$$Y = (X-\theta)^2 + \varepsilon \qquad (1)$$

$$E(\varepsilon) = 0 \qquad (2)$$

ϕ is joint dist. of (Y, X)

θ is target parameter

M? set of joint distributions of (ε, X) that satisfy (1) and (2)

$S(\phi, \theta)$ is non-empty when $E(Y - \overbrace{(X-\theta)^2}^{-(X^2 - 2\theta \cdot X + \theta^2)}) = 0$

$$E(Y) - E(X^2) + 2 \cdot \theta \cdot E(X) - \theta^2 = 0$$

↳ quadratic eq. in θ

$$\underbrace{\theta^2}_{a=1} \underbrace{- 2E(X)}_{b=-2E(X)} \cdot \theta + \underbrace{E(X^2) - E(Y)}_{c=E(X^2)-E(Y)} = 0$$

$$\theta_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a c}}{2a}$$

2 $D > 0$

1 $D = 0$

X $D < 0$

$$D = b^2 - 4 \cdot a \cdot c =$$

$$= 4 \cdot (E(X))^2 - 4 \cdot 1 \cdot (E(X^2) - E(Y))$$

$$= 4 \cdot \left[\underbrace{(E(X))^2 - E(X^2)}_{= -Var(X)} + E(Y) \right]$$

$$= 4 \cdot \left[E(Y) - Var(X) \right]$$

thus : if $E(Y) = Var(X)$ θ is point identified

if $E(Y) > Var(X)$ θ is set identified

if $E(Y) < Var(X)$ θ is not identified

$$E(Y) = \underbrace{E((X-\theta)^2)}$$

this quantity is minimized for $\theta = E(X)$
and it is equal to $Var(X)$

if $E(Y) < Var(X) = \min_{\theta} E((X-\theta)^2)$
we have no solution

$$E(Z.\varepsilon) = E(Z(Y - (X-\theta)^2)) =$$

$$= E(Z.Y) - E(Z.X^2) + 2\underbrace{E(Z.X)}_{\neq 0}\theta - \underbrace{E(Z)}_{=0}.\theta^2 = 0$$

$\Rightarrow$ (Y, X, Z)

$$\theta = \frac{E(Z.X^2) - E(Z.Y)}{2.E(Z.X)}$$

$$Y \sim \text{Pois}(\lambda) \quad \Rightarrow \quad P(Y=k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

$i:$ $\quad y_i, X_i \quad\quad 1, x_{i1}, \dots, x_{ip}$

$$L(\beta | y_i, X_i) = \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!} \Rightarrow L_n(\beta | y, X) = \prod_{i=1}^{n} \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!}$$

$$l_n(\beta | y, X) = \sum_{i=1}^{n} y_i \log(\lambda_i) - \lambda_i - \log(y_i!)$$

$$= \sum_{i=1}^{n} \left(y_i (\beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip}) - e^{\beta_0 + \dots + \beta_p x_{ip}} - \log(y_i!) \right)$$

$$\frac{\partial}{\partial \beta_k} l_n(\beta | y, X) = \sum_{i=1}^{n} \left(y_i x_{ik} - e^{\beta_0 + \dots + \beta_p x_{ip}} \cdot x_{ik} \right)$$

$$= \sum_{i=1}^{n} x_{ik} \left(y_i - e^{\beta_0 + \dots + \beta_p x_{ip}} \right)$$

IME

$$\frac{\partial^2}{\partial \beta_k \partial \beta_l} l_n(\beta | y, X) = - \sum_{i=1}^{n} x_{ik} x_{il} e^{\beta_0 + \dots + \beta_p x_{ip}}$$

$$\hat{\beta}_{ML} \overset{as}{\sim} N\left(\beta, \underbrace{\qquad\qquad}^{-1}\right)$$

$$\underbrace{\underbrace{\begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_{11} & x_{21} & \cdots & x_{n1} \\ & & \vdots & \\ x_{1p} & x_{2p} & \cdots & x_{np} \end{pmatrix}}_{X^T} \cdot \begin{pmatrix} e^{\beta_0+\dots+\beta_p x_{1p}} & & \\ & \ddots & \\ & & e^{\beta_0+\dots+\beta_p x_{np}} \end{pmatrix} \underbrace{\begin{pmatrix} 1 & x_{11} & x_{1p} \\ \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{np} \end{pmatrix}}_{X}}$$

$$\hat{\beta} \sim N\left(\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}, \underbrace{\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}}^{-1}\right) \qquad \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$$

$$\hat{\beta}_1 \sim N(\beta_1, \quad)$$