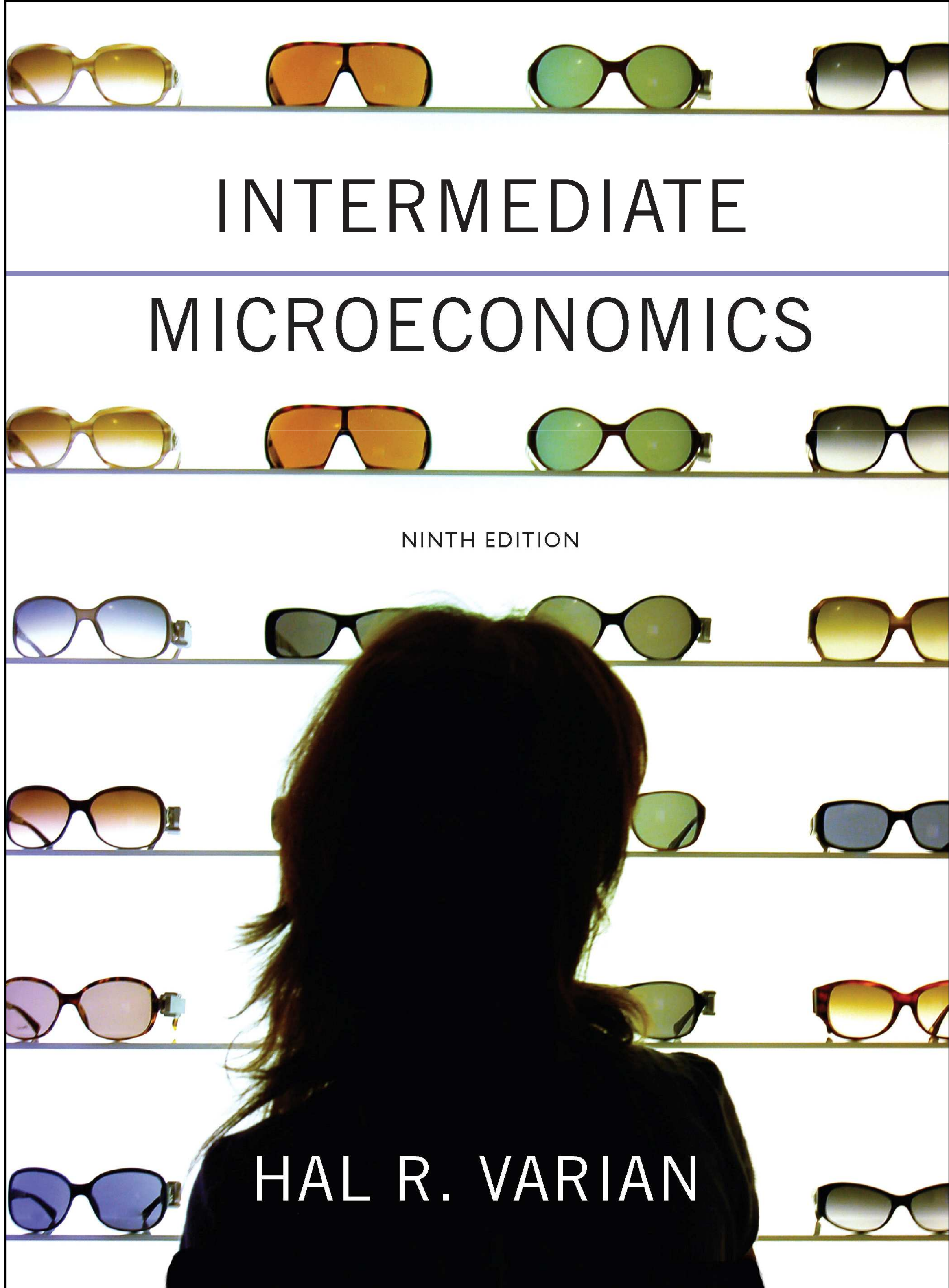


Chapter 12

Uncertainty

Uncertainty is Pervasive

- **What is uncertain in economic systems?**
  - **tomorrow's prices**
  - **future wealth**
  - **future availability of commodities**
  - **present and future actions of other people.**

Uncertainty is Pervasive

- **What are rational responses to uncertainty?**
  - **buying insurance (health, life, auto)**
  - **a portfolio of contingent consumption goods.**

States of Nature

- **Possible states of Nature:**
  - **"car accident" (a)**
  - **"no car accident" (na).**
- **Accident occurs with probability π_a, does not with probability π_{na} ;**

$$\pi_a + \pi_{na} = 1.$$

- **Accident causes a loss of \$L.**

Contingencies

- **A contract implemented only when a particular state of Nature occurs is state-contingent.**
- **E.g. the insurer pays only if there is an accident.**

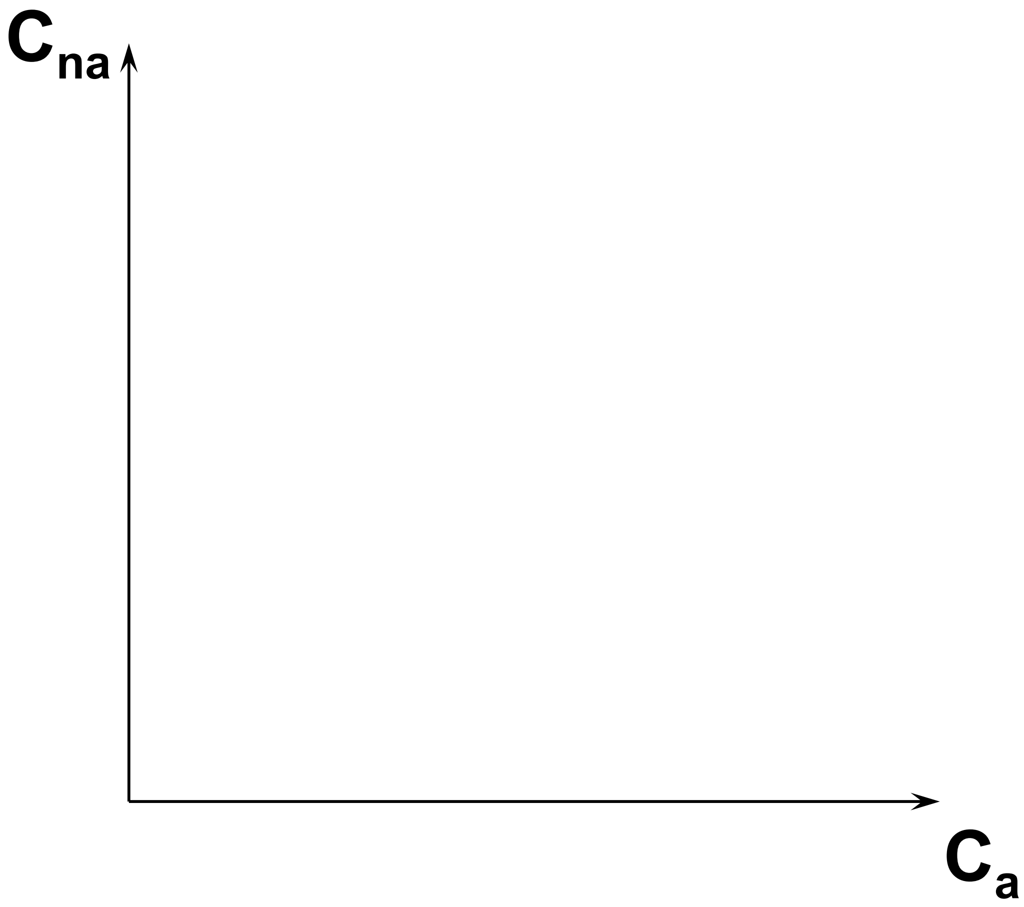
Contingencies

- **A state-contingent consumption plan is implemented only when a particular state of Nature occurs.**
- **E.g. take a vacation only if there is no accident.**

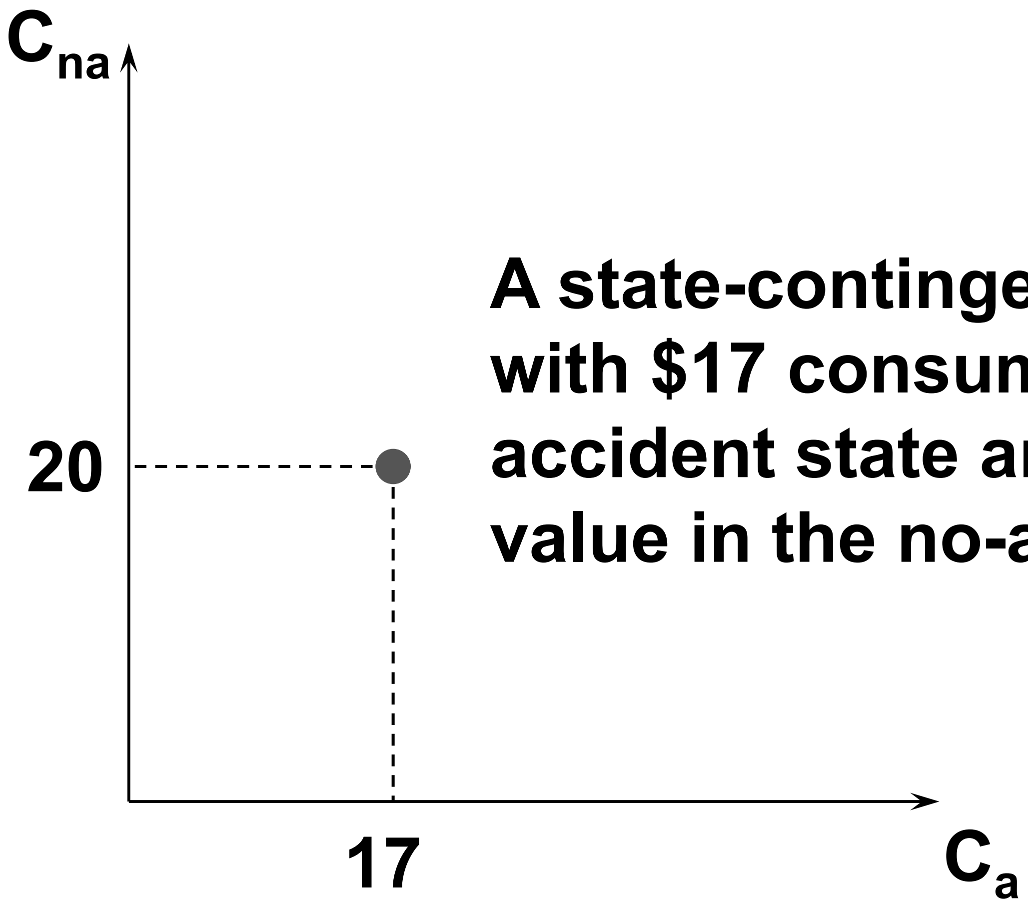
State-Contingent Budget Constraints

- Each $1 of accident insurance costs γ.
- Consumer has $m of wealth.
- C_{na} is consumption value in the no-accident state.
- C_a is consumption value in the accident state.

State-Contingent Budget Constraints



State-Contingent Budget Constraints

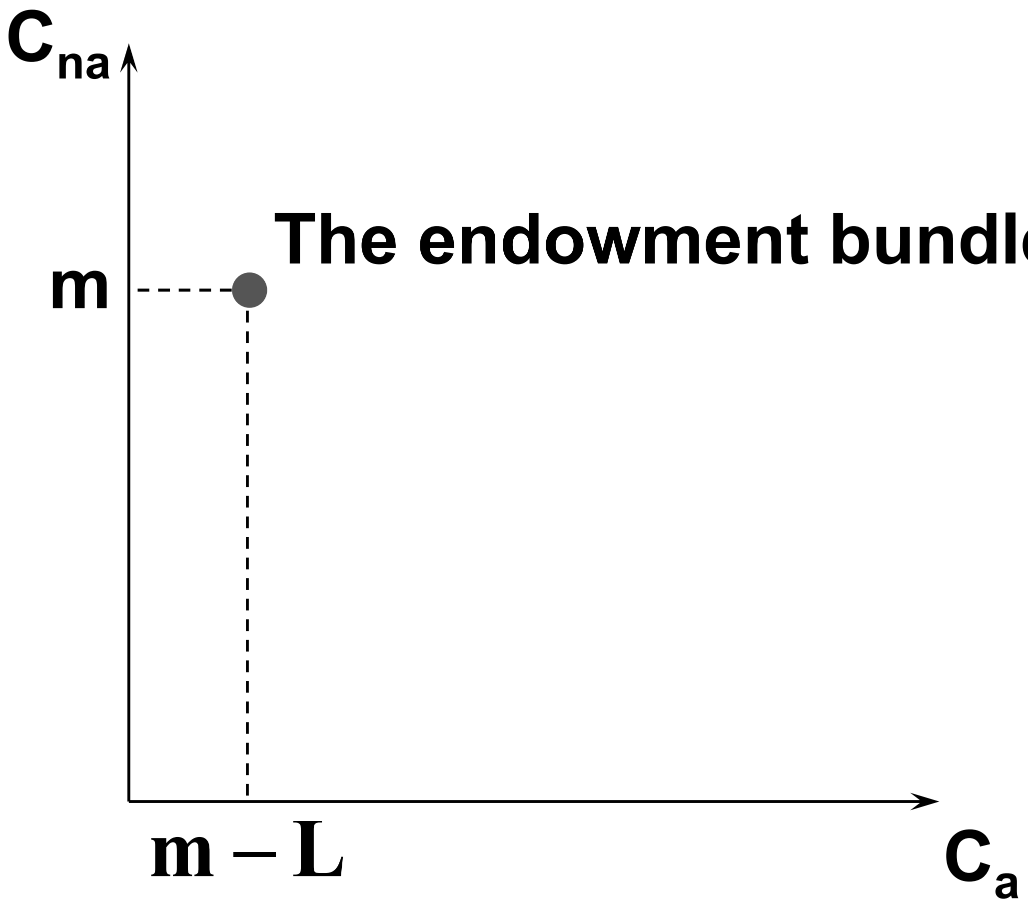


A state-contingent consumption with \$17 consumption value in the accident state and \$20 consumption value in the no-accident state.

State-Contingent Budget Constraints

- Without insurance,
- $C_a = m - L$
- $C_{na} = m.$

State-Contingent Budget Constraints



State-Contingent Budget Constraints

- **Buy \$K of accident insurance.**
- $C_{na} = m - \gamma K$.
- $C_a = m - L - \gamma K + K = m - L + (1- \gamma)K$.

State-Contingent Budget Constraints

- **Buy \$K of accident insurance.**
- $C_{na} = m - \gamma K$.
- $C_a = m - L - \gamma K + K = m - L + (1-\gamma)K$.
- So $K = (C_a - m + L)/(1-\gamma)$

State-Contingent Budget Constraints

- **Buy \$K of accident insurance.**
- $C_{na} = m - \gamma K$.
- $C_a = m - L - \gamma K + K = m - L + (1- \gamma)K$.
- So $K = (C_a - m + L)/(1- \gamma)$
- And $C_{na} = m - \gamma\,(C_a - m + L)/(1- \gamma)$

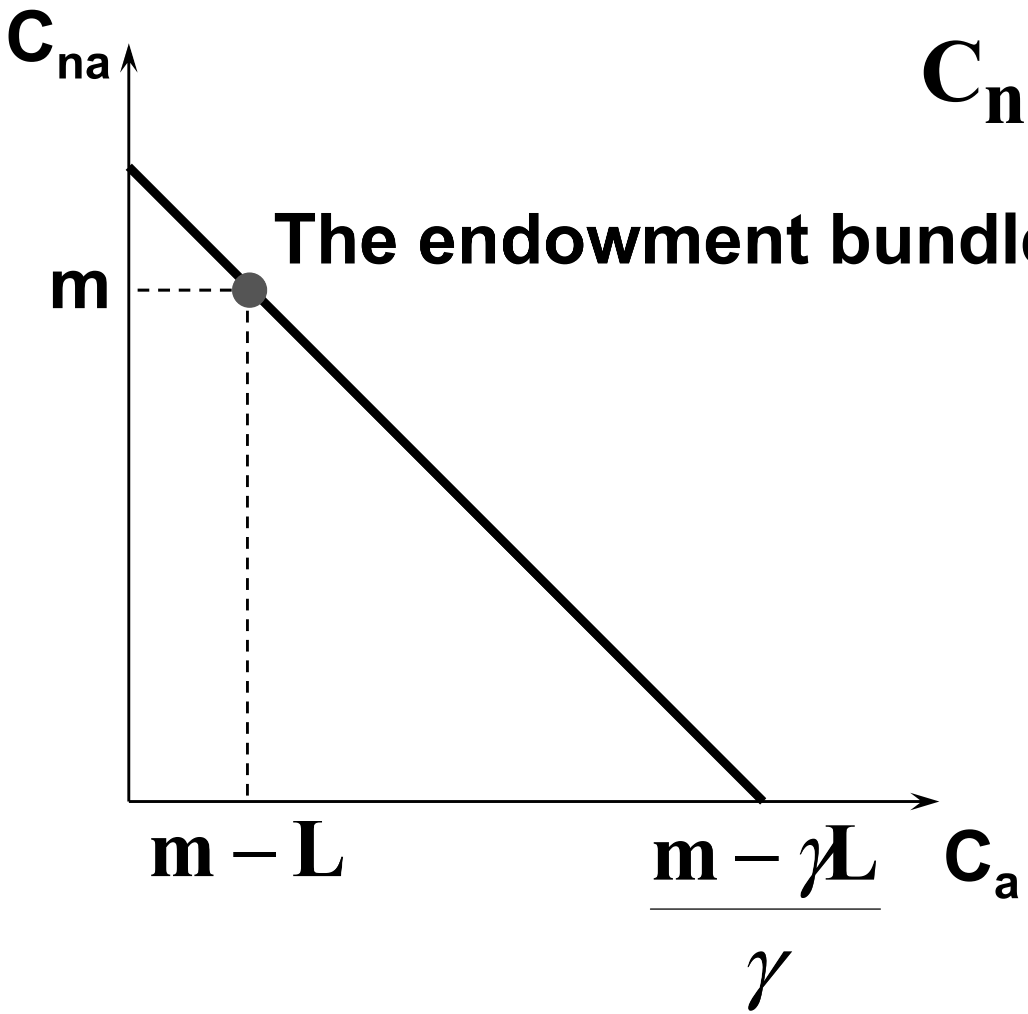
State-Contingent Budget Constraints

- **Buy \$K of accident insurance.**
- $C_{na} = m - \gamma K$.
- $C_a = m - L - \gamma K + K = m - L + (1-\gamma)K$.
- So $K = (C_a - m + L)/(1-\gamma)$
- And $C_{na} = m - \gamma (C_a - m + L)/(1-\gamma)$
- I.e.

$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$

State-Contingent Budget Constraints

$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$



State-Contingent Budget Constraints

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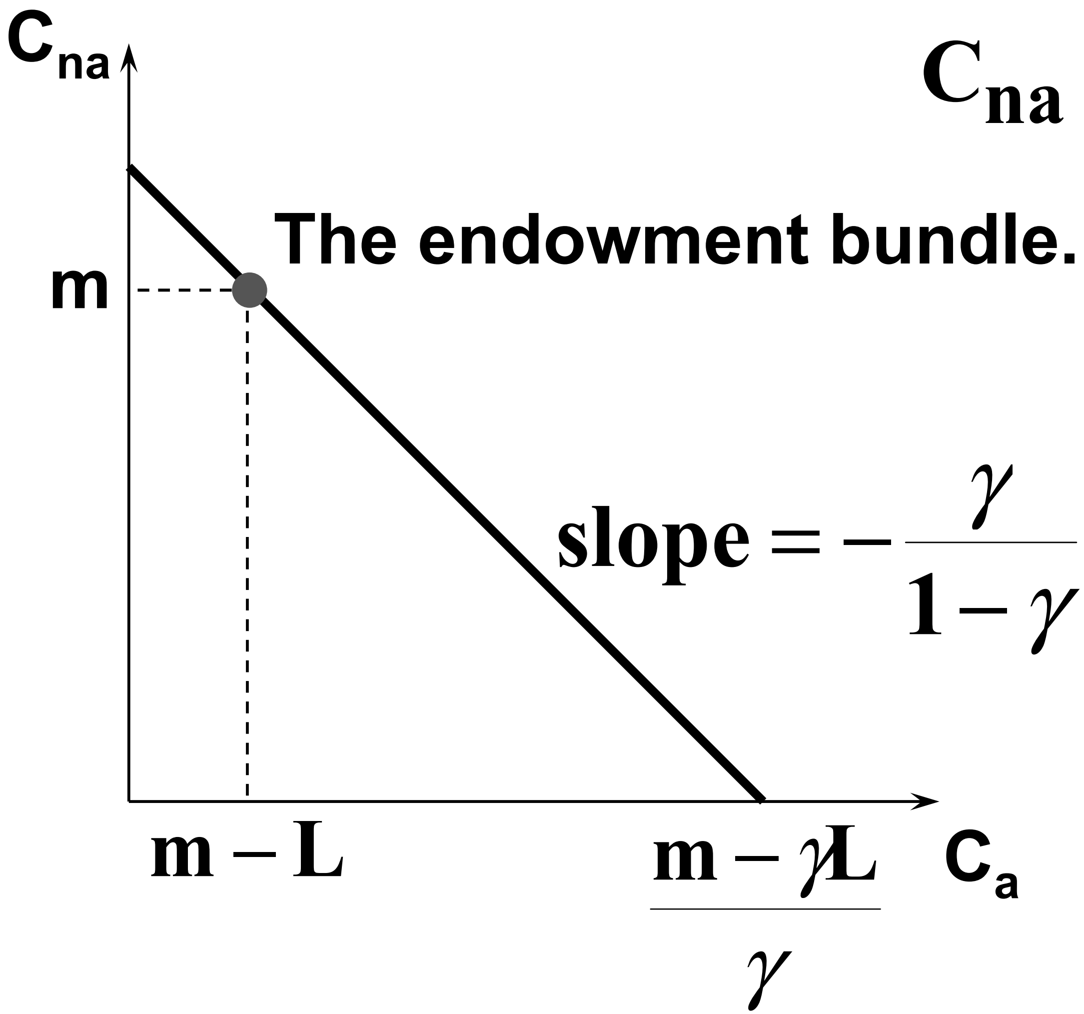
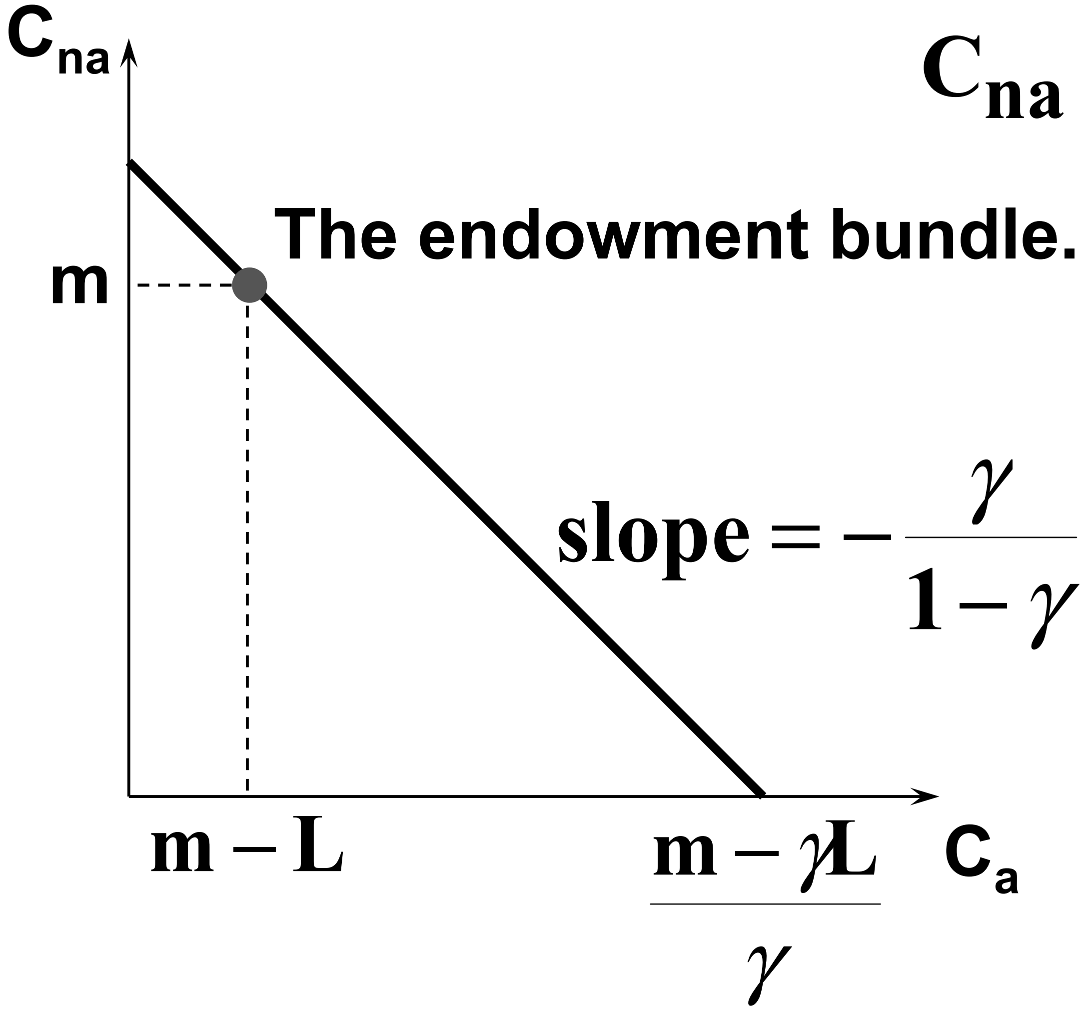


State-Contingent Budget Constraints

$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$



Where is the most preferred state-contingent consumption plan?

Preferences Under Uncertainty

- **Think of a lottery.**
- **Win \$90 with probability 1/2 and win \$0 with probability 1/2.**
- **U(\$90) = 12,   U(\$0) = 2.**
- **Expected utility is**

Preferences Under Uncertainty

- **Think of a lottery.**
- **Win \$90 with probability 1/2 and win \$0 with probability 1/2.**
- **U(\$90) = 12, U(\$0) = 2.**
- **Expected utility is**

$$\mathbf{EU} = \frac{1}{2} \times \mathbf{U(\$90)} + \frac{1}{2} \times \mathbf{U(\$0)}$$

$$= \frac{1}{2} \times 12 + \frac{1}{2} \times 2 = 7.$$

Preferences Under Uncertainty

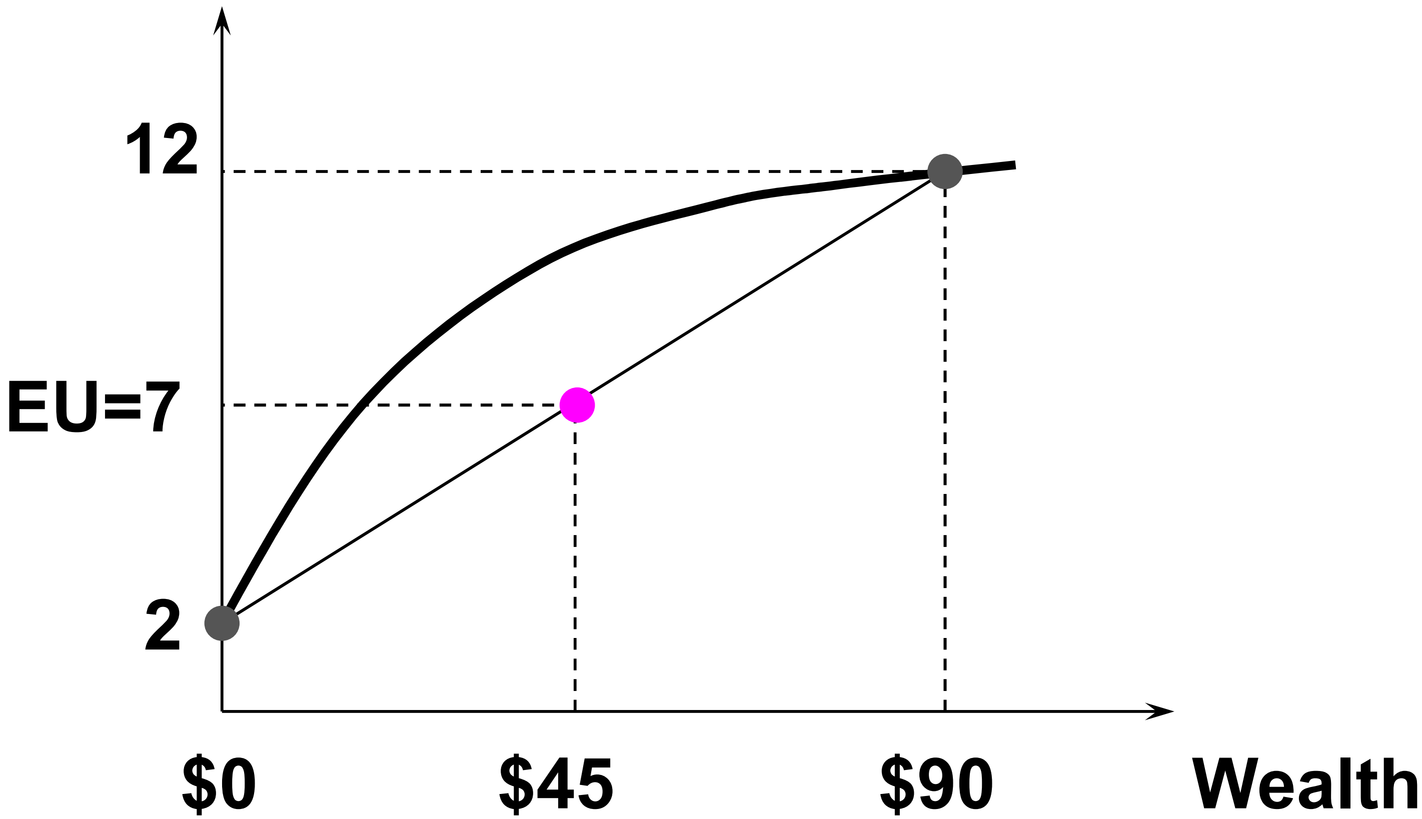
- **Think of a lottery.**
- **Win \$90 with probability 1/2 and win \$0 with probability 1/2.**
- **Expected money value of the lottery is**

$$\mathbf{EM} = \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{\$90} + \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{\$0} = \$\mathbf{45}.$$

Preferences Under Uncertainty

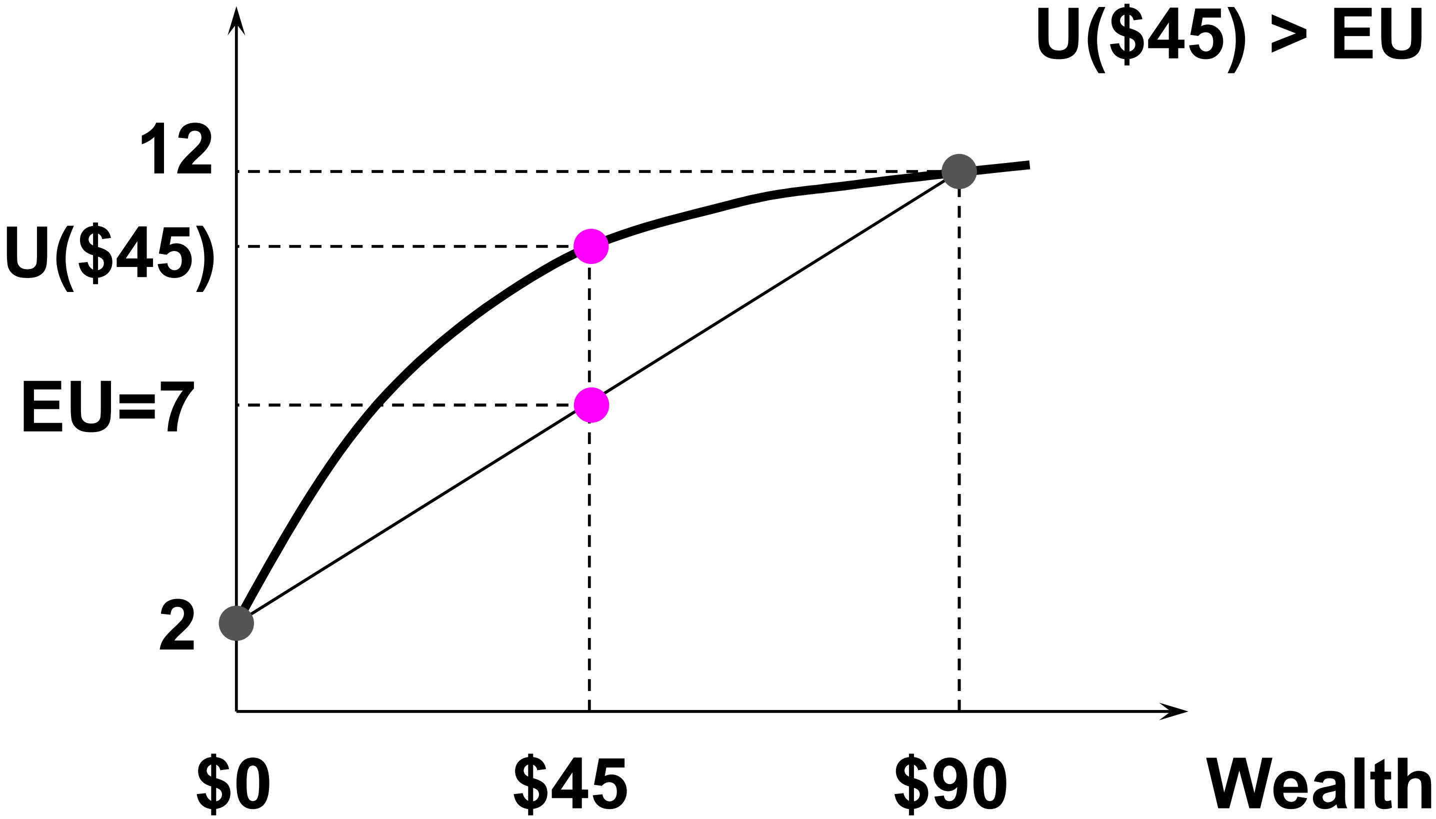
- **EU = 7 and EM = \$45.**
- **U(\$45) > 7 $\Rightarrow$ \$45 for sure is preferred to the lottery $\Rightarrow$ risk-aversion.**
- **U(\$45) < 7 $\Rightarrow$ the lottery is preferred to \$45 for sure $\Rightarrow$ risk-loving.**
- **U(\$45) = 7 $\Rightarrow$ the lottery is preferred equally to \$45 for sure $\Rightarrow$ risk-neutrality.**

Preferences Under Uncertainty

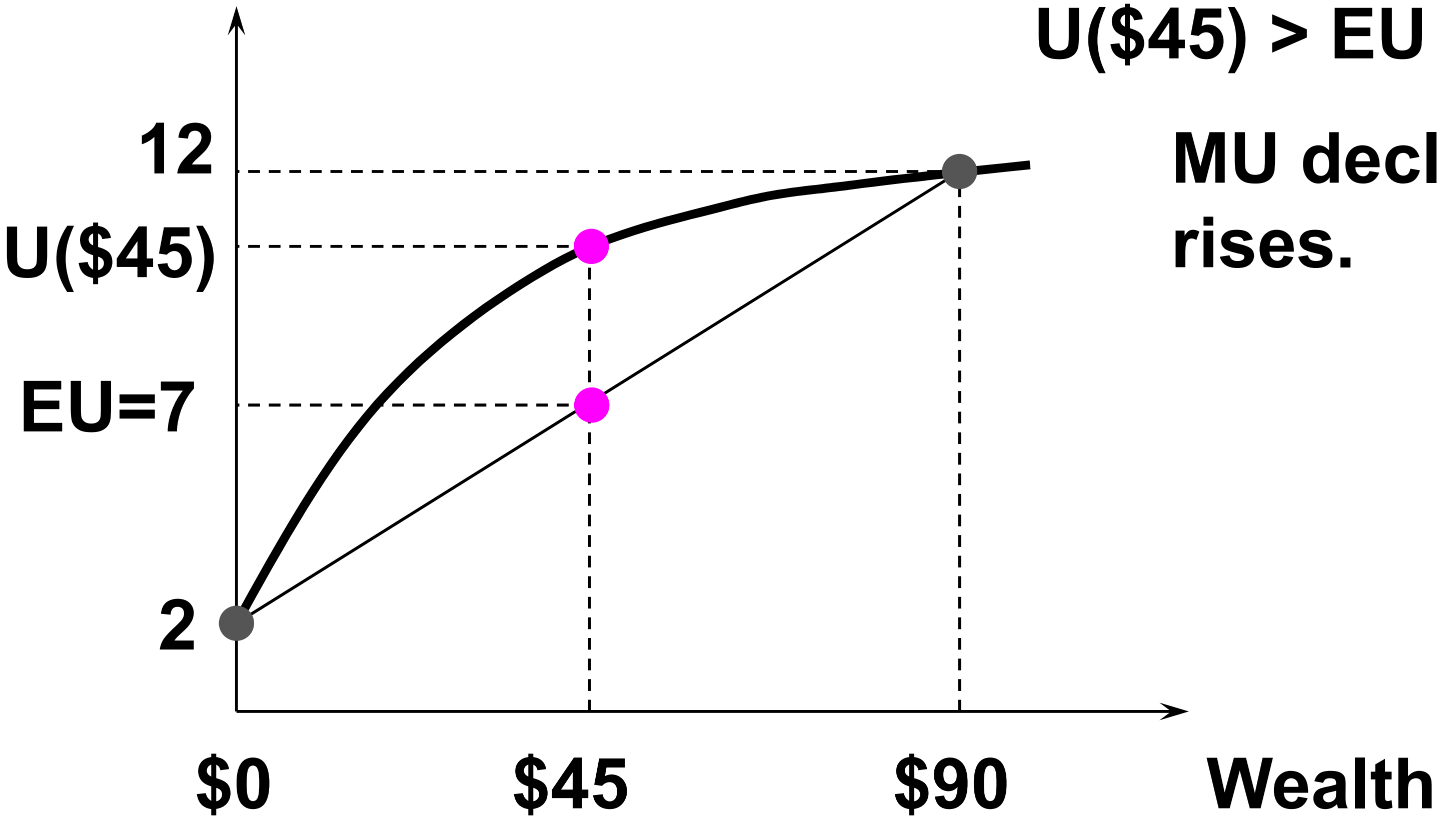


Preferences Under Uncertainty

U($45) > EU ⇒ risk-aversion.



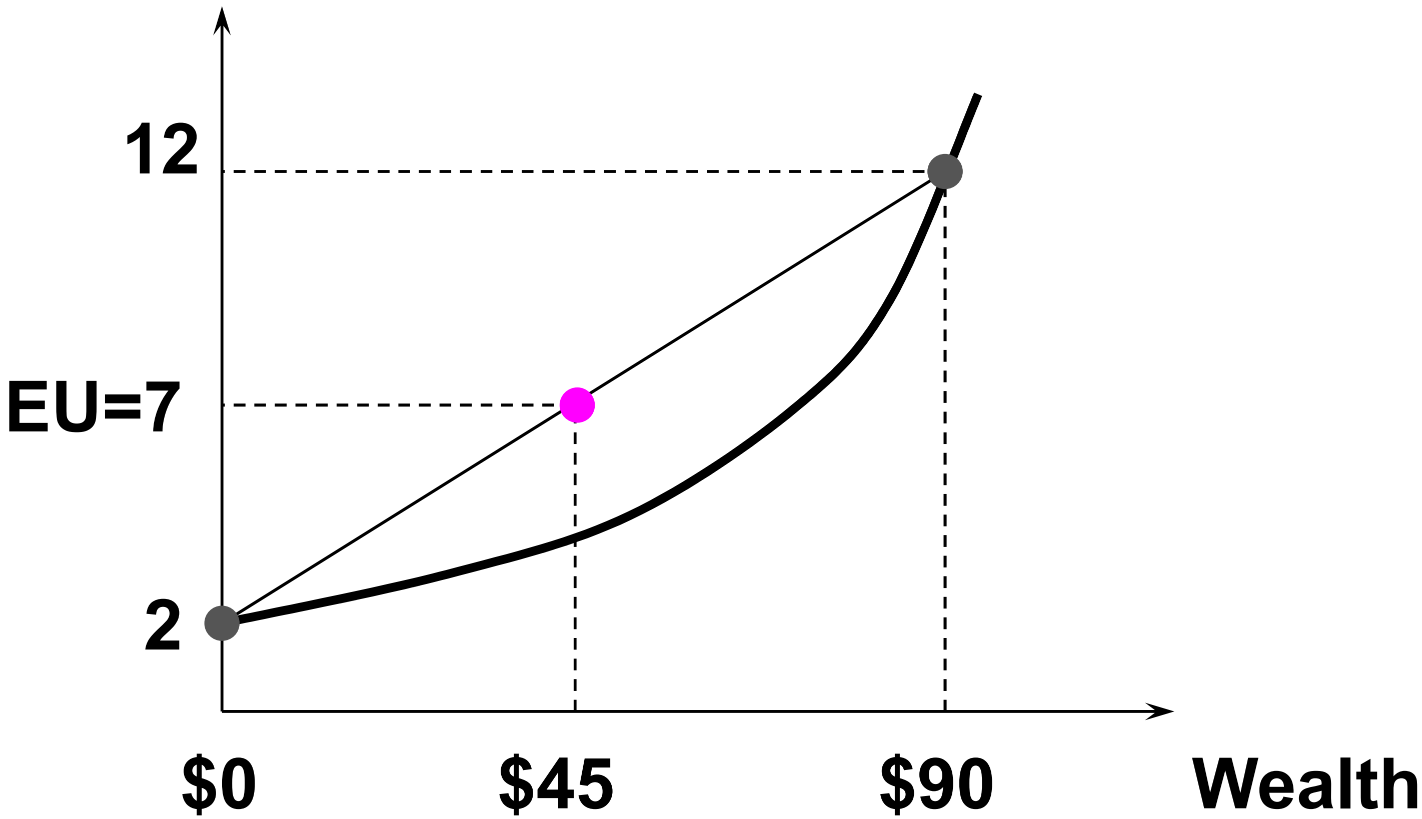
Preferences Under Uncertainty



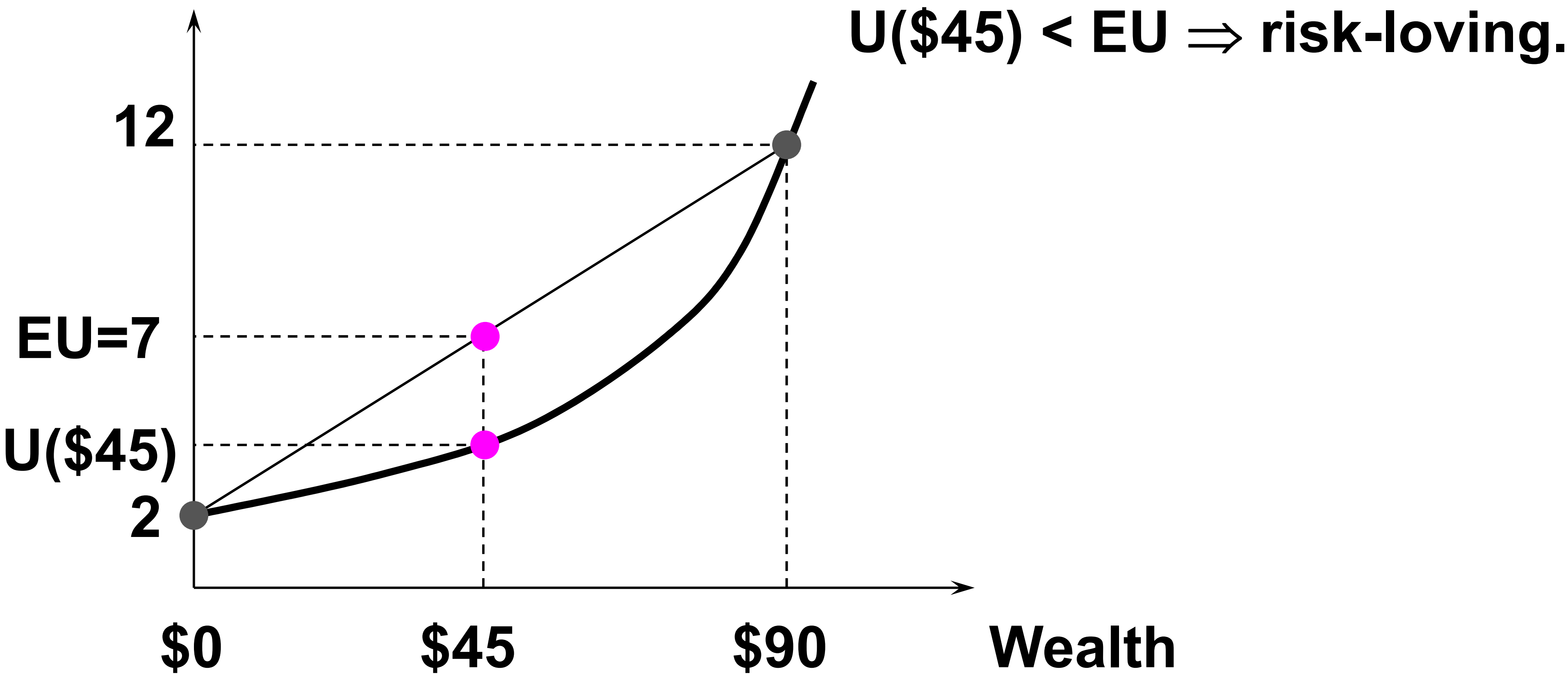
U($45) > EU $\Rightarrow$ risk-aversion.

MU declines as wealth rises.

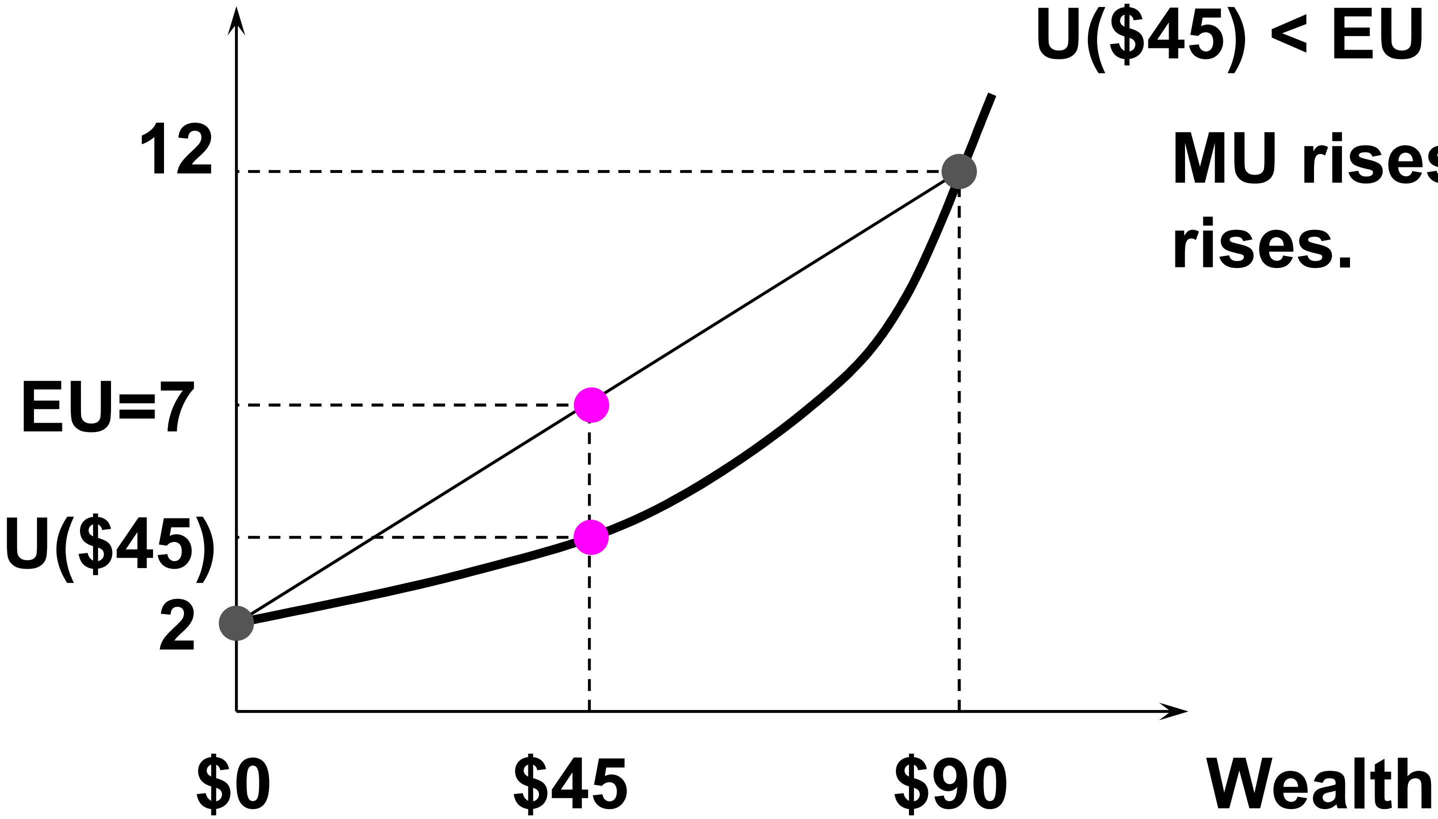
Preferences Under Uncertainty



Preferences Under Uncertainty



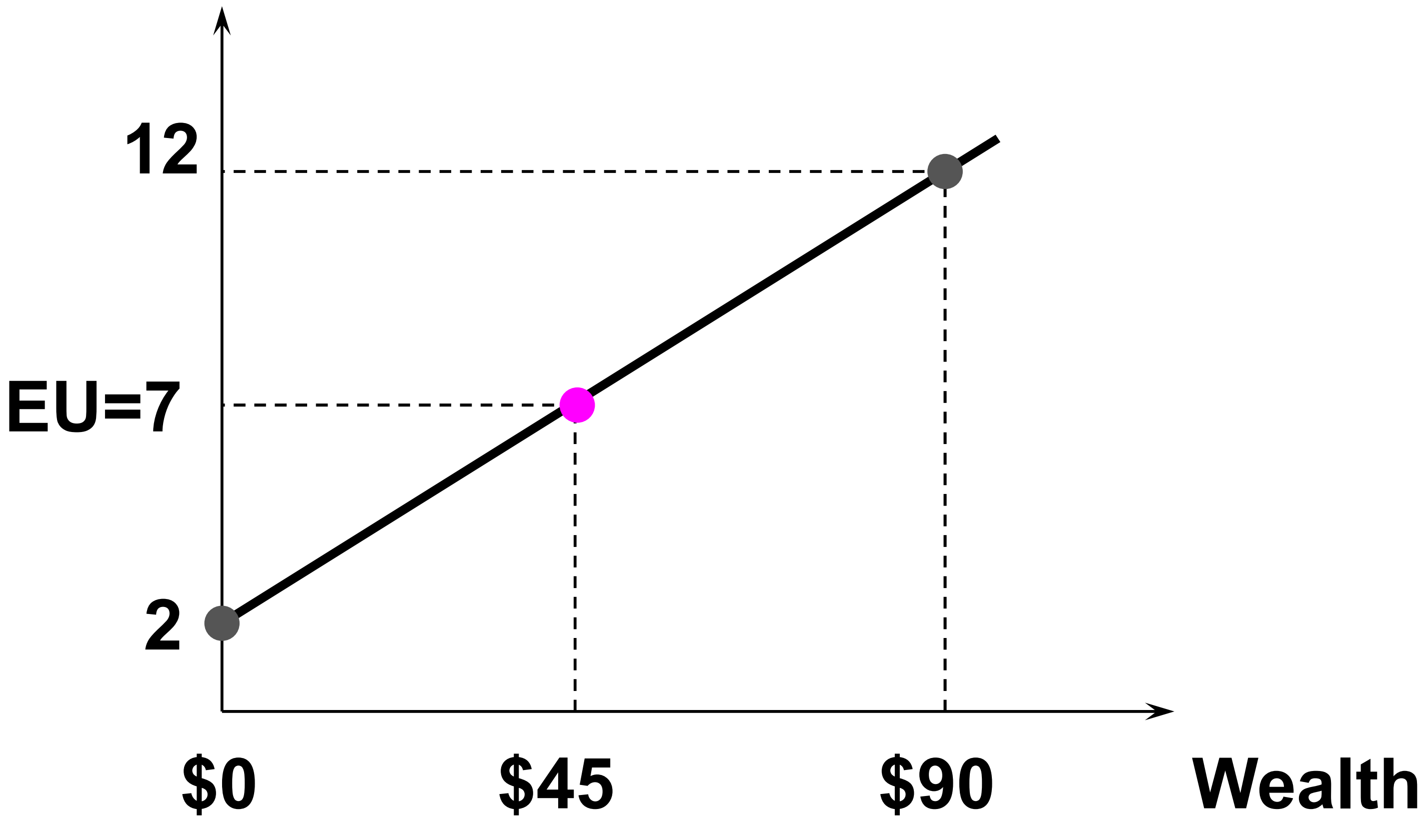
Preferences Under Uncertainty



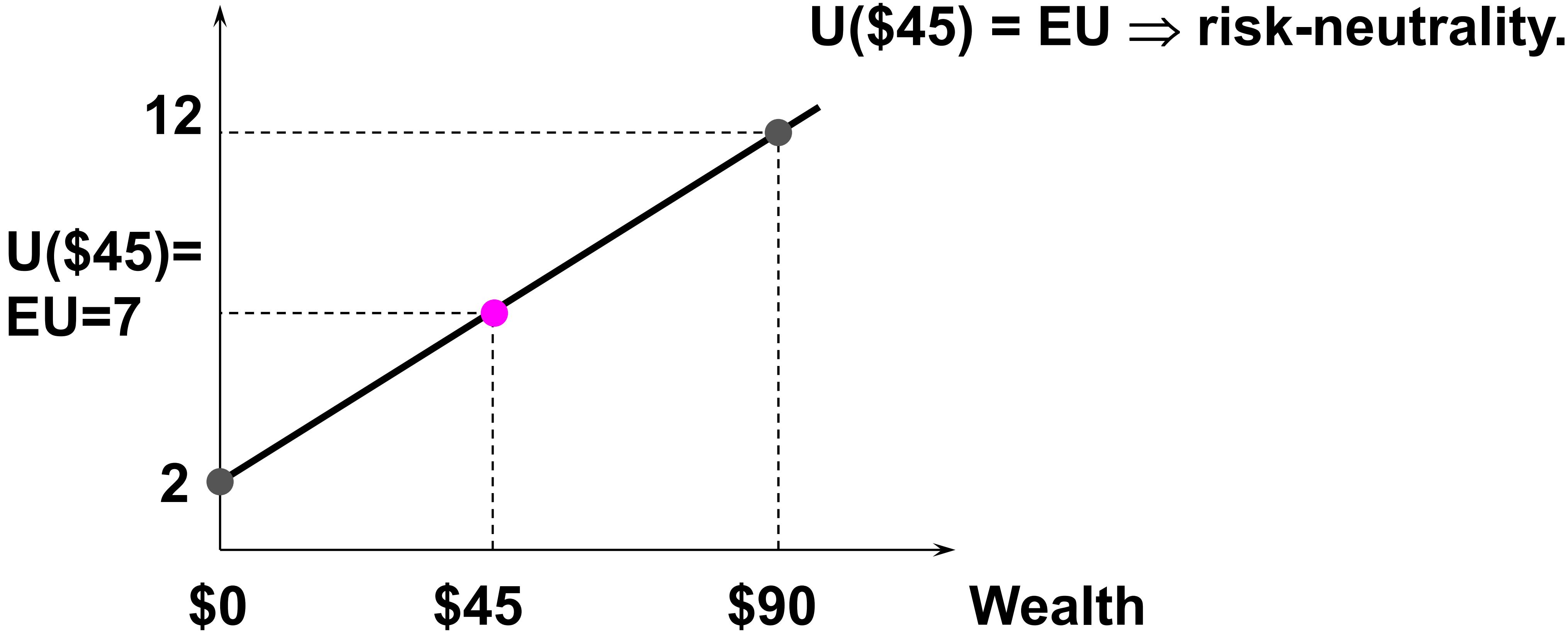
U($45) < EU ⇒ risk-loving.

MU rises as wealth rises.

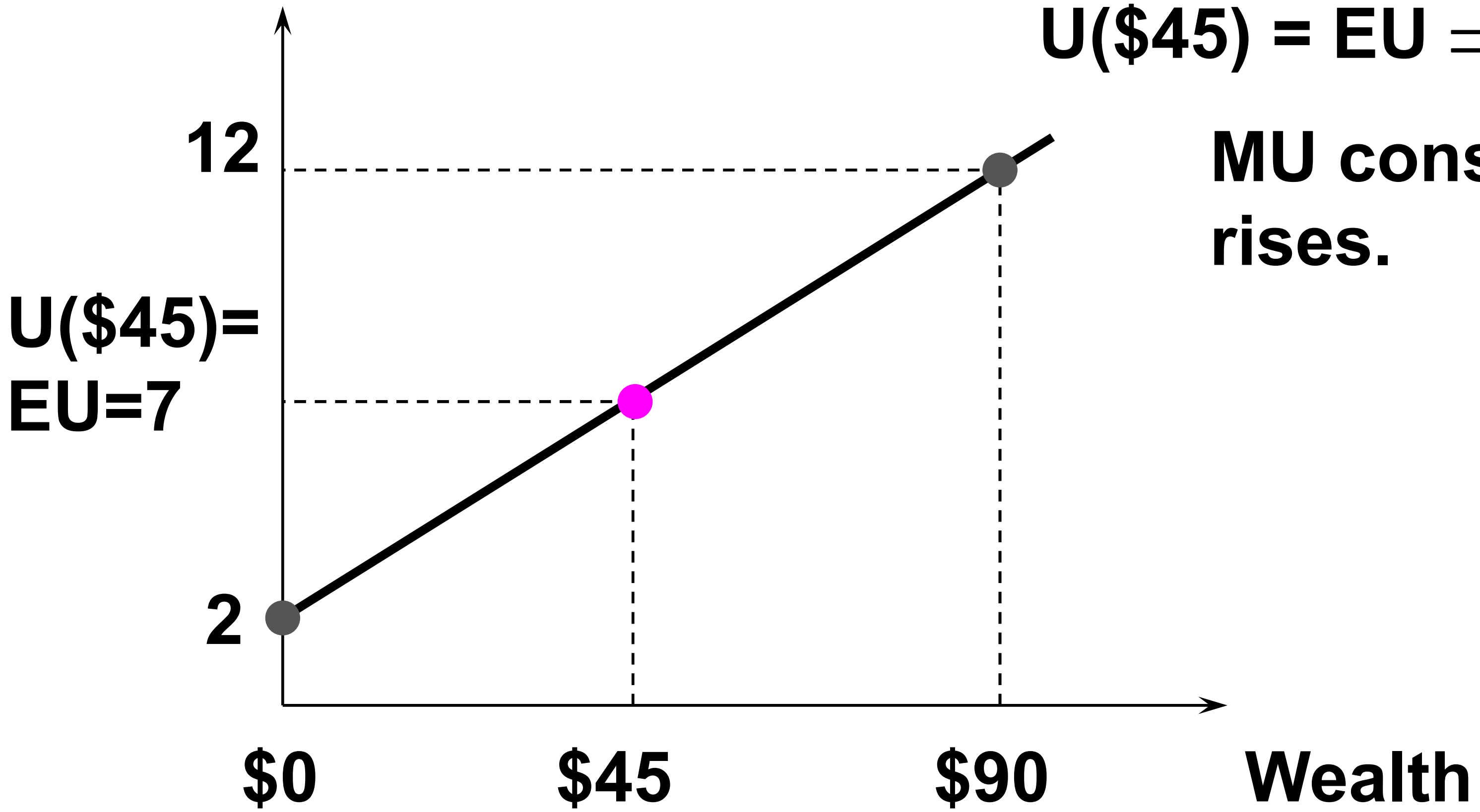
Preferences Under Uncertainty



Preferences Under Uncertainty



Preferences Under Uncertainty



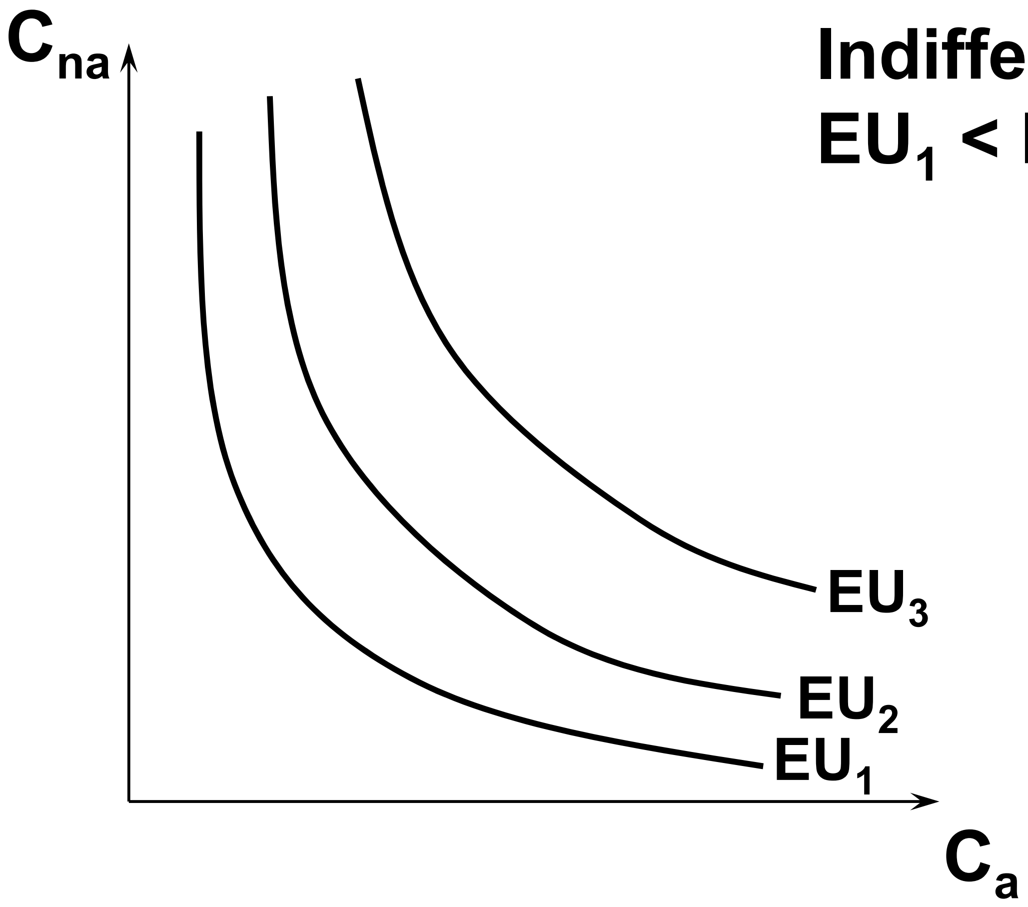
U($45) = EU ⇒ risk-neutrality.

MU constant as wealth rises.

Preferences Under Uncertainty

- **State-contingent consumption plans that give equal expected utility are equally preferred.**

Preferences Under Uncertainty



Indifference curves
$EU_1 < EU_2 < EU_3$

Preferences Under Uncertainty

- **What is the MRS of an indifference curve?**
- **Get consumption c_1 with prob. π_1 and c_2 with prob. π_2 ($\pi_1 + \pi_2 = 1$).**
- **$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$.**
- **For constant EU, dEU = 0.**

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

$$\mathbf{dEU} = \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 + \pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U(c_1)} + \pi_2 \mathbf{U(c_2)}$$

$$\mathbf{dEU} = \pi_1 \mathbf{MU(c_1)dc_1} + \pi_2 \mathbf{MU(c_2)dc_2}$$

$$\mathbf{dEU} = 0 \Rightarrow \pi_1 \mathbf{MU(c_1)dc_1} + \pi_2 \mathbf{MU(c_2)dc_2} = 0$$

Preferences Under Uncertainty

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$$\Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 = -\pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

Preferences Under Uncertainty

$$\mathbf{EU} = \pi_1 \mathbf{U}(\mathbf{c}_1) + \pi_2 \mathbf{U}(\mathbf{c}_2)$$

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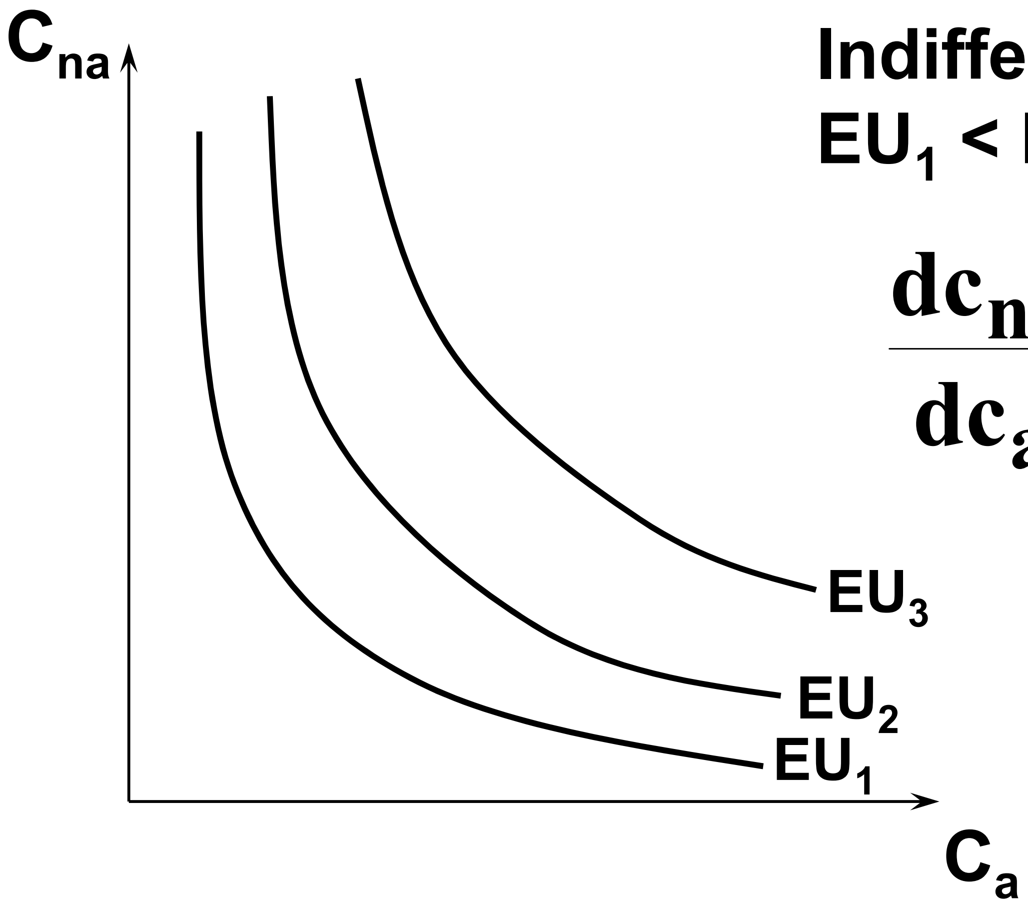
$$\Rightarrow \pi_1 \mathbf{MU}(\mathbf{c}_1)\mathbf{dc}_1 = -\pi_2 \mathbf{MU}(\mathbf{c}_2)\mathbf{dc}_2$$

$$\Rightarrow \frac{\mathbf{dc}_2}{\mathbf{dc}_1} = -\frac{\pi_1 \mathbf{MU}(\mathbf{c}_1)}{\pi_2 \mathbf{MU}(\mathbf{c}_2)}.$$

Preferences Under Uncertainty



Indifference curves
$EU_1 < EU_2 < EU_3$

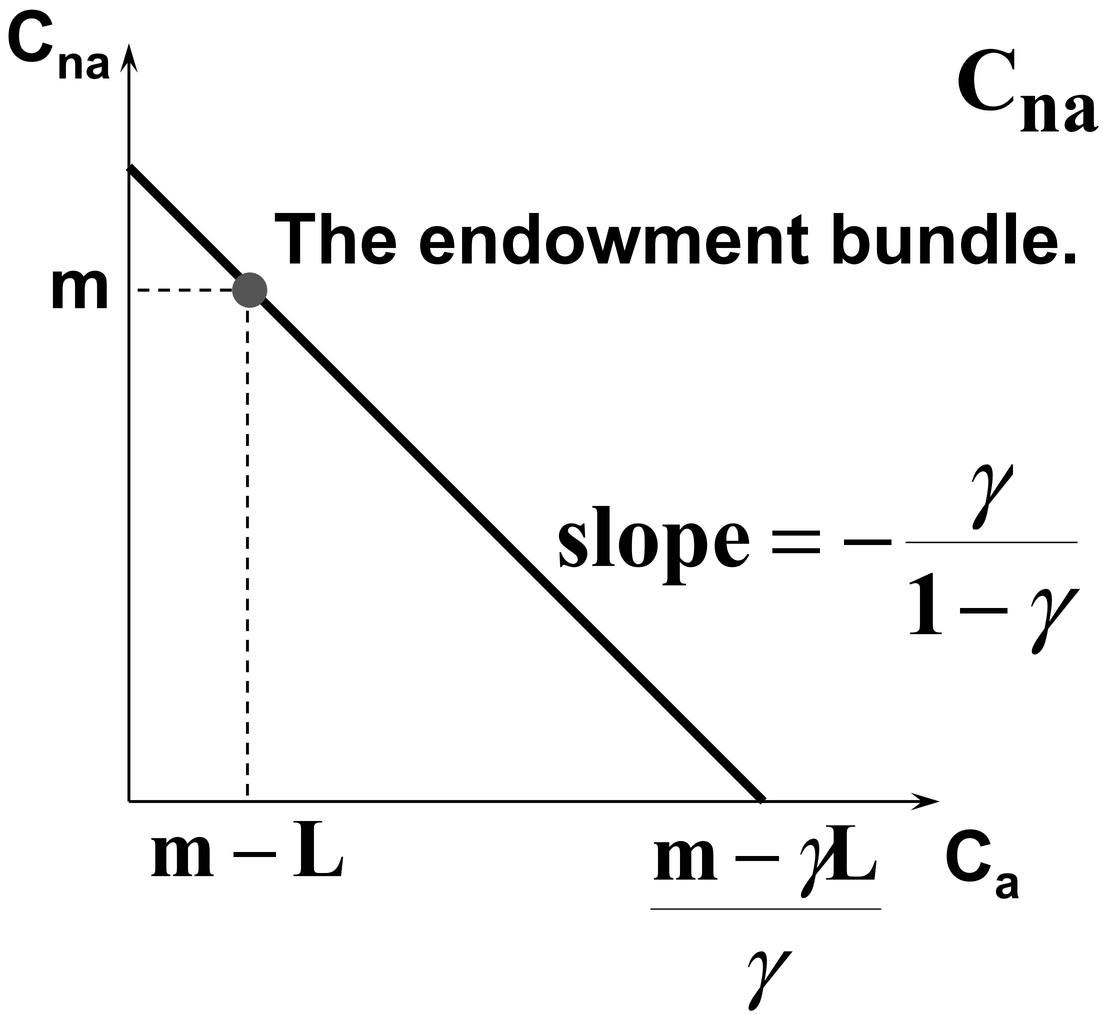
$$\frac{\mathbf{dc_{na}}}{\mathbf{dc_a}} = -\frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

Choice Under Uncertainty

- **Q: How is a rational choice made under uncertainty?**
- **A: Choose the most preferred affordable state-contingent consumption plan.**

State-Contingent Budget Constraints

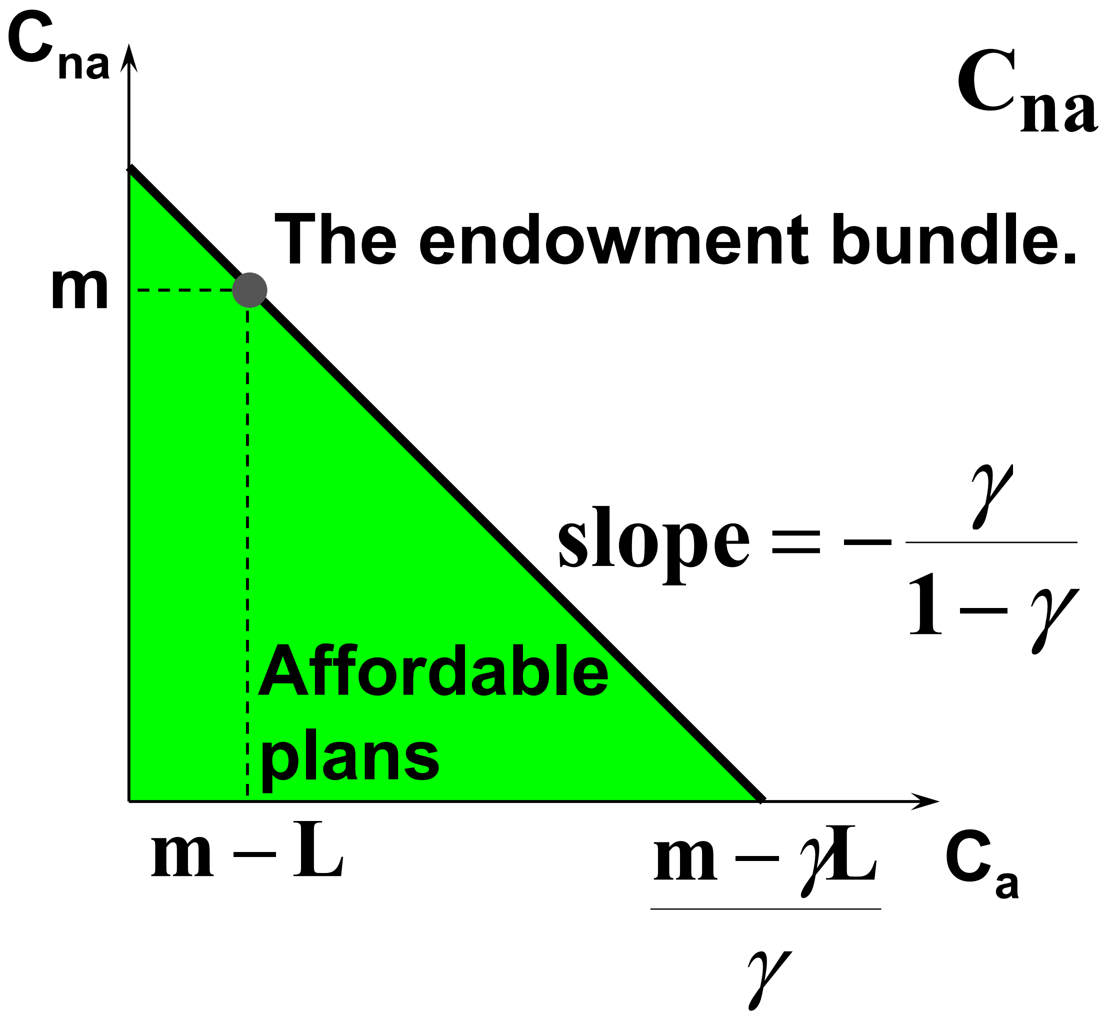
$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$



Where is the most preferred state-contingent consumption plan?

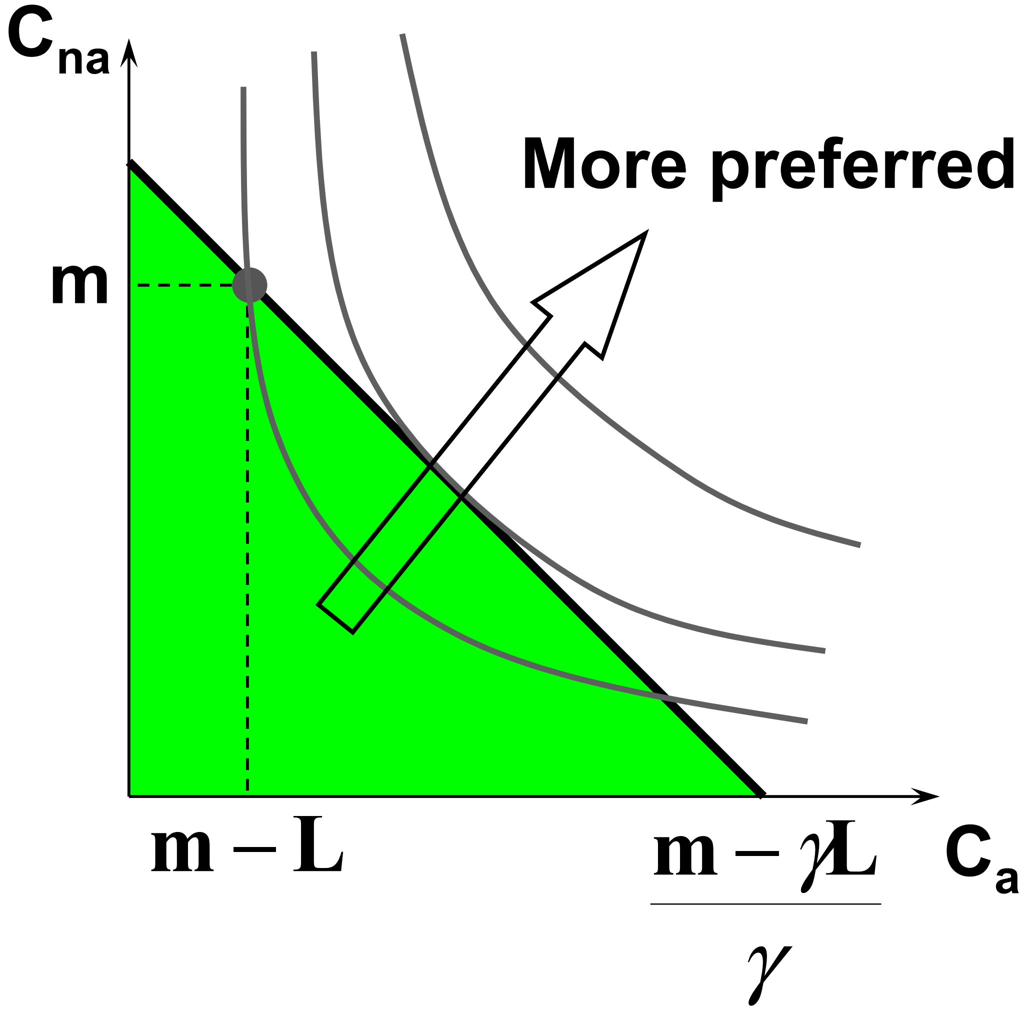
State-Contingent Budget Constraints

$$C_{na} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_a$$



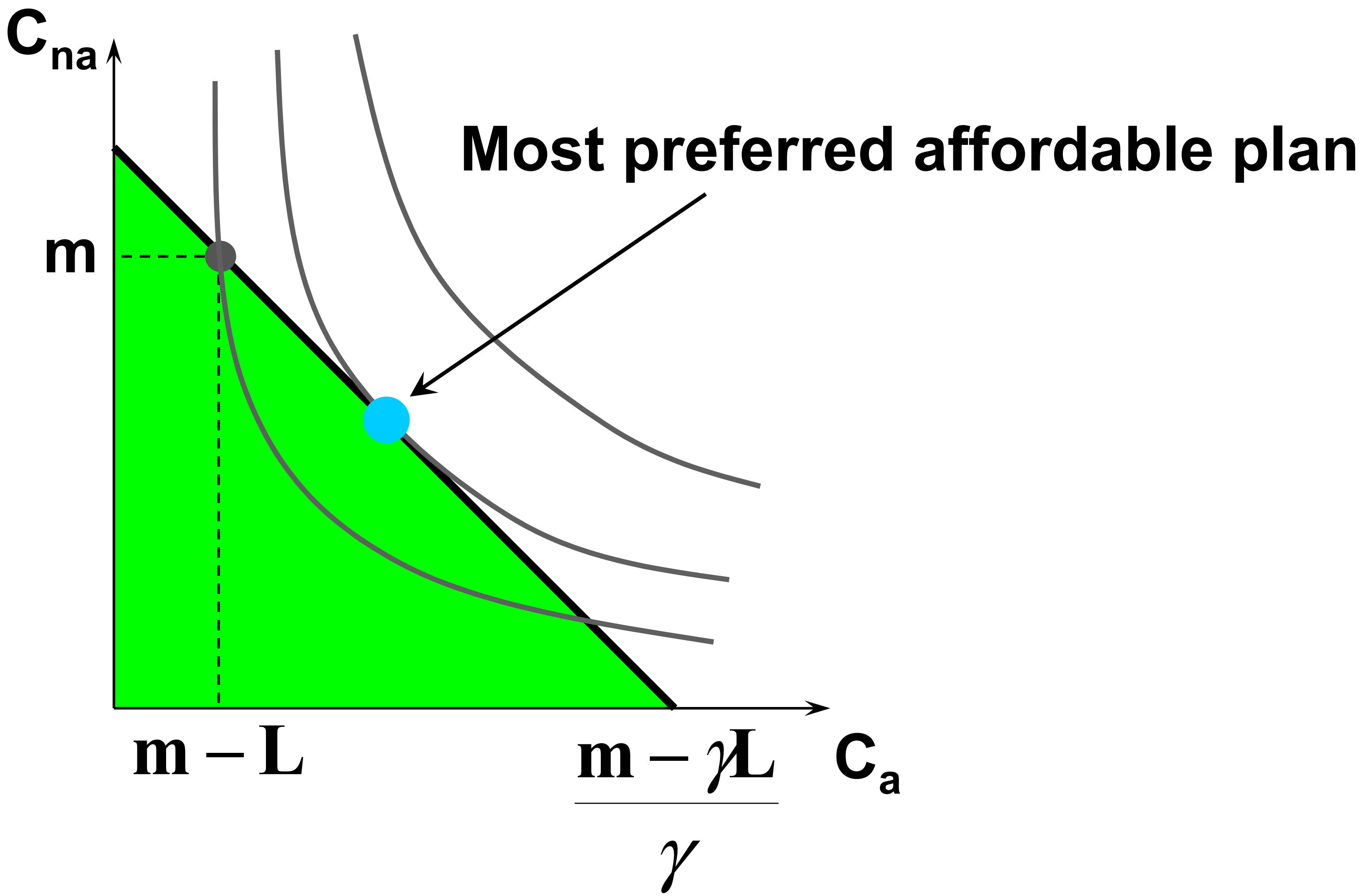
Where is the most preferred state-contingent consumption plan?

State-Contingent Budget Constraints

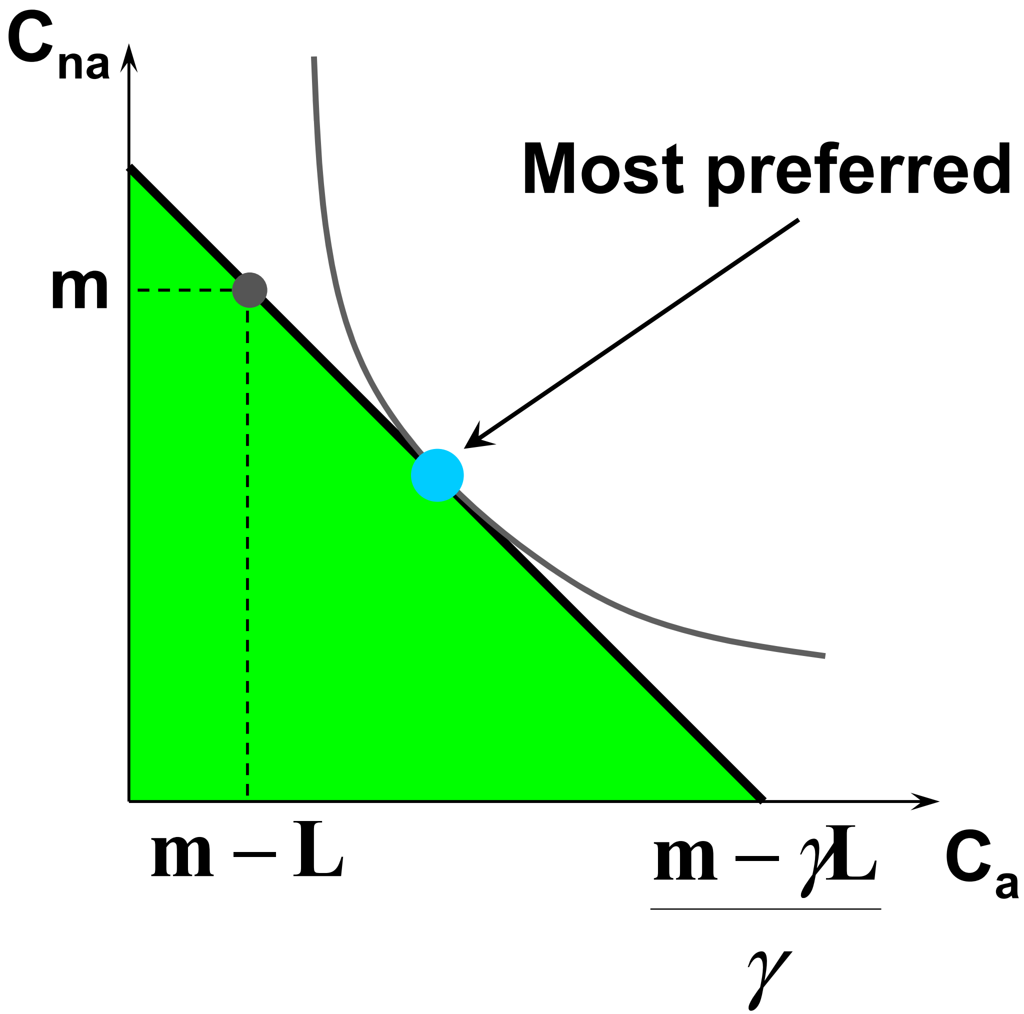


Where is the most preferred state-contingent consumption plan?

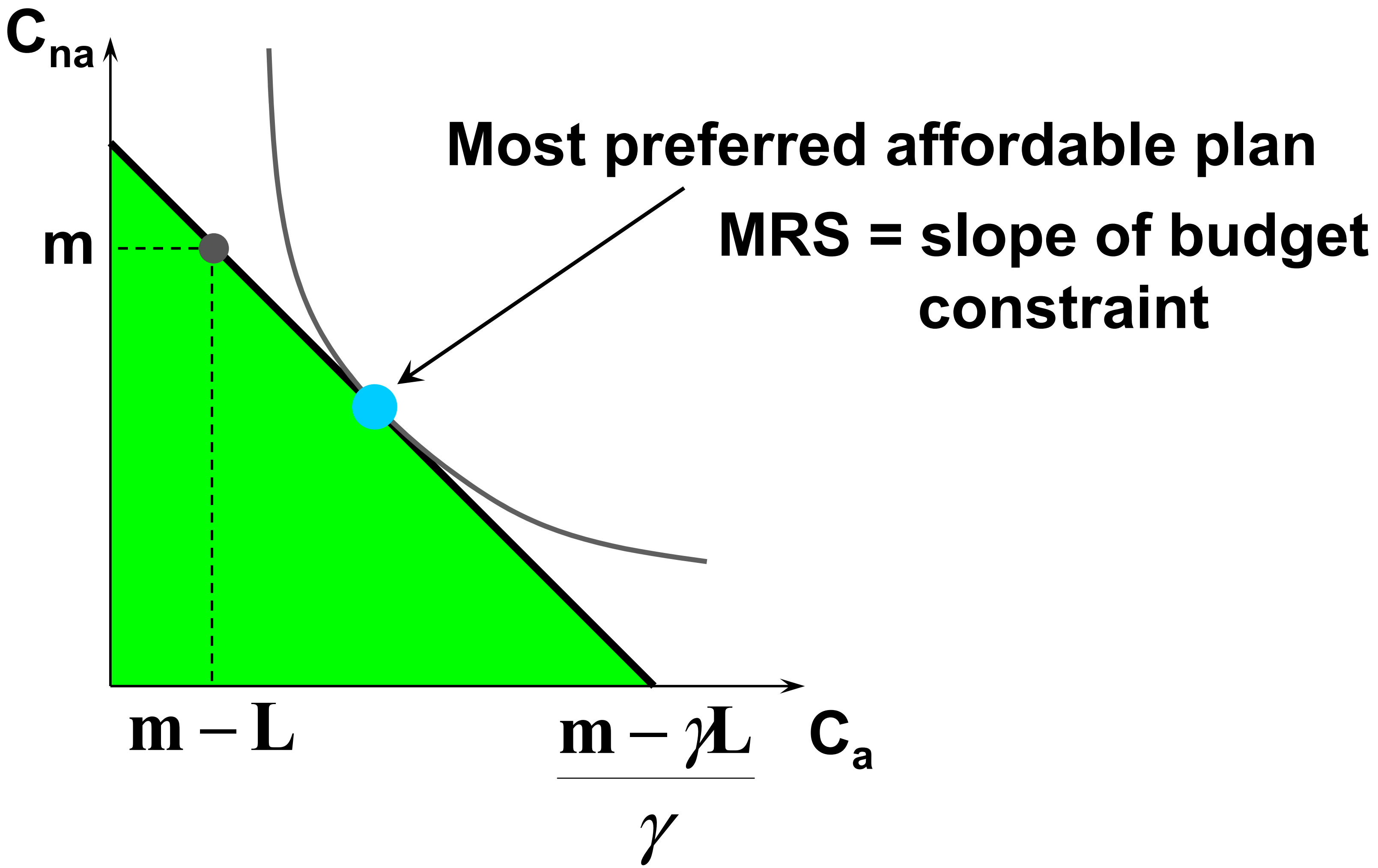
State-Contingent Budget Constraints



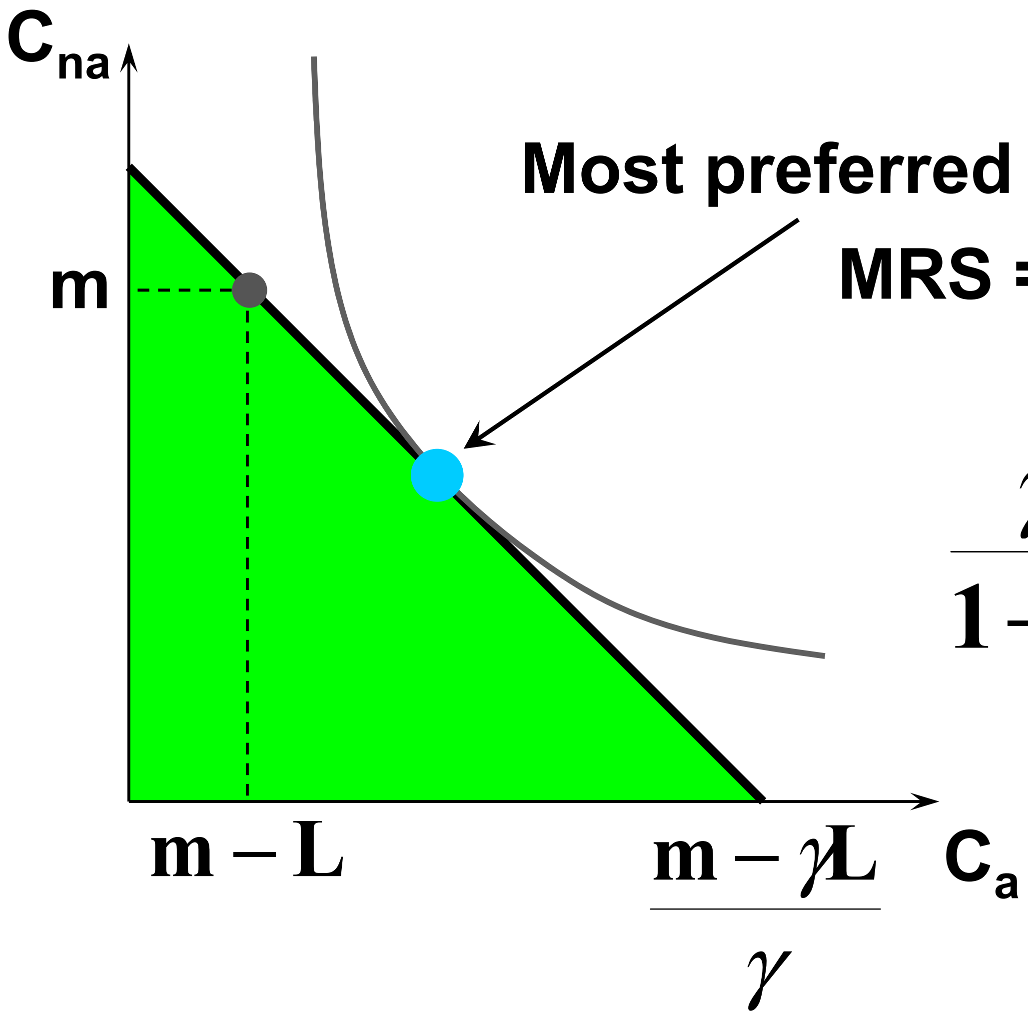
State-Contingent Budget Constraints



State-Contingent Budget Constraints



State-Contingent Budget Constraints



Most preferred affordable plan

MRS = slope of budget constraint; i.e.

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

Competitive Insurance

- **Suppose entry to the insurance industry is free.**
- **Expected economic profit = 0.**
- **I.e. $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K = 0$.**
- **I.e. free entry $\Rightarrow \gamma = \pi_a$.**
- **If price of $1 insurance = accident probability, then insurance is fair.**

Competitive Insurance

- **When insurance is fair, rational insurance choices satisfy**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

Competitive Insurance

- **When insurance is fair, rational insurance choices satisfy**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a \mathbf{MU}(\mathbf{c_a})}{\pi_{na} \mathbf{MU}(\mathbf{c_{na}})}$$

- **I.e.** $\mathbf{MU}(\mathbf{c_a}) = \mathbf{MU}(\mathbf{c_{na}})$

Competitive Insurance

- **When insurance is fair, rational insurance choices satisfy**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a \mathbf{MU}(\mathbf{c_a})}{\pi_{na} \mathbf{MU}(\mathbf{c_{na}})}$$

- **I.e.** $\mathbf{MU}(\mathbf{c_a}) = \mathbf{MU}(\mathbf{c_{na}})$
- **Marginal utility of income must be the same in both states.**

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

$$\mathbf{MU(c_a) = MU(c_{na})}$$

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

$$MU(c_a) = MU(c_{na})$$

- **Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.**

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

$$\mathbf{MU(c_a) = MU(c_{na})}$$

- **Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.**
- **Hence** $\mathbf{c_a = c_{na}}$.

Competitive Insurance

- **How much fair insurance does a risk-averse consumer buy?**

$$MU(c_a) = MU(c_{na})$$

- **Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.**
- **Hence** $c_a = c_{na}$.
- **I.e. full-insurance.**

“Unfair” Insurance

- **Suppose insurers make positive expected economic profit.**
- **I.e. $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K > 0$.**

"Unfair" Insurance

- **Suppose insurers make positive expected economic profit.**
- **I.e. $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K > 0$.**
- **Then $\Rightarrow \gamma > \pi_a \Rightarrow \dfrac{\gamma}{1-\gamma} > \dfrac{\pi_a}{1-\pi_a}$.**

“Unfair” Insurance

- **Rational choice requires**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathbf{MU}(\mathbf{c_a})}{\pi_{na} \mathbf{MU}(\mathbf{c_{na}})}$$

“Unfair” Insurance

- **Rational choice requires**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathbf{MU}(\mathbf{c_a})}{\pi_{na} \mathbf{MU}(\mathbf{c_{na}})}$$

- **Since** $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $\mathbf{MU}(\mathbf{c_a}) > \mathbf{MU}(\mathbf{c_{na}})$

“Unfair” Insurance

- **Rational choice requires**

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- **Since** $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $\mathbf{MU}(\mathbf{c_a}) > \mathbf{MU}(\mathbf{c_{na}})$

- **Hence** $\mathbf{c_a} < \mathbf{c_{na}}$ **for a risk-averter.**

“Unfair” Insurance

- **Rational choice requires**

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathbf{MU}(\mathbf{c_a})}{\pi_{na} \mathbf{MU}(\mathbf{c_{na}})}$$

- **Since** $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $\mathbf{MU}(\mathbf{c_a}) > \mathbf{MU}(\mathbf{c_{na}})$
- **Hence** $\mathbf{c_a} < \mathbf{c_{na}}$ **for a risk-averter.**
- **I.e. a risk-averter buys less than full “unfair” insurance.**

Uncertainty is Pervasive

- What are rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.

Uncertainty is Pervasive

- **What are rational responses to uncertainty?**
  - ✓ **buying insurance (health, life, auto)**
  - **a portfolio of contingent consumption goods.**

Uncertainty is Pervasive

- **What are rational responses to uncertainty?**
  - ✓ – **buying insurance (health, life, auto)**
  - ? – **a portfolio of contingent consumption goods.**

Diversification

- Two firms, A and B. Shares cost $10.
- With prob. 1/2 A's profit is $100 and B's profit is $20.
- With prob. 1/2 A's profit is $20 and B's profit is $100.
- You have $100 to invest. How?

Diversification

- **Buy only firm A's stock?**
- **$100/10 = 10 shares.**
- **You earn $1000 with prob. 1/2 and $200 with prob. 1/2.**
- **Expected earning: $500 + $100 = $600**

Diversification

- Buy only firm B's stock?
- \$100/10 = 10 shares.
- You earn \$1000 with prob. 1/2 and \$200 with prob. 1/2.
- Expected earning: \$500 + \$100 = \$600

Diversification

- **Buy 5 shares in each firm?**
- **You earn $600 for sure.**
- **Diversification has maintained expected earning and lowered risk.**

Diversification

- **Buy 5 shares in each firm?**
- **You earn $600 for sure.**
- **Diversification has maintained expected earning and lowered risk.**
- **Typically, diversification lowers expected earnings in exchange for lowered risk.**

Risk Spreading/Mutual Insurance

- **100 risk-neutral persons each independently risk a $10,000 loss.**
- **Loss probability = 0.01.**
- **Initial wealth is $40,000.**
- **No insurance: expected wealth is**

$$0 \cdot 99 \times \$40{,}000 + 0 \cdot 01(\$40{,}000 - \$10{,}000)$$

$$= \$39{,}900.$$

Risk Spreading/Mutual Insurance

- **Mutual insurance: Expected loss is**

$$0 \cdot 01 \times \$10{,}000 = \$100.$$

- **Each of the 100 persons pays \$1 into a mutual insurance fund.**
- **Mutual insurance: expected wealth is**

$$\$40{,}000 - \$1 = \$39{,}999 > \$39{,}900.$$

- **Risk-spreading benefits everyone.**