ 20 and $x_B =$ 10. Verify your answer by solving the two simultaneous equations given by his budget equation and the tangency condition.

(h) What is Charlie's utility if he consumes the bundle $(20, 10)$? 200.

(i) On the graph above, use red ink to draw his indifference curve through (20,10). Does this indifference curve cross Charlie's budget line, just touch it, or never touch it? Just touch it.

5.2 (0) Clara's utility function is $U(X, Y) = (X + 2)(Y + 1)$, where X is her consumption of good X and Y is her consumption of good Y.

(a) Write an equation for Clara's indifference curve that goes through the point $(X, Y) = (2, 8)$. $Y = \frac{36}{X+2} - 1$. On the axes below, sketch Clara's indifference curve for $U = 36$.



(b) Suppose that the price of each good is 1 and that Clara has an income of 11. Draw in her budget line. Can Clara achieve a utility of 36 with this budget? Yes.

(c) At the commodity bundle, (X, Y), Clara's marginal rate of substitution is $-\frac{Y+1}{X+2}$.

(d) If we set the absolute value of the MRS equal to the price ratio, we have the equation $\frac{Y+1}{X+2} = 1$.

(e) The budget equation is $X + Y = 11$.

(f) Solving these two equations for the two unknowns, X and Y, we find $X =$ 5 and $Y =$ 6.

5.3 (0) Ambrose, the nut and berry consumer, has a utility function $U(x_1, x_2) = 4\sqrt{x_1} + x_2$, where x_1 is his consumption of nuts and x_2 is his consumption of berries.

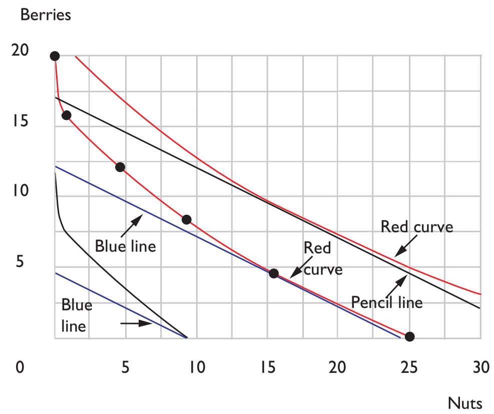
(a) The commodity bundle $(25, 0)$ gives Ambrose a utility of 20. Other points that give him the same utility are $(16, 4)$, (9, 8), (4, 12), (1, 16), and (0, 20). Plot these points on the axes below and draw a red indifference curve through them.

(b) Suppose that the price of a unit of nuts is 1, the price of a unit of berries is 2, and Ambrose's income is 24. Draw Ambrose's budget line with blue ink. How many units of nuts does he choose to buy? 16 units.

(c) How many units of berries? 4 units.

(d) Find some points on the indifference curve that gives him a utility of 25 and sketch this indifference curve (in red).

(e) Now suppose that the prices are as before, but Ambrose's income is 34. Draw his new budget line (with pencil). How many units of nuts will he choose? 16 units. How many units of berries? 9 units.



(f) Now let us explore a case where there is a "boundary solution." Suppose that the price of nuts is still 1 and the price of berries is 2, but Ambrose's income is only 9. Draw his budget line (in blue). Sketch the indifference curve that passes through the point $(9, 0)$. What is the slope of his indifference curve at the point $(9, 0)$? $-2/3$.

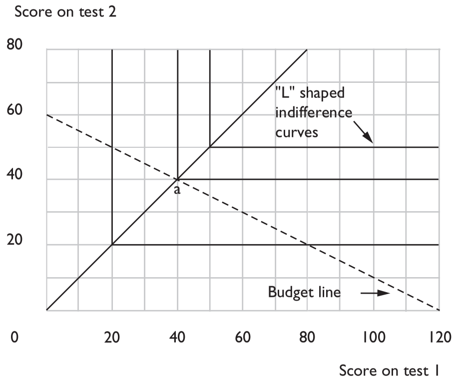
(g) What is the slope of his budget line at this point? $-1/2$.

(h) Which is steeper at this point, the budget line or the indifference curve? Indifference curve.

(i) Can Ambrose afford any bundles that he likes better than the point $(9, 0)$? No.

5.4 (1) Nancy Lerner is trying to decide how to allocate her time in studying for her economics course. There are two examinations in this course. Her overall score for the course will be the *minimum* of her scores on the two examinations. She has decided to devote a total of 1,200 minutes to studying for these two exams, and she wants to get as high an overall score as possible. She knows that on the first examination if she doesn't study at all, she will get a score of zero on it. For every 10 minutes that she spends studying for the first examination, she will increase her score by one point. If she doesn't study at all for the second examination she will get a zero on it. For every 20 minutes she spends studying for the second examination, she will increase her score by one point.

(a) On the graph below, draw a "budget line" showing the various combinations of scores on the two exams that she can achieve with a total of 1,200 minutes of studying. On the same graph, draw two or three "indifference curves" for Nancy. On your graph, draw a straight line that goes through the kinks in Nancy's indifference curves. Label the point where this line hits Nancy's budget with the letter A. Draw Nancy's indifference curve through this point.



(b) Write an equation for the line passing through the kinks of Nancy's indifference curves. $x_1 = x_2$.

(c) Write an equation for Nancy's budget line. $10x_1 + 20x_2 = 1,200$.

(d) Solve these two equations in two unknowns to determine the intersection of these lines. This happens at the point $(x_1, x_2) =$ $(40, 40)$.

(e) Given that she spends a total of 1,200 minutes studying, Nancy will maximize her overall score by spending 400 minutes studying for the first examination and 800 minutes studying for the second examination.

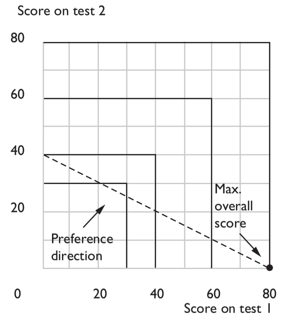
5.5 (1) In her communications course, Nancy also takes two examinations. Her overall grade for the course will be the *maximum* of her scores on the two examinations. Nancy decides to spend a total of 400 minutes studying for these two examinations. If she spends m_1 minutes studying

for the first examination, her score on this exam will be $x_1 = m_1/5$. If she spends m_2 minutes studying for the second examination, her score on this exam will be $x_2 = m_2/10$.

(a) On the graph below, draw a "budget line" showing the various combinations of scores on the two exams that she can achieve with a total of 400 minutes of studying. On the same graph, draw two or three "indifference curves" for Nancy. On your graph, find the point on Nancy's budget line that gives her the best overall score in the course.

(b) Given that she spends a total of 400 minutes studying, Nancy will maximize her overall score by achieving a score of 80 on the first examination and 0 on the second examination.

(c) Her overall score for the course will then be 80.



5.6 (0) Elmer's utility function is $U(x, y) = \min\{x, y^2\}$.

(a) If Elmer consumes 4 units of x and 3 units of y, his utility is 4.

(b) If Elmer consumes 4 units of x and 2 units of y, his utility is 4.

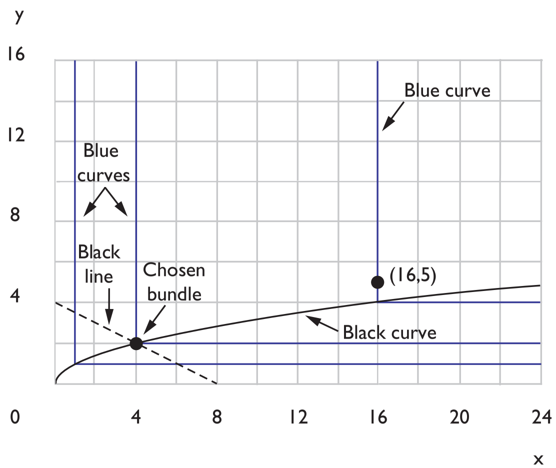
(c) If Elmer consumes 5 units of x and 2 units of y, his utility is 4.

(d) On the graph below, use blue ink to draw the indifference curve for Elmer that contains the bundles that he likes exactly as well as the bundle $(4, 2)$.

(e) On the same graph, use blue ink to draw the indifference curve for Elmer that contains bundles that he likes exactly as well as the bundle $(1, 1)$ and the indifference curve that passes through the point $(16, 5)$.

(f) On your graph, use black ink to show the locus of points at which Elmer's indifference curves have kinks. What is the equation for this curve? $x = y^2$.

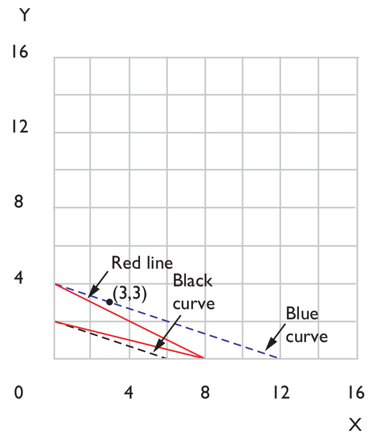
(g) On the same graph, use black ink to draw Elmer's budget line when the price of x is 1, the price of y is 2, and his income is 8. What bundle does Elmer choose in this situation? (4,2).



(h) Suppose that the price of x is 10 and the price of y is 15 and Elmer buys 100 units of x. What is Elmer's income? 1,150. (Hint: At first you might think there is too little information to answer this question. But think about how much y he must be demanding if he chooses 100 units of x.)

5.7 (0) Linus has the utility function $U(x, y) = x + 3y$.

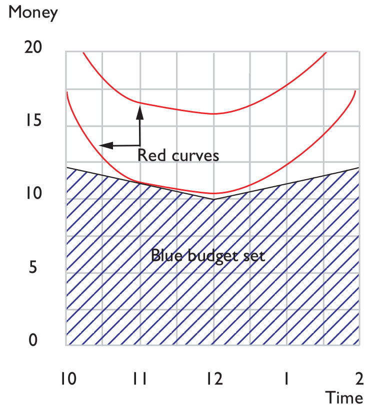
(a) On the graph below, use blue ink to draw the indifference curve passing through the point $(x, y) = (3, 3)$. Use black ink to sketch the indifference curve connecting bundles that give Linus a utility of 6.



(b) On the same graph, use red ink to draw Linus's budget line if the price of x is 1 and the price of y is 2 and his income is 8. What bundle does Linus choose in this situation? (0,4).

(c) What bundle would Linus choose if the price of x is 1, the price of y is 4, and his income is 8? (8,0).

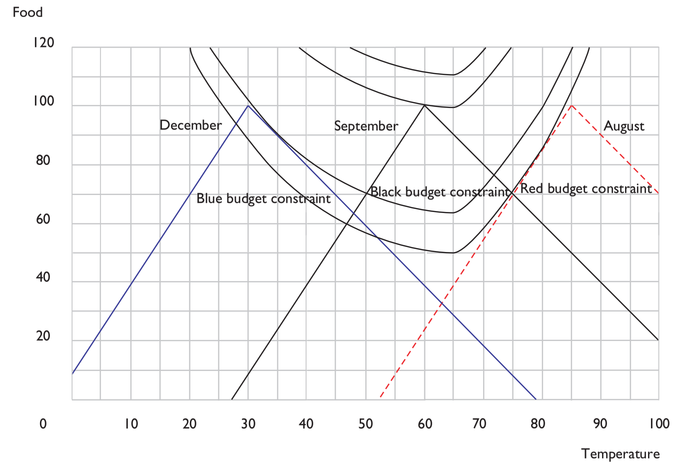
5.8 (2) Remember our friend Ralph Rigid from Chapter 3? His favorite diner, Food for Thought, has adopted the following policy to reduce the crowds at lunch time: if you show up for lunch t hours before or after 12 noon, you get to deduct t dollars from your bill. (This holds for any fraction of an hour as well.)



(a) Use blue ink to show Ralph's budget set. On this graph, the horizontal axis measures the time of day that he eats lunch, and the vertical axis measures the amount of money that he will have to spend on things other than lunch. Assume that he has \$20 total to spend and that lunch at noon costs \$10. (Hint: How much money would he have left if he ate at noon? at 1 P.M.? at 11 A.M.?)

(b) Recall that Ralph's preferred lunch time is 12 noon, but that he is willing to eat at another time if the food is sufficiently cheap. Draw some red indifference curves for Ralph that would be consistent with his choosing to eat at 11 A.M.

5.9 (0) Joe Grad has just arrived at the big U. He has a fellowship that covers his tuition and the rent on an apartment. In order to get by, Joe has become a grader in intermediate price theory, earning \$100 a month. Out of this \$100 he must pay for his food and utilities in his apartment. His utilities expenses consist of heating costs when he heats his apartment and air-conditioning costs when he cools it. To raise the temperature of his apartment by one degree, it costs \$2 per month (or \$20 per month to raise it ten degrees). To use air-conditioning to cool his apartment by a degree, it costs \$3 per month. Whatever is left over after paying the utilities, he uses to buy food at \$1 per unit.



(a) When Joe first arrives in September, the temperature of his apartment is 60 degrees. If he spends nothing on heating or cooling, the temperature in his room will be 60 degrees and he will have \$100 left to spend on food.

If he heated the room to 70 degrees, he would have \$80 left to spend on food. If he cooled the room to 50 degrees, he would have \$70 left to spend on food. On the graph below, show Joe's September budget constraint (with black ink). (Hint: You have just found three points that Joe can afford. Apparently, his budget set is not bounded by a single straight line.)

(b) In December, the outside temperature is 30 degrees and in August poor Joe is trying to understand macroeconomics while the temperature outside is 85 degrees. On the same graph you used above, draw Joe's budget constraints for the months of December (in blue ink) and August (in red ink).

(c) Draw a few smooth (unkinky) indifference curves for Joe in such a way that the following are true. (*i*) His favorite temperature for his apartment would be 65 degrees if it cost him nothing to heat it or cool it. (*ii*) Joe chooses to use the furnace in December, air-conditioning in August, and neither in September. (*iii*) Joe is better off in December than in August.

(d) In what months is the slope of Joe's budget constraint equal to the slope of his indifference curve? August and December.

(e) In December Joe's marginal rate of substitution between food and degrees Fahrenheit is -2. In August, his MRS is 3.

(f) Since Joe neither heats nor cools his apartment in September, we cannot determine his marginal rate of substitution exactly, but we do know that it must be no smaller than -2 and no larger than 3. (Hint: Look carefully at your graph.)

5.10 (0) Central High School has $60,000 to spend on computers and other stuff, so its budget equation is $C + X = 60{,}000$, where C is expenditure on computers and X is expenditures on other things. C.H.S. currently plans to spend $20,000 on computers.

The State Education Commission wants to encourage "computer literacy" in the high schools under its jurisdiction. The following plans have been proposed.

Plan A: This plan would give a grant of $10,000 to each high school in the state that the school could spend as it wished.

Plan B: This plan would give a $10,000 grant to any high school, so long as the school spent at least $10,000 *more* than it currently spends on computers. Any high school can choose not to participate, in which case it does not receive the grant, but it doesn't have to increase its expenditure on computers.

Plan C: Plan C is a "matching grant." For every dollar's worth of computers that a high school orders, the state will give the school 50 cents.

Plan D: This plan is like plan C, except that the maximum amount of matching funds that any high school could get from the state would be limited to $10,000.

(a) Write an equation for Central High School's budget if plan A is adopted. $C + X = 70{,}000$. Use black ink to draw the budget line for Central High School if plan A is adopted.

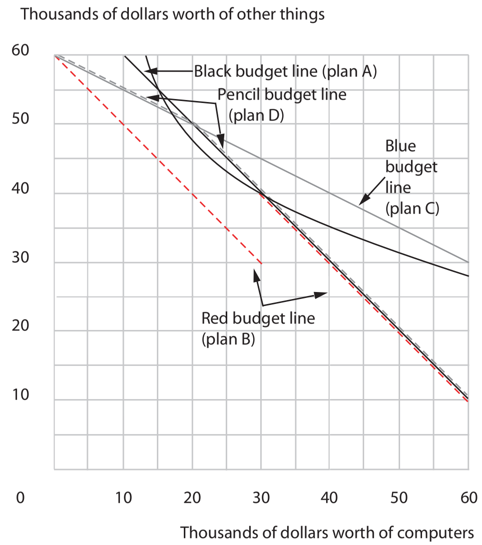
(b) If plan B is adopted, the boundary of Central High School's budget set has two separate downward-sloping line segments. One of these segments describes the cases where C.H.S. spends at least $30,000 on computers. This line segment runs from the point $(C, X) = (70{,}000, 0)$ to the point $(C, X) =$ (30,000, 40,000).

(c) Another line segment corresponds to the cases where C.H.S. spends less than $30,000 on computers. This line segment runs from $(C, X) =$ (30,000,30,000) to the point $(C, X) = (0, 60{,}000)$. Use red ink to draw these two line segments.

(d) If plan C is adopted and Central High School spends C dollars on computers, then it will have $X = 60{,}000 - .5C$ dollars left to spend on other things. Therefore its budget line has the equation .5C+X=60,000. Use blue ink to draw this budget line.

(e) If plan D is adopted, the school district's budget consists of two line segments that intersect at the point where expenditure on computers is 20,000 and expenditure on other instructional materials is 50,000.

(f) The slope of the flatter line segment is $-.5$. The slope of the steeper segment is -1. Use pencil to draw this budget line.



5.11 (0) Suppose that Central High School has preferences that can be represented by the utility function $U(C, X) = CX^2$. Let us try to determine how the various plans described in the last problem will affect the amount that C.H.S. spends on computers.

(a) If the state adopts none of the new plans, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. 20,000.

(b) If plan A is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. 23,333.

(c) On your graph, sketch the indifference curve that passes through the point (30,000, 40,000) if plan B is adopted. At this point, which is steeper, the indifference curve or the budget line? The budget line.

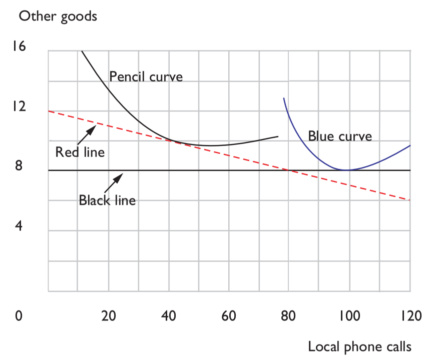
(d) If plan B is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. (Hint: Look at your graph.) 30,000.

(e) If plan C is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. 40,000.

(f) If plan D is adopted, find the expenditure on computers that maximizes the district's utility subject to its budget constraint. 23,333.

5.12 (0) The telephone company allows one to choose between two different pricing plans. For a fee of $12 per month you can make as many local phone calls as you want, at no additional charge per call. Alternatively, you can pay $8 per month and be charged 5 cents for each local phone call that you make. Suppose that you have a total of $20 per month to spend.

(a) On the graph below, use black ink to sketch a budget line for someone who chooses the first plan. Use red ink to draw a budget line for someone who chooses the second plan. Where do the two budget lines cross? $(80, 8)$.



(b) On the graph above, use pencil to draw indifference curves for someone who prefers the second plan to the first. Use blue ink to draw an indifference curve for someone who prefers the first plan to the second.

5.13 (1) This is a puzzle—just for fun. Lewis Carroll (1832-1898), author of *Alice in Wonderland* and *Through the Looking Glass,* was a mathematician, logician, and political scientist. Carroll loved careful reasoning about puzzling things. Here Carroll's Alice presents a nice bit of economic analysis. At first glance, it may seem that Alice is talking nonsense, but, indeed, her reasoning is impeccable.

"I should like to buy an egg, please." she said timidly. "How do you sell them?"

"Fivepence farthing for one—twopence for two," the Sheep replied.

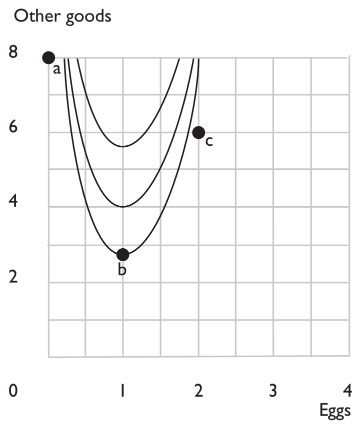
"Then two are cheaper than one?" Alice said, taking out her purse.

"Only you must eat them both if you buy two," said the Sheep.

"Then I'll have one please," said Alice, as she put the money down on the counter. For she thought to herself, "They mightn't be at all nice, you know."

(a) Let us try to draw a budget set and indifference curves that are consistent with this story. Suppose that Alice has a total of 8 pence to spend and that she can buy either 0, 1, or 2 eggs from the Sheep, but no fractional eggs. Then her budget set consists of just three points. The point where she buys no eggs is $(0, 8)$. Plot this point and label it A. On your graph, the point where she buys 1 egg is $(1, 2\frac{3}{4})$. (A farthing is 1/4 of a penny.) Plot this point and label it B.

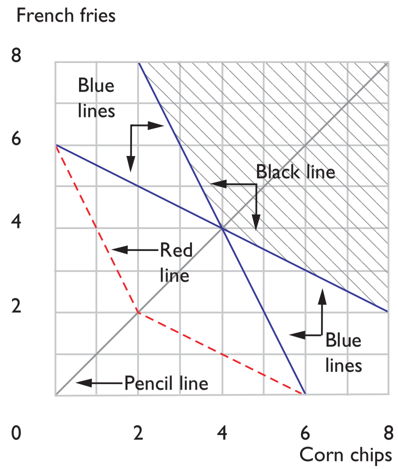
(b) The point where she buys 2 eggs is $(2, 6)$. Plot this point and label it C. If Alice chooses to buy 1 egg, she must like the bundle B better than either the bundle A or the bundle C. Draw indifference curves for Alice that are consistent with this behavior.



5.14 (1) You will remember Harry Mazzola, who consumes only corn chips and french fries. Harry's utility function is $u(x_1, x_2) = \min\{x_1 + x_2, 2x_1 + x_2\}$ where x_1 is his consumption of corn chips and x_2 is his consumption of french fries.

(a) On the graph below, use black ink to draw in the indifference curve along which Harry's utility is 6. Use red ink to draw the budget line for Harry if the price of corn chips is $p_1 = 3$, the price of french fries is $p_2 = 2$, and income is $m = 10$. How many units of corn chips and how many units of french fries should he consume to maximize his utility subject to this budget? `4 units of corn chips and 4 units of french fries`

(b) On the graph below use blue ink to draw Harry's budget line if the price of corn chips is 1, the price of french fries is 3, and Harry's income is 6. How many units of corn chips and how many units of french fries will Harry consume? `6 units of corn chips and no french fries`



(c) At what prices will Harry consume only corn chips and no french fries? `Whenever` $p_2 > 2p_1$ At what prices will he consume only french fries and no corn chips? `Whenever` $p_1 > 2p_2$

(d) At what price-income combinations does Harry choose to consume equal quantities of corn chips and french fries? `At any income, if` $1/2 < p_1/p_2 < 2$`, Harry will choose` $x_1 = x_2$.