CHAPTER 8

RESIDUAL INCOME VALUATION

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LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- calculate and interpret residual income, economic value added, and market value added;
- describe the uses of residual income models;
- calculate the intrinsic value of a common stock using the residual income model and compare value recognition in residual income and other present value models;
- explain fundamental determinants of residual income;
- explain the relation between residual income valuation and the justified price-to-book ratio based on forecasted fundamentals;
- calculate and interpret the intrinsic value of a common stock using single-stage (constant-growth) and multistage residual income models;
- calculate the implied growth rate in residual income, given the market price-to-book ratio and an estimate of the required rate of return on equity;
- explain continuing residual income and justify an estimate of continuing residual income at the forecast horizon, given company and industry prospects;
- compare residual income models to dividend discount and free cash flow models;
- explain strengths and weaknesses of residual income models and justify the selection of a residual income model to value a company's common stock;
- describe accounting issues in applying residual income models;
- evaluate whether a stock is overvalued, fairly valued, or undervalued based on a residual income model.

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1. INTRODUCTION

Residual income models of equity value have become widely recognized tools in both investment practice and research. Conceptually, residual income is net income less a charge (deduction) for common shareholders' opportunity cost in generating net income. It is the residual or remaining income after considering the costs of all of a company's capital. The appeal of residual income models stems from a shortcoming of traditional accounting. Specifically, although a company's income statement includes a charge for the cost of debt capital in the form of interest expense, it does not include a charge for the cost of equity capital. A company can have positive net income but may still not be adding value for shareholders if it does not earn more than its cost of equity capital. Residual income models explicitly recognize the costs of all the capital used in generating income.

As an economic concept, residual income has a long history, dating back to Alfred Marshall in the late 1800s.¹ As far back as the 1920s, General Motors used the concept in evaluating business segments.² More recently, residual income has received renewed attention and interest, sometimes under names such as economic profit, abnormal earnings, or economic value added. Although residual income concepts have been used in a variety of contexts, including the measurement of internal corporate performance, this reading will focus on the residual income model for estimating the intrinsic value of common stock. Among the questions we will study to help us apply residual income models are the following:

- How is residual income measured, and how can an analyst use residual income in valuation?
- How does residual income relate to fundamentals, such as return on equity and earnings growth rates?
- How is residual income linked to other valuation methods, such as a price-multiple approach?
- What accounting-based challenges arise in applying residual income valuation?

The reading is organized as follows: Section 2 develops the concept of residual income, introduces the use of residual income in valuation, and briefly presents alternative measures used in practice. Section 3 presents the residual income model and illustrates its use in valuing common stock. This section also shows practical applications, including the single-stage (constant-growth) residual income model and a multistage residual income model. Section 4 describes the relative strengths and weaknesses of residual income valuation compared to other valuation methods. Section 5 addresses accounting issues in the use of residual income valuation. The final section summarizes the reading and practice problems conclude.

2. RESIDUAL INCOME

Traditional financial statements, particularly the income statement, are prepared to reflect earnings available to owners. As a result, the income statement shows net income after deducting an expense for the cost of debt capital, that is, interest expense. The income statement does

¹Alfred Marshall, Book Two: Some Fundamental Notions, Chapter 4, "Income, Capital," in *Principles of Economics* (London; Macmillan and Co., Ltd., 1890).

 $^{^{2}}$ See, for example, Young (1999) and Lo and Lys (2000).

not, however, deduct dividends or other charges for equity capital. Thus, traditional financial statements essentially let the owners decide whether earnings cover their opportunity costs. Conversely, the economic concept of residual income explicitly deducts the estimated cost of equity capital, the finance concept that measures shareholders' opportunity costs. The cost of equity is the marginal cost of equity, which is also referred to as the required rate of return on equity. The cost of equity is a marginal cost because it represents the cost of additional equity, whether generated internally or by selling more equity interests. Example 1 illustrates, in a stylized setting, the calculation and interpretation of residual income.³

EXAMPLE 1 Calculation of Residual Income

Axis Manufacturing Company, Inc. (AXCI), a very small company in terms of market capitalization, has total assets of $\notin 2$ million financed 50 percent with debt and 50 percent with equity capital. The cost of debt is 7 percent before taxes; this example assumes that interest is tax deductible, so the after-tax cost of debt is 4.9 percent.⁴ The cost of equity capital is 12 percent. The company has earnings before interest and taxes (EBIT) of $\notin 200,000$ and a tax rate of 30 percent. Net income for AXCI can be determined as follows:

EBIT	€200,000
Less: Interest Expense	70,000
Pretax Income	€130,000
Less: Income Tax Expense	39,000
Net Income	€91,000

With earnings of €91,000, AXCI is clearly profitable in an accounting sense. But was the company's profitability adequate return for its owners? Unfortunately, it was not. To incorporate the cost of equity capital, compute residual income. One approach to calculating residual income is to deduct an **equity charge** (the estimated cost of equity capital in money terms) from net income. Compute the equity charge as follows:

Equity charge = Equity capital × Cost of equity capital = $\notin 1,000,000 \times 12\%$ = $\notin 120,000$.

³To simplify this introduction, we assume that net income accurately reflects *clean surplus accounting*, which will be explained later in this reading. The discussions in this reading assume that companies' financing only consists of common equity and debt. In the case of a company that also has preferred stock financing, the calculation of residual income would reflect the deduction of preferred stock dividends from net income.

⁴In countries where corporate interest is not tax deductible, the after-tax cost of debt would equal the pretax cost of debt.

As stated, residual income is equal to net income minus the equity charge:

Net Income	€91,000
Less: Equity Charge	120,000
Residual Income	€(29,000)

AXCI did not earn enough to cover the cost of equity capital. As a result, it has negative residual income. Although AXCI is profitable in an accounting sense, it is not profitable in an economic sense.

In Example 1, residual income is calculated based on net income and a charge for the cost of equity capital. Analysts will also encounter another approach to calculating residual income that yields the same results under certain assumptions. In this second approach, which takes the perspective of all providers of capital (both debt and equity), a **capital charge** (the company's total cost of capital in money terms) is subtracted from the company's after-tax operating profit. In the case of AXCI in Example 1, the capital charge is €169,000:

Equity charge	0.12 × €1,000,000 =	€120,000
Debt charge	0.07(1 - 0.30) × €1,000,000 =	49,000
Total capital charge		€169,000

The company's net operating profit after taxes (NOPAT) is $\notin 140,000 \ (\notin 200,000 - 30\%)$ taxes). The capital charge of $\notin 169,000$ is higher than the after-tax operating profit of $\notin 140,000$ by $\notin 29,000$, the same figure obtained in Example 1.

As illustrated in the following table, both approaches yield the same results in this case because of two assumptions. First, this example assumes that the marginal cost of debt equals the current cost of debt, that is, the cost used to determine net income. Specifically, in this instance, the after-tax interest expense incorporated in net income [\notin 49,000 = \notin 70,000 × (1 – 30%)] is equal to the after-tax cost of debt incorporated into the capital charge. Second, this example assumes that the weights used to calculate the capital charge are derived from the book value of debt and equity. Specifically, it uses the weights of 50 percent debt and 50 percent equity.

Approach 1		Reconciliation	Approac	n 2
Net income	€91,000	Plus the after-tax interest expense of €49,000	Net operating profit after tax	€140,000
Less: Equity charge	120,000	Plus the after-tax capital charge for debt of €49,000	Less: Capital charge	169,000
Residual income	€(29,000)		Residual income	€(29,000)

That the company is not profitable in an economic sense can also be seen by comparing the company's cost of capital to its return on capital. Specifically, the company's capital charge is greater than its after-tax return on total assets or capital. The after-tax net operating return on total assets or capital is calculated as profits divided by total assets (or total capital). In this example, the after-tax net operating return on total assets is 7 percent ($\leq 140,000/\leq 2,000,000$), which is 1.45 percentage points less than the company's effective capital charge of 8.45 percent ($\leq 169,000/\leq 2,000,000$).⁵

2.1. The Use of Residual Income in Equity Valuation

A company that is generating more income than its cost of obtaining capital—that is, one with positive residual income—is creating value. Conversely, a company that is not generating enough income to cover its cost of capital—that is, a company with negative residual income—is destroying value. Thus, all else equal, higher (lower) residual income should be associated with higher (lower) valuations.

To illustrate the effect of residual income on equity valuation using the case of AXCI presented in Example 1, assume the following:

- Initially, AXCI equity is selling for book value or €1 million with 100,000 shares outstanding. Thus, AXCI's book value per share and initial share price are both €10.
- Earnings per share (EPS) is €0.91 (€91,000/100,000 shares).
- Earnings will continue at the current level indefinitely.
- All net income is distributed as dividends.

Because AXCI is not earning its cost of equity, as shown in Example 1, the company's share price should fall. Given the information, AXCI is destroying \notin 29,000 of value per year, which equals \notin 0.29 per share (\notin 29,000/100,000 shares). Discounted at 12 percent cost of equity, the present value of the perpetuity is \notin 2.42 (\notin 0.29/12%). The current share price minus the present value of the value being destroyed equals \notin 7.58 (\notin 10 – \notin 2.42).

Another way to look at these data is to note that the earnings yield (E/P) for a no-growth company is an estimate of the expected rate of return. Therefore, when price reaches the point at which E/P equals the required rate of return on equity, an investment in the stock is expected to just cover the stock's required rate of return. With EPS of €0.91, the earnings yield is exactly 12 percent (AXCI's cost of equity) when its share price is €7.58333 (i.e., €0.91/€7.58333 = 12%). At a share price of €7.58333, the total market value of AXCI's equity is €758,333. When a company has negative residual income, shares are expected to sell at a discount to book value. In this example, AXCI's price-to-book ratio (P/B) at this level of discount from book value would be 0.7583. In contrast, if AXCI were earning positive residual income, then its shares should sell at a premium to book value. In summary, higher residual income is expected to be associated with higher market prices (and higher P/Bs), all else being equal.

Residual income (RI) models have been used to value both individual stocks⁶ and the Dow Jones Industrial Average.⁷ The models have also been proposed as a solution to measuring goodwill impairment by accounting standard setters (American Accounting Association

⁵ After-tax net operating profits as a percent of total assets or capital has been called **return on invested capital** (ROIC). Residual income can also be calculated as (ROIC – Effective capital charge) × Beginning capital.

⁶See Fleck, Craig, Bodenstab, Harris, and Huh (2001).

⁷See Lee and Swaminathan (1999) and Lee, Myers, and Swaminathan (1999).

Financial Accounting Standards Committee 2001). Recall that **impairment** in an accounting context means downward adjustment, and **goodwill** is an intangible asset that may appear on a company's balance sheet as a result of its purchase of another company.

Residual income and residual income models have been referred to by a variety of names. Residual income has sometimes been called **economic profit** because it is an estimate of the profit of the company after deducting the cost of all capital: debt and equity. In forecasting future residual income, the term **abnormal earnings** is also used. Under the assumption that in the long term the company is expected to earn its cost of capital (from all sources), any earnings in excess of the cost of capital can be termed abnormal earnings model and the **Edwards–Bell–Ohlson model** after the names of researchers in the field. This reading focuses on a general residual income model that can be used by analysts using publicly available data and nonproprietary accounting adjustments. A number of commercial implementations of the approach, however, are also very well known. Before returning to the general residual income model in Section 3, we briefly discuss one such commercial implementation and the related concept of market value added.

2.2. Commercial Implementations

One example of several competing commercial implementations of the residual income concept is **economic value added** (EVA).⁸ The previous section illustrated a calculation of residual income starting from net operating profit after taxes, and economic value added takes the same broad approach. Specifically, economic value added is computed as

$$EVA = NOPAT - (C\% \times TC)$$
(1)

where NOPAT is the company's net operating profit after taxes, C% is the cost of capital, and TC is total capital. In this model, both NOPAT and TC are determined under generally accepted accounting principles and adjusted for a number of items.⁹ Some of the more common adjustments include the following:

- Research and development (R&D) expenses are capitalized and amortized rather than expensed (i.e., R&D expense, net of estimated amortization, is added back to earnings to compute NOPAT).
- In the case of strategic investments that are not expected to generate an immediate return, a charge for capital is suspended until a later date.
- Deferred taxes are eliminated such that only cash taxes are treated as an expense.
- Any inventory LIFO (last in, first out) reserve is added back to capital, and any increase in the LIFO reserve is added in when calculating NOPAT.
- Operating leases are treated as capital leases, and nonrecurring items are adjusted.

⁸The acronym is trademarked by Stern Stewart & Company and is generally associated with a specific set of adjustments proposed by Stern Stewart & Co. The goal of these adjustments is to produce a value that is a good approximation of economic profit. For a complete discussion, see Stewart (1991) and Peterson and Peterson (1996).

⁹See, for example, Ehrbar (1998).

Because of the adjustments made in calculating EVA, a different numerical result will be obtained, in general, than that resulting from the use of the simple computation presented in Example 1. In practice, general (nonbranded) residual income valuation also considers the effect of accounting methods on reported results. Analysts' adjustments to reported accounting results in estimating residual income, however, will generally reflect some differences from the set specified for EVA. Section 5 of this reading will explore accounting considerations in more detail.

Over time, a company must generate economic profit for its market value to increase. A concept related to economic profit (and EVA) is market value added (MVA):

A company that generates positive economic profit should have a market value in excess of the accounting book value of its capital.

Research on the ability of value-added concepts to explain equity value and stock returns has reached mixed conclusions. Peterson and Peterson (1996) found that value-added measures are slightly more highly correlated with stock returns than traditional measures, such as return on assets and return on equity. Bernstein and Pigler (1997) and Bernstein, Bayer, and Pigler (1998) found that value-added measures are no better at predicting stock performance than are such measures as earnings growth.

A variety of commercial models related to the residual income concept have been marketed by other major accounting and consulting firms. Interestingly, the application focus of these models is not, in general, equity valuation. Rather, these implementations of the residual income concept are marketed primarily for measuring internal corporate performance and determining executive compensation.

3. THE RESIDUAL INCOME MODEL

In Section 2, we discussed the concept of residual income and briefly introduced the relationship of residual income to equity value. In the long term, companies that earn more than the cost of capital should sell for more than book value, and companies that earn less than the cost of capital should sell for less than book value. The **residual income model** of valuation analyzes the intrinsic value of equity as the sum of two components:

- the current book value of equity, and
- the present value of expected future residual income.

Note that when the change is made from valuing total shareholders' equity to directly valuing an individual common share, earnings per share rather than net income is used. According to the residual income model, the intrinsic value of common stock can be expressed as follows:

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{\mathrm{RI}_t}{(1+r)^t} = B_0 + \sum_{t=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}$$
(3)

where

 $V_0 = \text{value of a share of stock today } (t = 0)$ $B_0 = \text{current per-share book value of equity}$ $B_t = \text{expected per-share book value of equity at any time t}$ r = required rate of return on equity investment (cost of equity) $E_t = \text{expected EPS for period t}$ $\text{RI}_t = \text{expected per-share residual income, equal to } E_t - rB_{t-1}$

The per-share residual income in period t, RI_p is the EPS for the period, E_p minus the pershare equity charge for the period, which is the required rate of return on equity times the book value per share at the beginning of the period, or rB_{t-1} . Whenever earnings per share exceed the per-share cost of equity, per-share residual income is positive; and whenever earnings are less, per-share residual income is negative. Example 2 illustrates the calculation of per-share residual income.

EXAMPLE 2 Per-Share Residual Income Forecasts

David Smith is evaluating the expected residual income as of the end of August 2013 of Silver Wheaton Corporation (NYSE: SLW). Established in 2004 in Vancouver, British Columbia, Silver Wheaton is the largest precious metal streaming company in the world. The company has a number of agreements whereby, in exchange for an upfront payment, it has the right to purchase all or a portion of the silver (and sometimes gold) production from mines located around the globe. Using an adjusted beta of 1.50 relative to the TSX 300 Index, a 10-year government bond yield of 2.8 percent, and an estimated equity risk premium of 4.2 percent, Smith uses the capital asset pricing model (CAPM) to estimate Silver Wheaton's required rate of return, r, at 9.1 percent [2.8 percent + 1.50(4.2 percent)]. Smith obtains the following as of the close on 23 August 2013:

Current market price	\$27.70			
Book value per share as of 31 December 2012				
Consensus annual earnings estimates				
FY 2013 (ending December)	\$1.40			
FY 2014	\$1.60			
Annualized dividend per share forecast				
FY 2013	\$0.52			
FY 2014	\$0.60			

What is the forecast residual income for fiscal years ended December 2013 and December 2014?

Solution: Forecasted residual income and calculations are shown in Exhibit 1.

Year	<i>I</i> ear 2013		20	2014	
Forecasting book value per share					
Beginning book value (B_{t-1})		\$8.77		\$9.65	
Earnings per share forecast (E_t)	\$1.40		\$1.60		
Less dividend forecast (D_t)	0.52		0.60		
Add Change in retained earnings $(E_t - D_t)$		0.88		1.00	
Forecast ending book value per share $(B_{t-1} + E_t - D_t)$		\$9.65		\$10.65	
Calculating the equity charge					
Beginning book value per share	\$8.77		\$9.65		
Multiply cost of equity	× 0.091		$\times 0.091$		
Per-share equity charge $(r \times B_{t-1})$		\$0.80		\$0.88	
Estimating per share residual income					
EPS forecast	\$1.40		\$1.60		
Less equity charge	0.80		0.88		
Per-share residual income		\$0.60		\$0.72	

EXHIBIT 1 Silver Wheaton Corporation

The use of Equation 3, the expression for the estimated intrinsic value of common stock, is illustrated in Example 3.

EXAMPLE 3 Using the Residual Income Model (1)

Bugg Properties' expected EPS is \$2.00, \$2.50, and \$4.00 for the next three years. Analysts expect that Bugg will pay dividends of \$1.00, \$1.25, and \$12.25 for the three years. The last dividend is anticipated to be a liquidating dividend; analysts expect Bugg will cease operations after Year 3. Bugg's current book value is \$6.00 per share, and its required rate of return on equity is 10 percent.

- 1. Calculate per-share book value and residual income for the next three years.
- 2. Estimate the stock's value using the residual income model given in Equation 3.

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}$$

3. Confirm your valuation estimate in Part 2 using the discounted dividend approach (i.e., estimating the value of a share as the present value of expected future dividends).

Solution to 1: The book value and residual income for the next three years are shown in Exhibit 2.

EXF	HB	IΤ	2

Year	1	2	3
Beginning book value per share (<i>B</i> _{<i>t</i>-1})	\$6.00	\$7.00	\$8.25
Net income per share (EPS)	2.00	2.50	4.00
Less dividends per share (D)	1.00	1.25	12.25
Change in retained earnings (EPS – D)	1.00	1.25	-8.25
Ending book value per share $(B_{t-1} + EPS - D)$	\$7.00	\$8.25	\$0.00
Net income per share (EPS)	2.00	2.50	4.000
Less per-share equity charge (rB_{t-1})	0.60	0.70	0.825
Residual income (EPS – Equity charge)	\$1.40	\$1.80	\$3.175

Solution to 2: The value using the residual income model is

V = 6.00	1.40	1.80	3.175
$v_0 = 0.00 -$	$(1.10)^{-1}$	$(1.10)^2$	$+\frac{1}{(1.10)^3}$
= 6.00 -	+1.2727 -	+1.4876-	+ 2.3854
= \$11.1	5		

Solution to 3: The value using a discounted dividend approach is

$$V_0 = \frac{1.00}{(1.10)} + \frac{1.25}{(1.10)^2} + \frac{12.25}{(1.10)^3}$$
$$= 0.9091 + 1.0331 + 9.2036$$
$$= \$11.15$$

Example 3 illustrates two important points about residual income models. First, the RI model is fundamentally similar to other valuation models, such as the dividend discount model (DDM), and given consistent assumptions will yield equivalent results. Second, recognition of value typically occurs earlier in RI models than in DDM. In Example 3, the RI model attributes \$6.00 of the \$11.15 total value to the *first* time period. In contrast, the DDM model attributes \$9.2036 of the \$11.15 total value to the *final* time period. The rest of Section 3 develops the most familiar general expression for the RI model and illustrates the model's application.

3.1. The General Residual Income Model

The residual income model has a clear relationship to other valuation models, such as the dividend discount model. In fact, the residual income model given in Equation 3 can be derived from the DDM. The general expression for the DDM is

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \dots$$

The **clean surplus relation** states the relationship among earnings, dividends, and book value as follows:

$$B_t = B_{t-1} + E_t - D_t$$

In other words, the ending book value of equity equals the beginning book value plus earnings minus dividends, apart from ownership transactions. The condition that income (earnings) reflect all changes in the book value of equity other than ownership transactions is known as clean surplus accounting. By rearranging the clean surplus relation, the dividend for each period can be viewed as the net income minus the earnings retained for the period, or net income minus the increase in book value:

$$D_t = E_t - (B_t - B_{t-1}) = E_t + B_{t-1} - B_t$$

Substituting $E_t + B_{t-1} - B_t$ for D_t in the expression for V_0 results in:

$$V_0 = \frac{E_1 + B_0 - B_1}{(1+r)^1} + \frac{E_2 + B_1 - B_2}{(1+r)^2} + \frac{E_3 + B_2 - B_3}{(1+r)^3} + \dots$$

This equation can be rewritten as follows:

$$V_0 = B_0 + \frac{E_1 - rB_0}{(1+r)^1} + \frac{E_2 - rB_1}{(1+r)^2} + \frac{E_3 - rB_2}{(1+r)^3} + \dots$$

Expressed with summation notation, the following equation restates the residual income model given in Equation 3:

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{\mathrm{RI}_t}{(1+r)^t} = B_0 + \sum_{t=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}$$

According to the expression, the value of a stock equals its book value per share plus the present value of expected future per-share residual income. Note that when the present value of expected future per-share residual income is positive (negative), intrinsic value, V_0 , is greater (smaller) than book value per share, B_0 .

The residual income model used in practice today has its origins largely in the academic work of Ohlson (1995) and Feltham and Ohlson (1995) along with the earlier work of Edwards and Bell (1961), although in the United States this method has been used to value small businesses in tax cases since the 1920s.¹⁰ The general expression for the residual income model based on this work¹¹ can also be stated as:

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{(\text{ROE}_t - r)B_{t-1}}{(1+r)^t}$$
(4)

Equation 4 is equivalent to the expressions for V_0 given earlier because in any year, t, $RI_t = (ROE_t - r)B_{t-1}$. Other than the required rate of return on common stock, the inputs to the residual income model come from accounting data. Note that return on equity (ROE)

¹⁰In tax valuation, the method is known as the **excess earnings method**. For example, see Hitchner (2006) and US IRS Revenue Ruling 68-609.

¹¹See, for example, Hirst and Hopkins (2000).

in this context uses beginning book value of equity in the denominator, whereas in financial statement analysis ROE is frequently calculated using the average book value of equity in the denominator. Example 4 illustrates the estimation of value using Equation 4.

EXAMPLE 4 Using the Residual Income Model (2)

To recap the data from Example 3, Bugg Properties has expected earnings per share of \$2.00, \$2.50, and \$4.00 and expected dividends per share of \$1.00, \$1.25, and \$12.25 for the next three years. Analysts expect that the last dividend will be a liquidating dividend and that Bugg will cease operating after Year 3. Bugg's current book value per share is \$6.00, and its estimated required rate of return on equity is 10 percent.

Using the above data, estimate the value of Bugg Properties' stock using a residual income model of the form:

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{(\text{ROE}_t - r)B_{t-1}}{(1+r)^t}$$

Solution: To value the stock, forecast residual income. Exhibit 3 illustrates the calculation of residual income. (Note that Exhibit 3 arrives at the same estimates of residual income as Exhibit 2 in Example 3.)

EXHIBIT 3

Year	1	2	3
Earnings per share	\$2.00	\$2.50	\$4.00
Divided by beginning book value per share	÷ 6.00	÷7.00	÷ 8.25
ROE	0.3333	0.3571	0.4848
Less required rate of return on equity	-0.1000	-0.1000	- 0.1000
Abnormal rate of return (ROE $- r$)	0.2333	0.2571	0.3848
Multiply by beginning book value per share	× 6.00	$\times 7.00$	× 8.25
Residual income (ROE – r) × Beginning BV	\$1.400	\$1.800	\$3.175

Estimate the stock value as follows:

$$V_0 = 6.00 + \frac{1.40}{(1.10)} + \frac{1.80}{(1.10)^2} + \frac{3.175}{(1.10)^3}$$

= 6.00 + 1.2727 + 1.4876 + 2.3854
= \$11.15

Note that the value is identical to the estimate obtained using Equation 3, as illustrated in Example 3, because the assumptions are the same and Equations 3 and 4 are equivalent expressions:

$$V_0 = \frac{B_0 + \sum_{t=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}}{\text{Equation } 3} = \frac{B_0 + \sum_{t=1}^{\infty} \frac{(ROE_t - r)B_{t-1}}{(1+r)^t}}{\text{Equation } 4}$$

Example 4 showed that residual income value can be estimated using current book value, forecasts of earnings, forecasts of book value, and an estimate of the required rate of return on equity. The forecasts of earnings and book value translate into ROE forecasts.

EXAMPLE 5 Valuing a Company Using the General Residual Income Model

Robert Sumargo, an equity analyst, is considering the valuation of Google (NDQ:-GOOG), in late 2013 when a recent closing price is \$896.57. Sumargo notes that in general GOOG had a fairly high ROE during the past 10 years and that consensus analyst forecasts for EPS for the next two fiscal years reflect an expected ROE of around 19 percent. Sumargo expects that a high ROE may not be sustainable in the future. Sumargo usually takes a present value approach to valuation. As of the date of the valuation, GOOG does not pay dividends; although a discounted dividend valuation is possible, Sumargo does not feel confident about predicting the date of a dividend initiation. He decides to apply the residual income model to value GOOG, and uses the following data and assumptions:

- According to the CAPM, GOOG has a required rate of return of approximately 8.5 percent.
- GOOG's book value per share on 31 December 2012 was \$217.54.
- ROE is expected to be 21 percent for 2013. Because of competitive pressures, Sumargo expects GOOG's ROE to decline in the following years and incorporates an assumed decline of 0.5 percent each year until it reaches the CAPM required rate of return.
- GOOG does not currently pay a dividend. Sumargo does not expect the company to pay a dividend in the foreseeable future, so all earnings will be reinvested. In addition, Sumargo expects that share repurchases will approximately offset new share issuances.

Compute the value of GOOG using the residual income model (Equation 4).

Solution: Book value per share is initially \$217.54. Based on a ROE forecast of 21 percent in the first year, the forecast EPS would be \$45.68. Because no dividends are paid and the clean surplus relation is assumed to hold, book value at the end of the period is forecast to be \$263.22 (\$217.54 + \$45.68). For 2013, residual income is measured as projected EPS of \$45.68 minus an equity charge of \$18.49, or \$27.19. This is equivalent to the beginning book value per share of \$217.54 times the difference between ROE of 21 percent and r of 8.5 percent [i.e., \$217.54(0.21 - 0.085) = \$27.19]. The present value of \$27.19 at 8.5 percent for one year is \$25.06. This process is continued year by year as presented in Exhibit 4. The value of GOOG using this residual income model would be the present value of each year's residual income plus the current book value per share. Because residual income is zero starting in 2038, no forecast is required beyond that period. The estimated value under this model is \$920.24, as shown in Exhibit 4.

				Forecast				
Year	Projected Income EPS	Projected Dividend per Share	Book Value per Share	ROE (Based on Beginning Book Value)	Cost of Equity	Equity Charge	Residual Income (RI)	PV of BV and RI
	[Plus]	[Minus]	\$217.54					217.54
2013	\$45.68	\$0.00	263.22	21.0%	8.5%	18.49	27.19	25.06
2014	53.96	0.00	317.18	20.5	8.5	22.37	31.59	26.83
2015	63.44	0.00	380.62	20.0	8.5	26.96	36.48	28.56
2016	74.22	0.00	454.84	19.5	8.5	32.35	41.87	30.21
2017	86.42	0.00	541.26	19.0	8.5	38.66	47.76	31.76
2018	100.13	0.00	641.40	18.5	8.5	46.01	54.13	33.18
2019	115.45	0.00	756.85	18.0	8.5	54.52	60.93	34.42
2020	132.45	0.00	889.30	17.5	8.5	64.33	68.12	35.47
2021	151.18	0.00	1,040.48	17.0	8.5	75.59	75.59	36.27
2022	171.68	0.00	1,212.15	16.5	8.5	88.44	83.24	36.81
2023	193.94	0.00	1,406.10	16.0	8.5	103.03	90.91	37.06
2024	217.95	0.00	1,624.04	15.5	8.5	119.52	98.43	36.98
2025	243.61	0.00	1,867.65	15.0	8.5	138.04	105.56	36.55
2026	270.81	0.00	2,138.46	14.5	8.5	158.75	112.06	35.76
2027	299.38	0.00	2,437.84	14.0	8.5	181.77	117.62	34.60
2028	329.11	0.00	2,766.95	13.5	8.5	207.22	121.89	33.04
2029	359.70	0.00	3,126.66	13.0	8.5	235.19	124.51	31.11
2030	390.83	0.00	3,517.49	12.5	8.5	265.77	125.07	28.80
2031	422.10	0.00	3,939.59	12.0	8.5	298.99	123.11	26.13
2032	453.05	0.00	4,392.64	11.5	8.5	334.86	118.19	23.12
2033	483.19	0.00	4,875.83	11.0	8.5	373.37	109.82	19.80
2034	511.96	0.00	5,387.79	10.5	8.5	414.45	97.52	16.20
2035	538.78	0.00	5,926.57	10.0	8.5	457.96	80.82	12.38
2036	563.02	0.00	6,489.60	9.5	8.5	503.76	59.27	8.37
2037	584.06	0.00	7,073.66	9.0	8.5	551.62	32.45	4.22
2038	601.26	0.00	7,674.92	8.5	8.5	601.26	0.00	0.00
Total							\$920.24	

EXHIBIT 4 Valuation of GOOG Using the Residual Income Model

Note: PV is present value, and BV is book value. This table was created in Excel, so numbers may differ from what will be obtained using a calculator, because of rounding.

Example 5 refers to the assumption of clean surplus accounting. The residual income model, as stated earlier, assumes clean surplus accounting. The clean surplus accounting assumption is illustrated in Exhibit 4, for example, in which ending book value per share is computed as beginning book value plus net income minus dividends. Under International Financial Reporting Standards (IFRS) and US generally accepted accounting principles (US GAAP), several items of income and expense occurring during a period, such as changes in the market value of certain securities, bypass the income statement and affect a company's book value of equity directly.¹² Strictly speaking, residual income models involve all items of income and expense (income under clean surplus accounting). If an analyst can reliably estimate material differences from clean surplus accounting expected in the future, an adjustment to net income may be appropriate. Section 5.1 explores violations of the clean surplus accounting assumption in more detail.

3.2. Fundamental Determinants of Residual Income

In general, the residual income model makes no assumptions about future earnings and dividend growth. If constant earnings and dividend growth are assumed, a version of the residual income model that usefully illustrates the fundamental drivers of residual income can be derived. The following expression is used for justified price-to-book ratio (P/B) based on forecasted fundamentals, assuming the Gordon (constant growth) DDM and the sustainable growth rate equation, $g = b \times \text{ROE}$:¹³

$$\frac{P_0}{B_0} = \frac{\text{ROE} - g}{r - g}$$

which is mathematically equivalent to:

$$\frac{P_0}{B_0} = 1 + \frac{\text{ROE} - r}{r - g}$$

The justified price is the stock's intrinsic value ($P_0 = V_0$). Therefore, using the previous equation and remembering that residual income is earnings less the cost of equity, or (ROE × B_0) – ($r \times B_0$), a stock's intrinsic value under the residual income model, assuming constant growth, can be expressed as:

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0 \tag{5}$$

Under this model, the estimated value of a share is the book value per share (B_0) plus the present value $[(ROE - r)B_0/(r - g)]$ of the expected stream of residual income. In the case of a company for which ROE exactly equals the cost of equity, the intrinsic value is equal to the book value per share. Equation 5 is considered a single-stage (or constant-growth) residual income model.

In an idealized world, where the book value of equity represents the fair value of net assets and clean surplus accounting prevails, the term B_0 reflects the value of assets owned by the

 $^{^{12}}$ Items that bypass the income statement (dirty surplus items) are referred to as **other comprehensive income** (OCI). The relationship is Comprehensive income = Net income + Other comprehensive income.

¹³Note that the sustainable growth rate formula itself can be derived from the clean surplus relation.

company less its liabilities. The second term, $(ROE - r)B_0/(r - g)$, represents additional value expected because of the company's ability to generate returns in excess of its cost of equity; the second term is the present value of the company's expected economic profits. However, both IFRS and US GAAP allow companies to exclude some liabilities from their balance sheets, and neither set of rules reflects the fair value of many corporate assets. Internationally, however, a move toward fair value accounting is occurring, particularly for financial assets. Further, controversies, such as the failure of Enron Corporation in the United States, have highlighted the importance of identifying off-balance-sheet financing techniques.

The residual income model is most closely related to the P/B ratio. A stock's justified P/B ratio is directly related to expected future residual income. Another closely related concept is **Tobin's** q, the ratio of the market value of debt and equity to the replacement cost of total assets:¹⁴

Tobin's
$$q = \frac{\text{Market value of debt and equity}}{\text{Replacement cost of total assets}}$$

Although similar to P/B, Tobin's q also has some obvious differences. The numerator includes the market value of total capital (debt as well as equity). The denominator uses total assets rather than equity. Further, assets are valued at replacement cost rather than at historical accounting cost; replacement costs take into account the effects of inflation. All else equal, Tobin's q is expected to be higher the greater the productivity of a company's assets.¹⁵ One difficulty in computing Tobin's q is the lack of information on the replacement cost of assets. If available, market values of assets or replacement costs can be more useful in a valuation than historical costs.

3.3. Single-Stage Residual Income Valuation

The single-stage (constant-growth) residual income model assumes that a company has a constant return on equity and constant earnings growth rate through time. This model was given in Equation 5:

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0$$

EXAMPLE 6 Single-Stage Residual Income Model (1)

Joseph Yoh is evaluating a purchase of Canon, Inc. (NYSE: CAJ). Current book value per share is \$26.24, and the current price per share is \$34.68 (from Value Line, 26 July 2013). Yoh expects long-term ROE to be 11 percent and long-term growth to be 5.5 percent. Assuming a cost of equity of 9.5 percent, what is the intrinsic value of Canon stock calculated using a single-stage residual income model?

¹⁴See Tobin (1969) or more recent work such as Landsman and Shapiro (1995).

¹⁵Tobin theorized that q would average to 1 for all companies because the economic rents or profits earned by assets would average to zero.

Solution:

$$V_0 = \$26.24 + \frac{0.11 - 0.095}{0.095 - 0.55} \$26.24$$

= \\$36.08

Similar to the Gordon growth DDM, the single-stage RI model can be used to assess the market expectations of residual income growth—that is, an implied growth rate—by inputting the current price into the model and solving for *g*.

EXAMPLE 7 Single-Stage Residual Income Model (2)

Joseph Yoh is curious about the market-perceived growth rate, given that he is comfortable with his other inputs. By using the current price per share of 34.68 for Canon, Yoh solves the following equation for *g*:

$$34.68 = 26.24 + \frac{0.11 - 0.095}{0.095 - g} 26.24$$

He finds an implied growth rate of 4.84 percent.

In Examples 6 and 7, the company was valued at more than 1.3 times its book value because its ROE exceeded its cost of equity. If ROE was equal to the cost of equity, the company would be valued at book value. If ROE was lower than the cost of equity, the company would have negative residual income and be valued at less than book value. (When a company has no prospect of being able to cover its cost of capital, a liquidation of the company and redeployment of assets may be appropriate.)

In many applications, a drawback to the single-stage model is that it assumes the excess ROE above the cost of equity will persist indefinitely. More likely, a company's ROE will revert to a mean value of ROE over time, and at some point, the company's residual income will be zero. If a company or industry has an abnormally high ROE, other companies will enter the marketplace, thus increasing competition and lowering returns for all companies. Similarly, if an industry has a low ROE, companies will exit the industry (through bankruptcy or otherwise), and ROE will tend to rise over time. As with the single-stage DDM, the single-stage residual income model also assumes a constant growth rate through time. In light of these considerations, the residual income model has been adapted in practice to handle declining residual income. For example, Lee and Swaminathan (1999) and Lee, Myers, and Swaminathan (1999) used a residual income model to value the Dow 30 by assuming that ROE fades (reverts) to the industry mean over time. Lee and Swaminathan found that the residual income model had more ability than traditional price multiples to predict future returns. Fortunately, other models are available that enable analysts to relax the assumption of indefinite persistence of excess returns. The following section describes a multistage residual income model.

3.4. Multistage Residual Income Valuation

As with other valuation approaches, such as DDM and free cash flow, a multistage residual income approach can be used to forecast residual income for a certain time horizon and then estimate a terminal value based on continuing residual income at the end of that time horizon. Continuing residual income is residual income after the forecast horizon. As with other valuation models, the forecast horizon for the initial stage should be based on the ability to explicitly forecast inputs in the model. Because ROE has been found to revert to mean levels over time and may decline to the cost of equity in a competitive environment, residual income approaches often model ROE fading toward the cost of equity. As ROE approaches the cost of equity, residual income approaches zero. An ROE equal to the cost of equity would result in residual income of zero.

In residual income valuation, the current book value often captures a large portion of total value, and the terminal value may not be a large component of total value because book value is larger than the periodic residual income and because ROE may fade over time toward the cost of equity. This contrasts with other multistage approaches (DDM and DCF), in which the present value of the terminal value is frequently a significant portion of total value.

Analysts make a variety of assumptions concerning continuing residual income. Frequently, one of the following assumptions is made:

- residual income continues indefinitely at a positive level;
- residual income is zero from the terminal year forward;
- residual income declines to zero as ROE reverts to the cost of equity through time; or
- residual income reflects the reversion of ROE to some mean level.

The following examples illustrate several of these assumptions.

One finite-horizon model of residual income valuation assumes that at the end of time horizon T, a certain premium over book value $(P_T - B_T)$ exists for the company, in which case, current value equals the following:¹⁶

$$V_0 = B_0 + \sum_{t=1}^{T} \frac{\left(E_t - rB_{t-1}\right)}{\left(1+r\right)^t} + \frac{P_T - B_T}{\left(1+r\right)^T}$$
(6)

Alternatively,

$$V_0 = B_0 + \sum_{t=1}^{T} \frac{(\text{ROE}_t - r)B_{t-1}}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$$
(7)

The last component in both specifications represents the premium over book value at the end of the forecast horizon. The longer the forecast period, the greater the chance that the company's residual income will converge to zero. For long forecast periods, this last term may be treated as zero. For shorter forecast periods, a forecast of the premium should be calculated.

¹⁶See Bauman (1999).

EXAMPLE 8 Multistage Residual Income Model (1)

Diana Rosato, CFA, is considering an investment in Taiwan Semiconductor Manufacturing Ltd., a manufacturer and marketer of integrated circuits. Listed on the Taiwan Stock Exchange (Code: 2330), the company's stock is also traded on the New York Stock Exchange (NYSE: TSM). Rosato obtained the following facts and estimates as of August 2013:

- Current price equals TWD95.6.
- Cost of equity equals 12 percent.
- Taiwan Semiconductor's ROE has ranged from 18 percent to 22.9 percent during the period 2008–2012. The only time ROE was below 20 percent during that time period was in 2009.
- In 2012 the company paid a cash dividend of TWD2.9995.
- Book value per share was TWD28.8517 at the end of 2012.
- Rosato's forecasts of EPS are TWD7.162 for 2013 and TWD8.356 for 2014. She expects dividends of TWD2.9995 for 2013 and TWD3.2995 for 2014.
- Rosato expects Taiwan Semiconductor's ROE to be 25 percent from 2015 through 2019 and then decline to 20 percent through 2032.
- For the period after 2014, Rosato assumes an earnings retention ratio of 60 percent.
- Rosato assumes that after 2032, ROE will be 12 percent and residual income will be zero; therefore, the terminal value would be zero. Rosato's residual income model is shown in Exhibit 5.

Year	Book Value (TWD)	Projected Income (TWD)	Dividend per Share (TWD)	Forecasted ROE (Beg. Equity, %)	COE (%)	COE (TWD)	Residual Income (TWD)	Present Value of Residual Income (TWD)
2012	28.8517							28.85
2013	33.0142	7.1620	2.9995	24.82	12.00	3.4622	3.6998	3.30
2014	38.0707	8.3560	3.2995	25.31	12.00	3.9617	4.3943	3.50
2015	43.7813	9.5177	3.8071	25.00	12.00	4.5685	4.9492	3.52
2016	50.3485	10.9453	4.3781	25.00	12.00	5.2538	5.6916	3.62
2017	57.9008	12.5871	5.0349	25.00	12.00	6.0418	6.5453	3.71
2018	66.5859	14.4752	5.7901	25.00	12.00	6.9481	7.5271	3.81
2019	76.5738	16.6465	6.6586	25.00	12.00	7.9903	8.6562	3.92
2020	85.7626	15.3148	6.1259	20.00	12.00	9.1889	6.1259	2.47
2021	96.0541	17.1525	6.8610	20.00	12.00	10.2915	6.8610	2.47
2022	107.5806	19.2108	7.6843	20.00	12.00	11.5265	7.6843	2.47
2023	120.4903	21.5161	8.6065	20.00	12.00	12.9097	8.6065	2.47
2024	134.9492	24.0981	9.6392	20.00	12.00	14.4588	9.6392	2.47
2025	151.1431	26.9898	10.7959	20.00	12.00	16.1939	10.7959	2.47
								(continued)

EXHIBIT 5 Taiwan Semiconductor

EXHIBIT 5 (Continued)	
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Year	Book Value (TWD)	Projected Income (TWD)	Dividend per Share (TWD)	Forecasted ROE (Beg. Equity, %)	COE (%)	COE (TWD)	Residual Income (TWD)	Present Value of Residual Income (TWD)
2026	169.2802	30.2286	12.0914	20.00	12.00	18.1372	12.0914	2.47
2027	189.5938	33.8560	13.5424	20.00	12.00	20.3136	13.5424	2.47
2028	212.3451	37.9188	15.1675	20.00	12.00	22.7513	15.1675	2.47
2029	237.8265	42.4690	16.9876	20.00	12.00	25.4814	16.9876	2.47
2030	266.3657	47.5653	19.0261	20.00	12.00	28.5392	19.0261	2.47
2031	298.3296	53.2731	21.3093	20.00	12.00	31.9639	21.3093	2.47
2032	334.1291	59.6659	23.8664	20.00	12.00	35.7996	23.8664	2.47
					1	Present val	ue TWD	86.41
Termi	inal Premiu	m = 0.00						

The market price of TWD95.6 exceeds the estimated value of TWD86.41. The market price reflects higher forecasts of residual income during the period to 2032, a higher terminal premium than Rosato forecasts, and/or a lower cost of equity. If Rosato is confident in her forecasts, she may conclude that the company is overvalued in the current marketplace.

Lee and Swaminathan (1999) and Lee, Myers, and Swaminathan (1999) have presented a residual income model based on explicit forecasts of residual income for three years. Thereafter, ROE is forecast to fade to the industry mean value of ROE. The terminal value at the end of the forecast horizon (T) is estimated as the terminal-year residual income discounted in perpetuity. Lee and Swaminathan stated that this assumes any growth in earnings after T is value neutral. Exhibit 6 presents sector ROE data from Hemscott Americas, retrieved from Yahoo. com. (ROE data for specific industries can be retrieved from the same source.) In forecasting a fading ROE, the analyst should also consider any trends in industry ROE.

EXHIBIT 6 US Sector ROEs

Sectors	ROE(%)
Basic Materials	11.96
Conglomerates	22.60
Consumer Goods	14.54
Financial	9.78
Healthcare	18.60
Industrial Goods	15.57
Services	17.69
Technology	16.47
Utilities	5.69

Source: Based on Hemscott Americas data retrieved from http://biz.yahoo.com on 28 August 2013.

EXAMPLE 9 Multistage Residual Income Model (2)

Rosato's supervisor questions her assumption that Taiwan Semiconductor will have no premium at the end of her forecast period. Rosato assesses the effect of a terminal value based on a perpetuity of Year 2032 residual income. She computes the following terminal value:

TV = TWD23.8664/0.12 = TWD198.8867

The present value of this terminal value is as follows:

PV = TWD198.8867/(1.12)²⁰ = TWD20.6179

Adding TWD20.6179 to the previous value of TWD86.41 (for which the terminal value was zero) yields a total value of TWD107.03. Because the current market price of TWD95.6 is less than TWD107.03, market participants expect a continuing residual income that is lower than her new assumptions and/or are forecasting a lower interim ROE. If Rosato agrees with her supervisor and is confident in her new forecasts, she may now conclude that the company is undervalued.

Another multistage model assumes that ROE fades over time to the cost of equity. In this approach, ROE can be explicitly forecast each period until reaching the cost of equity. The forecast would then end, and the terminal value would be zero.

Dechow, Hutton, and Sloan (1999) presented an analysis of a residual income model in which residual income fades over time:¹⁷

$$V_0 = B_0 + \sum_{t=1}^{T-1} \frac{\left(E_t - rB_{t-1}\right)}{\left(1+r\right)^t} + \frac{E_T - rB_{T-1}}{\left(1+r-\omega\right)\left(1+r\right)^{T-1}}$$
(8)

This model adds a persistence factor, ω , which is between zero and one. A persistence factor of one implies that residual income will not fade at all; rather it will continue at the same level indefinitely (i.e., in perpetuity). A persistence factor of zero implies that residual income will not continue after the initial forecast horizon. The higher the value of the persistence factor, the higher the stream of residual income in the final stage, and the higher the valuation, all else being equal. Dechow et al. found that in a large sample of company data from 1976 to 1995, the persistence factor equaled 0.62, which was interpreted by Bauman (1999) as equivalent to residual income decaying at an average rate of 38 percent a year. The persistence factor considers the long-run mean-reverting nature of ROE, assuming that in time ROE regresses toward *r* and that resulting residual income fades toward zero. Clearly, the persistence factor varies from company to company. For example, a company with a strong market leadership position would have a lower expected rate of decay (Bauman, 1999). Dechow et al. provided

¹⁷See Dechow, Hutton, and Sloan (1999) and Bauman (1999).

insight into some characteristics, listed in Exhibit 7, that can indicate a lower or higher level of persistence.

EXHIBIT 7	Final-Stage Residual	Income Persistence
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Lower Residual Income Persistence	Higher Residual Income Persistence
Extreme accounting rates of return (ROE)	Low dividend payout
Extreme levels of special items (e.g., nonrecurring items)	High historical persistence in the industry
Extreme levels of accounting accruals	

Example 10 illustrates the assumption that continuing residual income will decline to zero as ROE approaches the required rate of return on equity.

EXAMPLE 10 Multistage Residual Income Model (3)

Rosato extends her analysis to consider the possibility that ROE will slowly decay toward r in 2033 and beyond, rather than using a perpetuity of Year 2032 residual income. Rosato estimates a persistence parameter of 0.60. The present value of the terminal value is determined as

$$\frac{E_T - rB_{T-1}}{(1+r-\omega)(1+r)^{T-1}}$$

with *T* equal to 21 and 2033 residual income equal to 26.7304 (23.8664×1.12), in which the 1.12 growth factor reflects a 12 percent growth rate calculated as the retention ratio times ROE, or (0.60)(20%) = 0.12.

$$\frac{26.73}{\left(1+0.12-0.60\right)\left(1.12\right)^{20}} = 5.33$$

Total value is TWD91.74 calculated by adding the present value of the terminal value, TWD5.33, to TWD86.41. Rosato concludes that if Taiwan Semiconductor's residual income does not persist at a stable level past 2032 and deteriorates through time, the shares are modestly overvalued at a price of TWD95.6.

4. RESIDUAL INCOME VALUATION IN RELATION TO OTHER APPROACHES

Before addressing accounting issues in using the residual income model, we briefly summarize the relationship of the residual income model to other valuation models.

Valuation models based on discounting dividends or on discounting free cash flows are as theoretically sound as the residual income model. Unlike the residual income model, however, the discounted dividend and free cash flow models forecast future cash flows and find the value of stock by discounting them back to the present by using the required return. Recall that the required return is the cost of equity for both the DDM and the free cash flows to equity (FCFE) model. For the free cash flow to the firm (FCFF) model, the required return is the overall weighted average cost of capital. The RI model approaches this process differently. It starts with a value based on the balance sheet, the book value of equity, and adjusts this value by adding the present values of expected future residual income. Thus, in theory, the recognition of value is different, but the total present value, whether using expected dividends, expected free cash flow, or book value plus expected residual income, should be consistent.¹⁸

Example 11 again illustrates the important point that the recognition of value in residual income models typically occurs earlier than in dividend discount models. In other words, residual income models tend to assign a relatively small portion of a security's total present value to the earnings that occur in later years. Note also that this example makes use of the fact that the present value of a perpetuity in the amount of *X* can be calculated as X/r.

EXAMPLE 11 Valuing a Perpetuity with the Residual Income Model

Assume the following data:

- A company will earn \$1.00 per share forever.
- The company pays out all earnings as dividends.
- Book value per share is \$6.00.
- The required rate of return on equity (or the percent cost of equity) is 10 percent.
- 1. Calculate the value of this stock using the DDM.
- 2. Calculate the level amount of per-share residual income that will be earned each year.
- 3. Calculate the value of the stock using a RI model.
- 4. Create a table summarizing the year-by-year valuation using the DDM and the RI model.

Solution to 1: Because the dividend, D, is a perpetuity, the present value of D can be calculated as D/r.

 $V_0 = D/r = \$1.00/0.10 = \10.00 per share

Solution to 2: Because each year all net income is paid out as dividends, book value per share will be constant at \$6.00. Therefore, with a required rate of return on equity of 10 percent, for all future years, per-share residual income will be as follows:

 $RI_t = E_t - rB_{t-1} = $1.00 - 0.10($6.00) = $1.00 - $0.60 = 0.40

Solution to 3: Using a residual income model, the estimated value equals the current book value per share plus the present value of future expected residual income (which in this example can be valued as a perpetuity):

 V_0 = Book value + PV of expected future per-share residual income

= \$6.00 + \$0.40/0.10= \$6.00 + \$4.00 = \$10.00

¹⁸See, for example, Shrieves and Wachowicz (2001).

Solution to 4: Exhibit 8 summarizes the year-by-year valuation using the DDM and the RI models.

D	vividend Discount	Model	Residual	Income Model
Year	D_t	$\overline{\text{PV of } D_t}$	$\overline{B_0 \text{ or } \operatorname{RI}_t}$	PV of B_0 or RI_t
0			\$6.00	\$6.000
1	\$1.00	\$0.909	0.40	0.364
2	1.00	0.826	0.40	0.331
3	1.00	0.751	0.40	0.301
4	1.00	0.683	0.40	0.273
5	1.00	0.621	0.40	0.248
6	1.00	0.564	0.40	0.226
7	1.00	0.513	0.40	0.205
8	1.00	0.467	0.40	0.187
Total		\$10.00		\$10.00

EXHIBIT 8 Value Recognition in the DDM and the RI Model

In the RI model, most of the total value of the stock is attributed to the earlier periods. Specifically, the current book value of \$6.00 represents 60 percent of the stock's total present value of \$10.

In contrast, in the DDM, value is derived from the receipt of dividends, and typically, a smaller proportion of value is attributed to the earlier periods. Less than \$1.00 of the total \$10 derives from the first year's dividend, and collectively the first five years' dividends (\$0.909 + \$0.826 + \$0.751 + \$0.683 + \$0.621 = \$3.79) contribute only about 38 percent of the total present value of \$10.

As shown earlier and illustrated again in Example 11, the dividend discount and residual income models are in theory mutually consistent. Because of the real world uncertainty in forecasting distant cash flows, however, the earlier recognition of value in a residual income approach relative to other present value approaches is a practical advantage. In the dividend discount and free cash flow models, a stock's value is often modeled as the sum of the present value of individually forecasted dividends or free cash flows up to some terminal point plus the present value of the expected terminal value of the stock. In practice, a large fraction of a stock's total present value, in either the discounted dividend or free cash flow models, is represented by the present value of the expected terminal value. Substantial uncertainty, however, often surrounds the terminal value. In contrast, residual income valuations typically are less sensitive to terminal value estimates. (In some residual income valuation contexts, the terminal value may actually be set equal to zero.) The derivation of value from the earlier portion of a forecast horizon is one reason residual income valuation can be a useful analytical tool. 4.1. Strengths and Weaknesses of the Residual Income Model

Now that the implementation of the residual income model has been illustrated with several examples, a summary of the strengths and weaknesses of the residual income approach follows:

The strengths of residual income models include the following:

- Terminal values do not make up a large portion of the total present value, relative to other models.
- RI models use readily available accounting data.
- The models can be readily applied to companies that do not pay dividends or to companies that do not have positive expected near-term free cash flows.
- The models can be used when cash flows are unpredictable.
- The models have an appealing focus on economic profitability.

The potential weaknesses of residual income models include the following:

- The models are based on accounting data that can be subject to manipulation by management.
- Accounting data used as inputs may require significant adjustments.
- The models require that the clean surplus relation hold, or that the analyst make appropriate adjustments when the clean surplus relation does not hold. Section 5.1 discusses the clean surplus relation (or clean surplus accounting).
- The residual income model's use of accounting income assumes that the cost of debt capital is reflected appropriately by interest expense.

4.2. Broad Guidelines for Using a Residual Income Model

The above list of potential weaknesses helps explain the reading's focus in the following section on accounting considerations. In light of its strengths and weaknesses, the following are broad guidelines for using a residual income model in common stock valuation.

A residual income model is most appropriate when:

- a company does not pay dividends, or its dividends are not predictable;
- a company's expected free cash flows are negative within the analyst's comfortable forecast horizon; or
- great uncertainty exists in forecasting terminal values using an alternative present value approach.

Residual income models are least appropriate when:

- significant departures from clean surplus accounting exist, or
- significant determinants of residual income, such as book value and ROE, are not predictable.

Because various valuation models can be derived from the same underlying theoretical model, when fully consistent assumptions are used to forecast earnings, cash flow, dividends, book value, and residual income through a full set of pro forma (projected) financial statements, and the same required rate of return on equity is used as the discount rate, the same estimate of value should result when using each model. Practically speaking, however, it may not be possible to forecast each of these items with the same degree of certainty.¹⁹ For example, if a company has near-term negative free cash flow and forecasts for the terminal value are uncertain, a residual income model may be more appropriate. But a company with positive, predictable cash flow that does not pay a dividend would be well suited for a discounted free cash flow valuation.

Residual income models, just like the discounted dividend and free cash flow models, can also be used to establish justified market multiples, such as price-to-earnings ratio (P/E) or P/B. For example, the value can be determined by using a residual income model and dividing by earnings to arrive at a justified P/E.

A residual income model can also be used in conjunction with other models to assess the consistency of results. If a wide variation of estimated value is found and each model appears appropriate, the inconsistency may lie with the assumptions used in the models. The analyst would need to perform additional work to determine whether the assumptions are mutually consistent and which model is most appropriate for the subject company.

5. ACCOUNTING AND INTERNATIONAL CONSIDERATIONS

To most accurately apply the residual income model in practice, the analyst may need to adjust book value of common equity for off-balance sheet items and adjust reported net income to obtain **comprehensive income** (all changes in equity other than contributions by, and distributions to, owners). In this section, we will discuss issues relating to these tasks.

Bauman (1999) has noted that the strength of the residual income model is that the two components (book value and future earnings) of the model have a balancing effect on each other, provided that the clean surplus relationship is followed:

All other things held constant, companies making aggressive (conservative) accounting choices will report higher (lower) book values and lower (higher) future earnings. In the model, the present value of differences in future income is exactly offset by the initial differences in book value. (Bauman 1999, page 31)

Unfortunately, this argument has several problems in practice because the clean surplus relationship does not prevail, and analysts often use past earnings to predict future earnings. IFRS and US GAAP permit a variety of items to bypass the income statement and be reported directly in stockholders' equity. Further, off-balance sheet liabilities or nonoperating and non-recurring items of income may obscure a company's financial performance. The analyst must thus be aware of such items when evaluating the book value of equity and return on equity to be used as inputs into a residual income model.

With regard to the possibility that aggressive accounting choices will lead to lower reported future earnings, consider an example in which a company chooses to capitalize an expenditure in the current year rather than expense it. Doing so overstates current-year earnings as well as current book value. If an analyst uses current earnings (or ROE) naively in predicting future residual earnings, the RI model will overestimate the value of the company.

¹⁹For a lively debate on this issue, see Penman and Sougiannis (1998), Penman (2001), Lundholm and O'Keefe (2001a), and Lundholm and O'Keefe (2001b).

Take, for example, a company with \$1,000,000 of book value and \$200,000 of earnings before taxes, after expensing an expenditure of \$50,000. Ignoring taxes, this company has a ROE of 20 percent. If the company capitalized the expenditure rather than expensing it immediately, it would have a ROE of 23.81 percent (\$250,000/\$1,050,000). Although at some time in the future this capitalized item will likely be amortized or written off, thus reducing realized future earnings, analysts' expectations often rely on historical data. If capitalization of expenditures persists over time for a stable size company, ROE can decline because net income will normalize over the long term, but book value will be overstated. For a growing company, for which the expenditure in question is increasing, ROE can continue at high levels over time. In practice, because the RI model uses primarily accounting data as inputs, the model can be sensitive to accounting choices, and aggressive accounting methods (e.g., accelerating revenues or deferring expenses) can result in valuation errors. The analyst must, therefore, be particularly careful in analyzing a company's reported data for use in a residual income model.

Two principal drivers of residual earnings are ROE and book value. Analysts must understand how to use historical reported accounting data for these items to the extent they use historical data in forecasting future ROE and book value. Other readings have explained the DuPont analysis of ROE, which can be used as a tool in forecasting, and discussed the calculation of book value. We extend these discussions below with specific application to residual income valuation, particularly in addressing the following accounting considerations:

- violations of the clean surplus relationship;
- balance sheet adjustments for fair value;
- intangible assets;
- nonrecurring items;
- aggressive accounting practices; and
- international considerations.

In any valuation, close attention must be paid to the accounting practices of the company being valued. The following sections address the above issues as they particularly affect residual income valuation.

5.1. Violations of the Clean Surplus Relationship

One potential accounting issue in applying a residual income model is a violation of the clean surplus accounting assumption. Violations of this assumption occur when accounting standards permit charges directly to stockholders' equity, bypassing the income statement. An example is the case of changes in the market value of "available-for-sale" investments under US GAAP and "equity instruments measured at fair value through other comprehensive income" under IFRS. Under both IFRS (IFRS 9 Financial Instruments, paragraph 5.7.5) and US GAAP (ASC 320-10-35-1), these categories of investments are shown on the balance sheet at market value. Any unrealized change in their market value, however, is reflected in other comprehensive income rather than as income on the income statement.

As stated earlier, comprehensive income is defined as all changes in equity during a period other than contributions by, and distributions to, owners. Comprehensive income includes net income reported on the income statement and *other comprehensive income*, which is the result

of other events and transactions that result in a change to equity but are not reported on the income statement. Items that commonly bypass the income statement include:²⁰

- unrealized changes in the fair value of some financial instruments, as discussed above;
- foreign currency translation adjustments;
- certain pension adjustments;
- portion of gains and losses on certain hedging instruments;
- changes in revaluation surplus related to property, plant, and equipment or intangible assets (applicable under IFRS but not under US GAAP); and
- for certain categories of liabilities, change in fair value attributable to changes in the liability's credit risk (applicable under IFRS but not under US GAAP.

Under both international and US standards, such items as fair value changes for some financial instruments and foreign currency translation adjustments bypass the income statement. In addition, under IFRS, which unlike US GAAP permits revaluation of fixed assets (IAS 16, paragraph 39–42), some changes in the fair value of fixed assets also bypass the income statement and directly affect equity.

In all of these cases in which items bypass the income statement, the book value of equity is stated accurately because it includes "accumulated other comprehensive income," but net income is not stated properly from the perspective of residual income valuation. The analyst should be most concerned with the effect of these items on forecasts of net income and ROE (which has net income in the numerator), and hence residual income.²¹ Because some items (including those listed above) bypass the income statement, they are excluded from historical ROE data. As noted by Frankel and Lee (1999), bias will be introduced into the valuation only if the present expected value of the clean surplus violations do not net to zero. In other words, reductions in income from some periods may be offset by increases from other periods. The analyst must examine the equity section of the balance sheet and the related statements of shareholders' equity and comprehensive income carefully for items that have bypassed the income statement. The analyst can then assess whether amounts are likely to be offsetting and can assess the effect on future ROE.

EXAMPLE 12 Evaluating Clean Surplus Violations

Excerpts from two companies' statements of changes in stockholders' equity are shown in Exhibits 9 and 10. The first statement, prepared under IFRS as of 31 December 2012, is for Nokia Corporation (NYSE: NOK), a leading manufacturer of mobile phones headquartered in Finland and with operations in four business segments: mobile phones, multimedia, enterprise solutions, and networks. The second statement, prepared under US GAAP as of 31 December 2012, is for SAP AG (NYSE: SAP), which is headquartered in Germany and is a worldwide provider of enterprise application software, including enterprise resource planning, customer relationship management, and supply chain management software.

²⁰See Frankel and Lee (1999).

²¹The analyst should more precisely calculate historical ROE at the aggregate level (e.g., as net income divided by shareholders' equity) rather than as earnings per share divided by book value per share, because such actions as share issuance and share repurchases can distort ROE calculated on a per-share basis.

EXHIBIT 9 Nokia Corporation Sta	tement of Ch	anges in S	hareholders	s' Equity (€	millions exce	pt number	of shares)				
						Fair Value	Reserve for		Before		
	Number		Share			and	Invested		Non-	Non-	
	of Shares	Share	Issue	Treasury	Translation	Other	NonRestrict.	Retained	Controlling	Controlling	
	(1,000's)	Capital	Premium	Shares	Differences	Reserves	Equity	Earnings	Interests	Interests	Total
Balance as of December 31, 2011	3,710,189	246	362	-644	771	154	3,148	7,836	11,873	2,043	13,916
Translation differences					40				40	-2	38
Net investment hedges, net of tax					-67				-67		-67
Cash flow hedges, net of tax						-67			-67	47	-20
Available-for-sale investments, net of tax						36			36		36
Other increase, net								7	7	3	10
Loss								-3,106	-3,106	-683	-3,789
Total comprehensive income					-27	-31		-3,099	-3,157	-635	-3,792
Share-based compensation			1								
Excess tax benefit on share-based compensation			ω								
Settlement of performance and restricted shares	296		ν. Γ	15			-12		-2		-2
Dividend								-742	-742	-22	-764
Convertible bond-equity component			85						85		85
Total of other equity movements	296		84	15			-12	-742	-655	-22	-677
Balance at December 31, 2012	3,710,985	246	446	-629	744	123	3,136	3,995	8,061	1,386	9,447
Source:www.nokia.com.											

			c		, , T					
				Equit	y Attributable to C	Owners of Par	ent			
					Other Componen	ts of Equity				
					Available-for-	Cash			Non-	
	Issued	Share	Retained	Exchange	Sale Financial	Flow	Treasury		Controlling	Total
	Capital	Premium	Earnings	Differences	Assets	Hedges	Sales	Total	Interests	Equity
December 31, 2011	1,228	419	12,466	-19	6	-27	-1,377	12,699	8	12,707
Profit after tax			2,823					2,823	0	2,823
Other comprehensive										
income			-8	-217	13	47		-165		-165
Comprehensive income	0	0	2,815	-217	13	47	0	2,658	0	2,658
Share-based payments		41						41		41
Dividends			-1,310					-1,310		-1,310
Issuance of shares under										
share-based payments	1	14						15		15
Purchase of treasury shares							-53	-53		-53
Reissuance of treasury shares										
under share-based payments		18					93	111		111
Other			2					2	0	2
December 31, 2012	1,229	492	13,973	-236	22	20	-1,337	14,163	8	14,171
Source: www.sap.com.										

EXHIBIT 10 SAP AG and Subsidiaries Statement of Changes in Shareholders' Equity (€ millions)

For Nokia, items that have bypassed the income statement in 2012 are those in the columns labeled "Share issue premium," "Translation differences," "Fair value and other reserves," and "Reserve for invested non-restricted equity." For SAP, the amounts that bypassed the income statement in 2012 are "Share premium," "Exchange differences," "Available-for-sale financial assets," and "Cash flow hedges."

To illustrate the issues in interpreting these items, consider the columns "Translation differences" (Nokia) and "Exchange differences" (SAP). The amounts in these columns reflect currency translation adjustments to equity that have bypassed the income statement. For Nokia, the adjustment for the year 2012 was -€27 million. Because this is a negative adjustment to stockholders' equity, this item would have decreased income if it had been reported on the income statement. The balance is not increasing, however; it appears to be reversing to zero over time. For SAP, the translation adjustment to stockholders' equity, this is a negative adjustment to stockholders equity, this is a negative adjustment to stockholders' equity, this item would have decreased income if the year 2012 was -€217 million. Again, because this is a negative adjustment to stockholders' equity, this item would have decreased income if it had been reported on the income statement. In this case, the negative balance appears to be accumulating: It does not appear to be reversing (netting to zero) over time. If the analyst expects this trend to continue and has used historical data as the basis for initial estimates of ROE to be used in residual income valuation, a downward adjustment in that estimated future ROE might be warranted. It is possible, however, that future exchange rate movements will reverse this accumulation.

The examples in this reading have used the actual beginning equity and a forecasted level of ROE (return on beginning equity) to compute the forecasted net income. Because equity includes accumulated other comprehensive income (AOCI), the assumptions about future other comprehensive income (OCI) will affect forecasted net income and thus residual income. To illustrate, Exhibit 11 shows a hypothetical company's financials for a single previous year, labeled year t-1, followed by three different forecasts for the following two years. In year t-1, the company reports net income of \$120, which is a 12 percent return on beginning equity of \$1,000. The company paid no dividends, so ending retained earnings equal \$120. In year t-1, the company also reports OCI of -\$100, a loss, so the ending amount shown in AOCI is a -\$100. (Companies typically label this line item "accumulated other comprehensive income (loss)," indicating that the amount is an accumulated loss when given in parentheses.)

All three forecasts in Exhibit 11 assume that ROE will be 12 percent and use this assumption to forecast net income for year t and t+1 by using the expression $0.12 \times$ Beginning book value. Each forecast, however, incorporates different assumptions about future OCI. Forecast A assumes that the company will have no OCI in year t or year t+1, so the amount of AOCI does not change. Forecast B assumes that the company will continue to have the same amount of OCI in year t and year t+1 as it had in the prior year, so the amount of AOCI becomes more negative each year. Forecast C assumes that the company's OCI will reverse in year t, so at the end of year t, AOCI will be zero. As shown, because the forecasts use the assumed ROE to compute forecasted net income, the forecasts for net income and residual income in year t+1 vary significantly.

Because this example assumes all earnings are retained, a forecast of 12 percent ROE also implies that net income and residual income will grow at 12 percent. Only the year t to

EXHIBIT 11 Hypothetical Com	1 pany Alternative	Forecasts with Diff	erent Assumptions	about Comprehens	ive Income		
	Actual	Forec	ast A	Forec	ast B	Forec	ast C
Year	t-1	t	<i>t</i> +1	t	¢+1	t	t+1
Beginning Balance Sheet							
Assets	\$1,000.00	\$1,020.00	\$1,142.40	\$1,020.00	\$1,042.40	\$1,020.00	\$1,242.40
Liabilities							
Common stock	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
Retained earnings		120.00	242.40	120.00	242.40	120.00	242.40
AOCI		(100.00)	(100.00)	(100.00)	(200.00)	(100.00)	
Total equity	1,000.00	1,020.00	1,142.40	1,020.00	1,042.40	1,020.00	1,242.40
Total liabilities and total equity	\$1,000.00	\$1,020.00	\$1,142.40	\$1,020.00	\$1,042.40	\$1,020.00	\$1,242.40
Net income	120.00	122.40	137.09	122.40	125.09	122.40	149.09
Dividends							I
Other comprehensive income	(100.00)			(100.00)	(100.00)	100.00	I
Ending Balance Sheet							
Assets	\$1,020.00	\$1,142.40	\$1,279.49	\$1,042.40	\$1,067.49	\$1,242.40	\$1,391.49
Liabilities							
Common stock	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00	1,000.00
Retained earnings	120.00	242.40	379.49	242.40	367.49	242.40	391.49
AOCI	(100.00)	(100.00)	(100.00)	(200.00)	(300.00)		
Total equity	\$1,020.00	\$1,142.40	\$1,279.49	\$1,042.40	\$1,067.49	\$1,242.40	\$1,391.49
Total liabilities and total equity	\$1,020.00	\$1,142.40	\$1,279.49	\$1,042.40	\$1,067.49	\$1,242.40	\$1,391.49
Residual income calculation based on beginning total equity							
Net income	120.00	122.40	137.09	122.40	125.09	122.40	149.09
Equity charge at 10 percent	100.00	102.00	114.24	102.00	104.24	102.00	124.24
Residual income	\$20.00	\$20.40	\$22.85	\$20.40	\$20.85	\$20.40	\$24.85

year *t*+1 under Forecast A, which assumes no future OCI, correctly reflects that relationship. Specifically, in Forecast A, both net income and residual income increase by 12 percent from year *t* to year *t*+1. Net income grows from \$122.40 to \$137.09, an increase of 12 percent [(\$137.09/\$122.40) - 1]; and residual income grows from \$20.40 to \$22.85, an increase of 12 percent [(\$22.85/\$20.40) - 1]. In contrast to Forecast A, neither Forecast B nor Forecast C correctly reflects the relationship between ROE and growth in income (net and residual). Growth in residual income from year *t* to year *t*+1 was 2.2 percent under Forecast B and 21.8 percent under Forecast C.

If, alternatively, the forecasts of future ROE and the residual income computation had incorporated total comprehensive income (net income plus OCI), the results of the residual income computation would have differed significantly. For example, suppose that in Forecast B, which assumes the company will continue to have the same amount of OCI, the estimated future ROE was 2.0 percent, using total comprehensive income [(\$120 - \$100)/\$1,000 = \$20/\$1,000]. If the residual income computation had then also used forecasted total comprehensive income at time *t*, the amount of residual income would be negative. Specifically, for time *t*, forecast comprehensive income would be \$22.40 (net income plus other comprehensive income), the equity charge would be \$102 (required return of 10 percent times beginning equity of \$1,020), and residual income would be -\$79.60 (comprehensive income of \$22.40 minus equity charge of \$102). Clearly, residual income on this basis significantly falls short of the positive \$20.40 when the violation of clean surplus is ignored. As this example demonstrates, using an ROE forecast or a net income forecast that ignores violations of clean surplus accounting will distort estimates of residual income. Unless the present value of such distortions net to zero, using those forecasts will also distort valuations.

What are the implications for implementing a residual-income-based valuation? If future OCI is expected to be significant relative to net income and if the year-to-year amounts of OCI are not expected to net to zero, the analyst should attempt to incorporate these items so that residual income forecasts are closer to what they would be if the clean surplus relation held. Specifically, when possible, the analyst should incorporate explicit assumptions about future amounts of OCI.

Example 13 illustrates, by reference to the DDM value, the error that results when OCI is omitted from residual income calculations (assuming an analyst has a basis for forecasting future amounts of OCI).²² The example also shows that the growth rate in residual income is generally not equal to the growth rate of net income or dividends.

EXAMPLE 13 Incorporating Adjustments in the Residual Income Model

Exhibit 12 gives per-share forecasts for Mannistore, Inc., a hypothetical company operating a chain of retail stores. The company's cost of capital is 10 percent.

²²See Lundholm and O'Keefe (2001a and 2001b), who show how RI model and DDM valuations will differ when the analyst fails to include OCI in residual income calculations or makes inconsistent assumptions about the growth rates of net income, dividends, and residual income.

			Year		
Variable	1	2	3	4	5
Shareholders' equity t-1	\$8.58	\$10.32	\$11.51	\$14.68	\$17.86
Plus net income	2.00	2.48	3.46	3.47	4.56
Less dividends	0.26	0.29	0.29	0.29	0.38
Less other comprehensive income	0.00	1.00	0.00	0.00	0.00
Equals shareholders' equity _t	\$10.32	\$11.51	\$14.68	\$17.86	\$22.04

EXHIBIT 12 Forecasts for Mannistore, Inc.

1. Assuming the forecasted terminal price of Mannistore's shares at the end of year 5 (time t = 5) is \$68.40, estimate the value per share of Mannistore using the DDM.

- 2. Given that the forecast terminal price of Mannistore's shares at the end of year 5 (time t = 5) is \$68.40, estimate the value of a share of Mannistore using the RI model and calculate residual income based on:
 - A. net income without adjustment, and
 - B. net income plus other comprehensive income.
- 3. Interpret your answers to Parts 2A and 2B.
- 4. Assume that a forecast of the terminal price of Mannistore's shares at the end of year 5 (time t = 5) is not available. Instead, an estimate of terminal price based on the Gordon growth model is appropriate. You estimate that the growth in net income and dividends from t = 5 to t = 6 will be 8 percent. Predict residual income for year 6, and based on that 8 percent growth estimate, determine the growth rate in forecasted residual income from t = 5 to t = 6.

Solution to 1: The estimated value using the DDM is:

$$V_0 = \frac{\$0.26}{(1.10)^1} + \frac{\$0.29}{(1.10)^2} + \frac{\$0.29}{(1.10)^3} + \frac{\$0.29}{(1.10)^4} + \frac{\$0.38}{(1.10)^5} + \frac{\$68.40}{(1.10)^5} = \$43.59$$

Solution to 2:

A. Calculating residual income as net income (NI) minus the equity charge, which is beginning shareholders' equity (SE) times the cost of equity capital (*r*), gives the following for years 1 through 5:

			Year		
	1	2	3	4	5
$RI = NI - (SE_{t-1} \times r)$	1.14	1.45	2.30	2.00	2.77

So, the estimated value using the RI model (using Equation 6), with residual income calculated based on net income, is:

$$V_{0} = \$8.58 + \frac{\$1.14}{(1.10)^{1}} + \frac{\$1.45}{(1.10)^{2}} + \frac{\$2.30}{(1.10)^{3}} + \frac{\$2.00}{(1.10)^{4}} + \frac{\$2.77}{(1.10)^{5}} + \frac{\$68.40 - \$22.04}{(1.10)^{5}}$$
$$V_{0} = \$8.58 + 35.84 = \$44.42$$

B. Calculating residual income as net income adjusted for OCI (NI + OCI) minus the equity charge, which equals beginning shareholders' equity (SE) times the cost of equity capital (*r*), gives the following for years 1 through 5:

			Year		
	1	2	3	4	5
$RI = (NI + OCI) - (SE_{t-1} \times r)$	\$1.14	\$0.45	\$2.30	\$2.00	\$2.77

So, the estimated value using the RI model, with residual income based on net income adjusted for OCI, is:

$$V_{0} = \$8.58 + \frac{\$1.14}{(1.10)^{1}} + \frac{\$.45}{(1.10)^{2}} + \frac{\$2.30}{(1.10)^{3}} + \frac{\$2.00}{(1.10)^{4}} + \frac{\$2.77}{(1.10)^{5}} + \frac{\$68.40 - \$22.04}{(1.10)^{5}}$$
$$V_{0} = \$8.58 + 35.01 = \$43.59$$

Solution to 3: The first calculation (2A) incorrectly omits an adjustment for a violation of the clean surplus relation. The second calculation (2B) includes an adjustment and yields the correct value estimate, which is consistent with the DDM estimate.

Solution to 4: Given the estimated 8 percent growth in net income and dividends in year 6, the estimated year 6 net income is $4.92 (4.56 \times 1.08)$, and the estimated amount of year 6 dividends is $0.42 (0.38 \times 1.08)$.

Residual income will then equal \$2.72 (which is net income of \$4.92 minus the equity charge of beginning book value of \$22.04 times the cost of capital of 10 percent). So, the growth rate in residual income is negative at approximately -2 percent (\$2.72/\$2.77 - 1).

Lacking a basis for explicit assumptions about future amounts of OCI, the analyst should nonetheless be aware of the potential effect of OCI on residual income and adjust ROE accordingly. Finally, as noted above, the analyst may decide that an alternative valuation model is more appropriate.

5.2. Balance Sheet Adjustments for Fair Value

To have a reliable measure of book value of equity, an analyst should identify and scrutinize significant off-balance sheet assets and liabilities. Additionally, reported assets and liabilities should be adjusted to fair value when possible. Off-balance sheet assets and liabilities may become apparent through an examination of the financial statement footnotes. Probably the most common example is the use of operating leases. Operating leases do not affect the amount of equity (because leases involve both off-balance sheet assets that offset the off-balance sheet liabilities) but can affect an assessment of future earnings for the residual income component of value. Other assets and liabilities may be stated at values other than fair value. For example, inventory may be stated at LIFO and require adjustment to restate to current value. (LIFO is not permitted under IFRS.) The following are some common items to review for balance sheet adjustments. Note, however, that this list is not comprehensive:

- inventory;
- deferred tax assets and liabilities;
- operating leases;
- reserves and allowances (for example, bad debts); and
- intangible assets.

Additionally, the analyst should examine the financial statements and footnotes for items unique to the subject company.

5.3. Intangible Assets

Intangible assets can have a significant effect on book value. In the case of specifically identifiable intangibles that can be separated from the entity (e.g., sold), it is generally appropriate to include these in the determination of book value of equity. If these assets have a finite useful life, they will be amortized over time as an expense. Intangible assets, however, require special consideration because they are often not recognized as an asset unless they are obtained in an acquisition. For example, advertising expenditures can create a highly valuable brand, which is clearly an intangible asset. Advertising expenditures, however, are shown as an expense, and the value of a brand would not appear as an asset on the financial statements unless the company owning the brand was acquired.

To demonstrate this, consider a simplified example involving two companies, Alpha and Beta, with the following summary financial information (all amounts in thousands, except per-share data):

	Alpha	Beta
Cash	€1,600	€100
Property, plant, and equipment	3,400	900
Total assets	€5,000	€1,000
Equity	5,000	1,000
Net income	€600	€150

Each company pays out all net income as dividends (no growth), and the clean surplus relation holds. Alpha has a 12 percent ROE and Beta has a 15 percent ROE, both expected to

continue indefinitely. Each has a 10 percent required rate of return. The fair market value of each company's property, plant, and equipment is the same as its book value. What is the value of each company in a residual income framework?

Using total book value rather than per-share data, the value of Alpha would be \notin 6,000, determined as follows:²³

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0 = 5,000 + \frac{0.12 - 0.10}{0.10 - 0.00} 5,000 = 6,000$$

Similarly, the value of Beta would be €1,500:

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0 = 1,000 + \frac{0.15 - 0.10}{0.10 - 0.00} 1,000 = 1,500$$

The value of the companies on a combined basis would be €7,500. Note that both companies are valued more highly than the book value of equity because they have ROE in excess of the required rate of return. Absent an acquisition transaction, the financial statements of Alpha and Beta do not reflect this value. If either is acquired, however, an acquirer would allocate the purchase price to the acquired assets, with any excess of the purchase price above the acquired assets shown as goodwill.

Suppose Alpha acquires Beta by paying Beta's former shareholders $\in 1,500$ in cash. Alpha has just paid $\in 500$ in excess of the value of Beta's total reported assets of $\in 1,000$. Assume that Beta's property, plant, and equipment is already shown at its fair market value of $\in 1,000$, and that the $\in 500$ is considered to be the fair value of a license owned by Beta, say an exclusive right to provide a service. Assume further that the original cost of obtaining the license was an immaterial application fee, which does not appear on Beta's balance sheet, and that the license covers a period of 10 years. Because the entire purchase price of $\in 1,500$ is allocated to identifiable assets, no goodwill is recognized. The balance sheet of Alpha immediately after the acquisition would be:²⁴

	Alpha
Cash	€200
Property, plant, and equipment	4,300
License	500
Total assets	€5,000
Equity	€5,000

Note that the total book value of Alpha's equity did not change, because the acquisition was made for cash and thus did not require Alpha to issue any new shares.

Making the assumption that the license is amortized over a 10-year period, the combined company's expected net income would be \notin 700 (\notin 600 + \notin 150 - \notin 50 amortization). If this net income number is used to derive expected ROE, the expected ROE would be 14 percent.

²³Results would be the same if calculated on a per-share basis.

²⁴For example, cash of €200 is calculated as €1,600 (cash of Alpha) + €100 (cash of Beta) – €1,500 (purchase price of Beta).

Under a residual income model, with no adjustment for amortization, the value of the combined company would be:

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0 = 5,000 + \frac{0.14 - 0.10}{0.10 - 0.00} 5,000 = 7,000$$

Why would the combined company be worth less than the two separate companies? If the assumption is made that a fair price was paid to Beta's former shareholders, the combined value should not be lower. The lower value using the residual income model results from a reduction in ROE as a result of the amortization of the intangible license asset. If this asset were not amortized (or if the amortization expense was added back before computing ROE), net income would be \notin 750 and ROE would be 15 percent. The value of the combined entity would be:

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0 = 5,000 + \frac{0.15 - 0.10}{0.10 - 0.00} 5,000 = 7,500$$

This amount, €7,500, is the same as the sum of the values of the companies on a separate basis.

Would the answer be different if the acquiring company used newly issued stock rather than cash in the acquisition? The form of currency used to pay for the transaction should not impact the total value. If Alpha used €1,500 of newly issued stock to acquire Beta, its balance sheet would be

	Alpha
Cash	€1,700
Property, plant, and equipment	4,300
License	500
Total assets	€6,500
Equity	€6,500

Projected earnings, excluding the amortization of the license, would be \notin 750, and projected ROE would be 11.538 percent. Value under the residual income model would be:

$$V_0 = B_0 + \frac{\text{ROE} - r}{r - g} B_0 = 6,500 + \frac{0.11538 - 0.10}{0.10 - 0.00} 6,500 = 7,500$$

The overall value remains unchanged. The book value of equity is higher but offset by the effect on ROE. Once again, this example assumes that the buyer paid a fair value for the acquisition. If an acquirer overpays for an acquisition, the overpayment should become evident in a reduction in future residual income.

Research and development (R&D) costs provide another example of an intangible asset that must be given careful consideration. Under US GAAP, R&D is generally expensed to the income statement directly (except in certain cases such as ASC 985-20-25, which permits the capitalization of R&D expenses related to software development after product feasibility has been established). Also, under IFRS, some R&D costs can be capitalized and amortized over time. R&D expenditures are reflected in a company's ROE, and hence residual income, over the long term. If a company engages in unproductive R&D expenditures, these will lower residual income through the expenditures made. If a company engages in productive R&D expenditures, these should result in higher revenues to offset the expenditures over time. In summary, on a continuing basis for a mature company, ROE should reflect the productivity of R&D expenditures without requiring an adjustment.

As explained in Lundholm and Sloan (2007), including and subsequently amortizing an asset that was omitted from a company's reported assets has no effect on valuation under a residual income model. Such an adjustment would increase the estimated equity value by adding the asset to book value at time zero but decrease the estimated value by an equivalent amount, which would include a) the present value of the asset when amortized in the future and b) the present value of a periodic capital charge based on the amount of the asset times the cost of equity. Expensing R&D, however, results in an immediately lower ROE vis-à-vis capitalizing R&D. But expensing R&D will result in a slightly higher ROE relative to capitalizing R&D in future years because this capitalized R&D is amortized. Because ROE is used in a number of expressions derived from the residual income model and may also be used in forecasting net income, the analyst should carefully consider a company's R&D expenditures and their effect on long-term ROE.

5.4. Nonrecurring Items

In applying a residual income model, it is important to develop a forecast of future residual income based on recurring items. Companies often report nonrecurring charges as part of earnings, which can lead to overestimates and underestimates of future residual earnings if no adjustments are made. No adjustments to book value are necessary for these items, however, because nonrecurring gains and losses are reflected in the value of assets in place. Hirst and Hopkins (2000) noted that nonrecurring items sometimes result from accounting rules and at other times result from "strategic" management decisions. Regardless, they highlighted the importance of examining the financial statement notes and other sources for items that may warrant adjustment in determining recurring earnings, such as:

- unusual items;
- extraordinary items (applicable under US GAAP but not under IFRS);
- restructuring charges;
- discontinued operations; and
- accounting changes.

In some cases, management may record restructuring or unusual charges in every period. In these cases, the item may be considered an ordinary operating expense and may not require adjustment.

Companies sometimes inappropriately classify nonoperating gains as a reduction in operating expenses (such as selling, general, and administrative expenses). If material, this inappropriate classification can usually be uncovered by a careful reading of financial statement footnotes and press releases. Analysts should consider whether these items are likely to continue and contribute to residual income in time. More likely, they should be removed from operating earnings when forecasting residual income.

5.5. Other Aggressive Accounting Practices

Companies may engage in accounting practices that result in the overstatement of assets (book value) and/or overstatement of earnings. We discussed some of these practices in the preceding

sections. Other activities that a company may engage in include accelerating revenues to the current period or deferring expenses to a later period.²⁵ Both activities simultaneously increase earnings and book value. For example, a company might ship unordered goods to customers at year-end, recording revenues and a receivable. As another example, a company could capitalize rather than expense a cash payment, resulting in lower expenses and an increase in assets.

Conversely, companies have also been criticized for the use of "cookie jar" reserves (reserves saved for future use), in which excess losses or expenses are recorded in an *earlier* period (for example, in conjunction with an acquisition or restructuring) and then used to reduce expenses and increase income in future periods. The analyst should carefully examine the use of reserves when assessing residual earnings. Overall, the analyst must evaluate a company's accounting policies carefully and consider the integrity of management when assessing the inputs in a residual income model.

5.6. International Considerations

Accounting standards differ internationally. These differences result in different measures of book value and earnings internationally and suggest that valuation models based on accrual accounting data might not perform as well as other present value models in international contexts. It is interesting to note, however, that Frankel and Lee (1999) found that the residual income model works well in valuing companies on an international basis. Using a simple residual income model without any of the adjustments discussed in this reading, they found that their residual income valuation model accounted for 70 percent of the cross-sectional variation of stock prices among 20 countries. Frankel and Lee concluded that there are three primary considerations in applying a residual income model internationally:

- the availability of reliable earnings forecasts;
- systematic violations of the clean surplus assumption; and
- "poor quality" accounting rules that result in delayed recognition of value changes.

Analysts should expect the model to work best in situations in which earnings forecasts are available, clean surplus violations are limited, and accounting rules do not result in delayed recognition. Because Frankel and Lee found good explanatory power for a residual income model using unadjusted accounting data, one expects that if adjustments are made to the reported data to correct for clean surplus and other violations, international comparisons should result in comparable valuations. For circumstances in which clean surplus violations exist, accounting choices result in delayed recognition, or accounting disclosures do not permit adjustment, the residual income model would not be appropriate, and the analyst should consider a model less dependent on accounting data, such as a FCFE model.

It should be noted, however, that IFRS is increasingly becoming widely used. As of 2012, approximately 120 nations and reporting jurisdictions permit or require IFRS for domestic listed companies, although approximately 90 countries have fully conformed with IFRS as promulgated by the IASB and include a statement acknowledging such conformity in audit reports. Furthermore, standard setters in numerous countries continue to work toward convergence between IFRS and home-country GAAP. In time, concerns about the use of different accounting standards should become less severe. Nonetheless, even within

²⁵See, for example, Schilit and Perler (2010).

a single set of accounting standards, companies make choices and estimates that can affect valuation.

SUMMARY

This reading has discussed the use of residual income models in valuation. Residual income is an appealing economic concept because it attempts to measure economic profits, which are profits after accounting for all opportunity costs of capital.

- Residual income is calculated as net income minus a deduction for the cost of equity capital. The deduction is called the equity charge and is equal to equity capital multiplied by the required rate of return on equity (the cost of equity capital in percent).
- Economic value added (EVA) is a commercial implementation of the residual income concept. $EVA = NOPAT (C\% \times TC)$, where NOPAT is net operating profit after taxes, C% is the percent cost of capital, and TC is total capital.
- Residual income models (including commercial implementations) are used not only for equity valuation but also to measure internal corporate performance and for determining executive compensation.
- We can forecast per-share residual income as forecasted earnings per share minus the required rate of return on equity multiplied by beginning book value per share. Alternatively, per-share residual income can be forecasted as beginning book value per share multiplied by the difference between forecasted ROE and the required rate of return on equity.
- In the residual income model, the intrinsic value of a share of common stock is the sum of book value per share and the present value of expected future per-share residual income. In the residual income model, the equivalent mathematical expressions for intrinsic value of a common stock are

$$V_0 = B_0 + \sum_{t=1}^{\infty} \frac{\text{RI}_t}{(1+r)^t} = B_0 + \sum_{t=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}$$
$$= B_0 + \sum_{t=1}^{\infty} \frac{(\text{ROE}_t - r)B_{t-1}}{(1+r)^t}$$

where

 V_0 = value of a share of stock today (t = 0)

 B_0 = current per-share book value of equity

 B_t = expected per-share book value of equity at any time t

- r = required rate of return on equity (cost of equity)
- E_t = expected earnings per share for period t
- RI_t = expected per-share residual income, equal to $E_t rB_{t-1}$ or to $(ROE r) \times B_{t-1}$
- In most cases, value is recognized earlier in the residual income model compared with other present value models of stock value, such as the dividend discount model.
- Strengths of the residual income model include the following:
- Terminal values do not make up a large portion of the value relative to other models.
- The models use readily available accounting data.
- The models can be used in the absence of dividends and near-term positive free cash flows.
- The models can be used when cash flows are unpredictable.

- Weaknesses of the residual income model include the following:
 - The models are based on accounting data that can be subject to manipulation by management.
 - Accounting data used as inputs may require significant adjustments.
 - The models require that the clean surplus relation hold, or that the analyst make appropriate adjustments when the clean surplus relation does not hold.
- The residual income model is most appropriate in the following cases:
 - A company is not paying dividends or if it exhibits an unpredictable dividend pattern.
 - A company has negative free cash flow many years out but is expected to generate positive cash flow at some point in the future.
 - A great deal of uncertainty exists in forecasting terminal values.
- The fundamental determinants or drivers of residual income are book value of equity and return on equity.
- Residual income valuation is most closely related to P/B. When the present value of expected future residual income is positive (negative), the justified P/B based on fundamentals is greater than (less than) one.
- When fully consistent assumptions are used to forecast earnings, cash flow, dividends, book
 value, and residual income through a full set of pro forma (projected) financial statements,
 and the same required rate of return on equity is used as the discount rate, the same estimate
 of value should result from a residual income, dividend discount, or free cash flow valuation.
 In practice, however, analysts may find one model easier to apply and possibly arrive at different valuations using the different models.
- Continuing residual income is residual income after the forecast horizon. Frequently, one of the following assumptions concerning continuing residual income is made:
 - Residual income continues indefinitely at a positive level. (One variation of this assumption is that residual income continues indefinitely at the rate of inflation, meaning it is constant in real terms.)
 - Residual income is zero from the terminal year forward.
 - Residual income declines to zero as ROE reverts to the cost of equity over time.
 - Residual income declines to some mean level.
- The residual income model assumes the clean surplus relation of $B_t = B_{t-1} + E_t D_t$. In other terms, the ending book value of equity equals the beginning book value plus earnings minus dividends, apart from ownership transactions.
- In practice, to apply the residual income model most accurately, the analyst may need to:
 - adjust book value of common equity for:
 - off-balance sheet items;
 - · discrepancies from fair value; or
 - the amortization of certain intangible assets.
 - adjust reported net income to reflect clean surplus accounting.
 - adjust reported net income for nonrecurring items misclassified as recurring items.

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