

$$Y = (X - \theta)^2 + \varepsilon \quad (1)$$

$$E(\varepsilon) = 0 \quad (2)$$

ϕ is joint dist. of (Y, X)

θ is target parameter

$\mathcal{M}^?$ set of joint distributions of (ε, X)

that satisfy (1) and (2) $-(x^2 - 2\theta x + \theta^2)$

$S(\phi, \theta)$ is non-empty when $E(Y - (X - \theta)^2) = 0$

$$E(Y) - E(X^2) + 2\theta \cdot E(X) - \theta^2 = 0$$

\hookrightarrow quadratic eq. in θ

$$\underbrace{\theta^2}_{a=1} - \underbrace{2E(X) \cdot \theta}_{b = -2E(X)} + \underbrace{E(X^2) - E(Y)}_{c = E(X^2) - E(Y)} = 0$$

$$\theta_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2	$D > 0$
1	$D = 0$
X	$D < 0$

$$D = b^2 - 4 \cdot a \cdot c =$$

$$= 4 \cdot (E(X))^2 - 4 \cdot 1 \cdot (E(X^2) - E(Y))$$

$$= 4 \cdot \left[\underbrace{(E(X))^2 - E(X^2)}_{= -\text{Var}(X)} + E(Y) \right]$$

$$= 4 \cdot [E(Y) - \text{Var}(X)]$$

thus : if $E(Y) = \text{Var}(x)$ θ is point identified
 if $E(Y) > \text{Var}(x)$ θ is set identified
 if $E(Y) < \text{Var}(x)$ θ is not identified

$$E(Y) = E((x - \theta)^2)$$

*this quantity is minimized for $\theta = E(x)$
 and it is equal to $\text{Var}(x)$*

if $E(Y) < \text{Var}(x) = \min_{\theta} E((x - \theta)^2)$
 we have no solution

$$E(z \cdot z) = E(z(Y - (x - \theta)^2)) =$$

$$= E(z \cdot Y) - E(z \cdot x^2) + \underbrace{2E(z \cdot x)}_{\neq 0} \theta - \underbrace{E(z)}_{=0} \cdot \theta^2 = 0$$

$$\Rightarrow \theta = \frac{E(z \cdot x^2) - E(z \cdot Y)}{2 \cdot E(z \cdot x)} \quad (Y, X, z)$$

$$Y \sim \text{Pois}(\lambda) \Rightarrow P(Y=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

$$i: \quad y_i, X_i \quad 1, x_{i1}, \dots, x_{ip}$$

$$L(\beta | y_i, X_i) = \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!} \Rightarrow L_n(\beta | y, X) = \prod_{i=1}^n \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!}$$

$$\ln L(\beta | y, X) = \sum_{i=1}^n y_i \log(\lambda_i) - \lambda_i - \log(y_i!)$$

$$= \sum_{i=1}^n \left(y_i (\beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip}) - e^{\beta_0 + \dots + \beta_p x_{ip}} - \log(y_i!) \right)$$

$$\frac{\partial}{\partial \beta_k} \ln L(\beta | y, X) = \sum_{i=1}^n \left(y_i x_{ik} - e^{\beta_0 + \dots + \beta_p x_{ip}} \cdot x_{ik} \right)$$

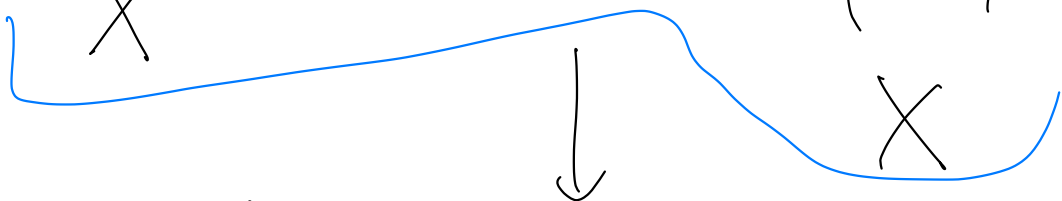
$$= \sum_{i=1}^n x_{ik} (y_i - e^{\beta_0 + \dots + \beta_p x_{ip}})$$

19E

$$\frac{\partial^2}{\partial \beta_k \partial \beta_l} \ln L(\beta | y, X) = - \sum_{i=1}^n x_{ik} x_{il} e^{\beta_0 + \dots + \beta_p x_{ip}}$$

$$\hat{\beta}_{MLE} \sim N(\beta, I^{-1})$$

$$X^T = \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_{11} & x_{21} & \dots & x_{n1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1p} & x_{2p} & \dots & x_{np} \end{pmatrix} \quad \begin{pmatrix} e^{\beta_0 + \beta_1 x_{11}} \\ \vdots \\ e^{\beta_0 + \beta_1 x_{np}} \end{pmatrix} \quad \begin{pmatrix} 1 & x_{11} & x_{1p} \\ \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{np} \end{pmatrix}$$



$$\hat{\beta} \sim N \left(\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}, \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}^{-1} \right)$$

$$\hat{\beta}_1 \sim N(\beta_1, \checkmark)$$