Chapter 6

NAME_______________

Demand

Introduction. In the previous chapter, you found the commodity bundle that a consumer with a given utility function would choose in a specific price-income situation. In this chapter, we take this idea a step further. We find demand *functions*, which tell us for *any* prices and income you might want to name, how much of each good a consumer would want. In general, the amount of each good demanded may depend not only on its own price, but also on the price of other goods and on income. Where there are two goods, we write demand functions for Goods 1 and 2 as $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.*

When the consumer is choosing positive amounts of all commodities and indifference curves have no kinks, the consumer chooses a point of tangency between her budget line and the highest indifference curve that it touches.

Example: Consider a consumer with utility function $U(x_1, x_2) = (x_1 + 2)(x_2 + 10)$. To find $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$, we need to find a commodity bundle (x_1, x_2) on her budget line at which her indifference curve is tangent to her budget line. The budget line will be tangent to the indifference curve at (x_1, x_2) if the price ratio equals the marginal rate of substitution. For this utility function, $MU_1(x_1, x_2) = x_2 + 10$ and $MU_2(x_1, x_2) = x_1 + 2$. Therefore the "tangency equation" is $p_1/p_2 = (x_2 + 10)/(x_1 + 2)$. Cross-multiplying the tangency equation, one finds $p_1x_1 + 2p_1 = p_2x_2 + 10p_2$.

The bundle chosen must also satisfy the budget equation, $p_1x_1 + p_2x_2 = m$. This gives us two linear equations in the two unknowns, x_1 and x_2. You can solve these equations yourself, using high school algebra. You will find that the solution for the two "demand functions" is

$$x_1 = \frac{m - 2p_1 + 10p_2}{2p_1}$$
$$x_2 = \frac{m + 2p_1 - 10p_2}{2p_2}.$$

There is one thing left to worry about with the "demand functions" we just found. Notice that these expressions will be positive only if $m - 2p_1 + 10p_2 > 0$ and $m + 2p_1 - 10p_2 > 0$. If either of these expressions is negative, then it doesn't make sense as a demand function. What happens in this case is that the consumer will choose a "boundary solution" where she consumes only one good. At this point, her indifference curve will not be tangent to her budget line.

* For some utility functions, demand for a good may not be affected by all of these variables. For example, with Cobb-Douglas utility, demand for a good depends on the good's own price and on income but not on the other good's price. Still, there is no harm in writing demand for Good 1 as a function of p_1, p_2, and m. It just happens that the derivative of $x_1(p_1, p_2, m)$ with respect to p_2 is zero.

When a consumer has kinks in her indifference curves, she may choose a bundle that is located at a kink. In the problems with kinks, you will be able to solve for the demand functions quite easily by looking at diagrams and doing a little algebra. Typically, instead of finding a tangency equation, you will find an equation that tells you "where the kinks are." With this equation and the budget equation, you can then solve for demand.

You might wonder why we pay so much attention to kinky indifference curves, straight line indifference curves, and other "funny cases." Our reason is this. In the funny cases, computations are usually pretty easy. But often you may have to draw a graph and think about what you are doing. That is what we want you to do. Think and fiddle with graphs. Don't just memorize formulas. Formulas you will forget, but the habit of thinking will stick with you.

When you have finished this workout, we hope that you will be able to do the following:

- Find demand functions for consumers with Cobb-Douglas and other similar utility functions.
- Find demand functions for consumers with quasilinear utility functions.
- Find demand functions for consumers with kinked indifference curves and for consumers with straight-line indifference curves.
- Recognize complements and substitutes from looking at a demand curve.
- Recognize normal goods, inferior goods, luxuries, and necessities from looking at information about demand.
- Calculate the equation of an inverse demand curve, given a simple demand equation.

6.1 (0) Charlie is back—still consuming apples and bananas. His utility function is $U(x_A, x_B) = x_A x_B$. We want to find his demand function for apples, $x_A(p_A, p_B, m)$, and his demand function for bananas, $x_B(p_A, p_B, m)$.

(a) When the prices are p_A and p_B and Charlie's income is m, the equation for Charlie's budget line is $p_A x_A + p_B x_B = m$. The slope of Charlie's indifference curve at the bundle (x_A, x_B) is $-MU_1(x_A, x_B)/MU_2(x_A, x_B) =$ $-x_B/x_A$. The slope of Charlie's budget line is $-p_A/p_B$. Charlie's indifference curve will be tangent to his budget line at the point (x_A, x_B) if the following equation is satisfied: $p_A/p_B = x_B/x_A$.

(b) You now have two equations, the budget equation and the tangency equation, that must be satisfied by the bundle demanded. Solve these two equations for x_A and x_B. Charlie's demand function for apples is $x_A(p_A, p_B, m) = \frac{m}{2p_A}$, and his demand function for bananas is $x_B(p_A, p_B, m) = \frac{m}{2p_B}$.

(c) In general, the demand for both commodities will depend on the price of both commodities and on income. But for Charlie's utility function, the demand function for apples depends only on income and the price of apples. Similarly, the demand for bananas depends only on income and the price of bananas. Charlie always spends the same fraction of his income on bananas. What fraction is this? 1/2.

6.2 (0) Douglas Cornfield's preferences are represented by the utility function $u(x_1, x_2) = x_1^2 x_2^3$. The prices of x_1 and x_2 are p_1 and p_2.

(a) The slope of Cornfield's indifference curve at the point (x_1, x_2) is $-2x_2/3x_1$.

(b) If Cornfield's budget line is tangent to his indifference curve at (x_1, x_2), then $\frac{p_1 x_1}{p_2 x_2} =$ 2/3. (Hint: Look at the equation that equates the slope of his indifference curve with the slope of his budget line.) When he is consuming the best bundle he can afford, what fraction of his income does Douglas spend on x_1? 2/5.

(c) Other members of Doug's family have similar utility functions, but the exponents may be different, or their utilities may be multiplied by a positive constant. If a family member has a utility function $U(x, y) = cx_1^a x_2^b$ where a, b, and c are positive numbers, what fraction of his or her income will that family member spend on x_1? a/(a+b).

6.3 (0) Our thoughts return to Ambrose and his nuts and berries. Ambrose's utility function is $U(x_1, x_2) = 4\sqrt{x_1} + x_2$, where x_1 is his consumption of nuts and x_2 is his consumption of berries.

(a) Let us find his demand function for nuts. The slope of Ambrose's indifference curve at (x_1, x_2) is $-\frac{2}{\sqrt{x_1}}$. Setting this slope equal to the slope of the budget line, you can solve for x_1 without even using the budget equation. The solution is $x_1 = \left(\frac{2p_2}{p_1}\right)^2$.

(b) Let us find his demand for berries. Now we need the budget equation. In Part (a), you solved for the amount of x_1 that he will demand. The budget equation tells us that $p_1 x_1 + p_2 x_2 = M$. Plug the solution that you found for x_1 into the budget equation and solve for x_2 as a function of income and prices. The answer is $x_2 = \frac{M}{p_2} - 4\frac{p_2}{p_1}$.

(c) When we visited Ambrose in Chapter 5, we looked at a "boundary solution," where Ambrose consumed only nuts and no berries. In that example, $p_1 = 1$, $p_2 = 2$, and $M = 9$. If you plug these numbers into the formulas we found in Parts (a) and (b), you find $x_1 =$ 16, and $x_2 = -3.5$. Since we get a negative solution for x_2, it must be that the budget line $x_1 + 2x_2 = 9$ is not tangent to an indifference curve when $x_2 \geq 0$. The best that Ambrose can do with this budget is to spend all of his income on nuts. Looking at the formulas, we see that at the prices $p_1 = 1$ and $p_2 = 2$, Ambrose will demand a positive amount of both goods if and only if $M >$ 16.

6.4 (0) Donald Fribble is a stamp collector. The only things other than stamps that Fribble consumes are Hostess Twinkies. It turns out that Fribble's preferences are represented by the utility function $u(s, t) = s + \ln t$ where s is the number of stamps he collects and t is the number of Twinkies he consumes. The price of stamps is p_s and the price of Twinkies is p_t. Donald's income is m.

(a) Write an expression that says that the ratio of Fribble's marginal utility for Twinkies to his marginal utility for stamps is equal to the ratio of the price of Twinkies to the price of stamps. $1/t = p_t/p_s$. (Hint: The derivative of $\ln t$ with respect to t is $1/t$, and the derivative of s with respect to s is 1.)

(b) You can use the equation you found in the last part to show that if he buys both goods, Donald's demand function for Twinkies depends only on the price ratio and not on his income. Donald's demand function for Twinkies is $t(p_s, p_t, m) = p_s/p_t$.

(c) Notice that for this special utility function, if Fribble buys both goods, then the total amount of money that he spends on Twinkies has the peculiar property that it depends on only one of the three variables m, p_t, and p_s, namely the variable p_s. (Hint: The amount of money that he spends on Twinkies is $p_t t(p_s, p_t, m)$.)

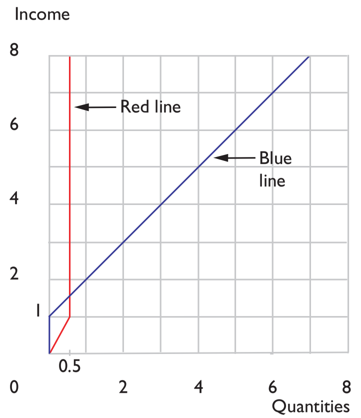
(d) Since there are only two goods, any money that is not spent on Twinkies must be spent on stamps. Use the budget equation and Donald's demand function for Twinkies to find an expression for the number of stamps he will buy if his income is m, the price of stamps is p_s and the price of Twinkies is p_t. $s = \frac{m}{p_s} - 1.$

(e) The expression you just wrote down is negative if $m < p_s$. Surely it makes no sense for him to be demanding negative amounts of postage stamps. If $m < p_s$, what would Fribble's demand for postage stamps be? $s = 0$ What would his demand for Twinkies be? $t = m/p_t.$ (Hint: Recall the discussion of boundary optimum.)

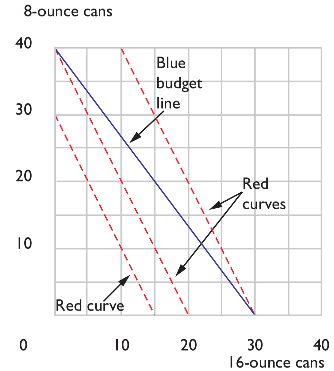
(f) Donald's wife complains that whenever Donald gets an extra dollar, he always spends it all on stamps. Is she right? (Assume that $m > p_s$.)

Yes.

(g) Suppose that the price of Twinkies is \$2 and the price of stamps is \$1. On the graph below, draw Fribble's Engel curve for Twinkies in red ink and his Engel curve for stamps in blue ink. (Hint: First draw the Engel curves for incomes greater than \$1, then draw them for incomes less than \$1.)



6.5 (0) Shirley Sixpack, as you will recall, thinks that two 8-ounce cans of beer are exactly as good as one 16-ounce can of beer. Suppose that these are the only sizes of beer available to her and that she has \$30 to spend on beer. Suppose that an 8-ounce beer costs \$.75 and a 16-ounce beer costs \$1. On the graph below, draw Shirley's budget line in blue ink, and draw some of her indifference curves in red.



(a) At these prices, which size can will she buy, or will she buy some of each? 16-ounce cans.

(b) Suppose that the price of 16-ounce beers remains \$1 and the price of 8-ounce beers falls to \$.55. Will she buy more 8-ounce beers? No.

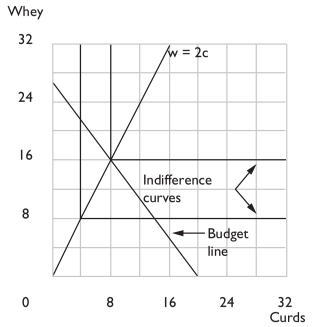
(c) What if the price of 8-ounce beers falls to \$.40? How many 8-ounce beers will she buy then? 75 cans.

(d) If the price of 16-ounce beers is \$1 each and if Shirley chooses some 8-ounce beers and some 16-ounce beers, what must be the price of 8-ounce beers? \$.50.

(e) Now let us try to describe Shirley's demand function for 16-ounce beers as a function of general prices and income. Let the prices of 8-ounce and 16-ounce beers be p_8 and p_{16}, and let her income be m. If $p_{16} < 2p_8$, then the number of 16-ounce beers she will demand is m/p_{16}. If $p_{16} > 2p_8$, then the number of 16-ounce beers she will demand is 0. If $p_{16} =$ 2 p_8, she will be indifferent between any affordable combinations.

6.6 (0) Miss Muffet always likes to have things "just so." In fact the only way she will consume her curds and whey is in the ratio of 2 units of whey per unit of curds. She has an income of \$20. Whey costs \$.75 per unit. Curds cost \$1 per unit. On the graph below, draw Miss Muffet's budget line, and plot some of her indifference curves. (Hint: Have you noticed something kinky about Miss Muffet?)

(a) How many units of curds will Miss Muffet demand in this situation? 8 units. How many units of whey? 16 units.



(b) Write down Miss Muffet's demand function for whey as a function of the prices of curds and whey and of her income, where p_c is the price of curds, p_w is the price of whey, and m is her income. $D(p_c, p_w, m) = \frac{m}{p_w + p_c/2}$. (Hint: You can solve for her demands by solving two equations in two unknowns. One equation tells you that she consumes twice as much whey as curds. The second equation is her budget equation.)

6.7 (1) Mary's utility function is $U(b, c) = b + 100c - c^2$, where b is the number of silver bells in her garden and c is the number of cockle shells. She has 500 square feet in her garden to allocate between silver bells and cockle shells. Silver bells each take up 1 square foot and cockle shells each take up 4 square feet. She gets both kinds of seeds for free.

(a) To maximize her utility, given the size of her garden, Mary should plant 308 silver bells and 48 cockle shells. (Hint: Write down her "budget constraint" for space. Solve the problem as if it were an ordinary demand problem.)

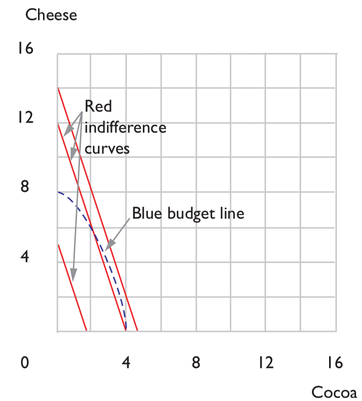
(b) If she suddenly acquires an extra 100 square feet for her garden, how much should she increase her planting of silver bells? 100 extra silver bells. How much should she increase her planting of cockle shells? Not at all.

(c) If Mary had only 144 square feet in her garden, how many cockle shells would she grow? 36.

(d) If Mary grows both silver bells and cockle shells, then we know that the number of square feet in her garden must be greater than 192.

6.8 (0) Casper consumes cocoa and cheese. He has an income of \$16. Cocoa is sold in an unusual way. There is only one supplier and the more cocoa one buys from him, the higher the price one has to pay per unit. In fact, x units of cocoa will cost Casper a total of x^2 dollars. Cheese is sold in the usual way at a price of \$2 per unit. Casper's budget equation, therefore, is $x^2 + 2y = 16$ where x is his consumption of cocoa and y is his consumption of cheese. Casper's utility function is $U(x, y) = 3x + y$.

(a) On the graph below, draw the boundary of Casper's budget set in blue ink. Use red ink to sketch two or three of his indifference curves.



(b) Write an equation that says that at the point (x, y), the slope of Casper's budget "line" equals the slope of his indifference "curve." $2x/2 = 3/1$. Casper demands 3 units of cocoa and 3.5 units of cheese.

6.9 (0) Perhaps after all of the problems with imaginary people and places, you would like to try a problem based on actual fact. The U.S. government's Bureau of Labor Statistics periodically makes studies of family budgets and uses the results to compile the consumer price index. These budget studies and a wealth of other interesting economic data can be found in the annually published *Handbook of Labor Statistics.* The tables below report total current consumption expenditures and expenditures on certain major categories of goods for 5 different income groups in the United States in 1961. People within each of these groups all had similar incomes. Group A is the lowest income group and Group E is the highest.

Table 6.1
Expenditures by Category for Various Income Groups in 1961

Income Group	A	B	C	D	E
Food Prepared at Home	465	783	1078	1382	1848
Food Away from Home	68	171	213	384	872
Housing	626	1090	1508	2043	4205
Clothing	119	328	508	830	1745
Transportation	139	519	826	1222	2048
Other	364	745	1039	1554	3490
Total Expenditures	1781	3636	5172	7415	14208

Table 6.2
Percentage Allocation of Family Budget

Income Group	A	B	C	D	E
Food Prepared at Home	26	22	21	19	13
Food Away from Home	3.8	4.7	4.1	5.2	6.1
Housing	35	30	29	28	30
Clothing	6.7	9.0	9.8	11	12
Transportation	7.8	14	16	17	14

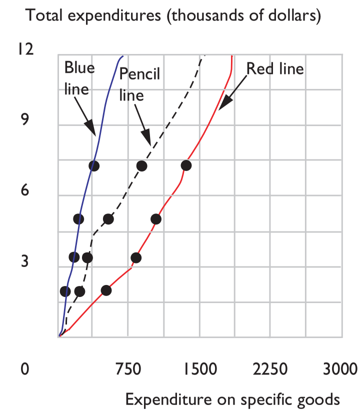
(a) Complete Table 6.2.

(b) Which of these goods are normal goods? All of them.

(c) Which of these goods satisfy your textbook's definition of *luxury goods* at most income levels? Food away from home, clothing, transportation.

(d) Which of these goods satisfy your textbook's definition of *necessity goods* at most income levels? Food prepared at home, housing.

(e) On the graph below, use the information from Table 6.1 to draw "Engel curves." (Use total expenditure on current consumption as income for purposes of drawing this curve.) Use red ink to draw the Engel curve for food prepared at home. Use blue ink to draw an Engel curve for food away from home. Use pencil to draw an Engel curve for clothing. How does the shape of an Engel curve for a luxury differ from the shape of an Engel curve for a necessity? The curve for a luxury gets flatter as income rises, the curve for a necessity gets steeper.



6.10 (0) Percy consumes cakes and ale. His demand function for cakes is $q_c = m - 30p_c + 20p_a$, where m is his income, p_a is the price of ale, p_c is the price of cakes, and q_c is his consumption of cakes. Percy's income is \$100, and the price of ale is \$1 per unit.

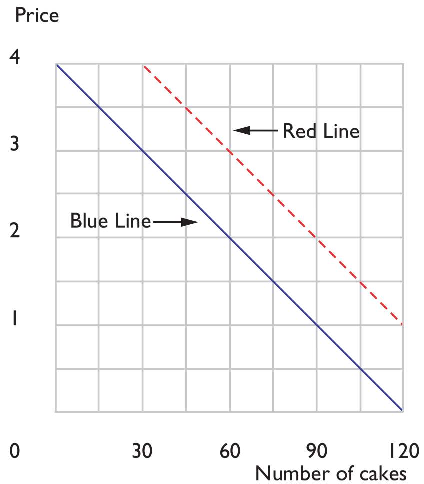
(a) Is ale a substitute for cakes or a complement? Explain. A substitute. An increase in the price of ale increases demand for cakes.

(b) Write an equation for Percy's demand function for cakes where income and the price of ale are held fixed at \$100 and \$1. $q_c = 120 - 30p_c$.

(c) Write an equation for Percy's inverse demand function for cakes where income is \$100 and the price of ale remains at \$1. $p_c = 4 - q_c/30$.

At what price would Percy buy 30 cakes? \$3. Use blue ink to draw Percy's inverse demand curve for cakes.

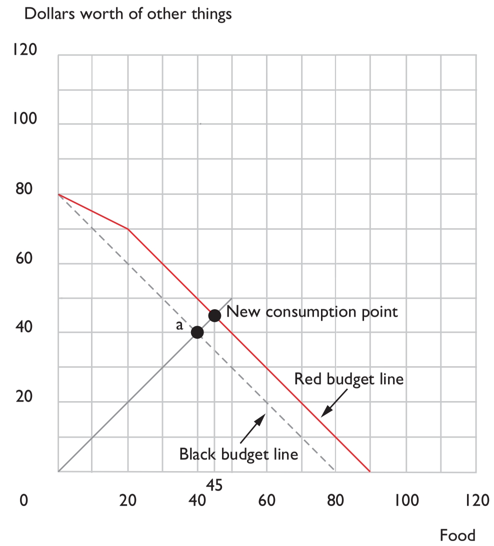
(d) Suppose that the price of ale rises to \$2.50 per unit and remains there. Write an equation for Percy's inverse demand for cakes. $p_c = 5 - q_c/30$. Use red ink to draw in Percy's new inverse demand curve for cakes.



6.11 (0) Richard and Mary Stout have fallen on hard times, but remain rational consumers. They are making do on \$80 a week, spending \$40 on food and \$40 on all other goods. Food costs \$1 per unit. On the graph below, use black ink to draw a budget line. Label their consumption bundle with the letter A.

(a) The Stouts suddenly become eligible for food stamps. This means that they can go to the agency and buy coupons that can be exchanged for \$2 worth of food. Each coupon costs the Stouts \$1. However, the maximum number of coupons they can buy per week is 10. On the graph, draw their new budget line with red ink.

(b) If the Stouts have homothetic preferences, how much more food will they buy once they enter the food stamp program? 5 units.



Calculus **6.12 (2)** As you may remember, Nancy Lerner is taking an economics course in which her overall score is the *minimum* of the number of correct answers she gets on two examinations. For the first exam, each correct answer costs Nancy 10 minutes of study time. For the second exam, each correct answer costs her 20 minutes of study time. In the last chapter, you found the best way for her to allocate 1200 minutes between the two exams. Some people in Nancy's class learn faster and some learn slower than Nancy. Some people will choose to study more than she does, and some will choose to study less than she does. In this section, we will find a general solution for a person's choice of study times and exam scores as a function of the time costs of improving one's score.

(a) Suppose that if a student does not study for an examination, he or she gets no correct answers. Every answer that the student gets right on the first examination costs P_1 minutes of studying for the first exam. Every answer that he or she gets right on the second examination costs P_2 minutes of studying for the second exam. Suppose that this student spends a total of M minutes studying for the two exams and allocates the time between the two exams in the most efficient possible way. Will the student have the same number of correct answers on both exams?

Yes. Write a general formula for this student's overall score for the course as a function of the three variables, P_1, P_2, and M: $S = \frac{M}{P_1+P_2}$. If this student wants to get an overall score of S, with the smallest possible total amount of studying, this student must spend $P_1 S$ minutes studying for the first exam and $P_2 S$ studying for the second exam.

(b) Suppose that a student has the utility function

$$U(S, M) = S - \frac{A}{2}M^2,$$

where S is the student's overall score for the course, M is the number of minutes the student spends studying, and A is a variable that reflects how much the student dislikes studying. In Part (a) of this problem, you found that a student who studies for M minutes and allocates this time wisely between the two exams will get an overall score of $S = \frac{M}{P_1+P_2}$. Substitute $\frac{M}{P_1+P_2}$ for S in the utility function and then differentiate with respect to M to find the amount of study time, M, that maximizes the student's utility. $M = \frac{1}{A(P_1+P_2)}$. Your answer will be a function of the variables P_1, P_2, and A. If the student chooses the utility-maximizing amount of study time and allocates it wisely between the two exams, he or she will have an overall score for the course of $S = \frac{1}{A(P_1+P_2)^2}$.

(c) Nancy Lerner has a utility function like the one presented above. She chose the utility-maximizing amount of study time for herself. For Nancy, $P_1 = 10$ and $P_2 = 20$. She spent a total of $M = 1,200$ minutes studying for the two exams. This gives us enough information to solve for the variable A in Nancy's utility function. In fact, for Nancy, $A = \frac{1}{36,000}$.

(d) Ed Fungus is a student in Nancy's class. Ed's utility function is just like Nancy's, with the same value of A. But Ed learns more slowly than Nancy. In fact it takes Ed exactly twice as long to learn anything as it takes Nancy, so that for him, $P_1 = 20$ and $P_2 = 40$. Ed also chooses his amount of study time so as to maximize his utility. Find the ratio of the amount of time Ed spends studying to the amount of time Nancy spends studying. 1/2. Will his score for the course be greater than half, equal to half, or less than half of Nancy's? Less than half.

6.13 (1) Here is a puzzle for you. At first glance, it would appear that there is not nearly enough information to answer this question. But when you graph the indifference curve and think about it a little, you will see that there is a neat, easily calculated solution.

Kinko spends all his money on whips and leather jackets. Kinko's utility function is $U(x, y) = \min\{4x, 2x + y\}$, where x is his consumption of whips and y is his consumption of leather jackets. Kinko is consuming 15 whips and 10 leather jackets. The price of whips is \$10. You are to find Kinko's income.

(a) Graph the indifference curve for Kinko that passes through the point (15, 10). What is the slope of this indifference curve at (15, 10)? -2. What must be the price of leather jackets if Kinko chooses this point? \$5. Now, what is Kinko's income? $15 \times 10 + 10 \times 5 = 200$.