Chapter 8

NAME_____

Slutsky Equation

Introduction. It is useful to think of a price change as having two distinct effects, a substitution effect and an income effect. The **substitution effect** of a price change is the change that would have happened *if* income changed at the same time in such a way that the consumer could exactly afford her old consumption bundle. The rest of the change in the consumer's demand is called the **income effect**. Why do we bother with breaking a real change into the sum of two hypothetical changes? Because we know things about the pieces that we wouldn't know about the whole without taking it apart. In particular, we know that the substitution effect of increasing the price of a good *must* reduce the demand for it. We also know that the income effect of an increase in the price of a good is equivalent to the effect of a *loss* of income. Therefore if the good whose price has risen is a normal good, then both the income and substitution effect operate to reduce demand. But if the good is an inferior good, income and substitution effects act in opposite directions.

Example: A consumer has the utility function $U(x_1, x_2) = x_1 x_2$ and an income of \$24. Initially the price of good 1 was \$1 and the price of good 2 was \$2. Then the price of good 2 rose to \$3 and the price of good 1 stayed at \$1. Using the methods you learned in Chapters 5 and 6, you will find that this consumer's demand function for good 1 is $D_1(p_1, p_2, m) = m/2p_1$ and her demand function for good 2 is $D_2(p_1, p_2, m) = m/2p_2$. Therefore initially she will demand 12 units of good 1 and 6 units of good 2. If, when the price of good 2 rose to \$3, her income had changed enough so that she could exactly afford her old bundle, her new income would have to be $(1 \times 12) + (3 \times 6) = \$30$. At an income of \$30, at the new prices, she would demand $D_2(1, 3, 30) = 5$ units of good 2. Before the change she bought 6 units of 2, so the substitution effect of the price change on her demand for good 2 is $5 - 6 = -1$ units. Our consumer's income didn't *really* change. Her income stayed at \$24. Her actual demand for good 2 after the price change was $D_2(1, 3, 24) = 4$. The difference between what she actually demanded after the price change and what she would have demanded if her income had changed to let her just afford the old bundle is the income effect. In this case the income effect is $4 - 5 = -1$ units of good 2. Notice that in this example, both the income effect and the substitution effect of the price increase worked to reduce the demand for good 2.

When you have completed this workout, we hope that you will be able to do the following:

- Find Slutsky income effect and substitution effect of a specific price change if you know the demand function for a good.
- Show the Slutsky income and substitution effects of a price change on an indifference curve diagram.
- Show the Hicks income and substitution effects of a price change on an indifference curve diagram.
- Find the Slutsky income and substitution effects for special utility functions such as perfect substitutes, perfect complements, and Cobb-Douglas.
- Use an indifference-curve diagram to show how the case of a Giffen good might arise.
- Show that the substitution effect of a price increase unambiguously decreases demand for the good whose price rose.
- Apply income and substitution effects to draw some inferences about behavior.

8.1 (0) Gentle Charlie, vegetarian that he is, continues to consume apples and bananas. His utility function is $U(x_A, x_B) = x_A x_B$. The price of apples is \$1, the price of bananas is \$2, and Charlie's income is \$40 a day. The price of bananas suddenly falls to \$1.

(a) Before the price change, Charlie consumed 20 apples and 10 bananas per day. On the graph below, use black ink to draw Charlie's original budget line and put the label A on his chosen consumption bundle.

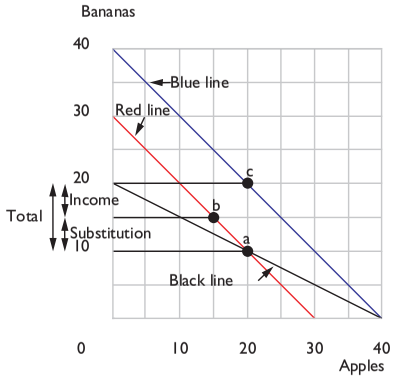
(b) If, after the price change, Charlie's income had changed so that he could exactly afford his old consumption bundle, his new income would have been 30. With this income and the new prices, Charlie would consume 15 apples and 15 bananas. Use red ink to draw the budget line corresponding to this income and these prices. Label the bundle that Charlie would choose at this income and the new prices with the letter B.

(c) Does the substitution effect of the fall in the price of bananas make him buy more bananas or fewer bananas? More bananas. How many more or fewer? 5 more.

(d) After the price change, Charlie actually buys 20 apples and 20 bananas. Use blue ink to draw Charlie's actual budget line after the price change. Put the label C on the bundle that he actually chooses after the price change. Draw 3 horizontal lines on your graph, one from A to the vertical axis, one from B to the vertical axis, and one from C to the vertical axis. Along the vertical axis, label the income effect, the substitution effect, and the total effect on the demand for bananas. Is the

blue line parallel to the red line or the black line that you drew before?

Red line.



(e) The income effect of the fall in the price of bananas on Charlie's demand for bananas is the same as the effect of an (increase, decrease) increase in his income of $ 10 per day. Does the income effect make him consume more bananas or fewer? More. How many more or how many fewer? 5 more.

(f) Does the substitution effect of the fall in the price of bananas make Charlie consume more *apples* or fewer? Fewer. How many more or fewer? 5 fewer. Does the income effect of the fall in the price of bananas make Charlie consume more apples or fewer? More. What is the total effect of the change in the price of bananas on the demand for apples? Zero.

8.2 (0) Neville's passion is fine wine. When the prices of all other goods are fixed at current levels, Neville's demand function for high-quality claret is $q = .02m - 2p$, where m is his income, p is the price of claret (in British pounds), and q is the number of bottles of claret that he demands. Neville's income is 7,500 pounds, and the price of a bottle of suitable claret is 30 pounds.

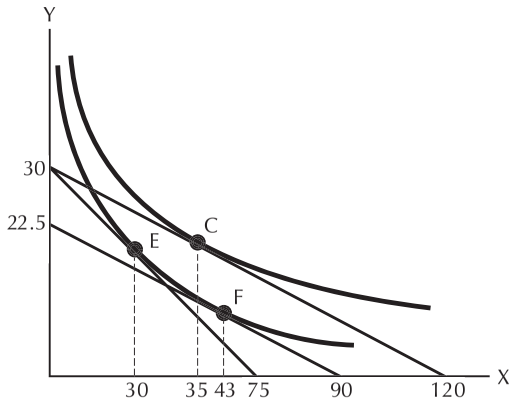
(a) How many bottles of claret will Neville buy? 90.

(b) If the price of claret rose to 40 pounds, how much income would Neville have to have in order to be exactly able to afford the amount of claret and the amount of other goods that he bought before the price change? 8,400 pounds. At this income, and a price of 40 pounds, how many bottles would Neville buy? 88 bottles.

(c) At his original income of 7,500 and a price of 40, how much claret would Neville demand? 70 bottles.

(d) When the price of claret rose from 30 to 40, the number of bottles that Neville demanded decreased by 20. The substitution effect (increased, reduced) reduced his demand by 2 bottles and the income effect (increased, reduced) reduced his demand by 18 bottles.

8.3 (0) *Note: Do this problem only if you have read the section entitled "Another Substitution Effect" that describes the "Hicks substitution effect".* Consider the figure below, which shows the budget constraint and the indifference curves of good King Zog. Zog is in equilibrium with an income of \$300, facing prices $p_X = \$4$ and $p_Y = \$10$.



(a) How much X does Zog consume? 30.

(b) If the price of X falls to \$2.50, while income and the price of Y stay constant, how much X will Zog consume? 35.

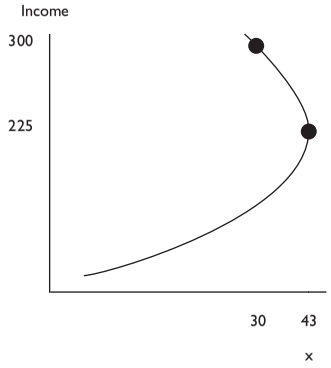
(c) How much income must be taken away from Zog to isolate the Hicksian income and substitution effects (i.e., to make him just able to afford to reach his old indifference curve at the new prices)? \$75.

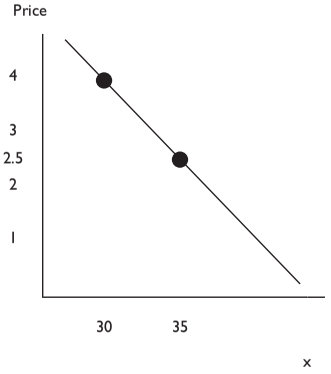
(d) The total effect of the price change is to change consumption from the point E to the point C.

(e) The income effect corresponds to the movement from the point F to the point C while the substitution effect corresponds to the movement from the point E to the point F.

(f) Is X a normal good or an inferior good? An inferior good.

(g) On the axes below, sketch an Engel curve and a demand curve for Good X that would be reasonable given the information in the graph above. Be sure to label the axes on both your graphs.



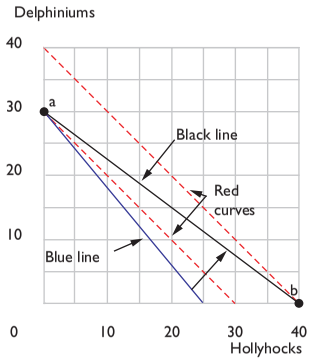


8.4 (0) Maude spends *all* of her income on delphiniums and hollyhocks. She thinks that delphiniums and hollyhocks are perfect substitutes; one delphinium is just as good as one hollyhock. Delphiniums cost \$4 a unit and hollyhocks cost \$5 a unit.

(a) If the price of delphiniums decreases to \$3 a unit, will Maude buy more of them? Yes. What part of the change in consumption is due to the income effect and what part is due to the substitution effect?

All due to income effect.

(b) If the prices of delphiniums and hollyhocks are respectively $p_d = \$4$ and $p_h = \$5$ and if Maude has \$120 to spend, draw her budget line in blue ink. Draw the highest indifference curve that she can attain in red ink, and label the point that she chooses as A.



(c) Now let the price of hollyhocks fall to \$3 a unit, while the price of delphiniums does not change. Draw her new budget line in black ink. Draw the highest indifference curve that she can now reach with red ink. Label the point she chooses now as B.

(d) How much would Maude's income have to be after the price of hollyhocks fell, so that she could just exactly afford her old commodity bundle A? \$120.

(e) When the price of hollyhocks fell to \$3, what part of the change in Maude's demand was due to the income effect and what part was due to the substitution effect? All substitution effect.

8.5 (1) Suppose that two goods are perfect complements. If the price of one good changes, what part of the change in demand is due to the substitution effect, and what part is due to the income effect? All income effect.

8.6 (0) Douglas Cornfield's demand function for good x is $x(p_x, p_y, m) = 2m/5p_x$. His income is \$1,000, the price of x is \$5, and the price of y is \$20. If the price of x falls to \$4, then his demand for x will change from

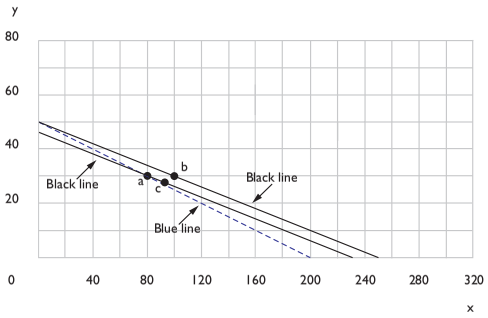
80 to 100.

(a) If his income were to change at the same time so that he could exactly afford his old commodity bundle at $p_x = 4$ and $p_y = 20$, what would his new income be? 920. What would be his demand for x at this new level of income, at prices $p_x = 4$ and $p_y = 20$? 92.

(b) The substitution effect is a change in demand from 80 to 92. The income effect of the price change is a change in demand from

92 to 100.

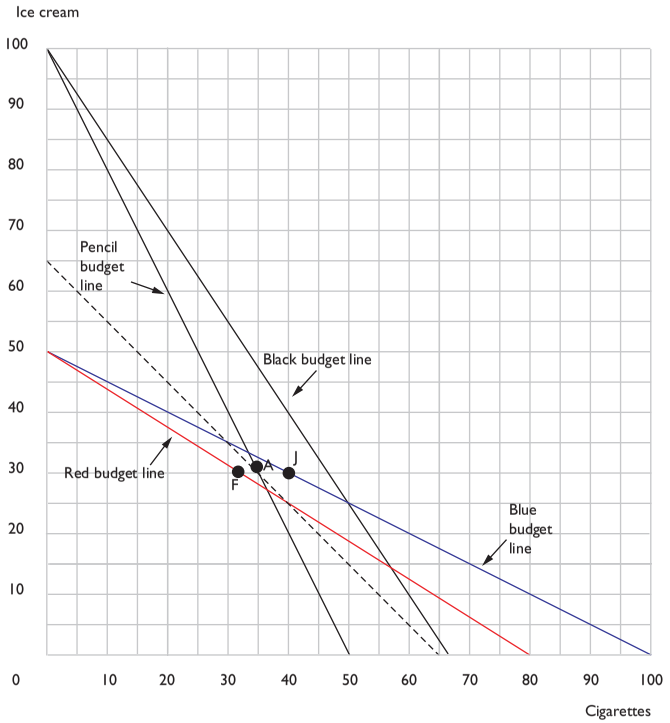
(c) On the axes below, use blue ink to draw Douglas Cornfield's budget line before the price change. Locate the bundle he chooses at these prices on your graph and label this point A. Use black ink to draw Douglas Cornfield's budget line after the price change. Label his consumption bundle after the change by B.



(d) On the graph above, use black ink to draw a budget line with the new prices but with an income that just allows Douglas to buy his old bundle, A. Find the bundle that he would choose with this budget line and label this bundle C.

8.7 (1) Mr. Consumer allows himself to spend \$100 per month on cigarettes and ice cream. Mr. C's preferences for cigarettes and ice cream are unaffected by the season of the year.

(a) In January, the price of cigarettes was \$1 per pack, while ice cream cost \$2 per pint. Faced with these prices, Mr. C bought 30 pints of ice cream and 40 packs of cigarettes. Draw Mr. C's January budget line with blue ink and label his January consumption bundle with the letter J.



(b) In February, Mr. C again had \$100 to spend and ice cream still cost \$2 per pint, but the price of cigarettes rose to \$1.25 per pack. Mr. C consumed 30 pints of ice cream and 32 packs of cigarettes. Draw Mr. C's February budget line with red ink and mark his February bundle with the letter F. The substitution effect of this price change would make him buy (less, more, the same amount of) less cigarettes and (less, more, the same amount of) more ice cream. Since this is true and the total change in his ice cream consumption was zero, it must be that the income effect of this price change on his consumption of ice cream makes him buy (more, less, the same amount of) less ice cream. The income effect of this price change is like the effect of an (increase, decrease)

decrease in his income. Therefore the information we have suggests that ice cream is a(n) (normal, inferior, neutral) normal good.

(c) In March, Mr. C again had \$100 to spend. Ice cream was on sale for \$1 per pint. Cigarette prices, meanwhile, increased to \$1.50 per pack. Draw his March budget line with black ink. Is he better off than in January, worse off, or can you not make such a comparison? Better off. How does your answer to the last question change if the price of cigarettes had increased to \$2 per pack? Now you can't tell.

8.8 (1) This problem continues with the adventures of Mr. Consumer from the previous problem.

(a) In April, cigarette prices rose to \$2 per pack and ice cream was still on sale for \$1 per pint. Mr. Consumer bought 34 packs of cigarettes and 32 pints of ice cream. Draw his April budget line with pencil and label his April bundle with the letter A. Was he better off or worse off than in January? Worse off. Was he better off or worse off than in February, or can't one tell? Better off.

(b) In May, cigarettes stayed at \$2 per pack and as the sale on ice cream ended, the price returned to \$2 per pint. On the way to the store, however, Mr. C found \$30 lying in the street. He then had \$130 to spend on cigarettes and ice cream. Draw his May budget with a dashed line. Without knowing what he purchased, one can determine whether he is better off than he was in at least one previous month. Which month or months?

He is better off in May than in February.

(c) In fact, Mr. C buys 40 packs of cigarettes and 25 pints of ice cream in May. Does he satisfy WARP? No.

8.9 (2) In the last chapter, we studied a problem involving food prices and consumption in Sweden in 1850 and 1890.

(a) Potato consumption was the same in both years. Real income must have gone up between 1850 and 1890, since the amount of food staples purchased, as measured by either the Laspeyres or the Paasche quantity index, rose. The price of potatoes rose less rapidly than the price of either meat or milk, and at about the same rate as the price of grain flour. So real income went up and the price of potatoes went down relative to other goods. From this information, determine whether potatoes were

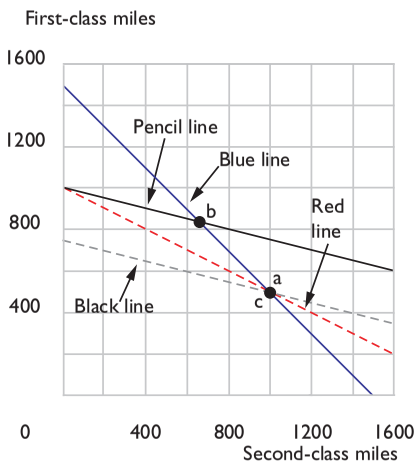
most likely a normal or an inferior good. Explain your answer.

If potatoes were a normal good, both the fall in potato price and the rise in income would increase the demand for potatoes. But potato consumption did not increase. So potatoes must be an inferior good.

(b) Can one also tell from these data whether it is likely that potatoes were a Giffen good? If potatoes were a Giffen good, then the fall in the price of potatoes would decrease demand and the rise in income would also decrease demand for potatoes. But potato demand stayed constant. So potatoes were probably not a Giffen good.

8.10 (1) Agatha must travel on the Orient Express from Istanbul to Paris. The distance is 1,500 miles. A traveler can choose to make any fraction of the journey in a first-class carriage and travel the rest of the way in a second-class carriage. The price is 10 cents a mile for a second-class carriage and 20 cents a mile for a first-class carriage. Agatha much prefers first-class to second-class travel, but because of a misadventure in an Istanbul bazaar, she has only $200 left with which to buy her tickets. Luckily, she still has her toothbrush and a suitcase full of cucumber sandwiches to eat on the way. Agatha plans to spend her entire $200 on her tickets for her trip. She will travel first class as much as she can afford to, but she must get all the way to Paris, and $200 is not enough money to get her all the way to Paris in first class.

(a) On the graph below, use red ink to show the locus of combinations of first- and second-class tickets that Agatha can just afford to purchase with her $200. Use blue ink to show the locus of combinations of first- and second-class tickets that are sufficient to carry her the entire distance from Istanbul to Paris. Locate the combination of first- and second-class miles that Agatha will choose on your graph and label it A.



(b) Let m_1 be the number of miles she travels by first-class coach and m_2 be the number of miles she travels by second-class coach. Write down two equations that you can solve to find the number of miles she chooses to travel by first-class coach and the number of miles she chooses to travel by second-class coach. $.2m_1 + .1m_2 = 200,\ m_1 + m_2 = 1,500.$

(c) The number of miles that she travels by second-class coach is 1,000.

(d) Just before she was ready to buy her tickets, the price of second-class tickets fell to $.05 while the price of first-class tickets remained at $.20. On the graph that you drew above, use pencil to show the combinations of first-class and second-class tickets that she can afford with her $200 at these prices. On your graph, locate the combination of first-class and second-class tickets that she would now choose. (Remember, she is going to travel as much first-class as she can afford to and still make the 1,500 mile trip on $200.) Label this point B. How many miles does she travel by second class now? 666.66. (Hint: For an exact solution you will have to solve two linear equations in two unknowns.) Is second-class travel a normal good for Agatha? No. Is it a Giffen good for her? Yes.

8.11 (0) We continue with the adventures of Agatha, from the previous problem. Just after the price change from \$.10 per mile to \$.05 per mile for second-class travel, and just before she had bought any tickets, Agatha misplaced her handbag. Although she kept most of her money in her sock, the money she lost was just enough so that at the new prices, she could exactly afford the combination of first- and second-class tickets that she would have purchased at the old prices. How much money did she lose?

\$50. On the graph you started in the previous problem, use black ink to draw the locus of combinations of first- and second-class tickets that she can just afford after discovering her loss. Label the point that she chooses with a C. How many miles will she travel by second class now?

1,000.

(a) Finally, poor Agatha finds her handbag again. How many miles will she travel by second class now (assuming she didn't buy any tickets before she found her lost handbag)? 666.66. When the price of second-class tickets fell from \$.10 to \$.05, how much of a change in Agatha's demand for second-class tickets was due to a substitution effect? None. How much of a change was due to an income effect? −333.33.