Chapter 7

NAME_____

Revealed Preference

Introduction. In the last section, you were given a consumer's preferences and then you solved for his or her demand behavior. In this chapter we turn this process around: you are given information about a consumer's demand behavior and you must deduce something about the consumer's preferences. The main tool is the *weak axiom of revealed preference*. This axiom says the following. If a consumer chooses commodity bundle A when she can afford bundle B, then she will never choose bundle B from any budget in which she can also afford A. The idea behind this axiom is that if you choose A when you could have had B, you must like A better than B. But if you like A better than B, then you will never choose B when you can have A. If somebody chooses A when she can afford B, we say that for her, A is *directly revealed preferred* to B. The weak axiom says that if A is directly revealed preferred to B, then B is not directly revealed preferred to A.

Example: Let us look at an example of how you check whether one bundle is revealed preferred to another. Suppose that a consumer buys the bundle $(x_1^A, x_2^A) = (2,3)$ at prices $(p_1^A, p_2^A) = (1,4)$. The cost of bundle (x_1^A, x_2^A) at these prices is $(2 \times 1) + (3 \times 4) = 14$. Bundle $(2,3)$ is directly revealed preferred to all the other bundles that she can afford at prices $(1,4)$, when she has an income of 14. For example, the bundle $(5,2)$ costs only 13 at prices $(1,4)$, so we can say that for this consumer $(2,3)$ is directly revealed preferred to $(1,4)$.

You will also have some problems about price and quantity indexes. A price index is a comparison of average price levels between two different times or two different places. If there is more than one commodity, it is not necessarily the case that all prices changed in the same proportion. Let us suppose that we want to compare the price level in the "current year" with the price level in some "base year." One way to make this comparison is to compare the costs in the two years of some "reference" commodity bundle. Two reasonable choices for the reference bundle come to mind. One possibility is to use the current year's consumption bundle for the reference bundle. The other possibility is to use the bundle consumed in the base year. Typically these will be different bundles. If the base-year bundle is the reference bundle, the resulting price index is called the *Laspeyres price index*. If the current year's consumption bundle is the reference bundle, then the index is called the *Paasche price index*.

Example: Suppose that there are just two goods. In 1980, the prices were $(1,3)$ and a consumer consumed the bundle $(4,2)$. In 1990, the prices were $(2,4)$ and the consumer consumed the bundle $(3,3)$. The cost of the 1980 bundle at 1980 prices is $(1 \times 4) + (3 \times 2) = 10$. The cost of this same bundle at 1990 prices is $(2 \times 4) + (4 \times 2) = 16$. If 1980 is treated as the base year and 1990 as the current year, the Laspeyres price ratio

is 16/10. To calculate the Paasche price ratio, you find the ratio of the cost of the 1990 bundle at 1990 prices to the cost of the same bundle at 1980 prices. The 1990 bundle costs $(2 \times 3) + (4 \times 3) = 18$ at 1990 prices. The same bundle cost $(1 \times 3) + (3 \times 3) = 12$ at 1980 prices. Therefore the Paasche price index is 18/12. Notice that both price indexes indicate that prices rose, but because the price changes are weighted differently, the two approaches give different price ratios.

Making an index of the "quantity" of stuff consumed in the two periods presents a similar problem. How do you weight changes in the amount of good 1 relative to changes in the amount of good 2? This time we could compare the cost of the two periods' bundles evaluated at some reference prices. Again there are at least two reasonable possibilities, the *Laspeyres quantity index* and the *Paasche quantity index*. The Laspeyres quantity index uses the base-year prices as the reference prices, and the Paasche quantity index uses current prices as reference prices.

Example: In the example above, the Laspeyres quantity index is the ratio of the cost of the 1990 bundle at 1980 prices to the cost of the 1980 bundle at 1980 prices. The cost of the 1990 bundle at 1980 prices is 12 and the cost of the 1980 bundle at 1980 prices is 10, so the Laspeyres quantity index is 12/10. The cost of the 1990 bundle at 1990 prices is 18 and the cost of the 1980 bundle at 1990 prices is 16. Therefore the Paasche quantity index is 18/16.

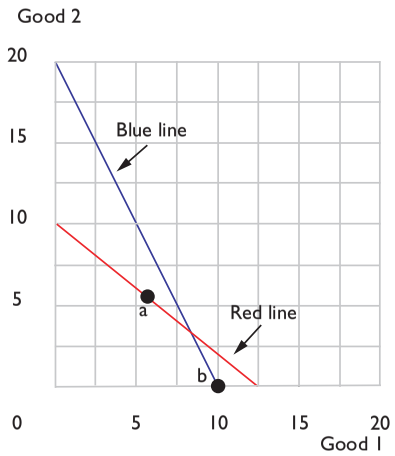
When you have completed this section, we hope that you will be able to do the following:

- Decide from given data about prices and consumption whether one commodity bundle is preferred to another.
- Given price and consumption data, calculate Paasche and Laspeyres price and quantity indexes.
- Use the weak axiom of revealed preferences to make logical deductions about behavior.
- Use the idea of revealed preference to make comparisons of well-being across time and across countries.

7.1 (0) When prices are $(4,6)$, Goldie chooses the bundle $(6,6)$, and when prices are $(6,3)$, she chooses the bundle $(10,0)$.

(a) On the graph below, show Goldie's first budget line in red ink and her second budget line in blue ink. Mark her choice from the first budget with the label A, and her choice from the second budget with the label B.

(b) Is Goldie's behavior consistent with the weak axiom of revealed preference? No.



7.2 (0) Freddy Frolic consumes only asparagus and tomatoes, which are highly seasonal crops in Freddy's part of the world. He sells umbrellas for a living, which provides a fluctuating income depending on the weather. But Freddy doesn't mind; he never thinks of tomorrow, so each week he spends as much as he earns. One week, when the prices of asparagus and tomatoes were each \$1 a pound, Freddy consumed 15 pounds of each. Use blue ink to show the budget line in the diagram below. Label Freddy's consumption bundle with the letter *A*.

(a) What is Freddy's income? \$30.

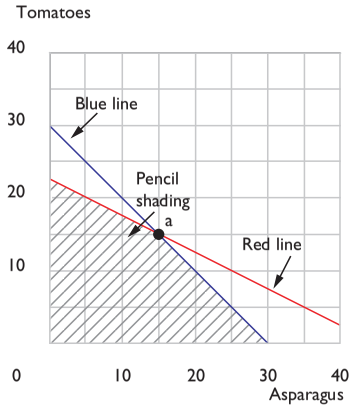
(b) The next week the price of tomatoes rose to \$2 a pound, but the price of asparagus remained at \$1 a pound. By chance, Freddy's income had changed so that his old consumption bundle of (15,15) was just affordable at the new prices. Use red ink to draw this new budget line on the graph below. Does your new budget line go through the point *A*? Yes.

What is the slope of this line? $-1/2$.

(c) How much asparagus can he afford now if he spent all of his income on asparagus? 45 pounds.

(d) What is Freddy's income now? \$45.

(e) Use pencil to shade the bundles of goods on Freddy's new red budget line that he definitely will *not* purchase with this budget. Is it possible that he would increase his consumption of tomatoes when his budget changes from the blue line to the red one? No.



7.3 (0) Pierre consumes bread and wine. For Pierre, the price of bread is 4 francs per loaf, and the price of wine is 4 francs per glass. Pierre has an income of 40 francs per day. Pierre consumes 6 glasses of wine and 4 loaves of bread per day.

Bob also consumes bread and wine. For Bob, the price of bread is 1/2 dollar per loaf and the price of wine is 2 dollars per glass. Bob has an income of \$15 per day.

(a) If Bob and Pierre have the same tastes, can you tell whether Bob is better off than Pierre or vice versa? Explain. Bob is better off. He can afford Pierre's bundle and still have income left.

(b) Suppose prices and incomes for Pierre and Bob are as above and that Pierre's consumption is as before. Suppose that Bob spends all of his income. Give an example of a consumption bundle of wine and bread such that, if Bob bought this bundle, we would know that Bob's tastes are not the same as Pierre's tastes. 7.5 wine and 0 bread, for example. If they had the same preferences,

this violates WARP, since each can afford but rejects the other's bundle.

7.4 (0) Here is a table of prices and the demands of a consumer named Ronald whose behavior was observed in 5 different price-income situations.

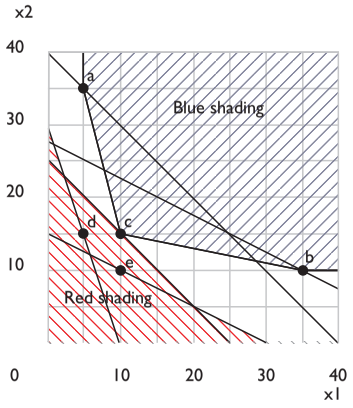
Situation	p_1	p_2	x_1	x_2
A	1	1	5	35
B	1	2	35	10
C	1	1	10	15
D	3	1	5	15
E	1	2	10	10

(a) Sketch each of his budget lines and label the point chosen in each case by the letters A, B, C, D, and E.

(b) Is Ronald's behavior consistent with the Weak Axiom of Revealed Preference? Yes.

(c) Shade lightly in red ink all of the points that you are certain are worse for Ronald than the bundle C.

(d) Suppose that you are told that Ronald has convex and monotonic preferences and that he obeys the strong axiom of revealed preference. Shade lightly in blue ink all of the points that you are certain are *at least as good as* the bundle C.



7.5 (0) Horst and Nigel live in different countries. Possibly they have different preferences, and certainly they face different prices. They each consume only two goods, x and y. Horst has to pay 14 marks per unit of x and 5 marks per unit of y. Horst spends his entire income of 167 marks on 8 units of x and 11 units of y. Good x costs Nigel 9 quid per unit and good y costs him 7 quid per unit. Nigel buys 10 units of x and 9 units of y.

(a) Which prices and income would Horst prefer, Nigel's income and prices or his own, or is there too little information to tell? Explain your answer.

Horst prefers Nigel's budget to his own. With Nigel's budget, he can afford his own bundle with money left over.

(b) Would Nigel prefer to have Horst's income and prices or his own, or is there too little information to tell? There is too little information to tell.

7.6 (0) Here is a table that illustrates some observed prices and choices for three different goods at three different prices in three different situations.

Situation	p_1	p_2	p_3	x_1	x_2	x_3
A	1	2	8	2	1	3
B	4	1	8	3	4	2
C	3	1	2	2	6	2

(a) We will fill in the table below as follows. Where i and j stand for any of the letters A, B, and C in Row i and Column j of the matrix, write the value of the Situation-j bundle at the Situation-i prices. For example, in Row A and Column A, we put the value of the bundle purchased in Situation A at Situation A prices. From the table above, we see that in Situation A, the consumer bought bundle $(2, 1, 3)$ at prices $(1, 2, 8)$. The cost of this bundle A at prices A is therefore $(1\times2)+(2\times1)+(8\times3) = 28$, so we put 28 in Row A, Column A. In Situation B the consumer bought bundle $(3, 4, 2)$. The value of the Situation-B bundle, evaluated at the situation-A prices is $(1 \times 3) + (2 \times 4) + (8 \times 2) = 27$, so put 27 in Row A, Column B. We have filled in some of the boxes, but we leave a few for you to do.

Prices/Quantities	A	B	C
A	28	27	30
B	33	32	30
C	13	17	16

(b) Fill in the entry in Row i and Column j of the table below with a D if the Situation-i bundle is directly revealed preferred to the Situation-j bundle. For example, in Situation A the consumer's expenditure is \$28. We see that at Situation-A prices, he could also afford the Situation-B bundle, which cost 27. Therefore the Situation-A bundle is directly revealed preferred to the Situation-B bundle, so we put a D in Row A, Column B. Now let us consider Row B, Column A. The cost of the Situation-B bundle at Situation-B prices is 32. The cost of the Situation-A bundle at Situation-B prices is 33. So, in Situation B, the consumer could not afford the Situation-A bundle. Therefore Situation B is *not* directly revealed preferred to Situation A. So we leave the entry in Row B, Column A blank. Generally, there is a D in Row i Column j if the number in the ij entry of the table in part (a) is less than or equal to the entry in Row i, Column i. There will be a violation of WARP if for some i and j, there is a D in Row i Column j and also a D in Row j, Column i. Do these observations violate WARP? No.

Situation	A	B	C
A	—	D	I
B	I	—	D
C	D	I	—

(c) Now fill in Row i, Column j with an I if observation i is *indirectly* revealed preferred to j. Do these observations violate the Strong Axiom of Revealed Preference? Yes.

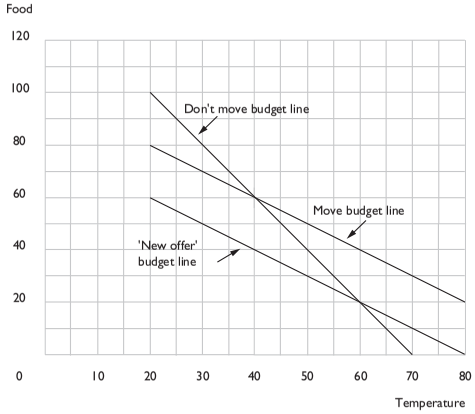
7.7 (0) It is January, and Joe Grad, whom we met in Chapter 5, is shivering in his apartment when the phone rings. It is Mandy Manana, one of the students whose price theory problems he graded last term. Mandy asks if Joe would be interested in spending the month of February in her apartment. Mandy, who has switched majors from economics to political science, plans to go to Aspen for the month and so her apartment will be empty (alas). All Mandy asks is that Joe pay the monthly service charge of \$40 charged by her landlord and the heating bill for the month of February. Since her apartment is much better insulated than Joe's, it only costs \$1 per month to raise the temperature by 1 degree. Joe

thanks her and says he will let her know tomorrow. Joe puts his earmuffs back on and muses. If he accepts Mandy's offer, he will still have to pay rent on his current apartment but he won't have to heat it. If he moved, heating would be cheaper, but he would have the \$40 service charge. The outdoor temperature averages 20 degrees Fahrenheit in February, and it costs him \$2 per month to raise his apartment temperature by 1 degree. Joe is still grading homework and has \$100 a month left to spend on food and utilities after he has paid the rent on his apartment. The price of food is still \$1 per unit.

(a) Draw Joe's budget line for February if he moves to Mandy's apartment and on the same graph, draw his budget line if he doesn't move.

(b) After drawing these lines himself, Joe decides that he would be better off not moving. From this, we can tell, using the principle of revealed preference that Joe must plan to keep his apartment at a temperature of less than 60 degrees.

(c) Joe calls Mandy and tells her his decision. Mandy offers to pay half the service charge. Draw Joe's budget line if he accepts Mandy's new offer. Joe now accepts Mandy's offer. From the fact that Joe accepted this offer we can tell that he plans to keep the temperature in Mandy's apartment above 40 degrees.



7.8 (0) Lord Peter Pommy is a distinguished criminologist, schooled in the latest techniques of forensic revealed preference. Lord Peter is investigating the disappearance of Sir Cedric Pinchbottom who abandoned his aging mother on a street corner in Liverpool and has not been seen

since. Lord Peter has learned that Sir Cedric left England and is living under an assumed name somewhere in the Empire. There are three suspects, R. Preston McAfee of Brass Monkey, Ontario, Canada, Richard Manning of North Shag, New Zealand, and Richard Stevenson of Gooey Shoes, Falkland Islands. Lord Peter has obtained Sir Cedric's diary, which recorded his consumption habits in minute detail. By careful observation, he has also discovered the consumption behavior of McAfee, Manning, and Stevenson. All three of these gentlemen, like Sir Cedric, spend their entire incomes on beer and sausage. Their dossiers reveal the following:

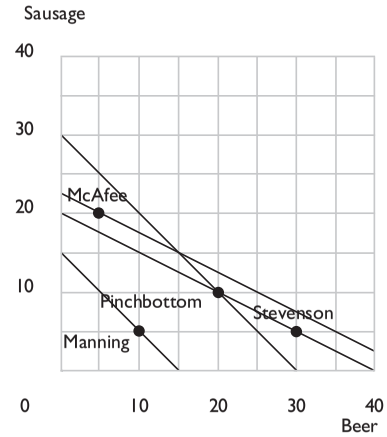
- **Sir Cedric Pinchbottom** — In the year before his departure, Sir Cedric consumed 10 kilograms of sausage and 20 liters of beer per week. At that time, beer cost 1 English pound per liter and sausage cost 1 English pound per kilogram.

- **R. Preston McAfee** — McAfee is known to consume 5 liters of beer and 20 kilograms of sausage. In Brass Monkey, Ontario beer costs 1 Canadian dollar per liter and sausage costs 2 Canadian dollars per kilogram.

- **Richard Manning** — Manning consumes 5 kilograms of sausage and 10 liters of beer per week. In North Shag, a liter of beer costs 2 New Zealand dollars and sausage costs 2 New Zealand dollars per kilogram.

- **Richard Stevenson** — Stevenson consumes 5 kilograms of sausage and 30 liters of beer per week. In Gooey Shoes, a liter of beer costs 10 Falkland Island pounds and sausage costs 20 Falkland Island pounds per kilogram.

(a) Draw the budget line for each of the three fugitives, using a different color of ink for each one. Label the consumption bundle that each chooses. On this graph, superimpose Sir Cedric's budget line and the bundle he chose.



(b) After pondering the dossiers for a few moments, Lord Peter announced. "Unless Sir Cedric has changed his tastes, I can eliminate one of the suspects. Revealed preference tells me that one of the suspects is innocent." Which one? McAfee.

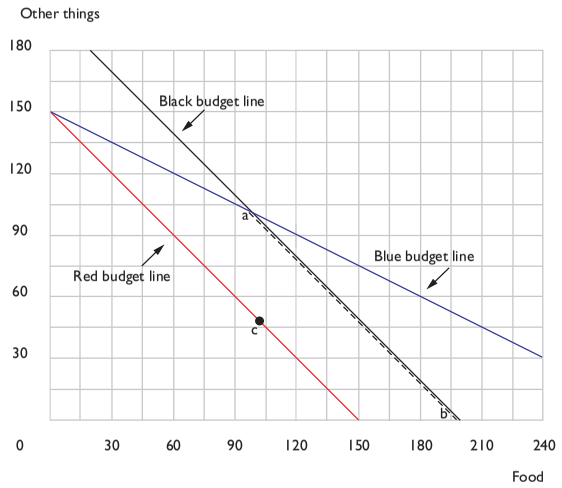
(c) After thinking a bit longer, Lord Peter announced. "If Sir Cedric left voluntarily, then he would have to be better off than he was before. Therefore if Sir Cedric left voluntarily and if he has not changed his tastes, he must be living in Falklands.

7.9 (1) The McCawber family is having a tough time making ends meet. They spend \$100 a week on food and \$50 on other things. A new welfare program has been introduced that gives them a choice between receiving a grant of \$50 per week that they can spend any way they want, and buying any number of \$2 food coupons for \$1 apiece. (They naturally are not allowed to resell these coupons.) Food is a normal good for the McCawbers. As a family friend, you have been asked to help them decide on which option to choose. Drawing on your growing fund of economic knowledge, you proceed as follows.

(a) On the graph below, draw their old budget line in red ink and label their current choice C. Now use black ink to draw the budget line that they would have with the grant. If they chose the coupon option, how much food could they buy if they spent all their money on food coupons?

\$300. How much could they spend on other things if they bought

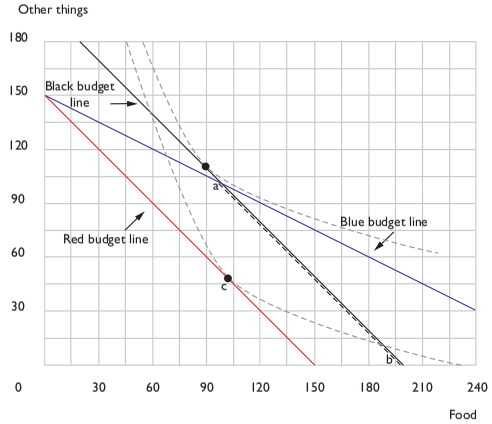
no food? $150. Use blue ink to draw their budget line if they choose the coupon option.



(b) Using the fact that food is a normal good for the McCawbers, and knowing what they purchased before, darken the portion of the black budget line where their consumption bundle could possibly be if they chose the lump-sum grant option. Label the ends of this line segment A and B.

(c) After studying the graph you have drawn, you report to the McCawbers. "I have enough information to be able to tell you which choice to make. You should choose the coupon because you can get more food even when other expenditure is constant.

(d) Mr. McCawber thanks you for your help and then asks, "Would you have been able to tell me what to do if you hadn't known whether food was a normal good for us?" On the axes below, draw the same budget lines you drew on the diagram above, but draw indifference curves for which food is not a normal good and for which the McCawbers would be better off with the program you advised them not to take.



7.10 (0) In 1933, the Swedish economist Gunnar Myrdal (who later won a Nobel prize in economics) and a group of his associates at Stockholm University collected a fantastically detailed historical series of prices and price indexes in Sweden from 1830 until 1930. This was published in a book called *The Cost of Living in Sweden.* In this book you can find 100 years of prices for goods such as oat groats, hard rye bread, salted codfish, beef, reindeer meat, birchwood, tallow candles, eggs, sugar, and coffee. There are also estimates of the quantities of each good consumed by an average working-class family in 1850 and again in 1890.

The table below gives prices in 1830, 1850, 1890, and 1913, for flour, meat, milk, and potatoes. In this time period, these four staple foods accounted for about 2/3 of the Swedish food budget.

Prices of Staple Foods in Sweden

Prices are in Swedish kronor per kilogram, except for milk, which is in Swedish kronor per liter.

	1830	1850	1890	1913
Grain Flour	.14	.14	.16	.19
Meat	.28	.34	.66	.85
Milk	.07	.08	.10	.13
Potatoes	.032	.044	.051	.064

Based on the tables published in Myrdal's book, typical consumption bundles for a working-class Swedish family in 1850 and 1890 are listed below. (The reader should be warned that we have made some

approximations and simplifications to draw these simple tables from the much more detailed information in the original study.)

Quantities Consumed by a Typical Swedish Family

Quantities are measured in kilograms per year, except for milk, which is measured in liters per year.

	1850	1890
Grain Flour	165	220
Meat	22	42
Milk	120	180
Potatoes	200	200

(a) Complete the table below, which reports the annual cost of the 1850 and 1890 bundles of staple foods at various years' prices.

Cost of 1850 and 1890 Bundles at Various Years' Prices

Cost	1850 bundle	1890 bundle
Cost at 1830 Prices	44.1	61.6
Cost at 1850 Prices	49.0	68.3
Cost at 1890 Prices	63.1	91.1
Cost at 1913 Prices	78.5	113.7

(b) Is the 1890 bundle revealed preferred to the 1850 bundle? Yes.

(c) The Laspeyres quantity index for 1890 with base year 1850 is the ratio of the value of the 1890 bundle at 1850 prices to the value of the 1850 bundle at 1850 prices. Calculate the Laspeyres quantity index of staple food consumption for 1890 with base year 1850. 1.39.

(d) The Paasche quantity index for 1890 with base year 1850 is the ratio of the value of the 1890 bundle at 1890 prices to the value of the 1850 bundle at 1890 prices. Calculate the Paasche quantity index for 1890 with base year 1850. 1.44.

(e) The Laspeyres price index for 1890 with base year 1850 is calculated using 1850 quantities for weights. Calculate the Laspeyres price index for 1890 with base year 1850 for this group of four staple foods. 1.29.

(f) If a Swede were rich enough in 1850 to afford the 1890 bundle of staple foods in 1850, he would have to spend 1.39 times as much on these foods as does the typical Swedish worker of 1850.

(g) If a Swede in 1890 decided to purchase the same bundle of food staples that was consumed by typical 1850 workers, he would spend the fraction .69 of the amount that the typical Swedish worker of 1890 spends on these goods.

7.11 (0) This question draws from the tables in the previous question. Let us try to get an idea of what it would cost an American family at today's prices to purchase the bundle consumed by an average Swedish family in 1850. In the United States today, the price of flour is about $.40 per kilogram, the price of meat is about $3.75 per kilogram, the price of milk is about $.50 per liter, and the price of potatoes is about $1 per kilogram. We can also compute a Laspeyres price index across time and across countries and use it to estimate the value of a current US dollar relative to the value of an 1850 Swedish kronor.

(a) How much would it cost an American at today's prices to buy the bundle of staple food commodities purchased by an average Swedish working-class family in 1850? $408.

(b) Myrdal estimates that in 1850, about 2/3 of the average family's budget was spent on food. In turn, the four staples discussed in the last question constitute about 2/3 of the average family's food budget. If the prices of other goods relative to the price of the food staples are similar in the United States today to what they were in Sweden in 1850, about how much would it cost an American at current prices to consume the same overall consumption bundle consumed by a Swedish working-class family in 1850? $919.

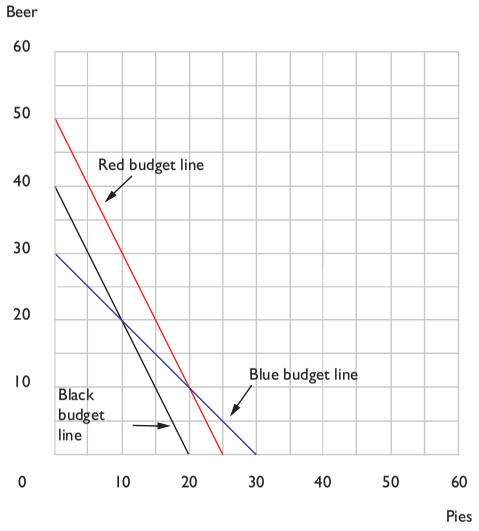
(c) Using the Swedish consumption bundle of staple foods in 1850 as weights, calculate a Laspeyres price index to compare prices in current American dollars relative to prices in 1850 Swedish kronor. 8.35. If we use this to estimate the value of current dollars relative to 1850 Swedish kronor, we would say that a U.S. dollar today is worth about .12 1850 Swedish kronor.

7.12 (0) Suppose that between 1960 and 1985, the price of all goods exactly doubled while every consumer's income tripled.

(a) Would the Laspeyres price index for 1985, with base year 1960 be less than 2, greater than 2, or exactly equal to 2? Exactly 2. What about the Paasche price index? Exactly 2.

(b) If bananas are a normal good, will total banana consumption increase? Yes. If everybody has homothetic preferences, can you determine by what percentage total banana consumption must have increased? Explain. Yes, by 50%. Everybody's budget line shifted out by 50%. With homothetic preferences, the consumption of each good increases in the same proportion.

7.13 (1) Norm and Sheila consume only meat pies and beer. Meat pies used to cost \$2 each and beer was \$1 per can. Their gross income used to be \$60 per week, but they had to pay an income tax of \$10. Use red ink to sketch their old budget line for meat pies and beer.



(a) They used to buy 30 cans of beer per week and spent the rest of their income on meat pies. How many meat pies did they buy? 10.

(b) The government decided to eliminate the income tax and to put a sales tax of \$1 per can on beer, raising its price to \$2 per can. Assuming that Norm and Sheila's pre-tax income and the price of meat pies did not change, draw their new budget line in blue ink.

(c) The sales tax on beer induced Norm and Sheila to reduce their beer consumption to 20 cans per week. What happened to their consumption of meat pies? Stayed the same--10. How much revenue did this tax raise from Norm and Sheila? \$20.

(d) This part of the problem will require some careful thinking. Suppose that instead of just taxing beer, the government decided to tax *both* beer and meat pies at the *same* percentage rate, and suppose that the price of beer and the price of meat pies each went up by the full amount of the tax. The new tax rate for both goods was set high enough to raise exactly the same amount of money from Norm and Sheila as the tax on beer used to raise. This new tax collects \$ .50 for every bottle of beer sold and \$ 1 for every meat pie sold. (Hint: If both goods are taxed at the same rate, the effect is the same as an income tax.) How large an income tax would it take to raise the same revenue as the \$1 tax on beer? \$20. Now you can figure out how big a tax on each good is equivalent to an income tax of the amount you just found.

(e) Use black ink to draw the budget line for Norm and Sheila that corresponds to the tax in the last section. Are Norm and Sheila better off having just beer taxed or having both beer and meat pies taxed if both sets of taxes raise the same revenue? Both. (Hint: Try to use the principle of revealed preference.)